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Nonlinear Adaptive Control
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Week-6
Lecture 31
Analysing of Parameter Varying Systems using General Integral Lemma - Part 2

Hello everyone, welcome to another week of our NPTEL on nonlinear and adaptive control, I am Srikant Sukumar from systems and control, IIT Bombay. We have as always this very nice motivating image on our background of this SpaceX satellite that is orbiting the Earth. And we are well on our way to developing algorithms and analysing them, that will eventually help drive systems such as this satellite autonomously. So, without delaying too much, we want to give a quick recap of what we were doing last week.

So, last week was sort of an excursion on so, last week, if you see here was sort of an excursion on the notions of persistence of excitation. So, we wanted to use persistence of excitation in order to prove stability of certain parameter identification systems. And that is what was the focus of almost the entire week, but of course, it was not just persistency of excitation itself, we introduced several new notions such as, these alternate exponential stability theorems, then we sort of connected persistence of excitation, so, we wanted to make this connection to exponential stability. So, we introduced the notion of uniform complete observability. And of course, connected that to exponential stability.

And, further, we went on to talk about the notions of exponential stability for linear time varying systems, which is what commonly appears in parameter identification for linear system. So, in model reference adaptive control, this is what you see, will we will see this. And then of course, UCO under output injection, and then we went on to prove how persistence allows us to guarantee that parameters converge to their true values.

So, of course, we also saw more general results of integral lemma and on parameter varying systems, we sort of preface this by saying that such parameter varying systems are in fact more common than purely time varying systems in adaptive control. And we will of course, again look at examples. So, we wanted to show this, how this parameter varying systems can also be analysed under this new notion of lambda uniform persistence of excitation.

So, what we will do today is, I mean starting today, of course, is start to look at our first adaptive control problem. So, we will spend this week actually introducing the notions of adaptive control to all of you, that is the idea here. So, but before we go on to begin this, I wanted to say a little bit more about how we were doing the proof in the very end of the last week sessions of stability of parameter varying systems. So, I want to sort of give an alternative approach to doing this proof, which makes a life a little bit more easier. So, that is sort of what I want to look at for a few moments first, and then we will move on to our material on adaptive control.

So, let us begin. So, if we go back, and you look at the system, we were analysing this was this very similar to this time varying system. The only thing, additional thing here was there was a additional parameter lambda. And we want all these nice exponential stability properties for the system, which do not depend on this lambda anymore, we want this lambda independent product. So, although this lambda is a parameter, which means once it is this, once the value is known, it is fixed. But in a lot of circumstances, we want to make claims about a system for many different values of lambda, or for a range of values of lambda.

And we would not want our stability properties or convergence properties to be impacted by values within this range. So even if the value changes from say,1 to 5 for example, I do not want the stability properties to alter within this range. And this is what is the purpose of doing uniform parameter uniform properties. So, this is the system. So simple scalar system, we already analysed this kind of a system where lambda was not present. But when lambda is present, we need this additional mechanism, this integral lemma and so on, and so forth, lambda uniform persistency of excitation, these were already introduced.

However, I want to slightly change how we analyse it. So, the general integral lemma required two things that the max of the infinity norm of the signal and some p norm is upper bounded by a constant times the norm of the initial state. And that is what we are going to try to use this. Because this gives us lambda uniform local exponential stability. Which means that uniform local exponential stability holds for all lambda in some domain. And of course, we can get a global version, of course, we can get a global version if this c exists for all initial conditions in R n, as simple as that. So, you want to, we want to try to satisfy these sorts of conditions for this system.

So, how do we go about it? As how we started about, started doing it last time, is that we sort of took a function V, let us not worry about it being a Lyapunov function, and so on and so forth, just a function V, which you can see is nice, radial and bounded and all that. But we are not going to comment on the Lyapunov theorems, because we are not really using the Lyapunov theorems here. So, we take a derivative, and it is pretty straightforward. It just comes out to be minus a squared times x squared, which I know is negative semi definite.

Now, because it is negative semi definite, I know that V is non increasing. And this immediately means that the square of x at t is less than equal to square of x at 0. And from here, I immediately get a relationship between the infinity norm of x at the initial condition, at the initial value. This is the first requirement. So, the infinity norm of x, I know is already bounded by norm of x 0. So that is what I have shown.

So now, in order to go forward to the rest of the steps, I am not going to use these. I am not going to use this. So, I am going to directly integrate this, I am going to try to directly integrate. So, I am going to write it in a sort of different colour. So that this is evident, so I know that this is equal to minus twice a squared times the V. So, because V is x squared by 2, so V dot is minus twice a squared times V. So now, this is a nice scalar system, and I can integrate it, what is the integral? So, if I integrate it, I will get something like dV over V is equal to and this is the integral from 0 to say some V of t.

And this is integral from 0 to t, I will get something like a minus 2a squared t lambda dt. Notice lambda is fixed, because it is a parameter. So, once I have chosen the value of the parameter it is fixed. So therefore, the integration is just with respect to time. So, what do I have here? So, if I solve it, I will get something like V t, is equal to let us see this is actually not 0, but V 0, is V 0 e to the power minus twice integral 0 to t a squared t lambda dt. And this is where I start to use my measure lemma, is where I start to use my measure lemma.

What does it say? I know that this a is lambda uniform PE. And, if there is a lambda uniformly persistent signal, then I know from this measure lemma that there exists a lower bounded finite time over which this signal has a nice lower bound value. So, if I expand this, if I actually try to expand this, this will be something like this is less than equal to V 0 e to the power minus twice 0 to some kT a square t lambda dt. So, all I have done is I have written this small t as some kT, plus delta. So, we did this even before t is kT plus delta and ignore delta, I can do this because it is like I have used this inequality here.

And once I have done that, I use this on each such interval, on each 0 to t and t to interval, I use this kind of an integration condition. So, and I am going to write this again as less than equal to V 0 again e to the power minus 2 summation over i equal to 1 to k, integral i, and this will be i minus 1 T, to iT a square t lambda dt. And this quantity is simply equal to V 0. So, I will just retain less than equal to just so that we are not confused the inequality is. So, this and this are just equal. So, this is well actually no there is still a less than equal to this is fine. So, this is less than equal to e to the power minus 2 summation i equals 1 to k, integral from i mu i a square t lambda dt.

So notice, what I am doing? What I am doing is I am taking this interval. See notice in any interval of t to t plus cap T, I am saying that there is a sub interval over which this quantity is at least this much value. And this is exactly what I am going to use. So, I am just going to look at the sub interval, and why is it okay to look at only the sub interval because this is a non-negative quantity. And so, even if I discard the rest of it, the rest of it is going to make at least some non-negative contribution. Therefore, I can, even if I do not consider the rest of the interval I am okay.

So, I am okay with considering a sub interval. And, now that I have considered this sub interval, I know that on this sub interval, I have this nice lower bound. And that is what I am going to use. So, this is mu over 2T phi M and all this, whatever this quantity. So, this is basically going to be less than equal to V 0 e minus 2, sum over i equal to 1 to k. This is going to be, I have to look at this expression again, mu over 2T phi M whole squared mu over 2T phi M whole squared times the measure of this. So basically, this is a lower bound, this quantity is a lower bound, so I can pull this out, I take the lower bound, I pull this out. So, the integral is just this dt, which means it is just the length of this interval.

And that length of this interval is at least this much. So, this again I copy to T mu over 2T phi M squared minus mu. So, this is just T mu divided by 2T phi M squared minus mu. So, this is what I get. And notice this summation means nothing much, because there is no dependence on i here. So, this summation can be erased, and replaced with k. Because I have k copies of the same thing. I hope this sort of makes sense. I will give you all a few moments, just think about it. And that is this sort of makes sense.

So now, if you see I have a T k and a T here. This is k and a T, which is equal to t minus delta. So, this k times T is just t minus delta. So, I can actually write this as such. This is just equal to V 0 e to the power minus 2 times I am going to call this some k 1 square. And the rest. Honestly, I am going to call this a some constant, gamma times t minus delta. So, I have combined all everything else into one constant gamma, you can do that. Or I can do that. And this is equal to V 0 e to the power gamma delta times e to the power minus gamma t.

So, what have we shown? So, what are we effectively shown? Let us look at this carefully. We have shown that x t squared, which is V of t is less than equal to x 0 square from here times e to the power gamma delta, which is a constant, times e to the power minus gamma t. Now, it is not difficult to integrate both sides, I will integrate both sides with respect to time. So integral x t squared dt from 0 to infinity has to be less than equal to 0 to infinity, x 0 squared e gamma delta e minus gamma t dt. Here only this is depending on time.

And what will I get here? I will just get e to the power gamma delta divided by gamma times 1 minus e. Just a second, let me be careful. This is going to be e to the power minus gamma t, minus gamma. This is just going to be 1, and I am going to get x 0 square here, and what is this quantity on the left? This is nothing but the square of the L 2 norm. So, this is essentially what I have computed is actually norm x 2 squared, there is just norm x 2 square is what I have computed. So, I have also gotten then what? So, I am sorry. So, this is.

So, what I proven is that norm x 2 square is less than equal to e to the power gamma delta divided by gamma norm x 0 square, because x 0 square and norm x 0 square is the same, they are scalar quantities. So, I have also gotten a relationship between a p-norm and upper bound on the p-norm that is the 2 norm in this case. And the initial condition one which is again what is this requirement. So, we also gotten a c here. So, I have got a I bounded the infinity norm by some value multiplying the x 0 norm, and the p-norm that is the 2 norm in this case, also by some multiple of the initial condition. And that is what we need for your integral lemma.

And so, this is enough to prove that. So, this highlighted quantity along with this two help satisfy the integral lemma. And this is enough to prove that your system is lambda uniformly, locally exponentially stable. In fact, in this case, I believe we can even claim lambda uniformly, globally exponentially stable. Because, I do not think it depends on there is any it is agnostic to the initial condition x 0. So, this is what sort of wraps up our discussion on persistence of excitation.

So, this is sort of what was remaining, we had done it with a different method. And, there were some queries left there on how we were integrating this and so on. So, I wanted to give you an alternate approach. In fact, this is very similar to what we did for the purely time varying case when there was no parameter. So, it is not too far from what we had done.

So, now we can sort of move forward and look at what is adaptive control? And how do we do adaptive control for a very basic system? And that is the idea. So, basically, we had already mentioned that adaptive control is the notion of designing a feedback control for system when some parameters are unknown. And we of course, want to achieve some tracking objective, we definitely want to achieve some tracking objective. So, you can think of again, we can think of robotic system, or space systems just like our nice motivational image.

Suppose your spacecraft is you want the spacecraft to track a particular orientation trajectory, or even a trajectory, or rate on the orbit. The idea is that you may not very well be able to model ever the entire spacecraft, it is not very easy to test a spacecraft while it is rotating on the earth, because of in general the sizes that are involved. So, we usually rely on some kind of robustness in our controllers, in order for our standard orientation controllers to work. So, adaptive controller typically does away with this by adding an estimator, or a parameter estimator.

For example, if the inertia is unknown, because of the fact that you did not model the space stock very well. You can apply an adaptive controller, which is agnostic to this error in the inertia parameter, and it will still achieve perfect tracking. So, this is critical, achieve perfect tracking. So anyway, so this is where adaptive control shines. And, we want to start off with like a first order nonlinear system. So, what is the first order nonlinear system? This is a system of the kind 2.1. Here there is an unknown parameter. So, this theta star is typically an unknown parameter, multiplied by some nonlinear function of state and time, and then there is also the control effort, which we are get to design.

So, this u of t is what we get to design. So, more and more, we started designing things. And, of course, there is some initial condition, we keep things very simple it is a scalar system. So, the states evolve in reals, the function f also maps the state and time to real numbers. And similarly, the control is also real number. And, theta star is some unknown constant parameter. So, this is the sort of setup, you will always have in adaptive control. Your, so, you basically you make a few different assumptions. So, we will of course, talk about the assumptions subsequently.

The first thing is that the control objective is typically for your state to track some smooth bounded trajectory r of t. Why does it have to be smooth? Because, I mean, you want, you do not want very jerky motion. For example, again, I think of a robot or a satellite, I do not want them to do a lot of jerks. I want them to move smoothly. So, it is not very unnatural to think of smooth trajectories. And of course, we want bounded trajectories because, again, we do not want the states to become unbounded, typically, I mean, those are would be rather very unusual application, where you want the states to become unbounded, and those are usually considered on a case-by-case basis. So, the objective is for the state x to track a trajectory r of t.

So, what are the standard assumptions? The first is that the model is linearly parameterised, what does it mean? It means that the unknown parameters appear linearly in the state space model. So, in this example, or the first order scalar system, you already see that the parameter is appearing linearly, this is the first assumption. Now, one may think that this is a rather restrictive assumption. But honestly, it is not. In all of the example, then we may we will look at some example, it is possible to deal with a nonlinear parameters also with this assumption, instead of identifying for example, if the parameter appears as theta squared instead of a theta, what an adaptive control will do is to identify theta square itself, rather than trying to identify theta.

And therefore, the parameter stay, the system stay linear in the parameters. So, this is a lot of cases can still be handled of non-linearity. So that is not such a big deal. So, the second assumption is that the unknown parameters are constant. So, this is again one might think rather restrictive assumption. Again, in a lot of cases, this works out quite well. When your unknowns are time or state dependent what we usually do is we consider basis functions. So, we assume that this function is parameterise linearly as a linear combination of basic functions, and then we try to identify the weights of the basic functions which are constant.

So, this is the in fact the basis for neural networks, where you have basis function, and you just try to identify the weights using an adaptive control type design. And therefore, again, these weights are constant, so in a lot of these cases, we identify constant unknown parameter only. And that is what we will look at in the entirety of this course. Of course, there are results which are more research oriented on slowly time varying parameters. So, it is very well known that if your parameters sort of vary during the operation very slowly for example, the parameters change maybe once in a day, or once in 2 days.

An adaptive controller is still able to readapt to the new parameter values, this is again something rather true, which is you cannot it is like a self-repair mechanism if you may, if you have a system which, for which something, there is some kind of a fault some issue, and then you sort of have a parameter which suddenly goes bonkers and changes values. An adaptive controller if implemented will still adapt to this new value. And it has been shown in experiments and in theory that you still get good performance. This is something that cannot be claimed by conventional nonlinear. So, this is again something special.

So, we assume that the model is linearly parametrised and we assume that the unknown parameters are constant. So, this is the setup. So, we will start from the next session, we are trying to design a controller, an adaptive controller for the system. What is an adaptive controller? It involves designing this control law to do tracking, and also an estimator and for this parameter theta star, so that is the idea.

So, what is what did we look at today? We spent most of our time to be honest trying to analyse this parameter varying system using notions of the general integral lemma, and lambda uniform persistency of excitation. And we started off on the first order scalar system, for which we will start designing an adaptive control law in the subsequent session. So, this is where we stop today. Thank you very much for joining.