Ordinary Differential Equations

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Lecture - 3
Examples Continued

So, in the last lecture you we were studying about interesting problems from population growth, and we first introduce the linear model in we are seen that linear model is not suitable when the population is high. And then we derived the non-linear model, it is also known as logistic model curve, curve we have not yet plotted.

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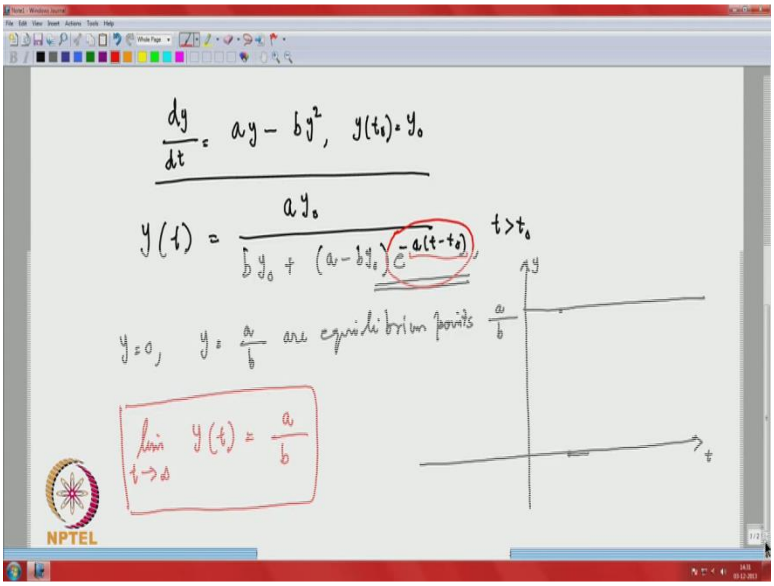

And then the equation which we have seen is d y by d t is equal to a of y minus b of y square with initial population y the t naught is equal to y naught. And we have seen that a solution under we have given provider few exercises, I hope you have already done the exercises by now. And you have produced a solution in the format y of t is equal to a of y naught by b of y naught plus a minus b y naught into e power minus a into t minus t naught is valid for t greater than t naught, this is what the solution you have obtain it.

And what do we have done is that, if you are recall the previous lecture, we have seen two equilibrium point this is the curve t, and this is the curve y, and y equal to 0 for this equation y equal to 0, and y equal to a by b are equilibrium points. You will see the exact definition of equilibrium points at a later time, but what the equilibrium showings are that ,y identically 0 is a solution to the system, similarly y identically y t equal to a by b the constants are that are solutions to the system.

And from the theory we you will eventually understand more and more that when you have two solutions it cannot cross each other, you will see all that. So, whenever you start a t here, y at t equal to 0 it will remain there only, so you will move in a move only this line. Similarly, if you mark y equal to a by b which is an equilibrium solution. So, if you start anything here it will remain there, and that is all we have explain in the last lecture, and we also look ask you to work out the details.

Now, if you look at here this particular term here as t tends to infinity, this is a minus term here, a is a positive quantity a is a positive quantity, and this is a positive quantity, and hence as t tends to infinity, this entire term goes to 0. Once it goes to 0, t tends to infinity this will be, so you will get this fact limit t tends to infinity, y t is nothing but a by b you see, so you have an analysis of that it.

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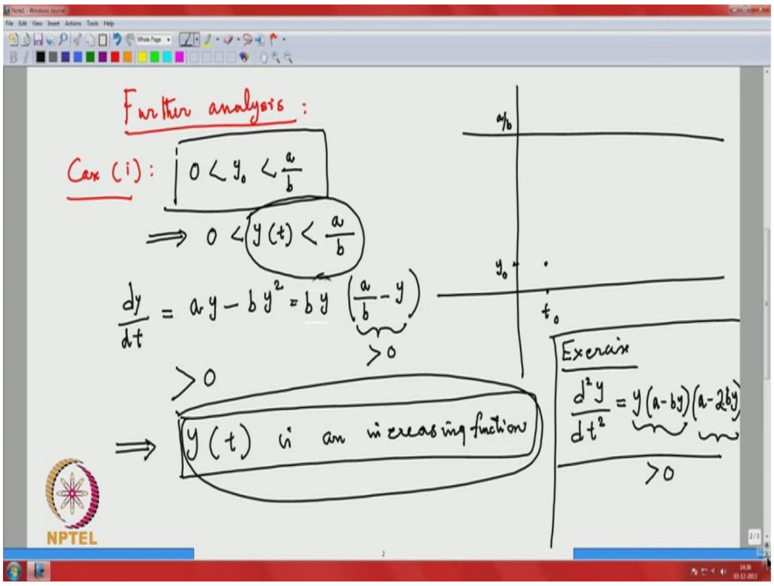

Now, we want to understand a different cases, so we will come to the next thing. So, we will what you do a some further analysis, we want to do some further analysis quickly. So, whatever I left out, you should go back home and work. So, consider the case1first case1. Case 1, so 0 less than y naught less than a by b, this a case situation is something like this here, so you have your a by b curve. So, this is your a by b, and at t naught you are starting the solution, this is your y naught.

And you look at the one of the thing here immediately what we have seen is that the solution, because this is y equal to a by b, y t equal to a by b is a constant solution, and y equal to 0 is also a constant solution as t moves y t has to remain here, this is for the exercises I have given in the last lecture. So, this is also true that 0 less than equal to y t less than a by b all the time, it will remain it can never touch these 2 lines. So, I want to understand that how the curve looks like, and we also seen that in the previous slide if you see you see, the limit of this one is also is a by b, the limit it approaches that, we will go there we will do that one.

So, what will happen? We want to understand. So, look at now this case, interesting case I want to the analysis we want to understand somehow how the curve, how is the shape of the curve. So, let us try to the to understand the curve better you have to understand it is derivative, you have to see the sign of it is derivative d y by d t is equal to a y minus b y square. So, this can be written as y I can take it out, I will take b also outside, so you will have a by b minus y you see y at t.

But, y at t for all time t is less than a by b, so that shows that this is always positive, y is already positive, therefore this is positive that immediately implies this is this case do not forget this thing, it is here somewhere here below a by b. So, this implies y t is an increasing function that is a one observation, so y t increases. So, you do not have any periodic solutions or something like it goes up increasing, but you want to understand little more about it what do you will do, for that here is an exercise for you, I will be keep on giving exercises.

So, one should do that one here is an exercise now for you in the same situation. The exercise is that I want to understand if you want to so I got some information that y t is an increasing function. So, I have an information that y t is an increasing function, but I want to understand little more about it, if you want to understand what do you have to understand more and more of it is derivative.

So, you just compute d square y by d t square you can immediately see that d square y by d t square. You just do computations it is a plane differentiation computation, you can write here it is in a nice way y into a minus b y into a minus 2 b y you see you can do this, this is simple computation which at the. Now, you observe some fact. So, let me at the what we have seen that, this is a positive number, this is positive. Now, to determine the sign of d square y by d t square it depends on the sign of a minus 2 b y.

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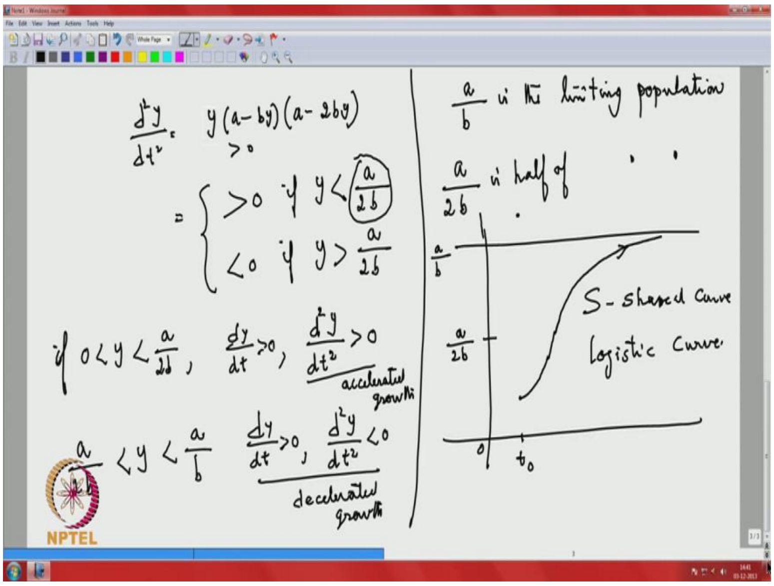

So, let me write d square y by d t square is equal to y into a minus b y into a minus 2 b y, since this is positive. The sign of this is determine by this will be positive d square y by d t square will be positive if y is less than a by 2 b, and it will be negative if y is greater than a by 2 b you see. So, the sign changes of the d square d y by d t is always positive by d square y by d t square changes sign when it reaches a by 2 b you see, a by 2 b is the half of the limiting population if you look at it, say what is a by b, a by b is the limiting population that is what you have seen.

The population cannot grow beyond a by b limiting population, in fact it cannot touch it. It like in linear case where unlike in linear case where in the linear case it can go to infinity, but in this case it cannot go to infinity. So, a by 2 b is half of limiting population. So, when you start the curve here, here a by b you have your a by b. And you mark here a by 2 b that is your half of your limiting population, and at t naught any time t naught if your population is something is here, till the population reaches the d square y by d t square is greater than 0 what does that indicate.

So, up to this area up to a by 2 b d y by d t is positive. So, if 0 less than y less than a by 2 b in this interval d y by d t is positive, and d square y by d t square is also positive, what does that indicates? It actually indicates it both are positive is not only the function is increasing, this is basically you can think it of it is acceleration. So, if you have an accelerated growth. So, the function is increasing in a very accelerated growth, but if you reach if the population reaches half of the population say a by 2 b. And then if the population reaches here up to it never reaches a by b.

In this situation you have d y by d t is still positive, that means the population is still growing, but then the growth of the right growth of the population the it is a decelerator growth. So, this is an accelerator growth here this it is a decelerator growth you see. So, you get basically get if the population starts from here, it will go very fast growing the growth is very fast. And then after that is still growing, but the growths slowly reduce the growth rate reduces. So, you get a logistic curve.

So, this is called S shaped curve or logistic curve you see. So, this is the more or less a complete analysis of this problem, when your initial point y the case 1where y naught is between 0 and a by b. So, we starts immediately the grow population down grow much, if the growth will be decelerator what will happen? If your initial population itself is huge what is the analysis? And that is an exercise for you.

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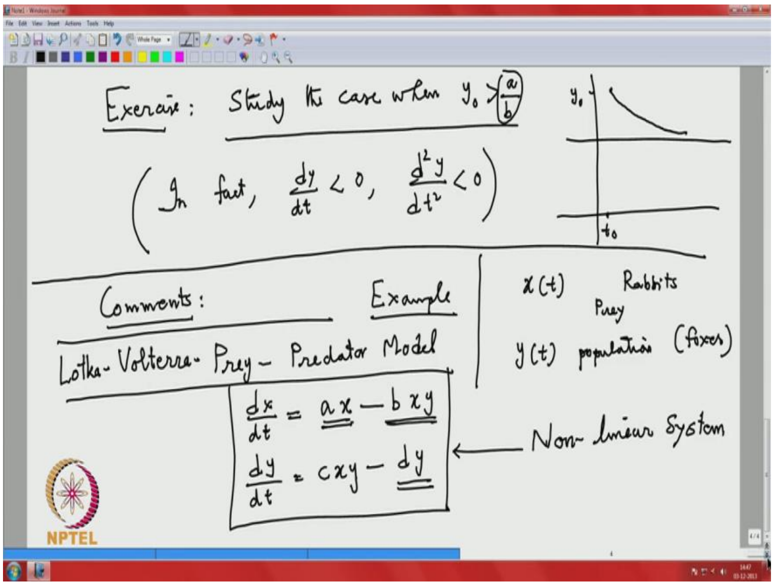

So, here is a final exercise in this problem, study the case when y naught is greater than a by b, since one more problem to be study. You can see immediately you can in fact see all that you can derive, in fact that is no separate at clear d y by d t is negative, and d square y by d t square is also negative, so that means it is a population decreases. So, the moment you have population above the limiting value, then the population decreases, and it is a very decelerator decrease. So, it is will be faster again you the limit will be still a by b, the limit of the population will still be a by b.

So, understand this case, so the once you understand this problem completely you can draw sketch the graph, the graph more or less looks like, but you prove it. So, if you start here initial population at time t naught is equal to y naught then the population will. So, you get that picture completely All right, so that some comments. As I said in the beginning of this example, the parameters a and b appearing that one is vital coefficients. And that varies this vital coefficients a and b varies according to which population you are studying.

So, that is not definite way of determining a and b, you have to study the population over a long period of time, and you have to do the kind of data analysis in that to determine a suitable a and b. And even this a and b will not remain the same for the same population as the time goes. So, for a certain period of time even for including the human population a certain a and b will be good enough, but then you have to revaluate the a and b after a certain period of time.

Secondly, the population model we are given where they have taken into account only the interaction between the a species themselves, that is the term you got it d y square, but in the natural thing that is not the case you have to live with the other species even within the single species, then the other issues like national disasters, epidemic and so many things in a population can come into picture. So, you how to define some model, according to the situation taking into these kinds of facts?

For example, let me tell you one example which you probably see in our course or you cannot that as I said initially, it is not possible to explain to every problem in this course, but let me tell you one important prey predator model. I will just state the equation, I will not give you anything much, but you can seen now since the other model is explain neatly, you will and it is also called Lotka volterra prey predator model. So, in this example you have 2 species x (t) is the species fund you can say the prey, you can think it is a rabbits, so prey.

And y (t) is the population, x (t) is the population of the rabbits prey, and y t is the population of the predators, foxes if you want is just one thing. So, the prey predator model one interesting this you can see that in analogy if the previous model, d x by d t you will have a x this is the population is growing linearly with respect to the prey. So, if you have a prey, if you do not have any other else it will think, but then there will be here I am not taking into account the interaction between the same species. Assuming that the rabbits would not kill rabbits or foxes do not kill rabbits, only the foxes will attack rabbits. So, you have a term the interaction between x and y.

So, you will have a term of this form, and the rate of population of the d y by d t you will have the C x y minus d x square d y square. So, you see this is an interesting again a non-linear model which we not d y square there is no, this is the growth corresponding to that one. It is because one is eating the other one, but to understand that one that is why this is positive sign, and this is the negative sign on the other hand this is the negative sign. So, this is the again a non-linear system, you will come across with some non-linear system if not this one some other non-linear system, and it is qualitative analysis will be studied in the coming lectures will do that.

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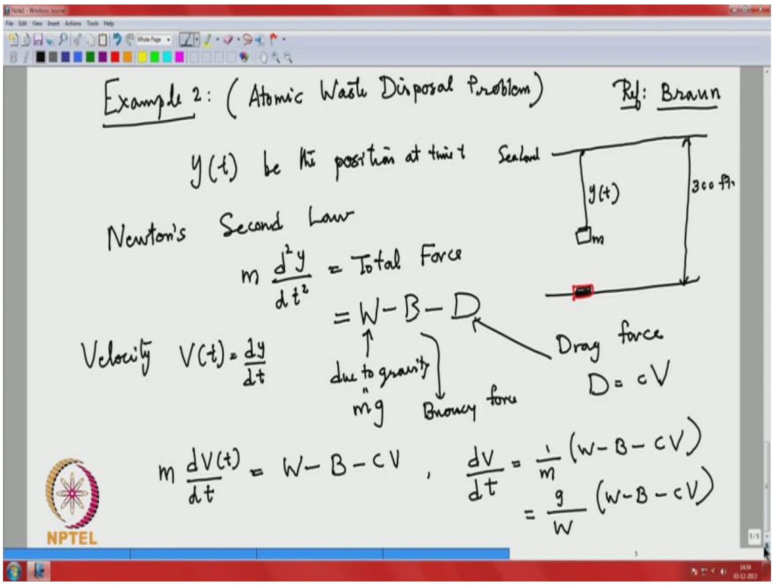

So, there is lot of interesting examples we can do it. Now, I want to give go to a next example as I told you, this lecture and next lecture will be examples from the standard nice example. Initially 2, 3 examples I will explain in detail, and then it is a very nice problem it is actually happen is called atomic waste disposal problem. As I told you all are these are all very standard problems probably to see these examples which are explained here, you can do the book by Braun which is given in the literature, which is by given in the references.

So, you can look into the book that there will be lot more examples. So, it is a very nice problem it is actually happen in US, where the environmental traisted issue about the disposal of the atomic waste, it is a very powerful atomic waste radioactive material, what they have what doing is that you have the atomic waste radioactive waste put it in drums. And sealed it very, very strong drums sealed it, and then they use to dispose it in the sea. So, you will have a dispose it sea, the environmentally this is the sea level.

So, what do I do doing it they were disposing this atomic waste in a place where the depth was around 300 feet, this was the total 300 feet. So, the issue was that when they were disposing these atomic wastes at the sea level, it will acquire the velocity due to gravity. And when the thing is that this drums will reach certain speed when the drums reaches at the bottom of the sea level. And it is possible that the impact the velocity the impact when the drum reaches the boundary may be so high that the drums can break.

So, the environmentalists were protesting against this one the disposal of the atomic waste problem. So, this problem was given how to understand this problem, and this is we want to see even this through this problem you can see even how you are attack the problem is also important. You may get the solution you mean as I said earlier getting the solution itself may not actually solve the problem ((Refer Time: 22:38)) has set out. So, let us calculate. So, calculate let y (t) be the position at time t.

So, you have disposing the thing, and you will the at time t your drums are here, this is this length is y (t). So, it is all Newton's second law, you will apply Newton's second law how do we apply? So, you have the mass of this one mass m, and then you have the acceleration d square y by d t square equal to the total force. So, it is a question of first computing the total force, how do I compute the total force; what are the forces acting here? What are the forces acting which you can immediately write, because any symbol physics which you have studied.

So, you have a gravitational force W, and then you will have a, because it is a water. So, there will be a buoucy force called. And this buoucy force will act against the motion that is what you have to understand. So, you have a buoucy, this one is coming under gravity down. So, there is a buoucy force, and then this acquiring the velocity and you will also have a dragged force. So, this is the due to gravity that is nothing but m g, and this is the buoucy force, and that you know for water this is the drag force, but normally the drag increases proportional to the velocity.

So, for introduce velocity here, so velocity introduce here if I introduce velocity V (t) that is equal to d y by d t, and the drag force will be proportional to V. So, the d will be something like c V for some parameter, so it will be proportional to the velocity, so you will get that one. So, how do I write this equation? So, if I combine these equations fully in the terms of V. So, you will have m into, so let me write instead of the second order in terms of the velocity, I can write it as the first order equation. So, m into d v by d t is equal to the total force, what is that one W minus B minus c V.

We shall write it in a suitable form, I want to write suitable form so you can solve it easily that is why. So, I will write d v by d t is equal to 1 over m into W minus B minus c V, so you see that you will have it. So, but I can write this is equal to m g is equal to W, because this form I need it that is why that you can see that, you can solve it from here itself, but this good form of writing. So, this is m g w. So, I can write multiply and divide by g you get m g. So, I get a W, W minus B minus c V, so this is it.

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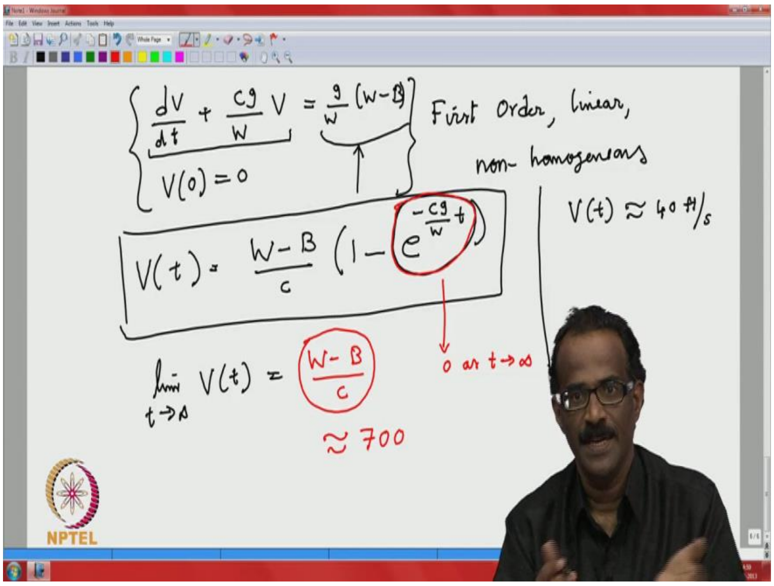

So, let me write it in a form many of your familiar, I will combine all that and I can write it in a nice way d v by d t plus c g by W into V is equal to g by W (W minus B), and the velocity are just dumpy need. So, initial velocity 0 at time t equal to 0, the velocity is equal to 0. So, you see this is a first order equation, first order linear equation. Of course, therefore attending an ODE this ODE course follow listening to this ODE course for the first time they may know how to solve this thing, but most of you I am familiar those who have seen some ODE notions you know how to solve it.

Anyway in our coming lectures we will see how to solve this equations, linear equation. So, let me not a spent time how to solve it here, because we are going to explain to you how to get the solution later. So, let me what I am going to do in that? I am just plainly writing my solution. So, the solution will be V (t) is equal to I can immediately this is also a non homogeneous equation, there is a non homogeneous term. So, it is a first order linear non homogeneous equation, linear equation.

So, this term in this form gives you the linearity this gives you nonlinear, and we will explain in detail how to solve it when we study the first order, second order all that equations, but let me write down the solution very nicely. The solution is W minus B by c (1minus e power minus c g by w t) this is the solution. So, what is our aim as for us our problem is concerned, the aim is to see that does velocity does not reach where the breaking label, and that was calculated that is calculated by the people.

If the velocity V (t) is approximately something like 40 feet per second, it is a computation done by the people environmentalist, either drum you drop the drum and reaches the bottom, by the time it reaches the bottom if the velocity reaches crosses 40, then there is a chance that these drums will break. So, we want to see that by the time it reaches the velocity will keep on increasing whether that is why the analysis is now required, whether if your solution never reaches 40 then there is no problem it is not going to reach and 40 and then.

But then, now you compute this as t the immediate analysis you can do, limit t tends to infinity V (t) is equal to this as t tends to infinity this term goes to 0. Let me mark it the color, this term goes to 0 as t tends to infinity. So, immediately you see that the limit goes to W minus B by c, and these are all known quantities. W, B are all and c these are all known quantities, because W is m g this the buoucy which is has a value, c has the proportionality to the drag force to the velocity, and this are all computed, but this is approximately happened to be 700 you see.

So, the velocity indeed will cross 40 as t tends to infinity, but there is nothing to completely ignore the problem here, it only says that the velocity can cross 40 at certain time. But that does not mean the velocity will reach 40 by the time, suppose it hits much earlier time the ground. If it reaches and hits the ground which is only 300 feet, at that time it may not reach 40 then you are fine. But from here from this computation we are unable to judge at that time it reaches I may, because we do not know the time what time it will heat?

Because, we are calculate the velocity in terms of the time, we have not calculate the velocity in terms of the distant that is where the a different approach. So, this approach even though it looks like your problem is solved completely, we are unable to predict what the actual problem was actual interest was about the problem? So, we have to view this problem, we have to analysis this problem in a slightly different way, and that is what we are going to do it now.

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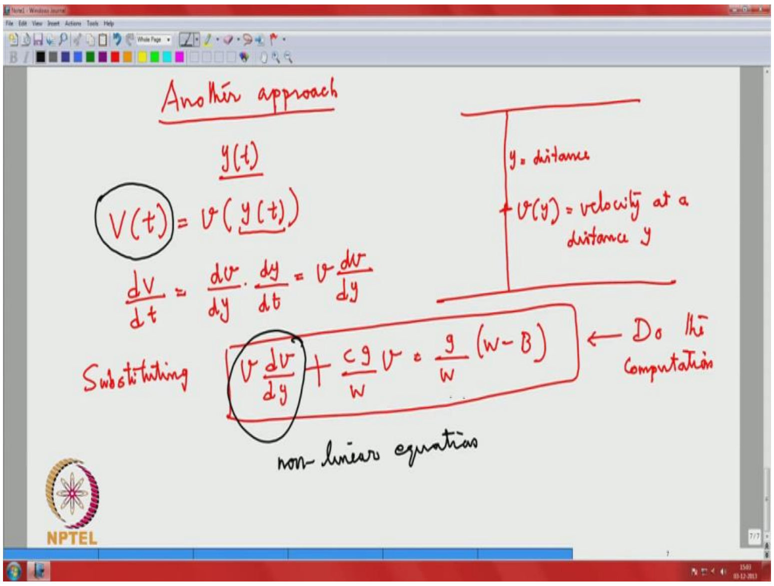

Another approach, so this is again to indicate solving just differential equations may not be the entire aim, and in fact solving may not give you what you want, and quite often it may not be necessary to solve a get the solution explicitly. The aim of the problem will be quite different which you have to learn it. So, the idea is that you have this problem, then you are damping it you want so far you calculate the velocity as a function of, so the velocity is your independent variable dependent variable, and time was your independent variable.

Now, what we are going to see is that your viewing this distance y is the distance indeed the distance earlier also, but you have seen the difference was that you viewed the distance 5 as a function of time. Now, I am trying to view y as an independent variable, and then you calculate the velocity, so it is going. So, you are calculating the velocity v of y velocity at distance y. So, the time is not coming into picture in this formulation now. So, but we need that one. So, v is the velocity at a distance y.

So, we are computing in the downward direction which you understand I hope at a distance y you see, so that is why. So, y is your independent variable now, v of y is the velocity as a function of the distance is the dependent variable. Of course, I tell t you already have, so y equal to t is the distance at time t that you still access, so if you come. So, if you derive my relations v is the velocity at time t, at time t your distance is y (t). So, v of t is nothing but v of y (t), so this relation you already have it you see. So, at time t you are at a distance y (t), and then the velocity at that time function v t, so you compute this thing.

So, it is a question of computing d v by d t is equal to nothing but you have to compute d v with respective y and into d y by d t, but d y by d t is the velocity that is nothing but v. So, this is nothing but v into d v by d y you see, you substitute, so you computed d v by d t in this equation, you substitute that equation substituting in the earlier equation. We get in substituting what you get it, v into d v by d y is equal to plus let me just write there equation the same equation I want to have c g by W v equal to g by W W minus B you can derive to this form. So, if you do not get it, you can get this equation.

These are all simple computations which you should do it you substituted you do the computation as I said do the computation. And I write this equation in a slightly here there is a problem, the problem is something serious now, because this is, so earlier in the equation you got was a linear equation, linear homogeneous first order equation by look at this term this is a non-linear term.

So, the moment non-linear equation, the moment we have changed the way you view it that is why the other model derived first step thinking that we give you the answer, but the required answer we did not get it. So, we are when we change the model, and view thus v as a function of the distance we have a non-linear model, and we have to analysis this model quickly nicely. So, you will I will give you some more exercise probably we.

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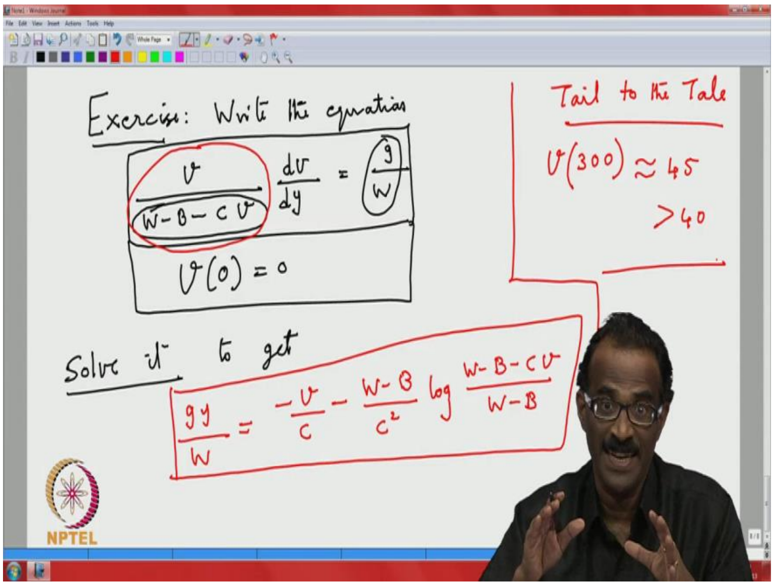

So, I will write this exercise, this is a simple computation you do not have to worry, but you should get familiarity with the computation. Write the equation as I want to form even though it is a non-linear equation I want to write it in the solvable form. Write the equation as this is easy to do it, equation as let me write it v by W minus B minus c v into d v by d y this is not difficult to write is equal to g by W. So, a non-linear equation solving is there are no general methods at all, so you have this equation. Now, this is an equation in v d v d y you can take it outs the other side, and of course the velocity initially it is an initial value problem, so you have a v 0 is equal to 0.

This equation can be integrated solve it, this is the solve it. This is standard integration problem it may look complicated does not matter, what you can add is that you can add you can make a numerator similar to this one, first you multiply something you can take c outside, and do all kinds of things. If you take c outside, you get W minus B by c, and v is the you add and subtract something. So, you can decompose it in a partial fraction or whatever form it is which you can solve it, and solve it to get.

If you do not get it at least after few lectures in differential equations after studying the basics of differential equation you should be able to solve it, but you need not have to know any differential equation to solve this one. So, this is an integral problem. So, you have to just integrate it which you can do it, because d y this is in an at least where even though it is non-linear it is a this were constant. So, g and g w this is the constant, so do not have to worry about it. So, d y you can take it, it is a separable form and integrate this function.

So, basically want to integrate this function you are in the you have to integrate this function, and substitute the this thing to get it. So, let me write down my solution in a get a solution in a implicit form g y by w, this is an implicit form equal to minus v by c minus W minus B by c square anyway do the computation if I have not make any error, I am not sure there may be some minor error of sign or something, but it is nice to correct the error of the lecturers, and that is the best way to learn in fact, so you see is that.

So, that is the final form of this solution, so earlier you have a better form you have a better thing, but this is a more complicated form of solution more complicated solution, but and you cannot separate it even you cannot see this an implicit equation, there is no easy way of representing v in terms of y. But, what is interesting they have done is that, that is the final tail, tail to the tale if you want. The both tails are different, what they have done some numerical computation, and what is your aim? Do not forget the aim.

They must compute v the velocity when it reaches a distance 300 that was the initial aim, you want to know what is the velocity when it reaches here, and then did some computation, it happened to be approximately 45 greater than 40 you see, you wanted that to be less than 40, but unfortunately it happened to be 45, and then eventual result was that they have to stop them make it. So, it was the protest from the environmental listed, and they have to see is how the mathematical simple analysis one has to deal with to solve problems.

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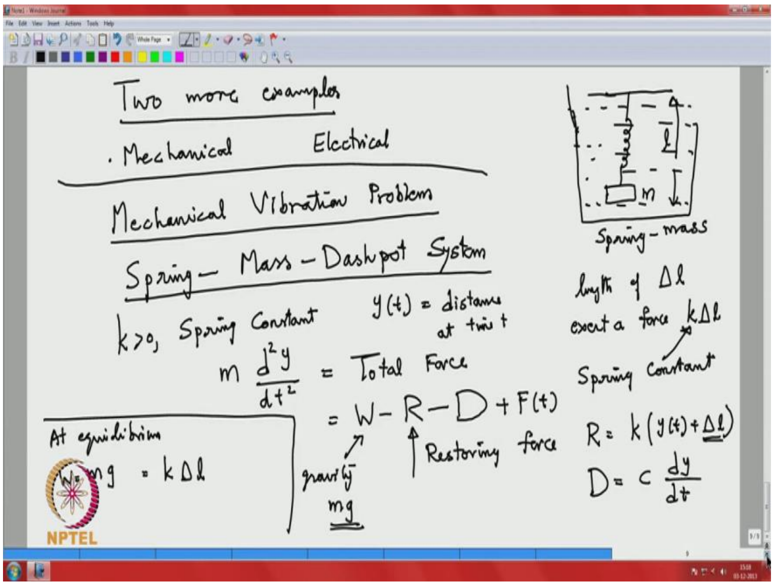

With this example I will now go to I will not give you the details here, I want to give you two more examples. I will not be able to complete the analysis, but these two more examples are extremely fundamental in two of the fundamental areas of engineering, one from mechanical engineering, and another from electrical. And this people the mechanical engineers, electrical engineers regularly use these examples. In fact, any automobile any things you do a do it any construction even civil whatever mechanical construction thing this examples will work.

So, I will today I will try to give these 2 examples how they are predict. The interesting aspect of this examples are that both leads to the same differential equation, if you recall my first lecture, the aim was the even though the differential equations may be coming as modeling of different physical phenomenon. The mathematicians always want to study in a very general perspective, it may have common features. This one of the interesting very simple example which most of your familiar those who are again take an course on ODE will be familiar.

It gives you a non homogeneous second order linear all simple equation with constant coefficient which you can solve it you see we both leads to. So, I will derive it that equation will show you that equations, how the equations are coming. And may be in the next lecture we will do an analysis of that, and this is the complete thing after that we will explains few more problems in the next lecture. But, we will not you will see some of the examples as we proceed the course thing. The thing is that if the mechanical vibration problem.

So, I will do this one mechanical vibration problem, this is a model which you are on a regular use which all of you see, when you drive a motorcycle wire when you dry and hit a hump, what is it doing especially the concept of the shock observe the basic thing is in the mechanical vibration model.

So, it is also called spring mass dashpot system you see that. So, it is a as I said is use observes in automobiles it is also bare a heavy guns, when you fire a gun you know that it is also comes back to hit you, and you have to understand that in a better way how to avoid that one, in the main thing is the shock observer. So, what is a basically a spring mass system? So, you have a spring natural spring attached with the mass here. So, you have a mass m, and you have a spring mass this call this is called this spring mass you see, how does it work?

Suppose you pull suppose it is length is l right now, this is l basically you pull it or push whatever it a length pull or push a length of delta l. Then this spring is spring has the thing of the restoring force that is what the spring will do. Spring will try to restore when you pull it spring will do that 1, and this delta thing it with exert a force proportional to delta l, and that is we call it k into delta l. And that is how the spring constant this k is called the dependence on the how strong your spring is called the spring constant k. So, it is an important parameter, so k positive spring constant.

So, you will have the vibrations by Newton’s second law or first law you have to say that when you have a spring, and if there are no external forces nothing else when you give start vibrating it should start vibrating without stopping. We should we want to see all that phenomenon there. Now, what are you how do create a spring mass thing, when you getting a shock when you are riding a vehicle in hitting a hump. And you get a shock you do not want the vehicle will feel a shock you want to reduce that shock on your body, you do not want to get it that is what shock absorbs will do it.

You want to remove that shock, you want to decay the vibrations created by enforcing circumstance by hitting or something like. Of course, if it is too much your shock absorber and other things will not take care of it, but you want to see that one you want to see that. So, what do you do is that? You put this spring mass system in oil whatever it is, you put it in oil whatever it is. So, you want to understand.

So, as soon as you put the spring mass system put it in oil, the oil will have its own affect on that spring mass system. When there is nothing by Newton's law it should start when you pull it, it will keep on oscillating, and that is what may suddenly create, but when you put it there you want to do that one. So, you again want to their Newton's second law of motion what you get it? You will have again if the mass is m, you will have d square y by d t square, and what is y (t) for you here, y t is the distance from the measured distance at time t distance at time t, measured in the downward direction measured at a distance at time t.

So, you have the total force that is what you have to understand, what are the total forces will come? Let me measure this one total force. So, you want to understand what are the forces, there will be a force due to gravity, and what are the other forces there will be a restoring force the spring has an effect I told you. The moment you pull it, this will have a restore in the spring will try to restore it to its original position, and that will against the motion. But for example, if you pull it, it will go to the position; it will push it up it will compare.

So, the restoring force will have apply against this is a gravity which is coming down, and then what are the other forces available there? You will have a drag force, and that also act against it, and then some external disturbance that is what more interest. So, you will have some external force. So, this is due to gravity. So, that is nothing but your m g, and then you have the restoring force, how do you compute that one force? So, you have your restoring force you have to write it, suppose it is y (t) and if you pull a distance of delta l, and so the restore it. So, y is the distance at time t, and you pull it at distance of delta l, the restoring force will be y (t) plus delta l.

So, y (t) at time you pull it that is our restore you first. And drag force as I tell you is normally proportional to the velocity. So, you will have c into d y by d t that is the velocity. So, these are the terms you have to apply, but now we are analyzing the situation at equilibrium. At equilibrium what will happen, say when it is an equilibrium the one create a due to this pulling should be canceled with your gravity force. So, you are exactly you will have it m g that is nothing but your W will be equal to the k into delta l, so that is what will happen. So, you have to write down the restoring forces I should it put sign here. So, basically you will have k into delta l.

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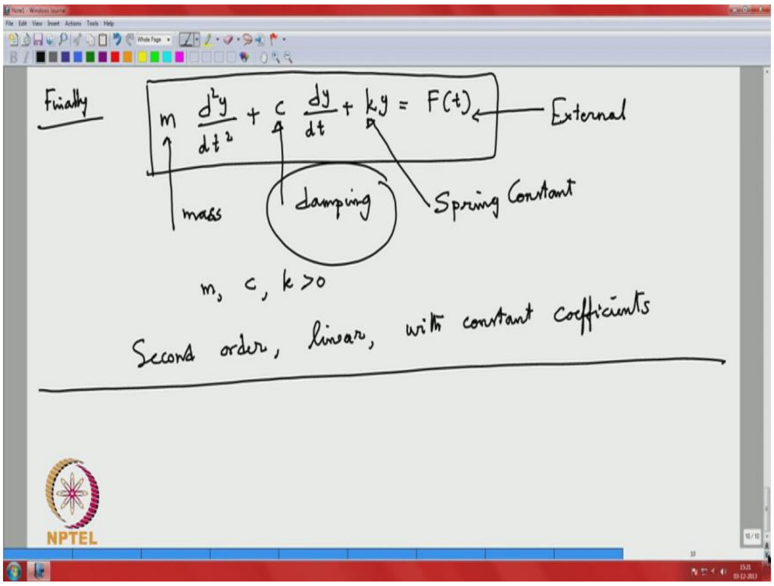

So, final equation if you rewrite all these equations comparing here you put it here everything, what you will happen is that after cancelation you will have equation with the form you rewrite derive, finally do the a bit of competition. Finally, m into d square y by d t square plus what you will have is that, c into d y by d t plus k y all these terms will come you see D is c into d y by d t will come, and I have taken the sign. So, it will come here with a minus sign, you will have a sign here that will come here, and the other terms will get cancel, only one term equal to F of t.

So, you have you see this is the, so you have to understand this terminology m is mass, this is called the damping basically you have putting it damping, you have to understand why mechanical system you need damping. If you do not have mechanical damping is exactly due to that you are putting it into the oil, you are putting this system in oil and creating a damping on the system. So, whenever you create any mechanical system you have to give enough damping otherwise the it will create oscillation, this is about the spring constant you see this a spring constant.

So, all these term terms m, c, k are positive, this of course it is an external force which you do not know, so you can happen external force. So, you have to analysis this system, you know how to solve suppose if t equal to 0 you know this is a second order as I said second order earlier equation for first order you will have a second order easier thing it is linear and with constant coefficients. So, you have all simple form with constant coefficient, and we know how to solve it we will do this. Solving later, but I will write down the solution, and I want to interpret.

Now from these last 2 examples you have understood just writing the solutions are not enough you have to write the solutions, and you have to interpret the solution what are the things happening in a mechanical system. And so, when you design a mechanical system you have to take into account how much damping you have to put into the system was the minimum thing required. So that you do not have the vibrations of your mechanical system, there are disasters due to the lack of damping.

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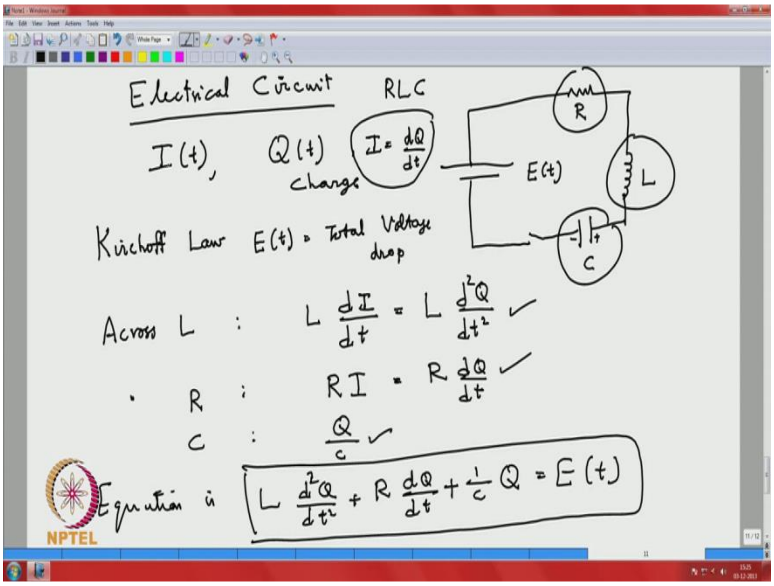

So, we will do that in the next lecture, but in the few minutes I am having I just want you to show one more example leading to the same example electrical circuit simple is a simple electrical circuit I want to do this. This is an RLC circuit simple RLC. So, you have a some E m f coming from here electromagnetic force coming and connected to a resistor and you have a inductance, and you have the capacitance. So, you have plus I put I hope the minus sign, if you want you can put a key this is simple RLC circuit.

So, you have your resistance here, and you have the corresponding inductance here, and you have your capacitance, and what you are what are the things you have it. You want to understand the current I (t) at time t when you have the charge, suppose Q (t) is the charge at the capacitor charge how do you model this one? This is a close circuit simple close circuit. So, you apply Kirchhoff law, I told you the physical laws you have to apply. So, you have the Kirchhoff law.

So, the Kirchhoff law you is exactly tells you that E (t) is the total voltage drops, you have to understand the voltage drops across all this things, and then you through you write down that one and that is simply. So, across L what is a voltage drop? L into d I by d t that is you know it this is from comes from how they design that if you know little bit of electrical circuits and equipments you can see that this is a kind of voltage that is nothing but are you have also a relation here, I (t) is I is nothing but d Q by d t, so you have that also this relation. So, this are standard things which when you study the basics of this one.

So, this is nothing but L into d square Q by d t square, and across R you have this that you know it nothing but R into I the voltage drop. This R I that is nothing but equal to R into d Q by d t, and across c is nothing but 1 by c is nothing but Q by c, and you have to write an integral from if you want to write the I, so it is will be Q by c.

So, if you add this three add these three your equation is you see equation is L into d square Q by d t square d by d t square plus R into d Q by d t plus Q by 1 by c Q is equal to E (t) you see it is a exactly the similar equation, as in mechanical is absolutely no different only the quantities are different, the interpretations are different.

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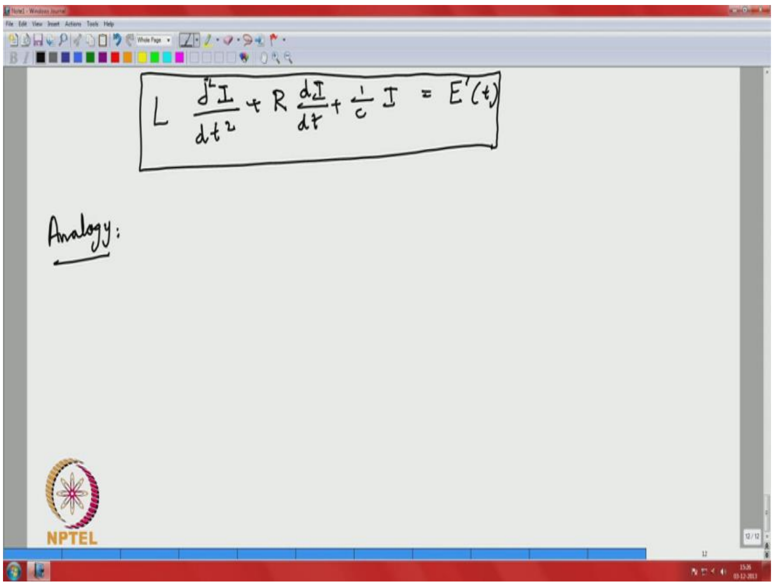

So, you have a similar equation. So, quit often the same equation the previous equation is all written in terms of a current, what do you want if you want to write it as a second order equation in terms of the current? You differentiate once more if you differentiate the once more, what you will get is that I in the, so you can write in this format also d square I by d t square plus R into d I by d t, so you can write this equation in this form plus 1 by c I, but you have to differentiate once more.

So, you will have basically E prime of t the derivative of this thing. So, you can see comparisons it is an analogy it is not really a comparison, but you can have analogy, because in the mechanical system I am going to do a detailed analysis, a similar things can be done for the electrical system also. The interesting thing is that in the mechanical system you can see that, so both places the just like you there we use damping, in electrical circuits you use a resistance.

So, an unwanted vibrations what you have seen you are there you are putting that thing to remove the vibrations, and so you want to have a stable system, but the use of that in electrical circuited we want to create that amplifications. So, the same phenomenon there you have vibrations and removing through damping which will see is used, you do not want unwanted vibrations in the mechanical system. So, at the bridges etcetera with this exactly have all happened to the Tacoma bridge collapse, which will explain later.

On the electrical systems the same things are used, the frequency model is used to amplify it can to correctly this is exactly the principle used to match the frequency, and to tune your radios and our televisions sets. So, how do you get all the signals are coming into your television system, and you are want to get that will explain this one, we do the analysis on this systems more, and in the next lecture and will continue it the next.

Thank you.