Ordinary Differential Equations
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Module - 3
Lecture - 12
Second Order Linear Equations

Welcome back to the present lecture. So, we are going to do the second order linear differential equations, both homogeneous and homogeneous cases in this next two or three lectures. The last two lectures in this module we have finished first order linear equations, and also we give some details about the differential equations which are exact; namely the exact differential equations.

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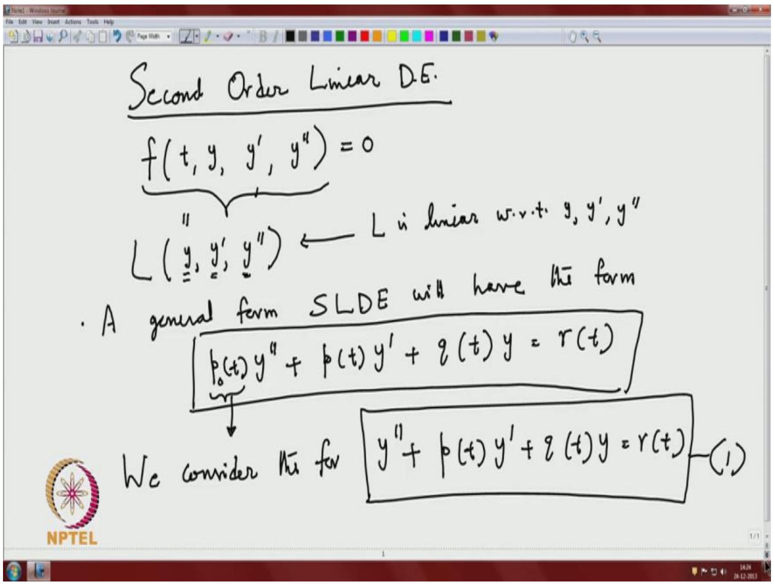

So, we will start with second order linear differential equations. A general second order differential equation will have the following form; f of t y y prime and y double prime equal to 0. So, this is the most general form which are relation connecting the unknown functions by equal y of t y prime of t and y double prime of t, and this a general form, but what we are going to see, is that as in the first order case, if treats this is as a function of, say y y prime y double prime, then we demand l is linear with respect to all these variables, with respect y y prime y double prime. Just like in first order case where there was no y double prime. We had only up to this part, up to y prime in the first order case and divide in the. We do not demand us. like in first order we do not demand any linearity with a respect to t. So, we have linearity with respect to y and y prime in y prime. Then from the linear algebra, is not a difficult thing, it is a general form of linear equation.

A general form of second order linear differential equation, will have the form p naught of t, because you do not have y double prime plus p of t y prime plus q t y equal to r of t. So, this will be the most general form of the second order linear equation. And again as I told you in the first order equation, whenever there is a coefficient in the y double prime which vanishes, which will be singular type differential equations. So, the moment is the highest order has coefficient which vanishes, which will not come in the regular category. So, in this thing, we are not going to deal with it. Later you may see some singular equations in some other module. So, the general forms we consider are more regular general form. We consider the form y double prime plus p of t y prime q of t y equal to r t. So, we will have this, is the equation which you are going to consider, which we keep on (( )). So, this is our main equation with coefficients p q and r.

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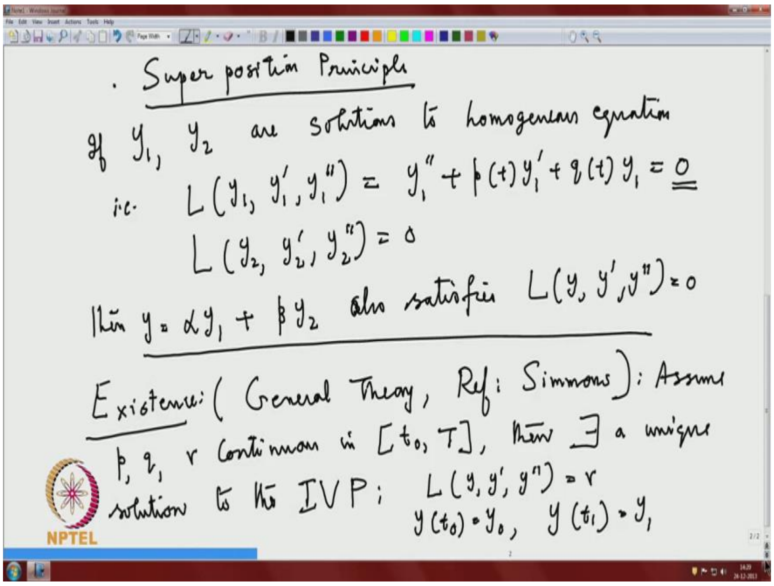

In fact, we have seen an interesting example in our module one, when given the introduction about the spring mass as from system and also r and c circuits; where p q r and t are constants, and see the impact of the second order and solution. All that results and how to find with constant coefficients. We will see it in this. So, one of the thing as I said again, in the first order equation, the important thing, is the super position principal, which we going to see, super position principal. What is the super position principal. If you have two solutions of the homogeneous equation; if y 1 y 2 are solutions to homogeneous equation; that is l of y 1 y 1 prime y 1 double prime, what is y 1 prime in this case. We have y 1 double prime plus p t into y 1 prime plus q t y 1 prime equal to 0; that is a homogeneous equation.

So, when the equation in this case is, here if r t equal to 0 one is called homogeneous, otherwise non homogeneous. So, that r t equal to 0 homogeneous equations. So, that is we are the taking, we are taking r t equal 0. So, similarly l of y 2 y 2 prime y 2 double prime equal to 0. So, if y 1 and y 2 are solutions to your homogeneous system equation, then alpha y 1 plus beta y 2 also satisfies y equal to alpha y 1 plus also satisfies l of y y prime y double prime. This is the typical nature of the linear system, so that is what it will be satisfied. We exploit this to understand the solution structure of the homogeneous equation and non-homogeneous equation. One of the difficulties is unlike in first order equation, where we could completely determine the solutions to your differential equation. There is no general recipe to find the solution to your second order equation. So, we want to understand the structure of the solutions space, but there is no way to determine the solutions, and that is what we are going to study in this situation, regarding existence.

This will follow from the general theory which will be studying general theory, or one can also refer any book. For example, you can refer the book by Simmons see to see a proof of existence. So, we are not going to prove the existence theory, but let me just state the existence theory. Assume p q are continuous in the interval t naught to t, then there exist a unique solution, unique solution to the problem. I will explain what is the initial value problem here. Initial value problem l of y y prime y double prime is equal to star with y at t naught is equal to some y naught l y at t 1 this equal to y 1 this y 1 is not the solution, y 1 is a constant. So, why this initial value problem is that naturally coming, let us look at given system. So, we will not prove the system, but system can be proved from the general theory or from any other situation one can prove that general theory, but let me see why this initial value problem is very natural. How do you get that one.

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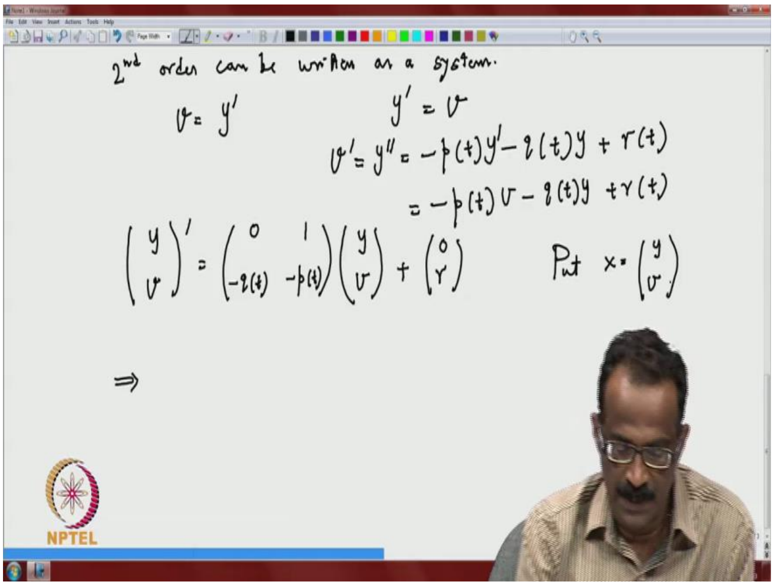

We can write second order equation, can be written as a system. These are also true for n'th order equation. So, that is why we want to specify. We will recall all these things later when we study general system. So, put v is equal to y prime you put v is equal to y prime, or in other wards y prime equal to v for the same. Then for the equation, you will get, if you substituting y double prime is equal to minus p t y plus q t, we are considering the homogeneous system. You can also write these also minus q t minus q t plus, this is y prime. So, this is y, you can put this is equal to r t you want it, non homogeneous system if you want to. So, if put the, in terms of v, this is equal to minus p t v minus q t y plus r t. So, if I write that this as a system y prime y double prime is nothing, but your v prime. So, you can write that, if you compare it as the system, so you have y v. So, you have a system a prime here. I can write it in the matrix form. So, you have your y here, you have your v here. So, what you want to get it. first to get it v here. So, you want y prime should be v. So, this will be 0, this will be 1. And then if you want v prime you want v here y here. So, you have minus q t here minus p t here plus you have your r t that will be of the form 0 r. So, you see here. So, this can be written in nice form. Later you will see, if you put your x is the vector y v, this is nothing but.

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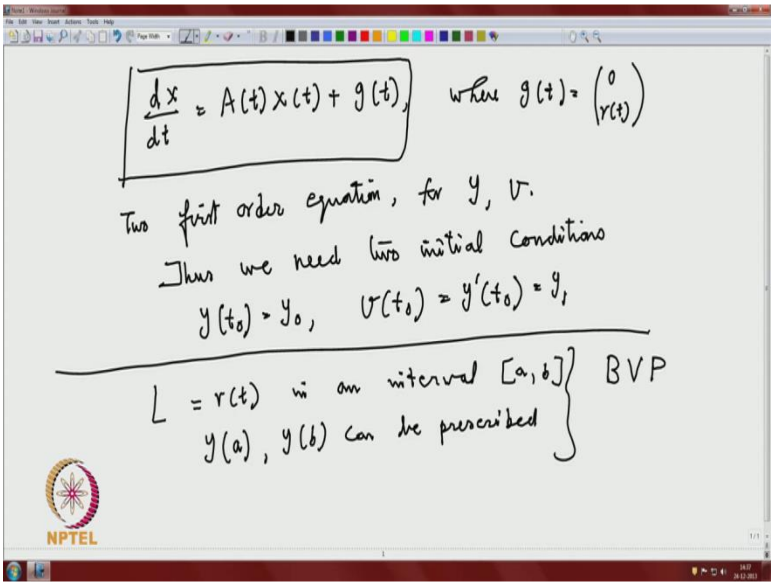

So, we will get a system of the form d x by d t is equal to a t x t plus some g t, where g t is equal to 0 or r t. So, you can write it as a system of equations, this can be done here. So, if you recall to first order equations you have 2. So, we have two first order equations, equations for y and v, this we need two initial conditions, conditions y at t naught j equal to y naught and v at t naught, is the same as y prime at t naught say y is equal to y 1. So, that is why the initial condition. And we have seen in the first order equation when you study, because of the integration, there is a constant when you integrate, you have to a constant. And if you want to fix that constant you need one condition, but you have two equations; two first order equations in principal you have to integrate, two unknowns y and v both requires, one condition each. So, the initial value problem with two conditions, for second order equations is quite natural.

But again there is a specialty in the second order equations. Second order equations can also be consider such equations l equal to say r t can also the consider in a interval, see a b in an interval a b and the conditions at the pin point conditions y a y at t can be prescribed. There are many interesting physical problems where we comes as second order equations in an interval with boundary conditions these are called the boundary value problem. And v may not go an elaborates study of boundary value problem in this course, but we make you two or three lectures about boundary value problems in some other thing, but in this particular module which I am going to do it, about the second order linear equations. We will not deal with boundary value problems, and this is the remark which I want to make it.

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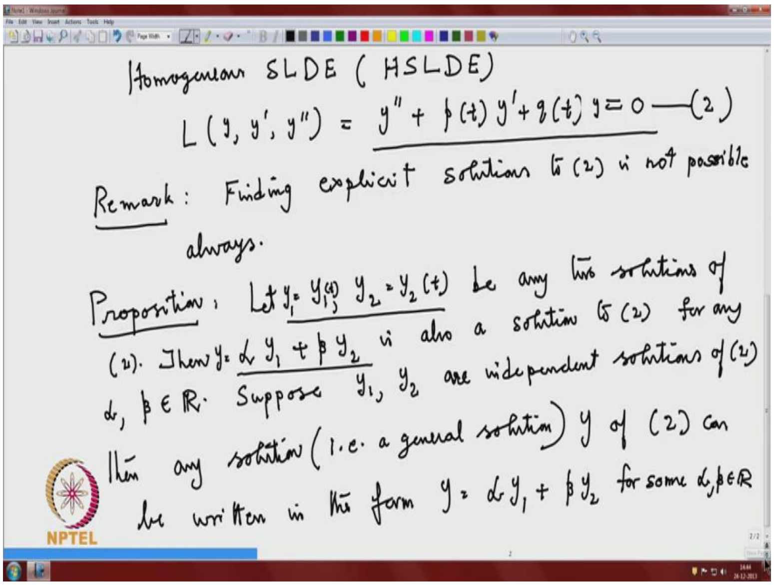

So, we will classify in two things; first will start with homogeneous second order linear differential equations. So, let me give a notation homogeneous second order linear differential equation. So, let me call once again, your differential equation for that l of y y prime y double prime is equal to y double prime plus p t y prime plus q t is equal to 0. So, that is why what we are going to study. So, I said the other remark, finding explicit solutions are difficult in general. Finding explicit solutions to this. We call this two, explicit solutions two, is not possible always. This was not a situation in first order equation, not possible always. What I am trying to say that, there is no general recipe to solve this equations in general with general p and q, but when p and q are constants, such equations are known as second order equations with constants coefficients, which we already seen in module one of that spring mass dash post system r l c circuits.

For that we will have an explicit method of solving it, but when the consciences are functions of t. Since there are no general methods do it, where there are no general recipes to write down the solutions, we need some methods. So, in this module, in the next lecture or something we will also introduced one or two methods to find it, but that may not covered everything, but it tells you in certain situations. We may be able to the find the solutions. So, the first main thing in this particular lecture, today's lecture, is to understand the solutions structure. Can you tell me, can you tell us, or can you find some property of the solutions space, and that is what we are going to do it, and we have the following proposition; that is what first. We mainly are going to prove this proposition in this lecture. So, let y 1, keep y 2 this is not the initial conditions. So, please y 1 and y 2 are functions of t y 1 equal to y is not the initial condition; y 1 equal to y 1 of t y t equal to y 2 of t, be any two solutions of two then this is nothing, but super position principal which I already explained.

Then alpha y 1 y equal to alpha y 1 plus beta y 2 is also a solution to two for any scalars, for any alpha beta belongs to r, is the super position principal which we already proved. But the converse is also converse in some sense. If suppose y 1 y 2 are independent solutions understood the concept of, independent solutions of 2. So, earlier case we are not demanding y 1 and y 2 are independent, any two solutions to you take it then it is linear combination is also a solution. what the converse is that y 1 and y 2 are independent solutions; y 1 that means as a function to function, set it as an independent set or independent solutions of two, then any solution. That means a general solution that is a general solution. We are not talking about the initial value problem. We are talking, we are discussing about this solutions of. There are no initial conditions prescribes so far. In solution y of 2 can be written in the form y equal to alpha y 1 plus beta y 2 for some y 1 by 2 for some alpha beta belongs to r. So, they shows that every solution y of this homogeneous equation, you can write it as the linear combination of two independent solutions, this actually let me make a remark before going to the proof.

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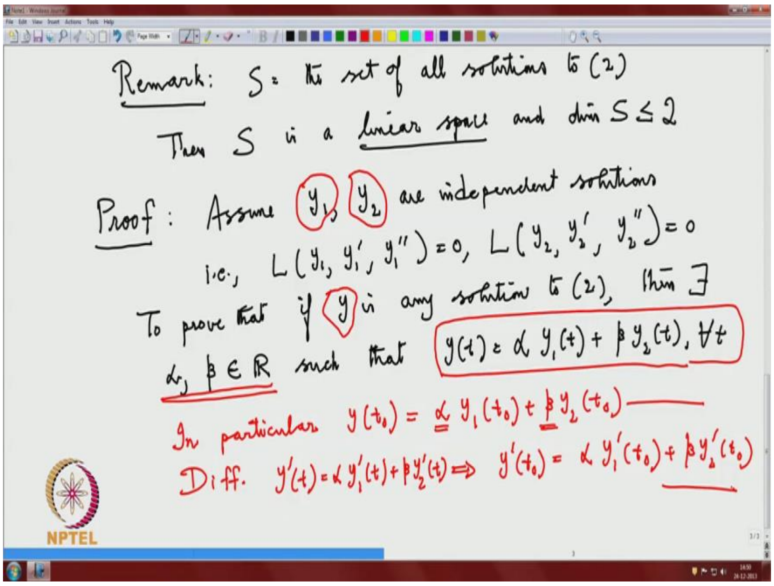

This is actually a very fascinating remarkable result, in the sense that even though solutions set, is a set of the any solution is something like twice differentiable functions, hence it will be a subset of some c 2 space, twice differentiable space. And first result what it says that, if y 1 and y 2 few take any function as a solution set. The first part of the result that alpha y 1 plus beta y 2, is also a solutions shows that, is the first part of the proposition tells you that that set is actually a linear subspace. The second thing says that if you can find two independent solutions from that set. Then every solution in that set can be a linear (( )). This is therefore shows that, even though the solution set infinite dimension space, it is an actual dimension is less than or equal to 2. So, this is the remark. If s is equal to the set of all solutions to 2, then s is a linear space; that is very important fact.

That is a super position principal that is how it is define. And remarkable other thing is that s is a linear space, and dimension of s is less than or equal to 2. What we are eventually after the proof of the theorem, we will see that in fact, dimension of s is equal to 2, but the proposition right now, do not say that this dimension s is less or equal to 2, because if you a seen there are two independent solutions, then that two independent solutions are good enough to determine any other solution. But we will actually produce using the unique existence. You can actually produce in theoretically to solutions, but the unfortunate or difficult part is that we do not know in general, or there is no general recipe, how to get that it two independent solutions; that is why in the second order (( ))

require methods and tricks, to find the solutions. So, with this remarks, let me go to the proof of the theorem proof is not really difficult only. We have to prove one part. So, assume y 1 y 2 are independence solutions; that is I am repeating again as in the l of y 1 y 1 prime y 1 double prime is equal to 0 l of y 2 y 2 prime y 2 double prime is equal to 0, so we have given that.

So, what we have to prove, we have to prove that, if y is any solution y is any solution to the homogeneous equation two, then you have to produce there exist alpha beta belongs to r, such that y is equal to, y means y of t is equal to alpha is a constant alpha into y 1 of t plus beta into y 2 of t for all t. So, what are the things given to you. y 1 is given to you y 2 is given to you, y is given to you, and you have to determine alpha and beta, and then that should satisfy for avidity. So, if (( )) should be satisfy avidity. it should also satisfy for a d t. So, in particular it should satisfy a t naught, in particular y at t naught. This is given to you y is given and y y at t naught is given, y at t naught should satisfy alpha at y 1 at t naught plus beta at y 2 of t naught. Say you see so, if have a. But you have to determine to two unknowns and alpha and beta.

So, you need two equations right now. You got one equation. But not differentiate the equation, differentiating equation by t. Differentiating this equation we get y prime of t equal to alpha y 1 of t prime of t plus beta y 2 prime of t; that implies y prime at t naught which is also given to you, because y t is given to you for all t is equal to alpha y 1 prime at t naught plus beta y 2 prime at t naught. So, we have two equations you see you have one equation here, and you have another equation here. So, you have two equations for two unknowns. So, the question is that whether we can solve for all alpha and beta, and that is what you are going to see.

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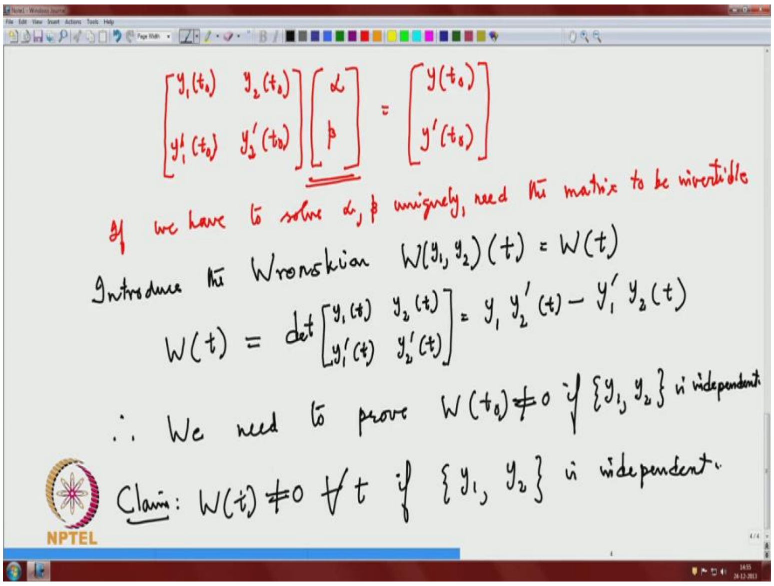

So, what are the equations. So, let me write down the equations in the matrix form. So, if you write down your equation in the matrix form, your job is to find alpha and beta. you have alpha here beta here, and right side you have y 1 at y at t naught, you have y prime at t naught in left side, you have y 1 at t naught y 2 at t naught, second entry here y 1 prime at t naught, and this is y 2 prime at t naught. So, if you want to solve for alpha beta uniquely, we need the matrix study invertible, the matrix to be invertible. This motivates us to introduce the Wronskian; we call this Wronskian, but introducing that matrix is the motivation introduce, the Wronskian w as a function of t, but dependence on y 1 and y 2. So, we are not treating w as a function of the y 1 and y 2. We are writing w of y 1 and y 2 to represent that. It depends on y 1 y 2. But as a function of t we took that what it is.

So, these also denoted by w t unless there is no confusion. we just introduce it at w t. So, what is your w t, w t is equal to (( )) y 1 at t y 2 at t y 1 prime at t y 2 prime at t, that is nothing, but y 1 y 2 prime at t everything at t minus y 1 prime y 2 prime. Note y 1 prime, this is y 2 prime and this is y 1 prime y 2 at t this is the Wronskian determine about that word. So, we need to prove. So, therefore, we need to prove w at t naught is not equal to 0, what is given to you if y 1 and y 2 are independent. So, under that assumption if y 1 y 2 is an independent set, is independent you see. So, as long as we have given this assumption, we are not used this assumption a going to, but interesting fact, in fact we prove claim which we are going to do soon, claim we not only prove y 1 and y 2 is independent. We not only prove that w t naught is not t equal to 0 at t naught, w t itself not equal to 0 for every t. In other words w t will be either identically 0 or w t is never 0. So, claim w is t not equal to 0, for all t together not just at t naught, every t we prove that whenever if; that is not very surprising, because independent at all point t y is independent. So, this is the two steps; first we will show that w is either identically 0. For that you do not need independent, either it will be identically 0 or it can be never 0. It will be seen immediately, and then we will show that in the case of independence, it is not so, that is what you have to show that.

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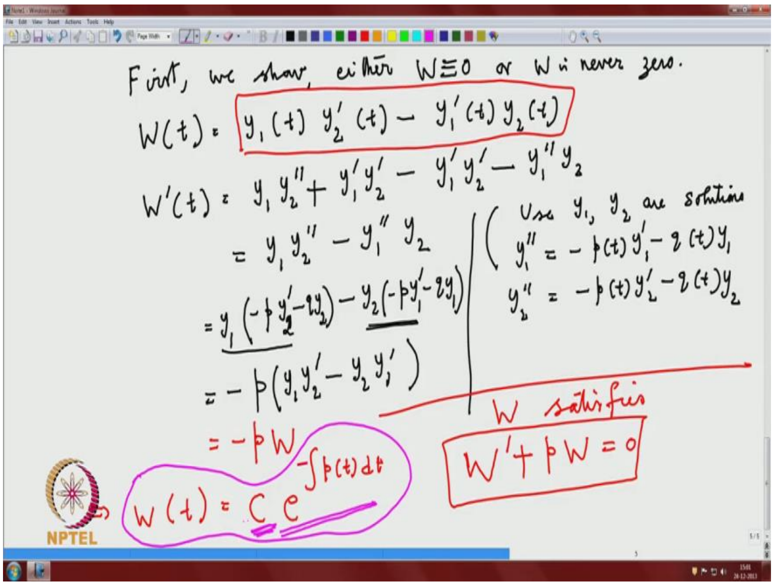

So, first we show either w identically 0 or w is never 0. So, here is a bit of thing. So, let us call w once again, what is your w t, w t is equal to y 1 t y 2 prime of t minus y 1 prime of t y 2. We differentiate with respect to t, you get w prime of t is equal to y 1 y 2 double prime minus y 1 prime y 2 prime here, y 1 prime y 2 prime minus, this is plus y 1 double prime y 2. So, this get cancel that is nothing but y 1 y 2 prime minus y 1 double prime y 2. Now use y 1 and y 2 are solutions, use. So, if you use that fact for example, you will get y 1 prime double prime is nothing, but if you go minus p t y 1 minus q t. If you take it to the right side and y 2 double prime, is equal to minus p t y 2 minus q t. So, if you substitute this here you get y 1 into minus p y 1 by skipping t minus q minus y 2 into y 1 double. This is y 2 and y 1 double prime is minus p y 1 minus q.

So, this term p this is plus p y 1 y 2 is plus p y 1 y 2, this is. Is something wrong I am sorry. So, there is a y 1 prime there, there is prime there. So, this is p y 2 prime, this is p y 1 prime now it is correct. So, there is y 1 prime is missing; a that is why. So, you have your there is a q y 1 here. So, I am missing that certain terms here, y 2 here. So, I have y 1 here y 2 here. So, let me write properly. So, you have q y 2 here, and here I have q y 1 you see. So, this second term is minus q y 1 y 2, here is the second term is plus q y 1 y 2 that get cancel. So, here you have minus p outside here, and you have y 1 y 2 prime this is plus y 2 y 1 prime that will be minus, you will have y 1 y 2 prime minus y, but what is this y 1 y 2 prime minus. If you look at the here y 1 y 2 prime minus y 2 y 1 prime is nothing,,, but t that is nothing but w. So, you have a nice equation for w, therefore w satisfies the Wronskian w, satisfies the first order equation w prime, w prime plus p w is equal to 0.

So, that shows immediately w is a first order equation homogeneous linear equation, and that implies immediately, your w t is in terms of the integral. Immediately you will see that w t is equal to c integral of p t d t, you see your correct equation be d 1. So, you see w satisfies this equation, and in this case, this part can never be 0. So, the w can be 0 only when you are constant is 0. So, when the constant is 0 w t will be identically 0, when the constant is non 0, then the w t can never be 0. So, the first part what will show that, the result that w is identically 0, or w is never 0. So, you see, so to prove the proposition, it to prove the proposition you want to show that this we need to prove w t naught, is not equal to 0. So, this claim is fine, so w is not equal to 0. Now you have to use this fact, what we had just proved that either w is identically 0 or w t is never 0. Now what you have to show you that w is identically 0, only if y 1 y 2 is dependent, and w is never 0 a y 1 y 2 is independent. So, that is in a next claim to be prove, that prove your theorem as well as the proposition everything is complete the movement we prove this proposition.

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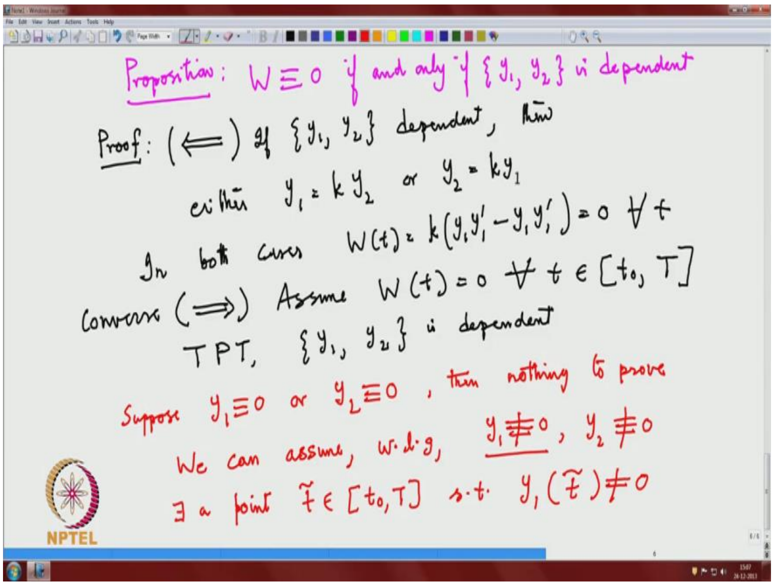

So, the first proposition is proved, if you prove this proposition, we use the uniqueness argument w is identically 0 if and only if y 1 y 2 is dependent we give a proof, because this requires one part is trivial; this part is trivial, why that part is trivial. If y 1 y 2 dependent y 1 y 2 dependent, then either y 1 is a multiple of y 2 or y 2 is a multiple of y 1 either or both; either y 1 is a multiple of y 2 k is constant or y 2 is a multiple of y 1 in both cases. In both cases you can immediately both cases w t is what w t it will be change same. So, it will be k into y 1 y 1 prime minus y 1 y 1 prime, because y 2 is a multiple. So, y 1 into y 2 prime is nothing, but k into y 1 prime is equal to 0, you see w for all t. So, the converse part is non-trivial part, that this part you have to prove it. So, assume w t is equal to 0 for all t, whatever it is in the interval, for all t in interval that is enough. You want to show that, say the interval t naught to t that is we have to prove it. So, to you prove that, this is for you to prove that y 1 y 2 is dependent.

So, how do you proceed. So, let us proceed slowly not very difficult, the proof. Suppose one of the (()) identically 0, then there is nothing more to prove. Suppose y 1 identically 0. So if I set contains 0 function that set is necessarily dependent y 1 identically 0, or y 2 identically 0. Then it will be dependent, then nothing to prove. Clear that point, because one of the measles 0 functions, we do not have to prove anything. So, we can assume with a loss of generality y 1 not identically. Not identically 0, I am not claiming that y 1 is never 0 y 1 can be 0 at some point. The identically 0 is not allowed, y 2 not identically 0. So, since y 1 is not identically 0, there exist some point. There exist a point, say some t tilde in this interval t naught to t; such that y 1 at t tilde not equal to 0, but now use the argument of continuity. If a function is not equal to 0 at one point, then that function is not equal to 0 in a small interval, so we will do that.

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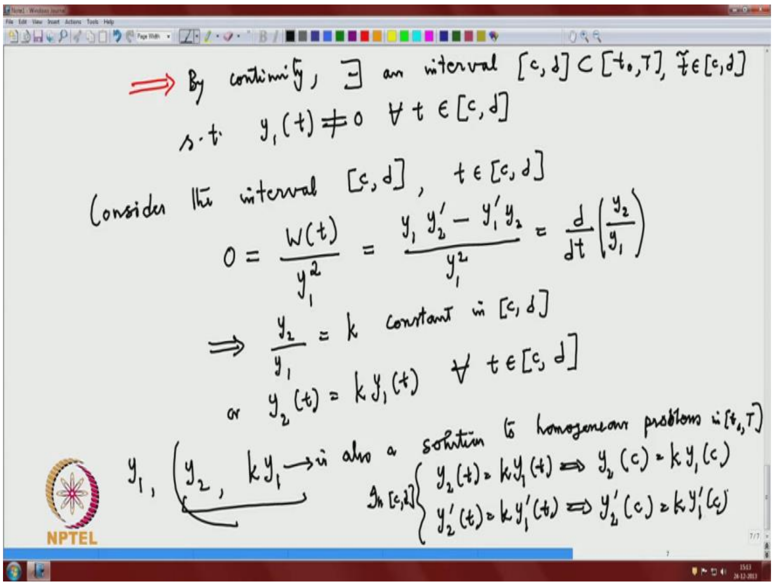

That implies by continuity there exist an interval c d. Of course, contain the t naught t wherever it is, and of course, t tilde. These all are not important right now in c d, because such that y 1 at t not equal to 0 for all t in c d, but we have using just continuity of the function y 1. If a function is not equal to 0 at one point it is not equal to 0 in neighborhood of that point, and that neighborhood I taken to be c d; that is all nothing more than that, you do not need anything to be taken; that is a simple analysis problem. You consider this is the only took we have to follow carefully. So, we are working in the interval c d nowhere else, consider the interval c d. So, we are doing all analysis in c d right now. w is identically 0 in everywhere; therefore, 0 is equal to w t by y 1 square, this is something a construction.

And this is allowed, because for t in c d on c d y 1 is not equal to 0, so this is allowed. So, now, you are right, this is y 1 y 2 prime minus y 1 prime y 2 y y 1 square; that is nothing, but that is where why we use that y. You can also work with y 2, there is nothing wrong d by d t of y 2 by y 1. Therefore, d by d t, this is why we need an interval, not a just one point claiming d by d t is not enough, you have to claim in an interval. So, d by d t of y 2 by y 1 is equal to 0 in interval that implies y 2 by y 1 is equal to a constant in c d; that is why important, constant in c d. Or y 2 of t is equal to k y 1 t for all t in c d. Now this is where you have to use cleverly the initial value problem has a unique solution. Now consider y 1 is a solution to the homogeneous problem, y 2 is a solution to your homogeneous problem. So, whenever you have a solution to your homogeneous problem. Any multiple of that is also has a solution to your homogeneous problem. Therefore, k y 1 is also a solution to your homogeneous problem; that is y 2, a solution to homogeneous problem, but y 2 is k y 1 in that interval.

So, k y 1 is a solution to your homogeneous problem. And y 2 is a solution. you take these 2 things. This is a very settle argument. k y 1 is a solution to your homogeneous problem in t naught to t. y 2 is also a solution to your homogeneous problem in t naught to t, but y 2 and k y 1 coincides in c d. since it coincides in c d, is derivative also coincides in c d. So, you have two solutions to your homogeneous problem, with same initial data. So, let me repeat that y 2 t in particular y 2 t is equal to k y 1 t in c d. So, y 2 t is equal to k y 1 t; that implies in particular y 2 at c anytime you can fix it. c is equal to k y 1 c. now this is equal in interval, I can differentiate this 1, y 2 prime of t is equal to k y 1 t. These are all happening in c d, but when I say that this is a solution, it is happening everywhere. So, y 2 and k y 1 is a solution to your homogeneous problem in everywhere, say t naught to t, but what is happening this is in a c d. So, that implies y 2 prime at c is equal to k y 2 at c. So, now you see you have two solutions y 2 and k y 1.

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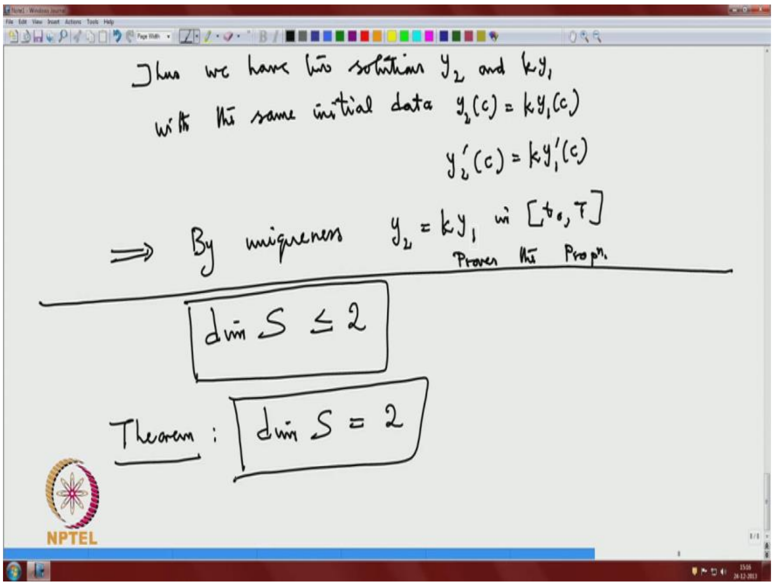

Thus we have two solutions y 2 and k y 1 with same initial data. What are the initial data you need to understand, two initial data y 2 at c. I am thinking c as my initial data; that is nothing wrong in that; y 1 at c and y 2 prime at c is equal to k y 1 at c. Therefore, by uniqueness y 2 and k y 1 has to be the same everywhere; that is what initial value problem tell you. You have two solutions in an k interval for the homogeneous equation with a same initial data. Initial data means y and y prime, then that solution is unique. Here we have produce two solutions y 2 and k y 1 in the full interval, but with a same initial data at c, by uniqueness. So, that is what is uniqueness or even properly uniqueness y 2 t is equal to k y 1 in the entire, wherever this exist t naught to t you see for all t. So, that proves the proposition, this proves the proposition. Once it proves the proposition.

So, let me recall, let me consolidates once again. What we have done so far in the proposition in this hour. If you take two independent solutions, then that generates all the solutions. In fact, what we have done is that dimension of s is less than or equal to 2; that is what we have proved that. But what we will be soon see, is that the dimension of the next proposition which you want to do that. Let me do that theorem before I complete this lecture; theorem, a c is the set of all solutions to the homogeneous equation, please keep that in mind dimension of s is equal to 2. The proof is now not difficult, the major part of the proof is already done, proving dimension of s is less than or equal to 2.

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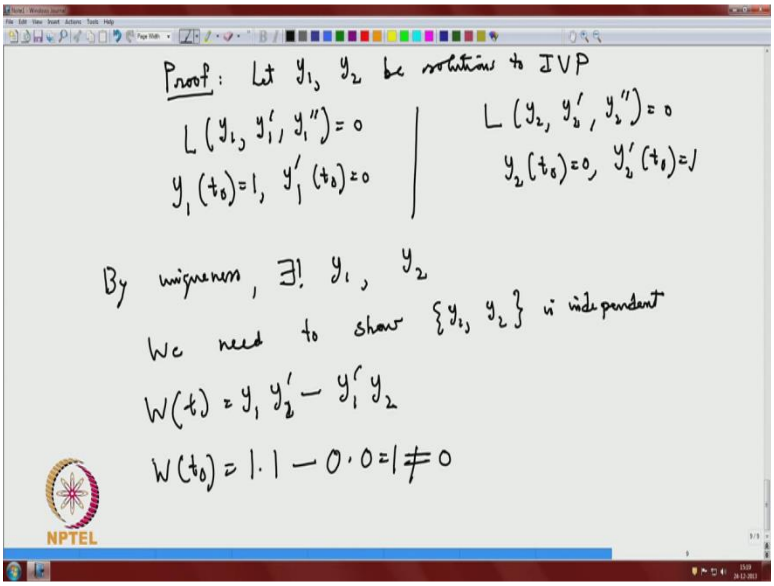

To proof, you have to just produce to independent solution. One gets confused when I producing two independent solution, is only linear theoretical production. It does not give you the method to how to find its. We can solve it, because we do not know how to solve the equation. So, let y 1 y 2 be solutions to the initial value problem. Now I am considering an initial. the idea is that you want two independent solution; thus method to get independent solution, you put your initial conditions are independent vectors, when you put your initial conditions are independent vector y 1 and t naught and y 1 at t naught y 1 prime at t naught and y 2 prime at t naught that will be independent, and hence the solution itself will be independent. So, what we will have l of y 1 y 1 prime y 1 double prime is equal to 0 with initial condition.

We put the basis element y 1 at t naught is equal to 1 y 2 at t naught is equal to 0, and then you prove l of y 1 2 y 2 prime y 2 double prime equal to 0 with y 2 at t naught is equal to 0, and y 2 prime at t naught is equal to 1. So, by uniqueness there exist y 1 and y 2. By uniqueness there exists unique y 1 y 2. We need to show y 1 y 2 is independent. How do you show y 1 y 2 is independent. This computes the Wronskian. So, you just compute the Wronskian. What is the Wronskian w at t. w at t is nothing, but y 1 y 1 prime y 2 prime minus y 1 prime y 2. And what is the w at t naught. w at t naught is equal to y 1 at t naught that 1 y 1 prime at t naught is equal to 1. What is y 1 and y 1 at t y 1 prime at y 1 prime at t naught is equal to 0 and y y 2 at t naught is equal to 0 that is one not equal to 0, so that is trivial.

So, you have your two independent solutions which satisfy the solutions. So, what we will be doing. So, we have shown that the dimension of s is exactly equal to 2. And interesting fact is that, this theory you can also extend to general n’th order equations, and you can see that n’th order linear equations with constant coefficients has n independent solution, and the dimension of this place will be end. We do not do it here, but the same procedure, you can take the basic elements. You take in n dimension you take the basic elements, can only basic elements e 1, e 2, etcetera e n for each e I you solve, and then you prove that the solution set is independent, which we will not do it, but the same principal works there, to see that solutions space is independent. So, tomorrow in the next class we will be giving few more things in the next two lectures. We may need it in this module to complete the second order linear equation. We will be doing equations with constant coefficients. And we will also discuss something about non-homogeneous situation, and we will try to understand the solution structure of the non-homogeneous equation.

Thank you.