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Nonlinear Adaptive Control
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Week- 5
Lecture 29
Parameter Identifier Convergence under PE Condition

Hello everyone, welcome to yet another session of our NPTEL on nonlinear and adaptive control, I am Srikant Sukumar of systems and control, IIT Bombay. We, are well into our, fifth week and we have been looking at rather interesting notions on how to prove convergence of parameter identification systems.

We of course motivated by these very nice background images that come up behind us and this one is on a spacecraft which is orbiting the Earth. And we hope that the algorithms that we design and the algorithms that we are analyzing, help us to develop autonomy for systems such as this. So, without delaying further, let us sort of look at what we were doing until the last session.

So, we moved on to trying to prove Exponential stability of a parameter standard parameter identification system. So, this system 7.1 that you see here is actually very, very standard in parameter update laws. So, parameter update laws and the derivatives typically look something like this in a lot of applications. So, we are trying to use persistence of excitation of the signal phi. In order to conclude that you have in fact stable parameter convergence and that is parameter converge to the true values, if you have some persistently exciting signal and so on.

So, in order to do that, we of course to Lyapunov candidate a valid Lyapunov candidate and then we computed a V dot, which we found was only negative semi definite, but when since we are interested in using this alternate exponential stability theorem, we are not too worry. So, we actually take the integral of this V dot and this is what this expression is.

So, I am going to highlight this again. So, that we can come back to this later. So, we are going to come back to this later. So, our subsequent attempts, like I said, are were to try to get a bound on this point, how did we begin we started from the connecting the persistence of phi to UCO of this system.

So, we showed that this system is in fact UCO, because of the persistence of phi itself got it and this is where we were. So, we want to first start from here.

And so, let me mark this, this is lecture 5.5. So, now that we have established UCO of a particular system, we want to use the idea of UCO being invariant under output injection in order to get to our original system that is really the idea here. So, what we do what do we do for that we consider rather interesting output injection term, which is basically this k of t is minus alpha phi. So, of course, this is a vector, it is a vector of dimension n for this k the gain itself is also a vector of dimension n. So, then if we compute, so, one of the requirements for the output interaction theorem was we had to compute a moving average bound. Suppose, we do that.

So, and that is what it is basically integral from t to t plus cap T of this quantity. And remember that we do have some kind of persistence bound on this quantity, there is an upper bound on this quantity and that is what we seek to use. It is well known that we use a rather well-known equality which is that norm or phi transpose phi for any vector for that matter, is the same as trace of phi phi transpose. This, of course, is norm phi squared. This is a very standard equality. And so here we have a norm of phi squared because the alpha square can be anyway be pulled out, it is just a scalar quantity.

And the norm of phi square can be written as the trace of phi phi transpose. Why do we do that? Because we have an outer product here and we want to see an outer product here. That is it. So, trace is just the sum of the two elements. So, now if you look at this guy, that this upper bounded by mu 2 I. So, the trace of this is upper bounded by mu 2 times n. So, why? Because, again we use the fact that trace of mu 2 times the identity matrix is just n mu 2. Because this is an identity matrix of dimension n. So, if you just sum mu 2 on all the diagonals, just get n times mu 2. And that is what we get here, alpha squared n times mu 2.

So, now that we have a nice upper bound on this sort of the output in, we know that A plus KC C is also UCO. So, in this case, basically k, just the k of t, the small k of t and C is basically just phi transpose of t. So, if I substitute so, what do I get A was 0 of course, the showed here, A is 0, C is phi transpose and K is minus alpha phi.

So, if we just substitute here, just from our injection output injection result it says that if you have a nice moving average bounded k, then you cannot destroy the UCO property. If you do this output injection on the dynamics, then essentially what we get is A plus KC C is UCO. And if I just substitute all these quantities in here, I will just get that minus alpha phi phi transpose comma phi transpose is in fact UCO.

So, what do we do? So, we go back to this nice integral, let make this smaller. So, you can see both go back to this integral in equation 7.3 and that is what is rewritten here. It is rewritten in the entire format. Here, I had just here, it is just written as a phi transpose x the whole square. But here, we expanded.

Here, we expanded, that is all and not just expanded, what else do we do? We write x in terms of the state transition matrices. So, which is what this is, I have written x as phi s t x t. So, x is phi s t times x t. This is basically standard way of computing solutions for linear time variant systems using state transition matrices. So, if you want the solution at s, you take the state transition matrix from s to t and then multiply by the value at x t. Now, the important thing to remember is that and I do that on both sides of course.

So, the important thing to remember is that these two quantities are now independent of the integration variable, and therefore I can move them outside the integration. That is what we do. And what we will left it inside is again our favorite UCO Grammian. So, this is what we like. We want to see the UCO Grammian everywhere. Why?

Because we are proving some kind of UCO condition and we went to leverage it to bound this thing. So, remember, so this is what we have for our expression here. The UCO Grammian and I also have that this system is now uniformly completely observable, which was obtained via via output injection through a nice gain.

Now, what is the system? So that is what we want do, we want to write this system. What is this dynamical system? There is dynamical system is just this and this. Because this is the A, this is the mu C, is the new A matrix, this is the new C matrix and is not the old A C but it is the new A and new C matrix. So, this the dynamical system and this is the output matrix.

What do I have? I have a system which looks like this I have a system which looks like this. Now, notice what is this system? This is in fact, the system that we are trying to prove exponential stability of. So, this system is exactly this guy. The same system. So, exactly the same system.

And this is an output this output is of course, you can see that this is not a real measurement. I mean, I do not think anybody would be able to claim that if I use the sensor, I would measure something exactly like this. So, this is not like a sensor measurement, this is sort of an artificially created output so that you can complete your stability. That is it. Because we need an output injection.

That is all the purpose of using this y. This y is not necessarily real measurement from a sensor. So, let us not sort of, we find ourselves and try to think that this is possibly some real measurement. It is not, I do not promise that it is any kind of a real measurement whatsoever. All I will say is this just a construction for the purpose of analyzing stability here. So, the only thing we need to focus on is sort of this system. And that is good, because this system is matching the system for which you want to prove stability.

Now, what is the UCO Grammian corresponding to this I know that this system is already UCO, when constants beta 1 bar beta 2 bar. So, what is the UCO Grammian? The UCO Grammian is t to t plus cap T phi transpose s tau sorry phi transpose s t, C transpose which is phi s C which is phi transpose s and the state transition matrix ds.

Let, us look at this and here the phi is corresponding to this system the phi correspond to this system the phi is the state transition matrix of this system. Now, if you look at this carefully this expression. So, what do I end up in before we do that, we know from the UCO criteria that this is less than equal to some beta 1 bar identity and this then equal to beta 2 bar identity. So, I have a bound on both sides and because of the UCO condition.

Now, if you look at the Grammian expression here, you see that it is exactly identical to here. This is exactly the same, because phi here is the state transition matrix corresponding to the original dynamics, which is this. And so, is this, this phi is also the state transition matrix corresponding to the same time and the matrix in the middle phi and phi transpose is of course, the same. So, now, the fact that these two are exactly identical helps us to bound this guy because of the UCO condition I have a lower bound on this and because there is a negative sign that is what we get, we need the lower bound.

So, what do we have? We use this lower bound to actually upper bound V dot as minus 2 alpha beta 1 bar norm x t squared. So, there is no numbering in this or maybe. So, earlier Grammian or maybe I will just number 7.35 let me call this equation 7.35. This is 7.35. So, I have an upper bound on the integral of V dot, and this is exactly of the form that we require in our alternate exponential stability theorem. And so, great.

I have in fact, prove that the equilibrium x equal to 0 for this outer parameter identifier system is in fact, exponentially stable. And this offer we have leveraged persistence of excitation UCO and UCO under output injection to do all this. So, and as I have mentioned before, the y that we actually see is purely for analysis purposes, there is no role of y, there is no real measurement, which can possibly resemble y. So, it is purely for the purpose of analysis. So, just think of it as an artificial construction.

Now, there are also extensions of these, which is again something which appears very commonly in model reference adaptive control parameter identifies. And that is, this is not a time invariant system. So, this is linear time varying system. We, look at this linear time varying system, which is again, something that is very common in modern reference adaptive control error here is basically some kind of a tracking error, or because it is a model reference system. So, e is the error with the model reference typically. So, that is what is mentioned here, is that all the norms. So, tracking error e and then you have a parameter estimation error theta tilde.

And usually, this is the dynamics and you find, you will find, like a sort of interesting dynamics, where you have an A is basically the coming from the original system itself or problem. B time phi is basically the dependence on the parameter. Because, there is a parameter error that is, it is an unknown parameter. So, there is no way I can cancel the parameter term. So, I sort of do the best and so, I did something in the parameter there.

And I get something in the update law, which is depending on the e on the tracking error, this is very standard very standard in model reference adaptive control when we get to that stage you will see that our system turns out to be of this form. Now, further if we consider. So, therefore, we have answering this linear time varying system.

But, further if we have A B to be a controllable pair, the standard assumption and A C to be an observable pair again a standard assumption, but there if there exists for a given positive definite symmetric Q there exists a positive definite symmetric P such that this Lyapunov equation is satisfied and this PB be equal to C transpose here both of these happen. But and phi is absolutely continuous in phi phi dot.

Then you have that the origin of this system is uniform being globally exponentially stable if and only if phi is persistently excited. So, this lot of different points here, so, these two I would say are like standard assumptions in linear systems, standard assumption in linear observer and control. So, nothing special about this is very standard assumption in linear systems observer and control design.

So, it is not like there is something too new that we are sort of like introducing here, the next condition here is like a, exactly like a Lyapunov equation corresponding to A being hurwitz. So, here A matrix is a hurwitz matrix and it is a stable matrix, then this sort of an equation is always satisfied. If, A is a stable matrix or hurwitz matrix, which is usually the case here, then this sort of a Lyapunov equation can always be satisfied.

This is the sort of Converse stability theorem for linear systems or conversely Lyapunov theorem for linear systems. Now, this condition is basically like a standard what I would call a matching condition. This is sort of additional, is the matching condition. And of course, this we are not really defined what absolute continuity is?

So, I will ask you to look up the definition, but the purpose of feeling absolutely continuous is just so that persistence excitation et cetera, are well defined, because in order to do persistent excitation, you need to take an integral and so on and so forth. So, essentially, we require phi to be absolutely continuous and we also require phi and it's derivative to be bounded. So, there is a standard is a very again like a more I would call a regularity assumption. So, that things are well defined, that is all so, that things are well it.

So, then if we have these four conditions, like, I would say relatively reasonable conditions, then you can actually claim that z equal to 0, that is the 0 equilibrium is in fact uniformly globally exponentially stable if and only if phi is persistently excited. So, this you can see is an extension of the previous result. The nice cool thing here is that you when you say z, z is basically both these states together, you are saying not only to the tracking errors converge, but the parameters also converge. So, this is not very common and adaptive control. Now, let me be honest, in a lot of cases, when we solve adaptive control problems, you will and we will do that very soon.

You will start to see that parameter convergence is not usually guaranteed and adaptive. All an adaptive control theorist will tell you is that in the presence of an uncertainty by designing a parameter estimator, what we can guarantee is that your tracking error that is your control objective, which is say for example, your robot wants to move with some kind of a sinusoidal shape.

So, basically, this kind of a tracking objective will be precisely met no problem, even with the parameter uncertainty, but identification of the set parameter may not happen. And, that is where this requirement of persistence of excitation and so on. And, also here to the theorems that you have, into the conditions that you have got, the theorems that you have are rather limited in terms of what kind of systems they can handle?

Because if you notice, the two results that we saw, are, in fact, only for linear systems, it is not really that easy to come up with such conditions when your dynamics is normally used. And, that is what we are usually dealing with, we are usually dealing with nonlinear systems. So, there is still a lot of open problems in trying to talk about parameter convergence for nonlinear systems.

So, adaptive identification of parameters that is precise identification of parameters is not guaranteed in adaptive control. There are only these few cases where in fact, you have some results, which under which typically work under persistence of excitation. And they guarantee that, your parameters will also converge to the true value.

On top of tracking errors going to 0. So, tracking error is going to 0 is guaranteed by all adaptive control. And that is what most practitioners are interested in anyway. They do not care about clearly identifying and noting down parameters. Sure, but in some cases, yes, you are also interested in doing that, because once you identify the parameters precisely enough, and if they stay fixed, you can implement simpler controllers subsequently and not need to implement adaptive controllers. So, this is sort of what the conundrum is in adaptive control. Of course, it provides a nice solution, but it also leaves something out, there open to work.

So, one of the other points, the final points on this additional result is that this condition three that you have that is this sort of condition, it actually helps you to know ensure that this Lyapunov function works. And, it is not very difficult to verify. And then we can actually take a very nice simple derivative here. So, V dot from here becomes e transpose P e dot. So, this is I believe a constant because A is absolute, A is a constant matrix.

So, we have when I am going to write it on e transpose P e dot plus e dot transpose P e plus theta tilde transpose theta tilde dot and this if I substitute for e dot from this guy here. So, what will I get? I get e transpose P A e plus B phi transpose theta tilde plus same thing, e transpose A transpose plus theta tilde transpose phi B transpose times P e plus theta tilde transpose and theta tilde dot is just minus phi C.

So, minus theta tilde transpose phi times C, as usual I have sort of gotten rid of all the time arguments. So, if you look at just these two together, this becomes e transpose PA plus A transpose P e. And, then I am left with now, then I am left with, plus twice phi time you are going to combine them theta tilde transpose phi B transpose P e. I missed an e is here. So, I do not think we should have an half here. And so, there should be a minus twice here. And so, this is minus twice theta tilde transpose phi time C time e. So, this of course, is nice minus e transpose Q e from our Lyapunov equation and these two in fact cancel out how?

This is by virtue of the fact that we have this set of a condition PB equal to C transpose. So, if you see this PB equal to C transpose. So, C is equal to B transpose. So, these two are the same. So, they actually cancel. So, we are left with this kind of minus C transpose Q e and this is our usual starting point if you remember, like this is where you have V dot is negative semi definite and you start to integrate V dot and you go on with the proof.

So, we are not completing the proof for this case, you can look up the proof. This work is by Narendra sometime in 1977-78, I believe. So, you can look up the proof if you wish. But this is the starting point you remember, we have a negative semi definite V dot, we start to integrate the V dot, start to use PE, UCO and all such conditions on Phi and then we sort of try to go on from here. So, I would strongly recommend that you try to see how you can complete this.

So, what did we look at today, we completed the proof for parameter identify convergence, under persistence of excitation. We saw how we could leverage the PE, UCO, UCO under output injection, this alternate exponential stability theorem. So, basically all the neat letters and results that we looked at from the beginning of this week, we could leverage that to actually prove that you have this parameter identifier convergence.

We also saw an extension of it, where you have this sort of system which contains the tracking error e from the model reference adaptive control setting, which you will look at the future and also the theta tilde. So, you had both the e dynamics and the theta tilde that is the parameter aerodynamics.

And we claim that theta and e tilde are exponential uniformly globally exponentially stable at 0. If this again this phi type of gain term is, phi t is in fact persistently excited. And we show what kind of conditions are required for that. So, anyway, we continue more on this when I mean try to wrap up this sort of material in the next session. And, that should be an interesting session. Thank you folks.