Ordinary Differential Equations
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Module - 3
Lecture - 13
Second Order Linear Equations Continued

Welcome back to this lecture, in the last lecture we were introduced the second order linear equations. We have introduce the concept of Wronskian and we also seen the solutions space of homogeneous equation is 2 dimension which is a in some a remarkable fact. But the only difficulty compared to your first order equations where the solutions could be easily founded by a integral calculus problem. There is no general method to find the 2 independent solutions.

So, if you have a procedure to find independent solution or if anybody can prove that independent solutions then, the engage solutions a space can be understood as a linear combination of 2 independent solutions. Since, there are no recipes I will begin with 2 methods to how to find the independent solution is not very general method and still we do not have a complete procedure.

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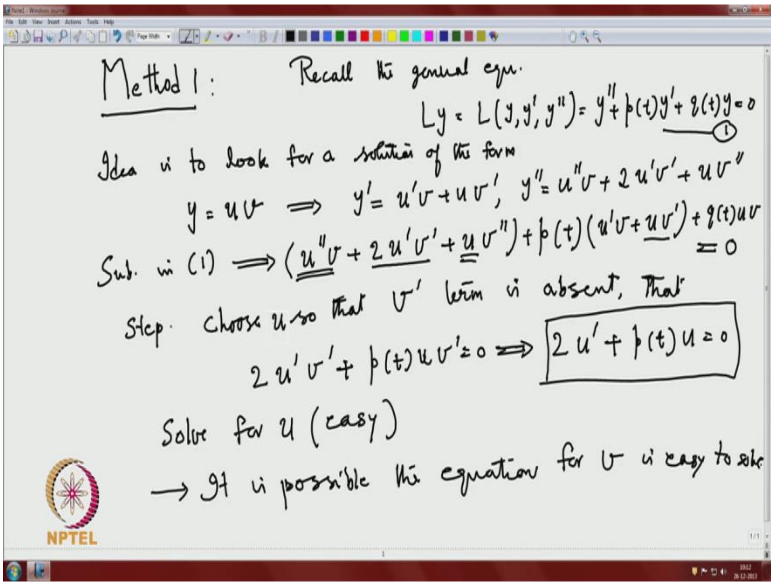

So, we will start with 1 easy method it may not work all the time it dependents on particular problems you are dealing with it might work or it might not work.

So, let me recall the equation the general equation recall the general equation that is nothing but L of y for shorter notation will put it or you can also put a L y y prime y double prime. That is equal to y double prime plus p t y prime plus q t y equal to 0. This is the general equation which we are going to introduce. So, the idea is to look for a solution of the form this there is now reason why such a solution may can look for. But then it was certain equations I will work out.

We are not claiming even we are solutions you are looking for a solution in the product form u and v. And we have fortunate enough we will be able to find the solution a find u and by v by solving some other easy equations. So, do a computation to that 1. So, you can do the computation I will not do the entire computation, but y equal to u prime v plus u v prime and then y double prime you can compute by double prime. When you compute by double prime you get u double prime v then u prime v prime 2 times will come 2 u prime v prime plus u v double prime.

Substitute in the equation let me call this equation to be 1 substitute in 1 you get a bigger equation you will give you a large equation. You can you have to substitute all that you get u double prime v plus 2 u prime v prime plus u v double prime plus p t y prime you substitute, you get u prime v plus u v prime plus q t u v equal to 0 that is what. So, the next step next step is to make the equation the make choose u in such a way that, you try to remove the v prime term from the equation. So, put in such a way that summation of this equation not this 1 the v prime term this 1 this combined term v prime is absent.

So, step a choose u so that, v prime term is absent is absent v prime v prime term is absent with. As I said idea is to look get equations for u and v which are simpler that is v prime term is absent you want to choose this way 2 u prime v prime plus p t u this v prime equal to0. That gives a equation for u prime is equal to 2 u prime plus p t u equal to 0. So, this is a easy equation this is an easy equation for u prime. So, you solve for u solve for u and that is easy this is an easy part; solving u for is an easy part.

Because, of it is a first order equation so this is that is not absolutely. But, the next result is that it is possible that because there is no v prime term, but then u is already determined. So, computing u double prime u etcetera is already known to you, but then there are no v prime terms. So, you will have an equation it is possible that the equation for v is not always true. If it is always true is a equation for v is easy to solve easy to solve.

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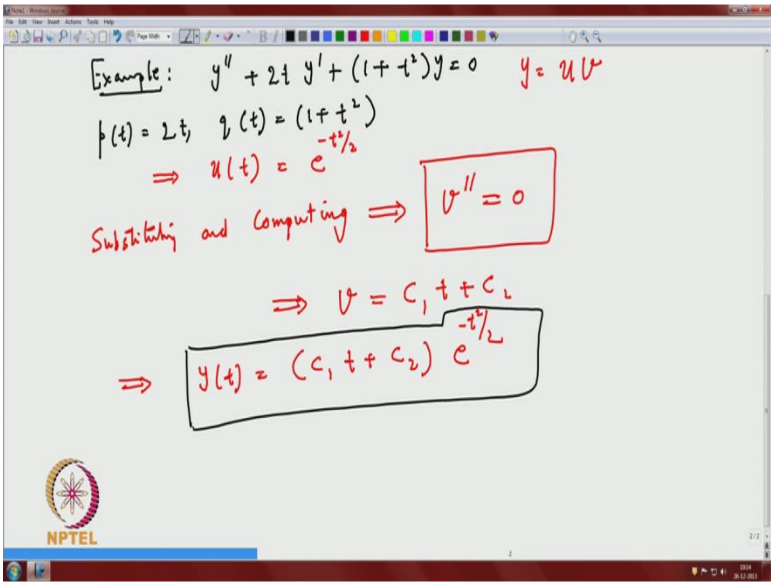

So, let see an example, if we are lucky you'll be able to do that. So let us see an simple example again y double prime plus 2 t y prime plus 1 plus t square by that will be plenty of equations to solve equal to 0. So, what is p t here p t is equal to 2 t 2 t is equal to we should also be get familiarized with how to solve equations that is important 1 plus t square. So, what you get it if you solve for u t you can write down the equation u t is equal to you can write down equation. So, equation for u t is this 1. So, this is what you are equation p t is easy. So, you can write down the equation for u and you can solve it. So, let me note spend time here.

So, let me write this will imply your u t is nothing but e power minus t square by 2 now substitute that is a easy part substituting in computing. Let me avoid a computation here because a that you can 1 can do it easily computing. Can write down equation you can actually see that will imply v prime will be double prime will be 0 you see. So, you have a equation this happens for this particular equation. So, you looking for a solution of the form y equal to u v and this will give you v is equal to c 1 t plus c 2. A general thing that will imply your y t is of the form c 1 t plus c 2 into e power minus t square by 2 you see. So, you have your solution is in a in this case. So, many examples can be d 1 that way.

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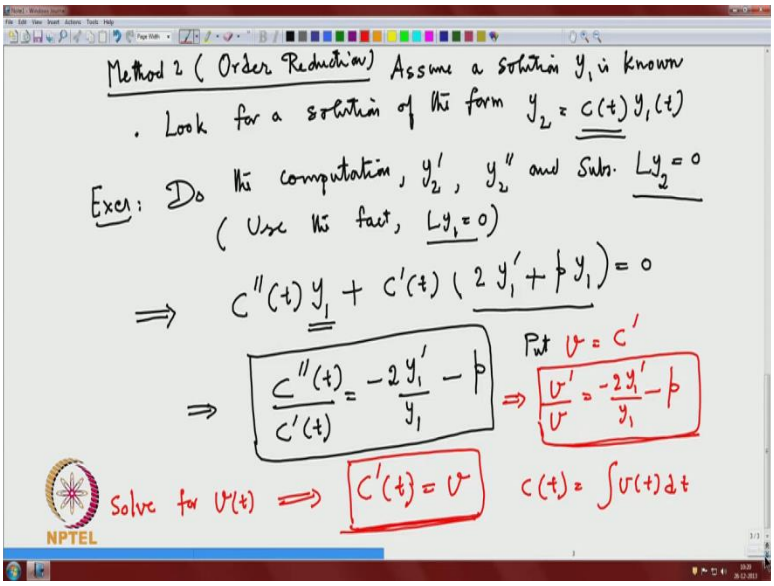

So, I will just show that example. Now, I will go to the next method, method 2 is called the reduction of order method 2 order reduction. This method is quite good, but then you have to pay a price. The idea of this method you need to know 1 solution. So, if you know 1 solution it gives method a systematic method to find the second independent solution and also.

So, assume a solution y1 is no. So, this is the price have to pay. So, still we do not have a this … So, if you are able to find 1 solution this method will give you a systematic procedure by reducing the order to find the second independent solution. What will happen? If y1 is a solution, a constant multiple cannot be a constant multiple will also be a solution, but then the constant multiple of y1 cannot be an independent solution. So, that is the first observation 1 has to make it and no constant multiple of y1 will give you a second solution.

So, the possible way to find a second solution, instead of multiplying by a constant you multiple by a function itself, so look for the solution of the form y2 equal to c t y t c 1 y1 t you see. Because, if you take c as a if you take c as a constant multiple we are not going to get any independent solution any constant multiple will always be a solution. But we want to have a solution y2 which is independent of the solution y 1. So, now, do the computation. So, you have to do a bit computation I will do.

Now, do the computation. So, the exercise is just to do the computation. What are the computation you have to use it to it fine the y1 prime y2 prime y2 double prime and substitute in the equation and substitute in L y2. So, you want l y to be to be a solution to that equation. So, you L y u 2 you also use the fact that use the fact l y1 equal to 0 because y1 is already given to be a solution to your homogeneous equation.

So, choose l y1 equal to 0 and you y2 to be a solution. So, use l y2 equal to 0. If you do the computation this will imply you will get an equation c prime of t y1, y1 is known to you plus you get c prime of t into 2 y1 prime plus p is of y1 is equal to 0. So, this is known to you because y1 is given to you this y1 is known to you though it looks like a second order c prime since second prime for c there is no c present here.

So, you can write down this in principle say double prime of t by c prime of t is equal to minus 2 y1 prime by y1 minus p you see. So, you have an equation even though looks like a second order equation, but if you put … So, l n use the color. So, we put v is equal to c prime will imply v prime by v is equal to minus 2 y1 prime minus y1 minus you see. So, that will give you can solve it. So, if you put v … So, solve for v first. So, that is why it is called an order reduction method of reduction of order.

So, your actually solving a first order equation here you see. So, if an first order equation solve for v which you can write it as explicitly because y1 is known to you. So, you can write for solve for v t and, but this imply once it get solve for v that is nothing but c prime of t is equal to v again we are solving. So, you have a second equation to solve for a c.

So, you see first you solve for v here using the first order equation and the v is given to you and you can just write c t and c t is nothing but integral of v t d t you see. And v can be obtained from here which is a function of t which you can be solved for that which will give you. So, that is a method of reduction.

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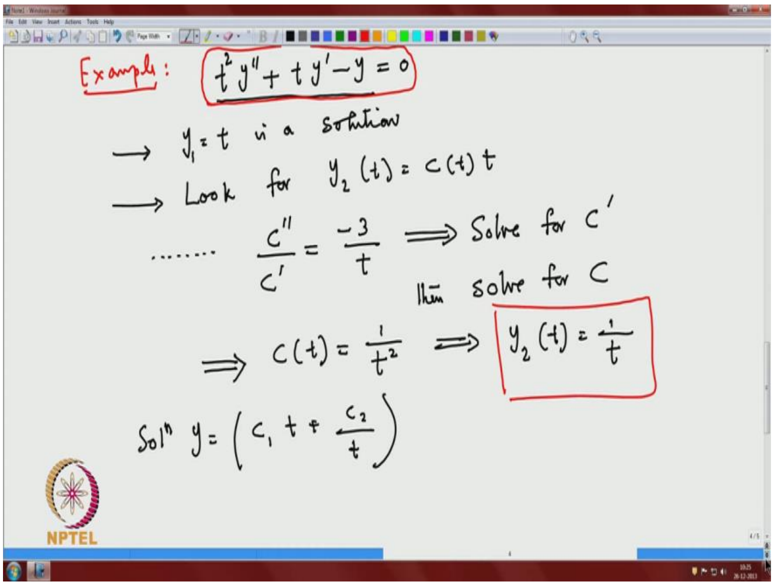

So, let me give you an example again to begin with an example for you. So, you an example a simple example I am trying to give you t y double prime plus t square y double prime t y prime minus y equal to 0. As I told you again there is still it does not give how to determine the 1 solution that you have to do by trial and error method or looking at equation you may have a some way of find it. For example, here … So, immediately looking at this equation properly if I choose y equal to t y prime will be 1 and this will get vanish and since y equal to c this will get vanished.

So, this immediately give you this will be easy if that is easy we have nothing more to do it here. So, y t equal to t is a solution you see you can see that y t is equal to 0 t is a solution and then how do you the order reduction we look for another look for. So, let me call it y1 equal to look for y2 t is equal to c t t look for a c t t. So, how do you do that 1? So, 1 more point I want to do here.

So, we have determined a solution y1 and y2 you can actually show that this y1 and y2 are actually y1 and y2 are independent solutions you have can compute the Wronskian from here. All that you can see that you can compute the Wronskian of a y1 and y2 and see that the Wronskian is nonzero. And so can actually prove that the solutions y1 and y2 are independent. So, which you can do which you can do it as a exercise again.

So, you compute this y2 prime and we if you do the computation a bit here do the computation here you can actually see that, c double prime by c prime is equal to provided by this 3 by t; we can do the computation. So, that is implies finally, you do the computation first compute c prime not compute solve for c prime solve for c prime then solve for c 1. And do if you do this a small exercise you can actually get that your c t is you can take c t or any constant multiple you can see that c t is equal to 1 over t you see. So, you have another solution.

So, with general solution can be written as a solution not c t c t you will get it as 1 over t square that will imply your y2 t is of the form 1 over t. You can take constant that does not matter for independent; so I get solution. So, other general solution is a your c 1 t plus c 2 by t. So, you see a y solution. So, you expect this singularity because, if you look at these equation this is actually a singular equation.

So, the solution singular have the problem. So, if t naught equal to 0 and if you are solving only on the positive side of t you have no problem with the t. As long as you have a singularity you except such kind of singular problem to show some singular behavior, which we have seen already in the previous examples. So, we will go to the here you can see that t n 1 based they are independent solutions this is the same situation in the earlier case also for general this a very general method to solve that 1.

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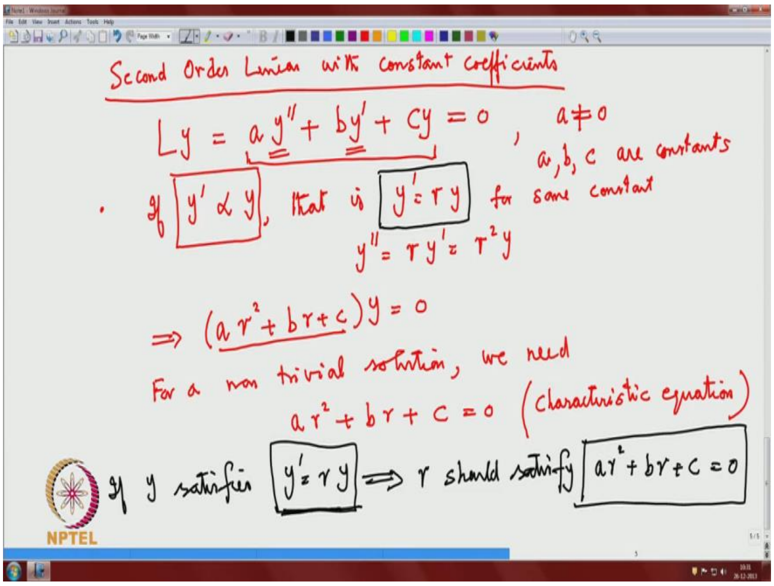

Now, we will go to a special class of equations, where a general method is available to the find the independent solution. This is a second order equations second order, linear second order linear with constant coefficients. We will do that second order constant equations.

So, how does a L y look like now your L y will be of the form I can put a constant here a not equal to 0. So, there is no singularity ay double prime plus by plus c equal to 0 you want to see that, so solutions to that 1. So, here how do you proceed with that 1 again observe the equation cy py prime plus cy equal to 0. So, look here is a fact this again we are trying to get some idea from using this equation a not equal to 0 a, b, c are constants now. Since, a not equal to 0 1 can divide by a, and you can write in the p q form itself that is up thing.

So, that is absolutely on singularity, but let me put it a here because this is the form you have already seen when you study the spring mass system, where a equal to m b equal to c the damping coefficients c is equal to k your spring constant. Now, you have also recall what we have done in the beginning in the module 1 when we have studied the spring mass system. If time permits me we I will do little about it recall what we have d 1 there in the module 1.

So, let us do it this equation the 1 observation is that if y prime is proportional to y, that is y prime is equal to some k y with for some constant k. That will immediately tell you y double prime is equal to k y prime, but y prime is k y. So, you can immediately write instead of k we have already used it for some spring constant in that example, so I want to preservers that. So, I will use here r y prime is equal to r square y prime what is shows that if y is proportional to y, y prime is proportional to y y double prime is also proportional to y with proportionality constant r square.

So, the immediately shows that the y prime is proportional to y, y double prime is also proportional to y. So, if the y comes out … So, if you substitute that tells you that you get a r square plus b r plus c of y equal to 0. Definitely this is a constant a r square plus b r plus c is constant. And you want y of course, is y is identically 0 this will satisfy and y identically 0 is a solution to your homogeneous system. But what we are interested is a nontrivial solution. So, if you want to get an nontrivial solution for an nontrivial solution we need we need a r square plus b r plus c equal to 0. And this is called the characteristic equation.

So, immediately if you are … So, starting with a solution of the form … So, what we have start the thing is that you started with a solution of the not started with a solution of the form if you are looking for a assuming that y prime is equal to alpha y that is nothing but y prime is equal to r y. So, if y is solution to thing equation y prime equal to r y then r will satisfy this 1. That is if y satisfies y prime equal to r y that will imply r should satisfy a r square plus b r plus c equal to 0. That character, that is why we study all this characteristic equation with constant coefficients.

So, the moment this has solution a r square plus b r plus c equal to 0 is a solution and then you if you look for a solution to this form that will be a solution to your second order equation. So, what are the solutions to this equation y prime equal to r?

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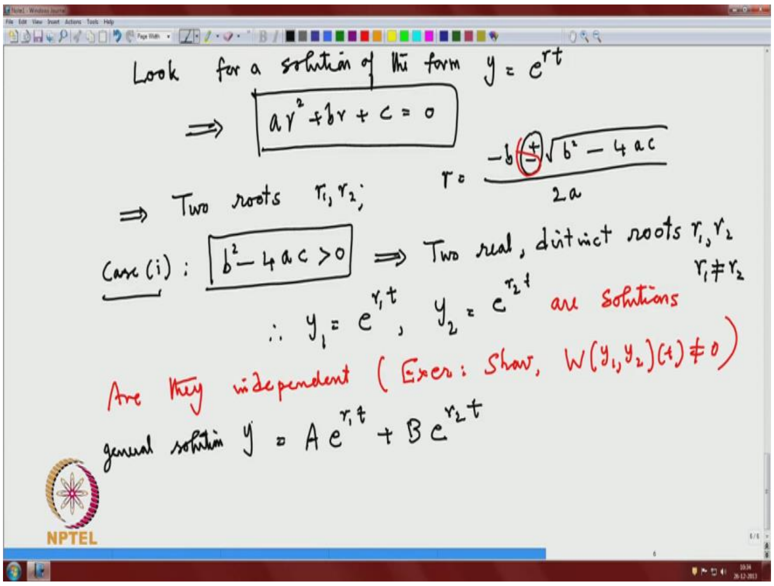

So, that is a reason so look for a solution of the form look for a solution what you study of the form y equal to e power r t you see. So, this is the just a motivate most of the time its introduce that look for a solution of the form y equal to r t. This gives you a some motivation why we looking for this solution and that immediately implies a r square plus b r plus c equal to 0.

So, you look for a solution of this form and of course, this has roots this gives you 2 roots: r 1, r 2 solve it what are the roots you get it r is equal to minus b plus or minus square root of b square minus 4 a c by 2 a you see. So, this gives with a different situations; so the cases. So, we will start with a different case, case 1: is the case which of easier 4 a c positive this is the first case. In these case 2 real distinct roots that is very important then it if it is not distinct you do not get it to solution distinct roots. Therefore, y1 equal to e power r 1 t what is r 1 with a plus sign here with a plus sign here and y2 equal to e power r 2 t with a minus sign here, you get that e power r 2 t is r 1 is with plus r 2 t equal to t.

So, you have are solutions are solutions are they independent are they independent? Simple exercises for you are they independent a simple exercise show how do you show. We have a good method to show independent show w of y1 y2 is at t not equal to simple computation because, you know y1 you know y2 you compute y1 y2 prime minus y2 y1 prime y2. You can see that it is 0 if r 1 and r 2 at distinct 2 distinct to roots r 1 and r 2 that is a important r 1 r 2 and r 1 not equal to r 2 is a just a computation for you needs to, but you can do it. So, there is absolutely no problem. So, you have … So, what would be the general solution in this case? General solution y is equal to a a e power r 1 t plus b e power r 2 t you see. So, we have that.

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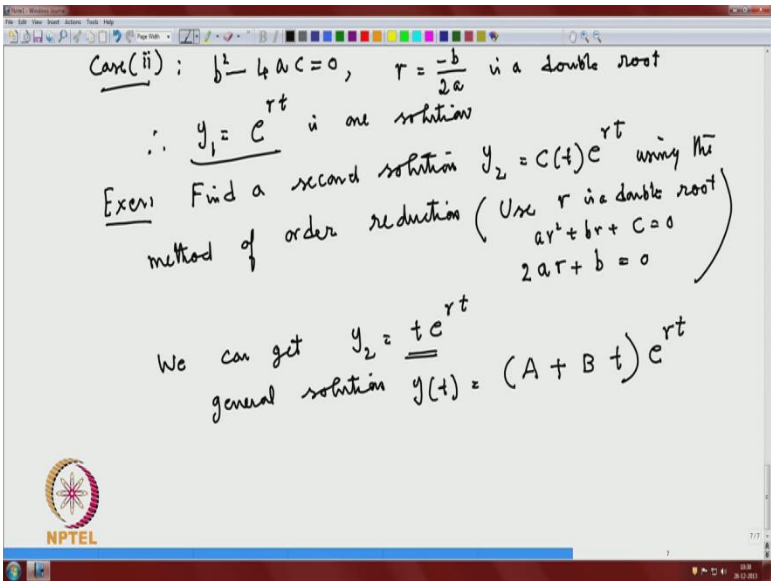

Now, case 2 b square main has 4 a c equal to 0 we said double root. So, therefore, r is equal to minus b by a minus b by 2 a is a double root. Therefore, y1 we get only 1 solution double root does not give you 2 independent solution e power r t is 1 solution. You see the moment you get a 1 solution then you know the method of reduction of order to find a solution.

So, use here is a simple again exercise for you I know have to about that that is an easy thing and is also nice to do it. So, by that method of reduction can be learn properly fins a second root fins a second solution y2. Second solution y2 using y2 of the form c c 1 c t say solution c t into e power r t. So, you have a solution c t into r e power r t using the method of reduction or using the method of order reduction. Very simple computation absolutely no difficult use the fact which is a double root the only thing that use r is a double root. You have to use this. What is the meaning of double root? That is means a r square plus b r plus c is equal to 0 since, it is a double root its derivatives is also 0.

So, you have this condition 2 a r plus b is also equal to 0. So, t is this both the facts. So, you use this fact and you get it again we can get y2 of the form t e power r t. So, see you can solve c t to see that it is nothing but t; it is a method general solution. So, again so this t is not coming in a very arbitrary 1 it comes as a second root. So, it will be general solution y t y t is equal to c 1 plus. So, you have c 1 into e power r t and c 2 into this 1. So, that is c 1 c 2 t or let me use a b. So, that you do not get confused with that c a plus b t e power r t you use a nice t.

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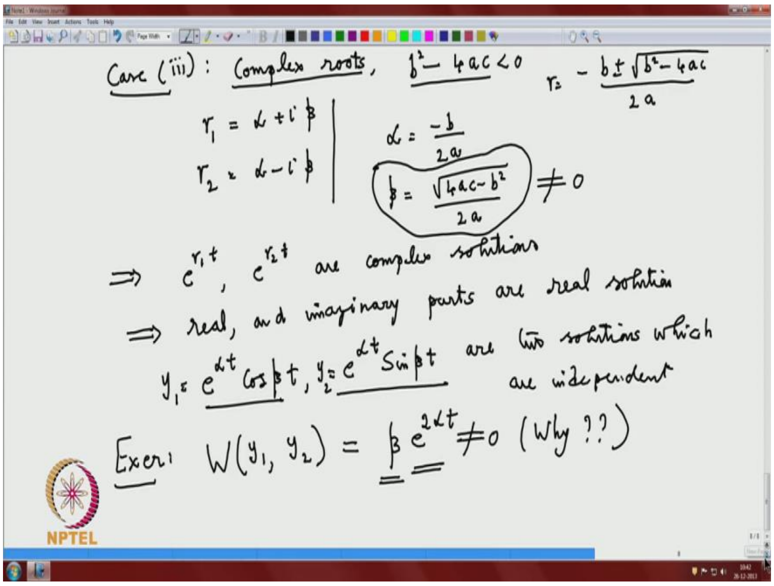

So, now we will go to the case 3 with complex root. Case 3 complex roots all are the complex roots, here b square minus 4 a c is negative. So, you have 2 complex roots r 1. So, what are the let me the thing minus b r is equal to minus b plus or minus square root of b square minus 4 a c by 2 a, so it is a negative. So, you will have r 1 is of the form something like alpha plus I beta 2 complex root r 2 is equal to alpha minus I beta.

So, what is alpha? Alpha is nothing but minus b by 2 a and beta is of the form a square root of 4 a c minus b square. So, this is a the minus we take it out by 2 a. So, that will give you e power r 1 t e power r 2 t are complex solutions, But it is an easy factor to see that when you have complex solutions it will immediately give you real and imaginary parts are all real solutions real and imaginary parts are real solutions.

So, and that will immediately give you e power alpha t into cos theta t e power alpha t sine beta t are 2 solutions. Yes, write down e power n 1 t you get it these 2 things e power alpha t and e power are 2 solutions. What about their independents are they independent? Yes, they are independent 2 solutions which are independent how do you compute the same track there is no problem independent a simple a computation is not real exercise as, but it is a simple computation which 1 have to vary. So, if you compute … So, this is your y1 and this is your y2 here.

So, you’ll have your complete e power w of y1 w of y2 if you compute you will get you will get nothing but you compute this you get beta into e power 2 alpha t maybe. So, you will have a computation that is not a problem because, you can just compute y1 y2 prime and y2 y1 prime y2 you get it. This is not equal to 0 why, why? That is easy because this is non 0 which is exponential and beta is non 0 because beta is a we are in the situation of complex roots since b square minus 4 a c is negative. So, this is non 0 you see this is a non 0 complexity.

So, if it is a complex roots its complex roots your beta should be non 0 because beta equal to 0 is the case were you have the real roots you see. So, that proves immediately they are independent. So, this completely gives you an the complete analysis of your equations with constant coefficient. You can always determine if the equation with constant coefficients that 2 roots.

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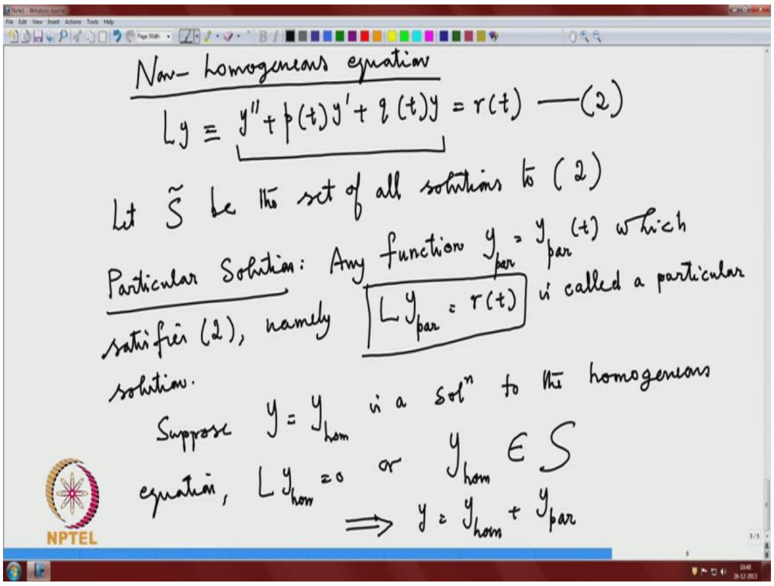

With this thing now we will go to the general case of inhomogeneous equation. And today first in the first lecture in this lecture homogeneous equation, in this particular lecture we will give you the thing.

So, what is a inhomogeneous equation you have L of y general case it is not the whatever we are going to give you is about the general case not necessarily with constant coefficients. I want to understand the solutions structure of this case L y is equal to y double prime this is L y by definition plus p t y prime plus q t y is equal to some r t. You already seen that using the existence uniqueness theory an initial value problem has a unique solution under the that is proof which we already stated in the beginning. When, p q and r are continuous, the initial value problem as a unique solution to the homogeneous equations. But the idea is to understand the solution structure.

Naturally, we cannot expect the linear space now, because if you have 2 solutions to this space y1 and y2 y1 plus y2 cannot be a solution to that 1. You have the linear structure here. So, if you have y1 and y2 are solutions in and you add you get a L of y1 plus L of y2 is 2 r t not r t. So, it does not have the linear structure here, but we will see a some a fine structure. So, let us let s tilde be the set of also I do not call it as space be the set of all solutions. We are, when as I say that it still is the set of whole solution we are not looking in to the initial value problem. Initial value problem has a unique solution.

So, here general solution we are looking at it be the set of all solutions to the inhomogeneous system so let me call it 2. Now, so set of all solutions 2 this we will see that. Suppose, a particular solution first you want to understand any solution is called a particular solution. Any solution that is why any particular solution let me particular any solution particular any function or any function y particular equal to of t which satisfies 3. Namely, L of y particular is equal to r t is called the particular solution, no constants there called a particular solutions.

So, L y particular solutions that is why this solution. And this is L y is a set of all is called the a particular solution there is no 3 here. So, it is root 3 it is 2 any solution to 2 is called the there is no 3 here yeah we are not introduce 3 yet so is called a particular solution. So, now, look at these an interesting fact.

So, suppose y let me to distinguish a homogeneous equation suppose y equal to y homogeneous is a solution to the homogeneous equation which we already know homogeneous equation. What does that mean l of y homo is equal to 0 or equally y homo belongs to s lets we call this notation where, s is the set of all solutions to your way homogeneous equation.

And we know that s has 2 independent solutions only and the dimension of s is equal to 2. Then immediately this will give you immediately what about y is equal to y homo plus y particular. In other words, what I am trying to do then taking a solution from your homogeneous equation and 1 any particular solution.

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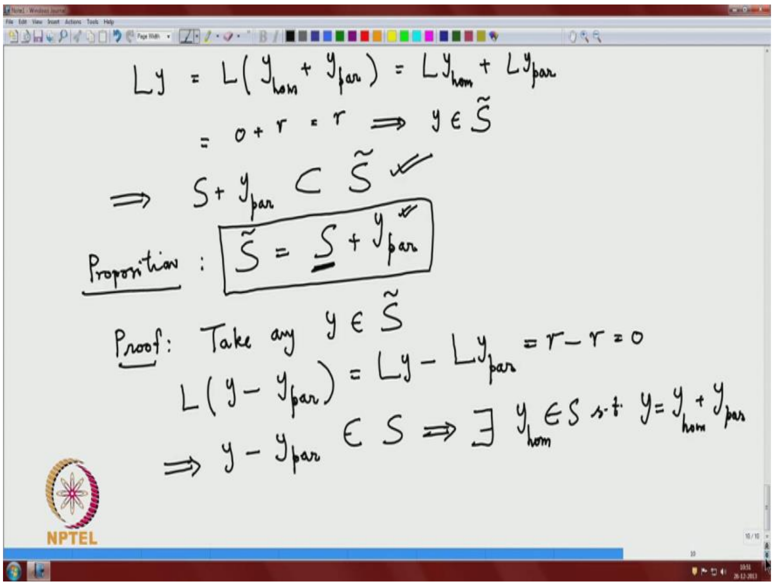

Then what would be L of y? L of y is equal to L of y homo plus y particular here is where you use the fact that L is a linear operator. Since, L is a linear operator there is L of y homo plus L of y particular. But, what is L of y homo? L of y homo is equal to 0, L of y particular is equal to r of t that immediately tells you this is nothing but 0 plus r t that is equal to r there y is a that implies y belongs to s tilde.

So, you start with a solution in a particular solution of your choice you take any s; the solution from s add you get a new solution to your. So, this immediately tells you that s tilde if you take s and if you take any particular solution this is always contained in d tilde. So, what the interesting proposition is that it can immediately prove a proposition in a with a 1 line proof actually s tilde. So, that is what I sets it in the of y particular solution you see proof is 1 line, but let me give you a proof. So, you take any s in s tilde. So, we already proved 1 inclusion you have to prove the other inclusion take any y belongs to s tilde.

Then y particular is given to you that is what I am saying. So, if you can find 1 particular solution and then take any solutions, so that gives you the complete solutions structure. So, any solution to your non homogeneous equation can be written as the general solutions can be written in this form and that is what I essentially this proposition tells you. That implies, now consider L of y minus y particular again and consider this that implies, L of y minus L of y particular, but what is L of y y is in s tilde.

So, L of y is nothing but r, but L of y particular is a particular solution and that is also equal to r. And hence is equal to 0 that implies y minus y particular belongs to s tilde s it is a solution. So, y minus y particular is a solution to your homogeneous equation hence that implies there exists y homo ES such that y is equal to y homo plus y particular. So, what this tells is corollary which is …

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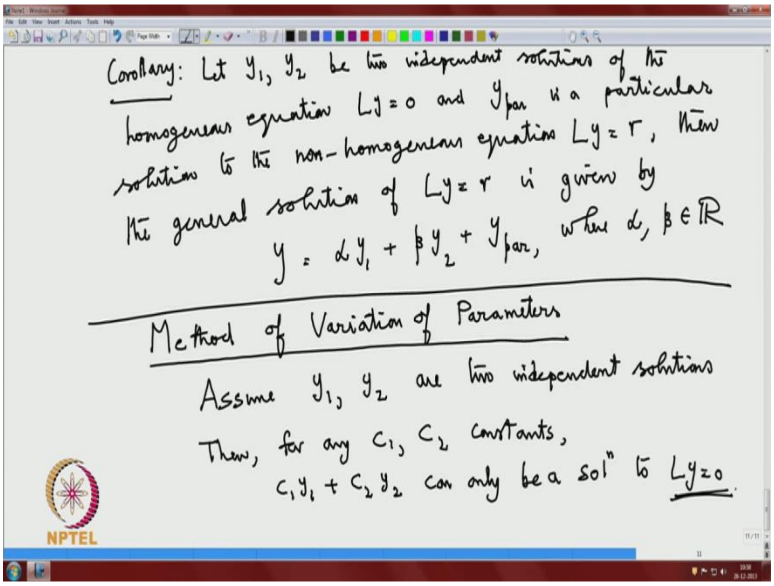

So, let me state it another corollary there is nothing new I am rewriting this same proposition, but because this is away probably in introduced in the you know so it is a and other books. The so this is the theorem the proposition which I have stated is basically rewrite it let y1 y2 be 2 independent solutions to be 2 independent solutions to your homogeneous equation L y equal to 0.

Then y particular is a particular solution to the non homogeneous equation. Then non homogeneous equation l y equal to r, then the general solution then of L y equal to r is given by given by y is equal to y is equal to alpha y1 plus beta y2 plus y particular where, alpha beta are constants you see. So, that gives you the complete structure of the solutions to you homogeneous non homogeneous equation.

So, the problem here is that, if you can find 2 independent solutions to your homogeneous equation and 1 particular solution whatever form it is you can find you can write down your general solution. And if you want to solve your initial value problem find the general solution use the 2 conditions of the initial values substitute and determine the thing. And that is why is method is again as you see finding 2 particular 2 independent solutions is the difficult task. And what the reduction order tells you that, if you can find 1 solution to your homogeneous equation then, using the method of reduction order you can find the second solution.

What we are going to quickly probably describe and a method to find solution find a particular solution even the 2; if you have 2 independent solutions to your homogeneous system. That can be used using the method of variation of parameter to determine a particular solution. So, at the end of it if you look at it in theoretically you may have difficulty in integrating of the functions, but end of it, the fundamental difficulty is finding 1 solutions to your homogeneous equation. If that is there you are more or less near the general solution to your non homogeneous equation as well. So, let me I have few more minutes in this class.

So, let me just introduce the method of variation of parameters. So, we will do that in this class. So, again look a how do you the idea of method of variation parameters is to determine a particular solution using the 2 known if at all solutions to your homogeneous equations that is what we want to do it. So, assume so we are basically assume y1 y2 are 2 independent solutions. The idea we are going to use is the same as we have used in the reduction order in the reduction order if you know 1 solution y, a second solution cannot be a constant multiple of that equation.

So, if varied that constant as a function of t and fortunately the reduce to the first order equation. Now, we have 2 solutions y1 and y2, we use this 2 solutions if you take a linear combination of a y1 and y2 in the form c 1 y1 plus c 2 y2. That is going to be only a solution to your homogeneous equation. So, if you want a solution to if you all right looking for a solution to your non homogeneous equation using the linear combination of y1 and y2 with constants on the coefficients it is not possible.

So, the idea is vary both c 1 and c 2 in the coefficient what I am trying to say is that then, for any c 1 and c 2 constants c 1 y1 plus c 2 y2. Then for the then for any constants is also is only as solution can only be a solution to L y equal to 0; you are not by putting any linear combination of y1 and y2 you are not going to get any solution to the non homogeneous equation. You will get solutions to only homogeneous equation.

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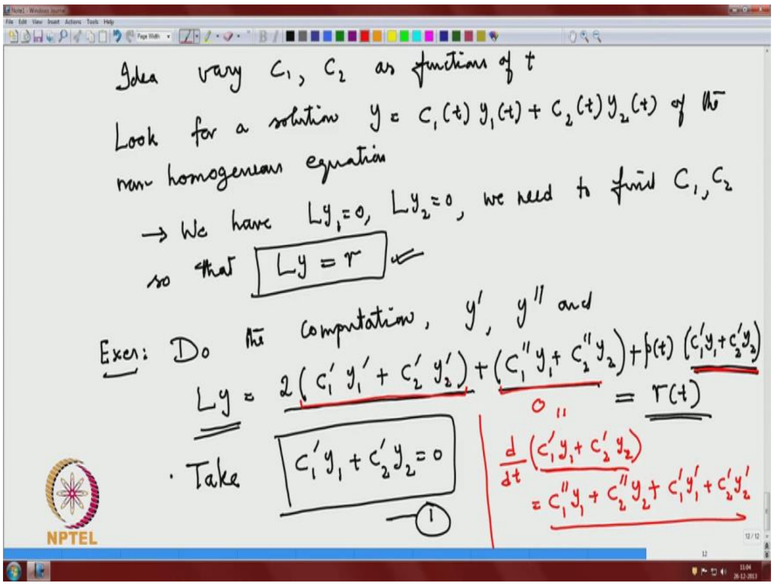

So, the ideas vary c 1, c 2 as functions of functions of r t function of t. So, look for a solution look for a solution y equal to c 1 t into y1 t plus c 2 t y2 t of the non homogeneous equation. So, what are the things we have? So, we have, so let us consolidate the facts. So, we have L y1 equal to 0, L y equal to 0 that is a solution to homo and we need use this thing. We need to find c 1 c 2 so that, L y is equal to r. So, you need that is important thing that is what we are trying to that is it possible to find.

So, we need to find c 1 and c 2; 2 unknown functions stratifying this equation l y equal to r. So, indeed we need to conditions and that is what you are to. So, I again what I want to be do is that the exercise is to do the computations. What are the computations you have to do it? With these you find y prime y double prime and compute L of y substitute it in the operator L of y. So, I will write down the expression here you will get 2 times 2 you have to see where the computation comes here.

Let’s see 2 into c 1 prime y1 prime plus c 2 prime y2 prime to the computation plus you get c 1 double prime y1 plus c 2 double prime y2. You have a factor here and you will have p t into c 1 prime y1 plus c 2 prime y2. And the q term will get absent because you have y1 and y2. Use the fact that l of y1 you have to use the fact that l of y1 is equal to 0 l of y2 equal to 0. Now, our idea is to get a 2 equations for c 1 and c 2 soluble. So, the first step is to take … So, we are trying to make is this is you want l of y equal to 0 and we want this to be r t that is more important.

So, I want that equation to be r t because you want l of y to be the solution to you non homogeneous equation. So, we are trying to make the simple of thing first you make this quantity 0. So, these are all impossible that you can. So, you will have c 1 prime y1 plus c 2 prime y2; y1 and y2 are given to you. So, you have 1 equation. So, you have a equation 1 then is called this equation 1. Now, look at these expressions, I take there are 2 terms here 1 term here if you do a computation there, if I calculate my d by d t of here c 1 prime y1 plus c 2 prime y1 you see.

This will give you if I compute here c 1 double prime y1 1 more term is there I will write that 1 from here I will get c 2 prime y2. And then from here I get c 1 prime y1 prime plus c 2 prime y2 prime. So, if I use this thing, so this is similar terms if I take among the 2 term if I combine these term with 1 term here that is nothing but d by d t of this 1. But, this is already chosen by 0. So, this is nothing but 0. So, only term if I choose this is the advantage if I choose here I get all this term plus 1 more term will get canceled. So, only remaining term will be 1 more term.

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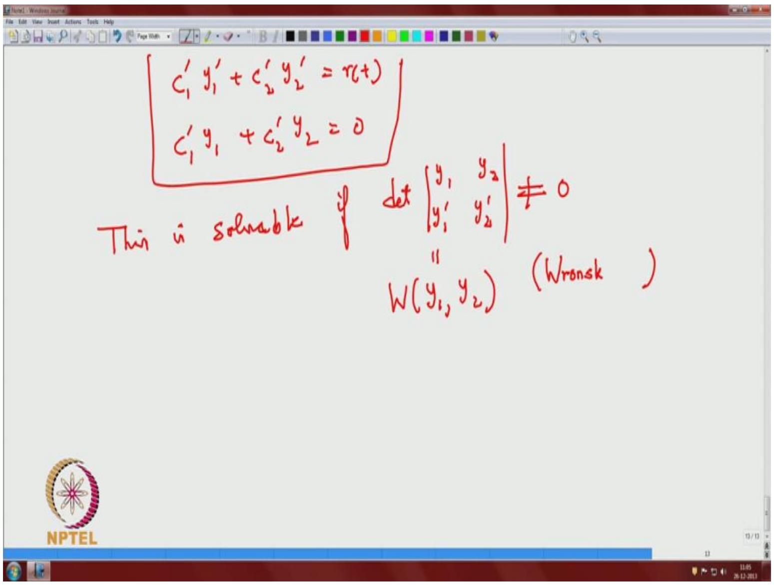

So, what I will get is that in that case I will get the c 1 prime y1 prime plus c 2 prime y2 prime is equal to r t and the first equation already I get a c 1 prime y1 plus c 2 prime y2 is equal to 0. So, you have 2 equation in 2 unknowns and this is soluble when, this is soluble if this determining of y1 y2 y1 prime y2 prime is not equal to 0. But what is this determining? This is nothing but your Wronskian if you recall it, this is the Wronskian you see. Maybe I will, what I will do is that I will start from here in the next class and I will do the is a more or less is done. But I do not want to hurry up, but essentially tells you that, you have a Wronskian which will be non 0 because y1 and 2 are independent solution. And hence you can solve it for c 1 and c 2 c 1 prime and c 2 prime.