Advanced Strength of Materials
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Lecture - 1
Structure of Materials Part – I

In your first course, you have seen how to calculate stresses in component slide, bar, shafts, subjected to actual loading, bending loading and tonsorial loading. These are simple situations, most often. Components in machines are having complicated geometry. They will have three dimensional configurations like this.

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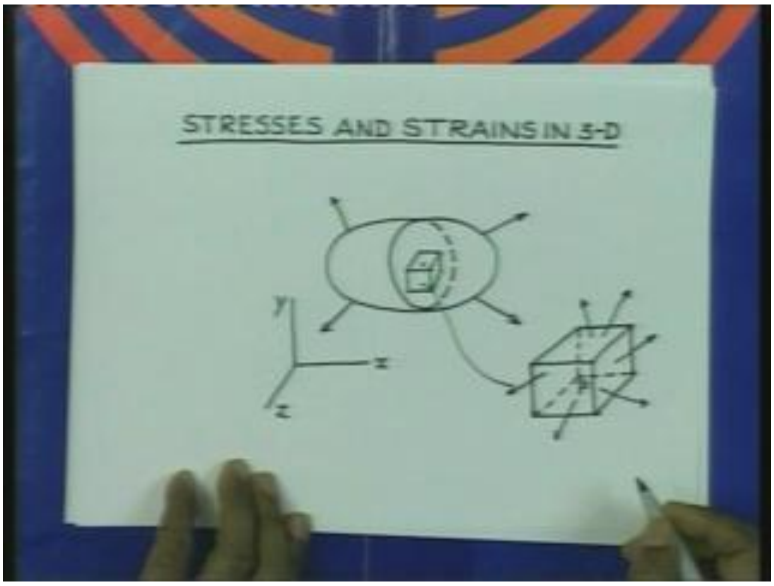

They are subjected to loading, under the action of the external loading, internal forces developed. Each and every point, undergo deformation and there are also internal forces at each and every point. Our objective, today is to see, how this internal forces, can be quantified at a point and also, how the strains can be quantified at a point on the body. In order to do that, we are interested in quantifying the stresses and strain at a point, let us say p.

And what we consider, we try to enclose this point by a very small cube. And when we try to segregate this cube, from the rest of the body and I will draw it to a larger scale here. So, this point around the point p. This body is subjected to forces by the rest of the body. Each of this, six phases of the body, will be subjected to some forces, internal forces, if we can show by the forces like this. So, there will be six stresses and six forces coming up on each of the phases.

We can express these forces, for unit area. And as these size of this cube is shrunk to 0. We will find that, the stresses or force intensity at the point p will be obtained. So, we are going to see, how the stresses can be obtained from these forces acting on the six phases of this cube.

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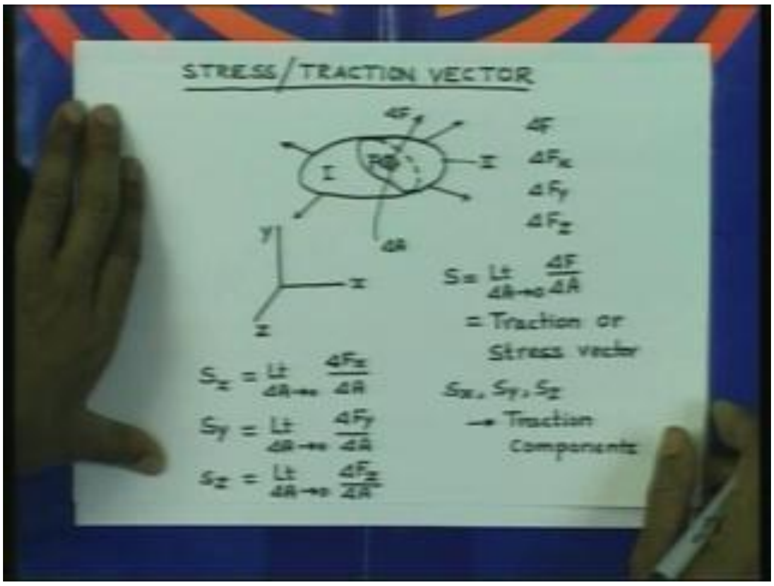

So, will focus on looking into the stress first, we will see, what is known as stress or traction vector. We consider again the body. Now, if we divide this body into two parts by splitting it along a surface. Let us say, that we split this body into two parts by considering this split. It has now got divided into two parts, 1 and 2. Typically, if you consider a point on this surface, the force applied by part 2, on part 1, at this point p, let us say, will have a typical magnitude and direction.

So, will consider that, this force is equal to delta f. Now, if you just consider an area around this point. Let us say these area, which will represent by delta a. Now, delta f will have components. It can be resolved into components in the three Cartesian directions, x, y and z. Let us say, that these components are in the x direction is delta f x, in the y direction is delta f y and in the z direction delta f z.

Now, when we try to take the intensity of these force delta f, for unit area, when you try to calculate the intensity, that is delta f by delta a. And if delta a tends to 0. We get one limit, which is indicated by s. So, therefore, this intensity of s, given by this ratio, that is defined at s. And this is known as traction or stress vector. Now, we can also define limit is like the following s x is given by delta a tending to 0 delta f x by delta a. That is the component of the traction index, x direction. So, also we can have s y limit delta a tending to 0, delta f y by delta a. Similarly, s z limit delta a tending to 0, delta f z by delta a. So, these components s x, s y, s z; they are known as traction components.

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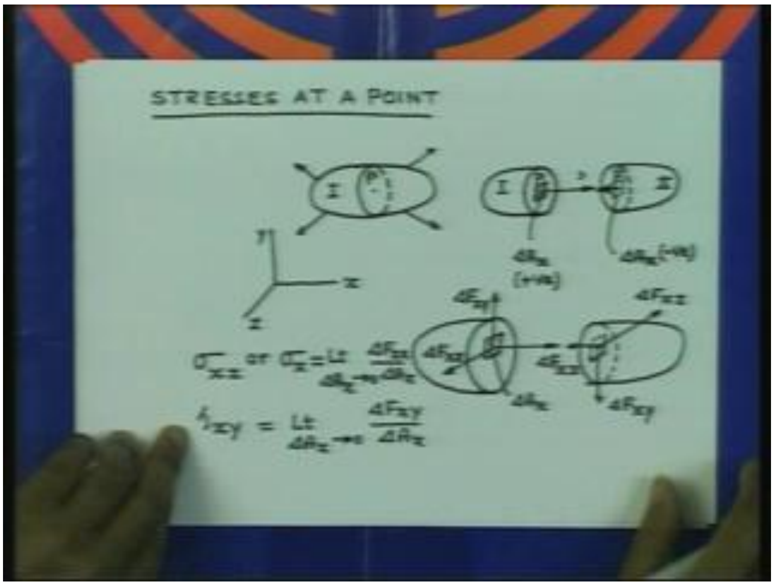

Now, will try to go into considering the stresses at a point, the body that we have considered will again consider the same body. And now, we would like to consider splitting this body into two parts by considering a plane passing through over point p. And it is perpendicular to the x axis, so will have our Cartesian directions, as before. So, we now consider a plane, which divides this body into two parts and this is the typical point here on concentrating on p point.

Now, the outer normal to this plane, if you consider this part 1; so we will write separately. So, this is out part 1. And the outer normal on this plane is oriented in the positive x direction. And now, if we consider an area, there is small area around this point. Let us say that area is equal to delta a x. Now, this area is delta a, but we give this suffix a, to indicate that the outer normal to this area is directed in the positive x direction.

Now, if you try to consider the part 2 of the body and the same plane. So, this is the other part of the body. So, this is part 1, this is part 2. Now, if you consider the point p on this area and the outer normal to this area is now directed in the opposite direction. So, this point is, how the normal to this plane is now directed exactly in the opposite direction. So, therefore, if we now considered this area, here on this surface, it will be also indicated at delta a x.

But, we consider this area to be negative. Because, it is outer normal is directed in the negative x direction. And this area, it is outer normal is directed in the positive x direction, that is delta a x and this is positive. Now, the force, which is applied by part 2, on this is going to have arbitrary orientation. And this can be resolved into three components, so to draw it little to a larger scale. So, therefore, I would like to draw this again and let us draw it to a larger scale.

So, we can have the force resolved into components like, delta f, on the x plane in the x direction delta f, x x, that be the core. In the x direction and the component in the y direction, it is x plane acting in the y direction. And the third component is delta f, x z. so these are the three components. Now, the other side, because just look again into the forces on the other side.

So, on this point again, I will have the forces, which are going to be just opposite. It will be directed in the negative x direction and this component will be delta f, x y. It is again directed in negative direction and the z component is going to be like this that is delta f, x z. Now; obviously, these components are mirror reflection of these components. So, in fact, on the negative area, please note this point, on this negative area, the forces that are going to act. They are just going to be directed in the opposite direction with respect to this area, which is positive.

Now, we can define the limits, that if the area delta a x tends to 0. Then, the intensity of this force, that is force per unit area as delta x ax tends to 0. That is what we call it to the stress in the x direction sigma x x. Sometime, we also represent it by sigma x and this sigma x is equal to limit delta a x tending to 0, delta f x x divided by delta a x. So, the intensity of the stress delta, intensity of the force delta f, x x as delta x, a x tends to 0. That gives the stress on this area at the point p in the x direction.

Similarly, this delta f, x y can be also considered for unit area and as delta x tends to 0, we get one component, which is represented by delta x y. And it is given by delta a x tending to 0, delta f, x y divided by delta a x. So, this is tau x y and it is acting parallel to this area. So, it is a shear stress. Similarly, we can consider the intensity of delta f, x z for unit area and as this area tends to 0, we get the intensity at the point p. And that gives us the shear stress for x z delta a x tending to 0 delta f, x z divided by delta a x.

Obviously, the stresses on this phase are going to have the same magnitude. But, they are going to be directed in the opposite direction. So, please note this, that on this positive phase, we are going to have the stresses, given by these values. And the stresses on the negative phase, this area is going to be given by the same magnitude. But, they are directed in the opposite direction. And note that, these sigma x x is acting normal to the area. So, therefore, it is known as normal stress.

And the other two components, tau x y, tau x z, which has arising out of delta f, x y delta f, x z respectively, they are acting parallel to the area. So, they are known as shear stresses. We can now consider a plane, dividing the body, which is perpendicular to the positive y direction.

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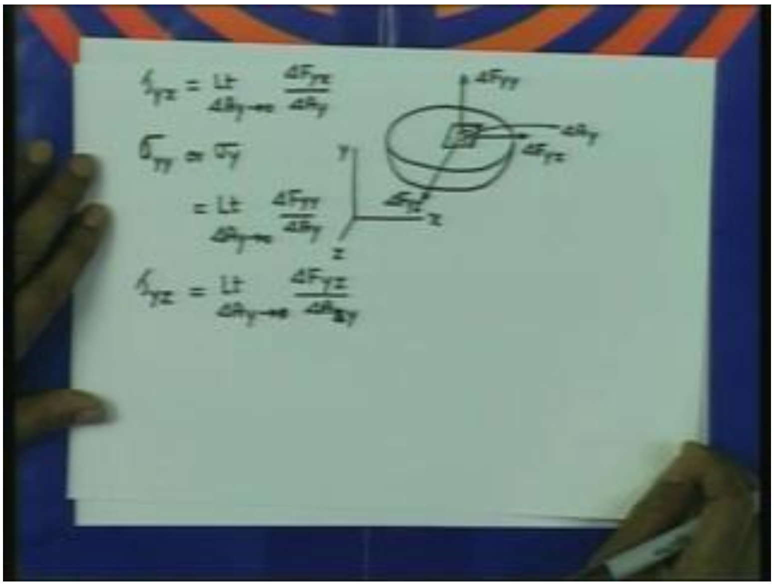

So, if you consider the bottom part of this body, after we have divided it by a plane, perpendicular to the y direction. This is the bottom part. And now the point p, we have really considered, this plane passing through the point p and if I now consider a small area. So, just I think, I will redraw. So, if you consider this bottom part, this is the point p. And now, you consider the area, around this point.

This area, you would like to indicate by delta a, y, why suffix indicate that the outer normal to this area is it directed in the positive z direction, I am sorry, positive y direction, because this is our y axis. Now, the force, which is applied by the second part on the first one, at this point, would have again some arbitrary orientation. And it can be resolved into components like this.

We will have one component, which is delta f, y y. Then, will have one component, which is parallel to the x axis. So, therefore, this is acting on the y phase in the x direction. So, it is delta f y x and then we are also going to get components in the z direction. So, this is delta f, it is y plane, z direction. So, therefore, this is delta f.

Now, if I consider the intensity of these forces for unit area. And then as you take the area to shrunk to 0, we get delimits. We get the stresses like this, delta, I am sorry, tau y x limit, delta a y tending to 0, delta f y x by delta a y. So, that gives us the shear stress acting in the x direction on this area. Similarly, if we consider the limit delta a y tending to 0, delta f y y by delta a y, that gives us the stress in the y direction, sigma y y or sigma y.

Similarly, if I take the third force and if I consider again similar limit, you get the stress tau y z, delta a y. So, these are the three stresses acting on this plane, which is positive y plane passing through the point a. So, we have got again three components of stresses acting at this point on a plane y. If we consider a plane, passing through the point, which is perpendicular to that z plane.

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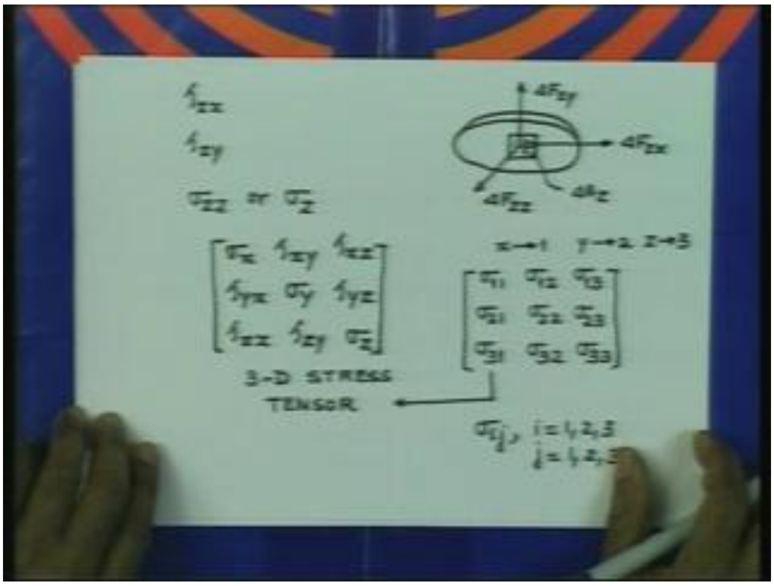

So, will have again the division of the body like this and the point p is here. If we consider again an area, enclosing this point, that area will be indicated as delta a z. It is outer normal is directed in the upward direction. Now, the force acted upon or force acting on this area, will have components in the three coordinate directions. We can resolve them by e t, components parallel to z axis delta f z z. And the component, it is parallel to the y axis is delta f z y and the component delta f z x.

So, these are the three components of forces, which are acting at this point, because of the external loading. And from these again, you can calculate the three components of stresses, which will be represented by tau z x, arising out of this force, tau z y, arising out of this force and sigma z z or sigma z arising out of this force. So, thus, you can have three planes passing through the point and as you calculate the intensity of these forces, internal forces and as the area is shrunk to 0, we get the stresses at the point p.

And we have seen that, there are three planes on each of the planes, there are three stresses. Therefore, we will have now nine components. If you can write at one place, sigma x is the stress. Normal stress in the x direction, tau x y is the shear stress. In the y direction on the x plane, tau x z is the shear stress on the x plane in the z direction. Similarly, for the y plane, we will have stresses like y plane in the x direction, then normal stress in the y direction. And then shear stress on the y plane in the z direction.

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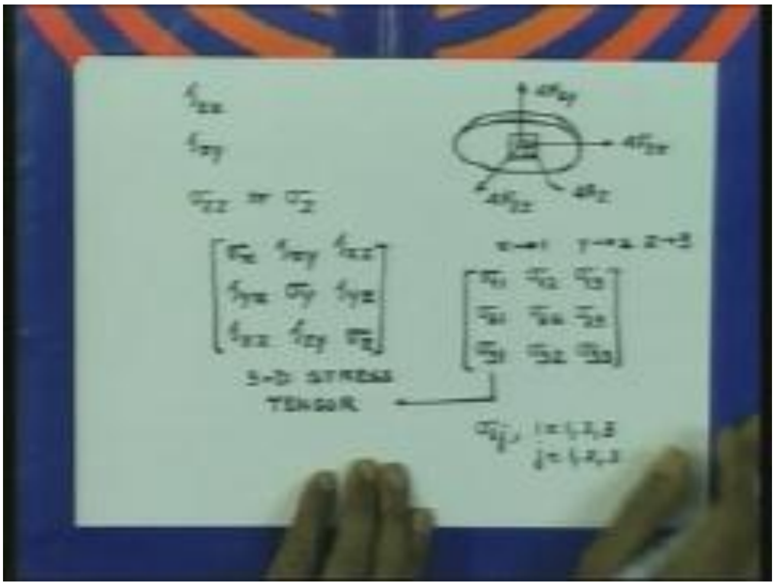

Similarly, for the z plane, we will have tau z x, tau z y and sigma z. So, these are the nine components. These are nothing but, the component of 3 d stress tensor at the point p. If we make use of the initial notations, if we indicate the x direction by 1, y direction by 2 and z direction by 3. Then, in that case, we can represent these stresses in the following form, sigma 11, sigma 12, sigma 13 on the plane 1, sigma 21, sigma 22, sigma 23 on the plane 2 and sigma 31, sigma 32 and sigma 33 on the plane 3. So, this is also the way that stress tensor in three dimensions are indicated. In fact, by using tensor notation, we can indicate these stress components by simply by sigma i j, with the implicit assumption, that I takes up value 1, 2, 3. So, also j takes up value 1, 2, 3.

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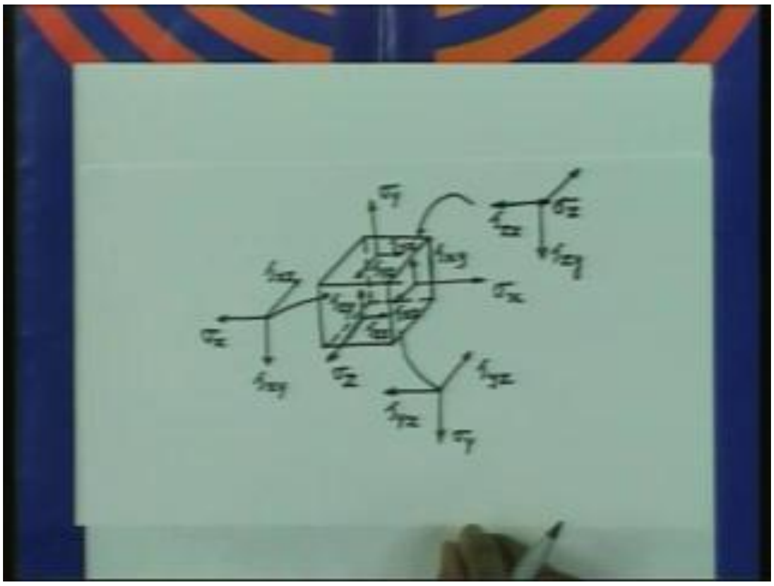

Now, to indicate the stresses at a point, we try to consider, as I have said earlier. A unit cube or infinity symbol q and the intensity of the forces acting on these phases are nothing but, the stresses at the point. Intensity of the forces, acting on the phases is nothing but, the stresses at the point. Now, you can see that, we are trying to say that, these stress at a point is nothing but, the intensity of the internal forces, as this i's of this cube is shrunk to 0.

And now, we have seen that this stresses acting on the phase here is nothing but, sigma x stress. And in this direction it is nothing but, tau x y and the stress in this direction is nothing but, tau x z. Similarly, the stresses acting on this phase, which is perpendicular to the y is nothing but, sigma y. Then, we have nothing, but y x and this is nothing but, tau y z. And the stress, which is acting on this plane, they are again having components like, sigma z and the component in this direction is tau z x and the component here is tau z y.

So, these are the component from the three positive phases. And now, the components, which are acting on the negative phases, that can be obtained by considering the mirror reflection of the component from the positive phases. I would like you to concentrate again on one diagram, which I have shown you earlier. That this stresses on the negative phase here are nothing but, the mirror reflection of the stresses on this phase.

So, therefore, they are as the dimension of the body is shrunk to 0. These stresses on the negative phase are going to be nothing but, mirror reflection of the phases on the positive phase. So, we can now draw these stresses on the positive phase. So, if we want to show these stress from this phase, it is going to be nothing but sigma x directed in the negative x direction. And shear stress is going to be tau x y, which is directed in the negative y direction and the shear stress in the z direction is going to be directed like this.

Similarly, on the negative z phase, we will have this stresses given by the shear stress is going to be directed like this. This is going to be directed in the negative direction, because it is negative z phase. So, therefore, this is tau z x and we are going to have the shear stress here tau z y. And the normal stress is going to be directed like this, which is nothing but, sigma z stress.

And the stress at the bottom point or stress at the bottom surface is going to be again given by sigma y acting in the negative direction. And the shear stress, it is going to be tau y x and the shear stress on this phase is tau y z. So, that is how, you are going to get the stresses coming up on all the six phases and under the action of these stresses the body is in equilibrium. So, therefore, this is how the stresses are acting at a point in the body under external loading in three dimensions. So, you see that there are nine components of stresses would define the state of stress at a point in three dimensions.

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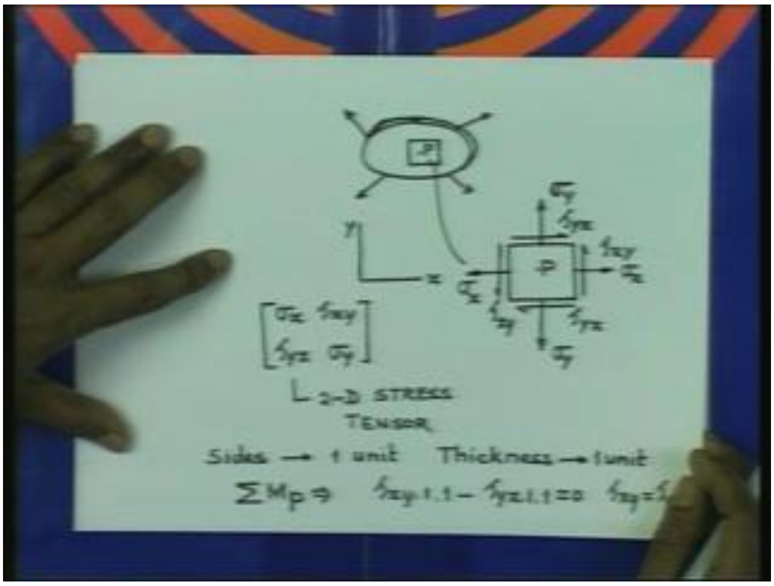

If we consider a two dimensional body, which you are quite familiar consider a thin disk like this. So, this disk is subjected to say external forces in it is own plane. There could be loading also coming from internal point. Under the action of these forces, we will find that at a point on this body; let us say p will again have stresses. And will try to show it by considering an infinite square, around the point and the stresses acting on the edges of this square will be the stresses.

So, therefore, if you draw to a larger scale, this is the element around the point p. And if we select our axis to coincide with the plane of the body, then we are going to get the stresses acting on this will be positive phase. So, there would be sigma x stress acting here and there will be shear stress acting on this phase, which is x stress y direction. Similarly, the stresses on the opposite phase, which will be given by the mirror reflection, so this is sigma x and this is going to be the shear stress tau x y.

Similarly, the stresses on this plane, which is positive y plane, it is going to be sigma y. And the shear stress is going to be tau y x and on the negative plane, it is going to be sigma y and the shear stress is tau y x. So, these are the four components of stresses around this point, components of stresses acting at the point p. And the two dimensional stress tensor is therefore indicated by these components, tau sigma x, tau x y, tau y x sigma y. So, these are the four components in two dimensions. So, this is nothing but, 2 d stress tensor.

Let us now consider case, that the sides of this cube are of unit dimensions and the thickness. So, sides are 1 unit each, thickness is also 1 unit. Now, will see that, these stresses are such, this element is in equilibrium. The forces acting on this phase is nothing but, sigma x into one forces acting on this phase sigma x into 1. So, they are equal. Therefore they are balancing. Similarly, these force is nothing but, tau y x into 1 and this is tau y x into 1 again, that is balancing. So, there is no net force in the x direction.

Similarly, in the y direction, this force is going to balance with this force. Similarly, this force is going to balance with this force. And if you now consider the moment about the point p, will find that the distributed force sigma x. It is resultant is going to pass through this point, if p is at the center. Similarly, resultant of this force is going to pass through the center. So, it will not produce any moment.

Similarly, the resultant of sigma y and this sigma y will pass through the point p and it will not produce any moment. The moment will be produced by forces like, tau x y into 1 and this tau x y into 1 and so tau y x into 1, tau y x into 1. So, if you now consider the moment about the point p equal to, if we calculate the moment, then we will find that, this moment, these two forces are of magnitude tau x y into 1. And it is acting at a distance of 1. So, therefore, moment is this. It is anti clockwise.

Similarly, the moment produced by tau y x and this tau y x is going to be nothing but, tau y x and area is 1 and distance is 1. So, therefore, this is the moment and this is equal to it must be an equilibrium. Therefore, it is 0. And hence, you will find that tau x y is equal to tau y x. So, this is very important, that these shear stresses acting on the two perpendicular planes, they are equal. So, they are known as complementary shear stresses. So, if this is working, this has got over and that is why, this is known as complementary to this. Similarly, on this side this compliment. So, if we extend this consideration to three dimensions, we will find that we will have tau y z is equal to tau z y.

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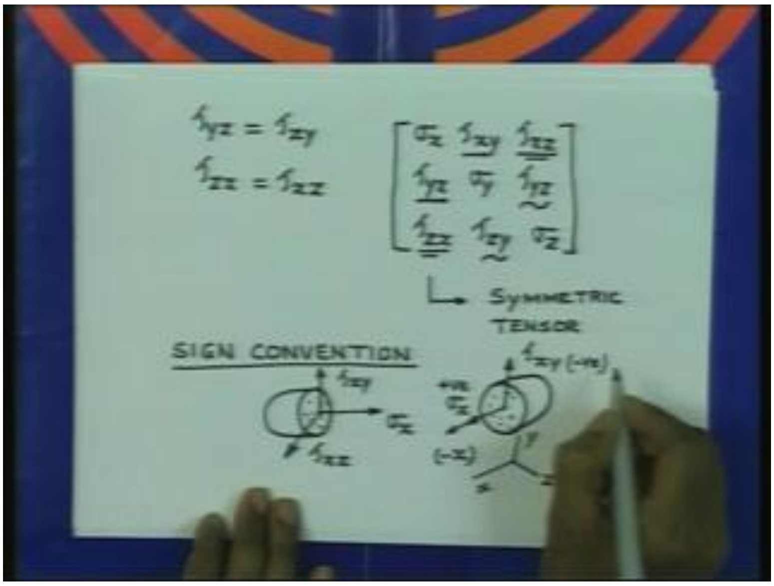

Similarly, tau z x is equal to tau x z so these shear stresses are equal. And hence, I would like you to look at the 3 d tensor, stress tensor once again. So, we have sigma x, tau x y, tau x z, tau y x, sigma y, tau y z, tau z x, tau z y and sigma z. So, in this stress tensor, what we find that, this component is equal to this component. So, they are equal in magnitude.

Similarly, this component is equal in magnitude of this and this component is equal in magnitude of this one. So, all of diagonal terms are having similarity and this similarity gives rise to a symmetric tensor. So, this 3 d stress tensor is a symmetric tensor. Now, we like to look into the sign convention, what is a positive stress, what is a negative stress. So, therefore, we will now consider sign convention for stresses.

If the phase, acting on a positive phase, let us say, you consider this phase and if the stress acting on this phase, this outer normal is directed in a positive x direction. So, therefore, it is a positive phase. And if the normal phase is acting in the positive x direction, then in that case, it is a positive stress. Positive phase and the stress is directed in the positive direction, it will be a positive stress.

Similarly, the shear stress, if it is acting on this positive area and it is directed in the positive direction. So, this is tau x y, it is also positive stress. And this is also the stress x z, which is acting on the positive phase in the positive direction, directed in the positive direction. So it is a positive. So, all this stresses are positive and inside that, when the normal stress is positive, it is tensile in nature.

Now, if you considered a negative phase, let us say, that we will consider this component like this. Now, it is outer normal, let us say that this is directed in the negative x direction. And now, if the stresses are acting like this, that the stress is acting now in this direction, normal stress, this is sigma x. So, which is negative phase and it is directed in the negative x direction.

Let us say, this is x direction, this is y and this is z direction. Then, you see that, this stress is acting in the negative direction. So, it is negative phase and negative direction of this phase. So, therefore, this stress is again positive. And suppose, we have the negative phase and the shear stress, which is acting here, it is tau x y, it is negative phase. But, the shear stress is acting in the positive direction. So, it is a negative stress.

So, therefore, this is a positive stress. So, this is positive, whereas, this is negative stress. So, therefore, if area is negative, but the direction of the stress is positive, then it is negative stress. And if both are negative, if area is negative, direction of the stress is negative, then it is again the positive stress. And you will see that, positive normal stress is going to be always tensile and it will cause tension on the area.

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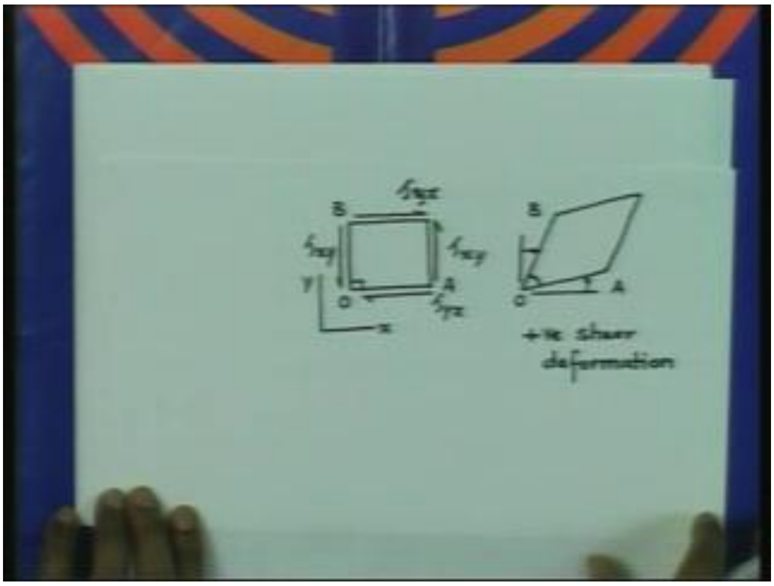

Now, positive shear stress, I would like to give you some physical consideration for the positive stress. If you have two dimensional problems, we can immediately consider that these are all coordinate directions. Now, the positive shear stresses, this is positive tau x y, this is also positive tau x y and this is positive tau y x. This is positive tau y, tau y x, this is the positive, these are all positive stresses.

Now, you will see that, under the action of the stresses like this. It is quite natural to expect, that the element will deform in the following manner. We can deform like this. So, here in, what you find is that, if you consider that this is o, this is a, this is b, o a and o b are initially at 90 degree. But, because of this stresses the angle between o a and o b becomes less than 90 degree.

So, here in, what we find is that, the direction o a, has rotated like this. Direction o b has rotated like this and therefore, this causes a reduction in the original angle. And this is known as positive shear deformation. In fact, positive shear stresses will give rise to the reduction in the angle in the first quadrant. And it will lead to increase in the angle in the 4th or 2nd quadrant. Here, in you see that, this is positive x direction, this is negative x direction, so this angle has increased. So, therefore, thumb rule, as a thumb rule, you can take that positive shear stresses, will give rise to reduction in the angle.

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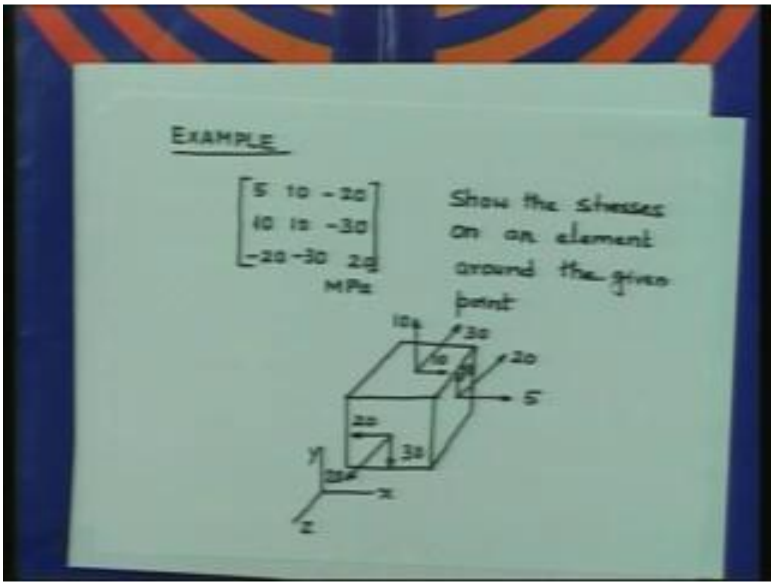

Now, we will consider example of showing stresses. So, let us consider one example of, say the stress tensor at a point is given by, so these are the stresses at a point. And this is in given in terms of unit Mega Pascal. Now, we have to show the stresses. So, show the stresses on an element, around given point. So, if you consider now the infinite decimal cube around the point and I will just show the stresses on the positive phases. And we will have the same convention for the axis. So, these are our axis.

Now, you see that on the x plane, I have stress like, this is 5 MPa and this is acting in the positive direction. It is 10 MPa, so this is 10 and this is acting in the negative direction. So, therefore, it is going to be directed like this, so this is 20 MPa. So, these are the stresses on the x plane. Similarly, here on the y plane, we have normal stress is 10; acting in the positive direction and the shear stress is acting in the positive direction.

So, therefore, this is 10 and this shear stress is acting in the negative direction. So, therefore, this is going to be equal to the direction of the shear stress indicator. Similarly, here on this phase, we have the direct stress. It is in the positive z direction, which is given by 20 and the shear stress, here in the x direction is in the negative direction. So, therefore, this is 20 and this shear stress is acting in the negative direction. It is of magnitude 30. So, these are the stresses as for this particular stress tensor and you can show these stresses on the negative phases by considering the mirror reflection of this.