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Non-Linear Adaptive Control
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Week-6
Lecture No: 34
Parameter Convergence in Adaptive Control Design

Hello everyone, welcome to yet another session of our NPTEL on non-linear and adaptive control. I am Srikant from Systems and Control IIT Bombay. So we have this very nice motivating image in the background which is of this nice SpaceX satellite which is orbiting the earth and we are moving towards designing algorithms which will help drive systems such as these autonomously.

So we are, well to the sixth week of this course and with the halfway mark if you may and we have already started to look at design elements, so we have already started to see how to design adaptive controllers for a first order scalar system and that is what we have been doing.

So last time we had already done the design in the session before last and we had essentially arrived at a point where we had a negative semi-definite V dot for the unknown parameter case. So we had a control law we had designed and we also had an update law for the unknown parameter.

So the control of course depended on this unknown parameter update, so with this when we analyze the system using this Lyapunov function that we constructed in fact the update law was obtained using the said Lyapunov function and we got a negative semi definite V dot and what we did last time was to start from this one this V dot and use the signal chasing in Barbalat's Lemma method of analysis which we had learned in the analysis part of the course in the first few weeks and we use this V and V dot in order to prove that well we are already proved that we have uniform stability for the states which are the error that is the tracking error and also for the parameter estimation error.

So what we wanted to do next was to prove that the tracking errors go to 0, converge to 0 so we already had a stability result we wanted to obtain a convergence result because stability is not enough. So that is what we did we were able to prove in fact here from step number 4 if you see we were able to prove that the error goes to 0 as t goes to infinity. So inspite of the fact that we do not know a parameter of the system we were able to show that we can do accurate tracking just by adding the parameter update and so this is the magic of adaptive control this is what we did.

So I hope you could see that there is this rather cool element in adaptive control and so now beyond that we of course want to see if the other state which is the parameter error can also be driven to 0 and we try to do that but honestly we are not successful, because we reached a point where we found that the product of theta tilde and this function f goes to 0 as t goes to infinity.

Now if you sort of have something very simplistic so this is where we start let us say lecture 6.4 going to mark it here and look at this and then if you can make a very simplistic sort of assumption which is probably not a very viable assumption to be honest I will tell you why very soon. So if we assume that f let us see limit as t goes to infinity f x t, t is equal to 0, is not equal to 0 then there is hope that theta tilde does not actually goes to 0 can hope theta tilde goes to 0, which is also not a guarantee I mean so this is a rather complicated thing to talk about because we are talking about things converging to 0 and I mean as t goes to infinity and things like that so we are talking about objects in the limit.

Now this itself may not be a very viable assumption in all scenarios it may be in the tracking scenario, so but notice that your dynamics of the system was something like x dot is a f sorry I apologize was something like theta star f x t plus a control u. Now typically if I assume that 0 is an equilibrium then what you would have is that to ensure that 0 is equilibrium typically you would have f 0 t would be equal to 0 for all t.

If that is not the case then you can see that this right hand side being 0 and x equal to 0 and u equal to 0 and typically at the equilibrium the control is also expected to be 0. So the right hand side will not be 0 and it will not be an equilibrium 0 will not be equilibrium. So you would expect f 0 t to take on the value 0 for all that all time. Now if that is the case here you see I am requiring f 0 t to be equal to 0 and here I am requiring that as t goes to infinity f x t t is not equal to 0 looks like opposite requirements. Now if I am looking at trajectories of r which actually go to 0 as t goes to infinity this is possible.

There is many possibilities if I am looking at say trajectories like this is actually going to 0 for large t for large t for the trajectory going to 0 then this is t and this is r of t then this is difficult I mean there is a difficult situation where you see that if r goes to 0 then as t goes to infinity with the error tracking error going to 0 x itself is also going to 0. So you can imagine that as t goes to infinity this by standard continuity type arguments this function itself has to be rather close to 0 in fact we go to 0 itself. So this is not satisfied so this condition is not going to be satisfied for all trajectories which are going to 0 also for the stabilization problem where r of t is actually 0 for all time, even in that case this condition cannot be satisfied.

So it is not very easy to have satisfaction of a condition of this kind, so anyway this is a naive requirement I would say, so this is the lesson in adaptive control and what is the lesson is that typically you can show that the state follows the desired trajectory accurately this is guaranteed no problem, however parameter convergence or parameter estimation is never guaranteed in fact you need persistence of excitation for the parameters to converge to eventual values.

So now of course in the previous week we spent a lot of time before we go on to the second order system in the previous week we spent quite a bit of time looking at persistence and notions of persistence and so what I want to do is try to give a I mean we almost do not in most circumstances analyze parameter convergence in adaptive control and whatever we see in subsequent lectures because parameter identification or any identification or system identification is a complete a problem that predates adaptive control.

I mean you can imagine if I want to identify for example parameters of a manipulator a robotic manipulator you are expected to move the manipulator in all sorts of interesting or rich trajectory if I may so that you can identify the parameters of this manipulator; so you move it in all sorts of rich trajectories that is the idea.

Now this may not always be possible because we are talking about doing a control task and we are not talking about just identification in itself when you are doing a control task you are expected to follow a certain trajectory. For example, a robot has to say move along a constant trajectory that has to say move along a straight line and not oscillate all around just to identify the parameters that is not very okay as far as the operation of the system goes and so it may not be possible to do good identification of parameters.

So most engineers and adaptive control theorists are happy to have good tracking performance and if they do get parameter identification in the process excellent, however since we did do persistence and we studied persistence of excitation in the last week now we do let us just try to look at what we can get out of this.

So let us see let us see what we have let us first write out the system equations so what is e dot, let us use this parameter so e dot was minus k e plus theta tilde times f x t, I think that was it, this is it minus k e plus theta delta f x t this is what was e dot and what was theta dot theta tilde dot theta tilde dot is just minus theta hat dot which is this guy, so let us write it out. So, theta tilde dot equals minus theta hat dot equals minus 1 over gamma times f x t times e. So if I write this in the nice matrix form I get e theta tilde let us see if this is correct or not, yes one more gamma e f x t excellent and this is like this is it is equal to minus k then I have f x t then I have minus 1 over gamma f x t and then I have 0 here multiply e and theta tilde.

Now if you remember the lectures on persistence of excitation you will remember that we had a very similar looking system when we were looking at this this object if you look at this result again you look at this particular system where we had these of course these nice requirements say AB is controllable, AC is observable, A had to be a Hurwitz matrix because then I can have this Lyapunov equation and this phi function had some nice absolute continuity problem and of course phi phi dot was L infinity then we could claim uniform global exponential stability for this entire system.

So what does it mean here, so this system that we have here is very identical to what we got right now this system so let us try to compare. So first of all first of all what I am going to do is let us assume for the time being that f x t is f of t and that it is independent of the state and let us make our lives easy and assume that it is independent of the state. So that is one, so I am going to nicely mark it because this is a big assumption because here we were talking about dependence with respect to x and all that stuff but here suppose I do make this assumption. So then it starts to match this because it is exactly a function of time. Now let us go back and try to write what is B, C and A matrices are?

So in this case the A matrix is just equal to minus k just a scalar, the B matrix is just 1, the C matrix is 1 over gamma. So let me check if I got the signs right, C matrix is 1 over gamma, now let us see is AB a controllable pair, A and B are both scalars so trivially controllable, yes. Again AC are both scalars trivially controllable observable, no problem. Is A Hurwitz matrix?

Yes, obviously it is just a minus k is a stable system so that is to say that e dot equal to minus k times e is a stable system it is in fact Hurwitz no problem and finally if we assume further that this is f is absolutely continuous and f f dot is bounded then the e theta tilde system is uniformly globally exponentially stable at the origin if and only is this f of t is persistently exciting and that is what it says here.

So if I have all these conditions and we have this we have shown AB is controllable, we have shown AC is observable, we already know A is Hurwitz because it is just a scalar minus k if we assume that this phi which is f of t in our case is absolutely continuous and both the function and its derivative are bounded then the system is UGS if and only if phi is persistently so that is exactly what you require that f is persistently excited.

So what does persistent excitation mean if you you can recall the definition for persistence of excitation you do not need the signal to be strictly positive at all times so a signal can be persistent even if it is going through zeros even if it goes through zeros like signals like these which pass through zeros are still persistently exciting, so that is the nice thing.

Now if you want to go back to the case where f in fact depends also on the state and not just a function of time so I will just make a remark here not actually try to prove it because it is a little bit more involved and complicated so if f x g, if f also function of x then what do you do? You write f as you still want to write as a function of time so you write it as a solution t at some t0 x0 and t this is how you write f and in this case if you go back again we had this more novel result or the more general result it is the integral lemma result and what did it say what it is help us show it helps us to show for parameter varying systems also we can have similar results, so we can have similar results for parameter varying systems also, this is just the parameter dependent output injection and so on and so forth.

So, it can be shown very easily using this integral lemma we did not show but this results are available in standard literature that this a result that you have can be shown also when phi depends on some parameter lambda and can also be shown when phi depends on some parameter lambda, can also be shown when phi depends on some parameter lambda in addition to time and then this is the result you need to use here. So use lambda u PE plus integral lemma, what is the deal with lambda u PE and integral lemma see notice that once I write this x here as the solution which depends on time and the initial time and initial state that is if you make our life simple and keep the initial time at 0 then this is a function of just the initial state parameter.

Just one parameter if you keep the time also initial time also then it is a function of these two parameters. Now you want a result which is independent of these parameters and how do we do that by using lambda uniform PE because that is what gives you uniform, lambda uniform global exponential stability that you get all the properties are independent of the parameter. So basically you start treating the initial conditions as parameters of the system because they are I mean once you fix an initial time and initial state it does not change for the entire duration of your run your simulation or your actual hardware experiment.

If I choose say three seconds when I start my robot and I say that my robot starts in the xy plane at say at the point 1 comma 2 in some cartesian coordinate then it is fixed for the entire duration of my hardware that is in fact a parameter there is no time varying there is no time dependence there.

So we have reduced this problem to a problem of looking at parameter dependent systems and this is where the integral lemma and lambda PE we said that we introduced these more complicated notions and we were worried maybe some of you might have thought why am I taking this excursion into more complicated things and what is the purpose and so on but this is the purpose but in most real scenarios you will have functions that depend not just on time but also on state, so what we do in those cases is we simply write the state as an evolution as a solution and then it depends only on these parameters which is the initial data of the system and then there is initial data is the parameters and we just want to give properties which are independent of these parameters called uniform with respect to these parameters, so that is the idea.

So in this case you see that under conditions such as persistence of excitation and uniform persistence of excitation or lambda uniform persistence of excitation you can in fact prove things like exponential stability that is you can get convergence of both the tracking error and also the parameter error.

So usually it is not very evident here but not if f is just a pure function of time but if f is also a function of the state just like we have here and your state x is trying to track the trajectory r it should be obvious to you that if the trajectory r is rich enough. So this condition reduces to if you look at this lambda u PE condition this condition this will reduce to r of t being sufficiently rich, so reduced to r of t being sufficiently rich or containing sufficient number of frequency components.

So when we do simulations and tests that is what we do and we want to see r with sufficiently large number of frequency components which is what makes the r of t signal rich and if the r of t signal is rich then the x solution is rich because it is eventually going to track the r signal, so lots of oscillatory signals then you have oscillatory x solutions and you have much higher chance of identifying the parameter.

But like I said already before this is not guaranteed in real applications in a lot of real applications the trajectories you want to track are not super oscillatory with tons of frequencies and all that you want to do nice simple things in fact that is what you are designing controllers for you do not want them to do crazy oscillatory movements I mean you do not want your robot to be doing this is just to follow a straight line. So you want to do relatively simple things and therefore adaptive control theorists do not care about parameter identification, so if you want to do the system identification it is a separate question.

If you have the freedom to do system identification if your parameters are not going to change with time then absolutely you can do your system identification and then feed these parameters to a normal controller but if you do not have the luxury of doing these kinds of experiments for example if you have one ton spacecraft which you cannot do so many oscillations of into system identification experiments of and there is a chance that these parameters may change in space conditions or high temperature conditions or very low temperature conditions then you have no choice but to resort to adaptive control where you do not necessarily get parameter identification but you are guaranteed to get precise tracking.

So once we understand what is happened in the scalar case and for the scalar system we want to move on to using our wisdom, the same wisdom that we have already learned to try to design adaptive controllers for the second order system. So we are just taking very baby steps one step at a time so and and please do not think that these are very trivial steps most mechanical systems that you see around can be written as second order systems but the thing is here we assume that the system is scalar that is each state is in fact a scalar value but for mechanical system these may be vector value but that does not change too many things it does not change the fundamental ideas that go behind the adaptive control design.

So please do not worry about things being very trivial and very messy you it is very important that we get a handle on this first order and second order scalar systems very well, so that we understand how to do the design what the issues are and so on so fourth.

What is the setup? the setup is that you had this system you had this right hand side already you are just adding an integrator to it you are just adding an integrator layer to it. So if you make a block diagram I would make the original earlier system and then I will add an integrator to it is as simple as that. You have x1 dot is x2 and then x2 dot is given by this, so the control appears here in the second layer and as I said the states are evolving in the real number space, the function f takes again the states and time and gives you some real numbers, theta star is the constant unknown parameter and u is as usual a scalar value. The standard second order scalar system this is the setup and what is the aim?

The aim is now to design a control such that the first state if you think of it as a mechanical system motion is the position you want the position state to track a smooth bounded trajectory, so we do not specify the second state that is x2 state because it is intrinsically related to x1 state by the derivative therefore I cannot specify x2 arbitrarily and therefore if I want x1 to follow r, x2 has to follow its derivative there is no two ways about it. I cannot have instead of r and r dot I cannot have r and 2 r dot, I cannot have r dot by 2, I cannot have any variation I cannot have r dot squared I cannot have any arbitrary condition on the trajectory that x 2 forms.

So to make it simple we refer to these conditions as matching conditions that is because these trajectories have to somehow be consistent with the dynamics of the system. So x2 desired has to equal the derivative of x1 desired is the desired value of x1 has to, if you take the desired value of x1 you should get the desired value of x2. So this is because it is dictated by the dynamics here.

So what did we look at today, so what we looked at today is we already had completed the analysis for trajectory tracking we were successful, so of course we are very happy but we could not prove anything about parameter convergence. So we tried to leverage what we had learnt in the last week to see under what conditions we can get parameter convergence and so we saw that if you have only functions of time appearing in the dynamics then you can use persistence of excitation and if you have functions of both state and time f x t then you have to use parameter dependent that is lambda u persistence excitation and then we gave the setup for the second order scalar system that is we are going to do adaptive control design for in the subsequent lectures. So this is where we stop today, thank you for joining.