Advanced Matrix Theory and Linear Algebra for Engineers

Prof. R. Vittal Rao

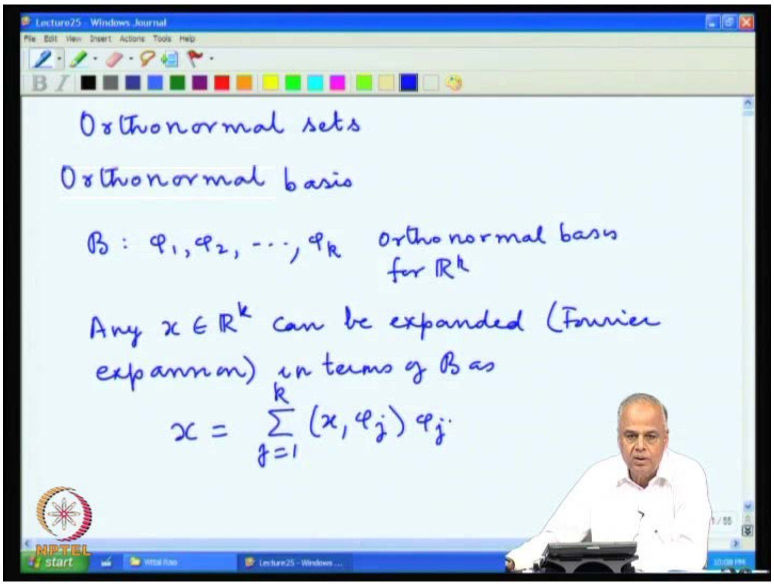
Centre for Electronics Design and Technology

Indian Institute of Science, Bangalore

Lecture No. # 25

Inner Product and Orthogonality- Part 4

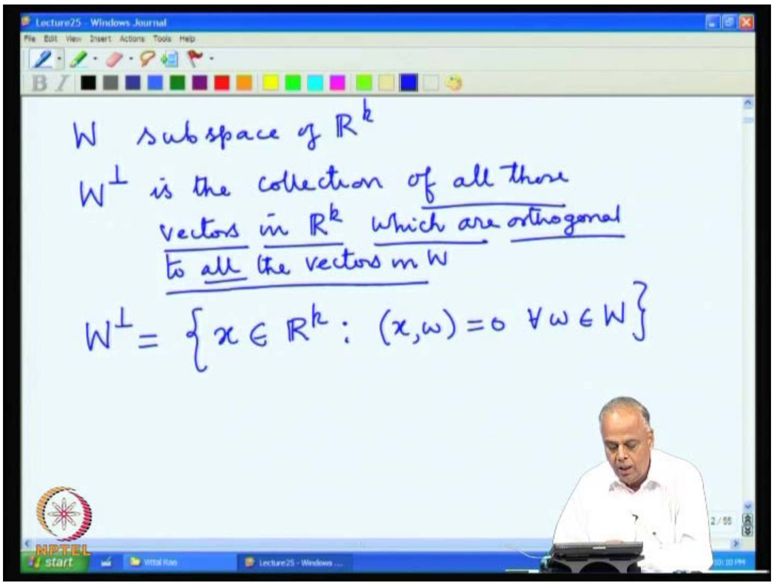
(Refer Slide Time: 00:20)



We have been discussing the notion of orthonormality are in the space R k. We look at the notion of orthonormal sets that is these are sets, in which any two vectors are orthonormality to each other and every vectors length 1. Then, we look at the notion of an orthonormal basis, and important outcome of this notion of orthonormal basis was that, if you had an orthonormal basis phi 1, phi 2, phi k for so this is an orthonormal basis. Suppose, this is an orthonormal basis for R k, so whenever we had an orthonormal basis for R k, we can expand any x in R k, and that expansion was called the Fourier expansion, expanded this is called Fourier expansion. Expanded in terms of this basis, as x equal to summation j equal to 1 to k x phi j phi j, where x phi j remember denotes the inner product. So, if you recall the inner product x phi j is also the same as phi j transpose x.

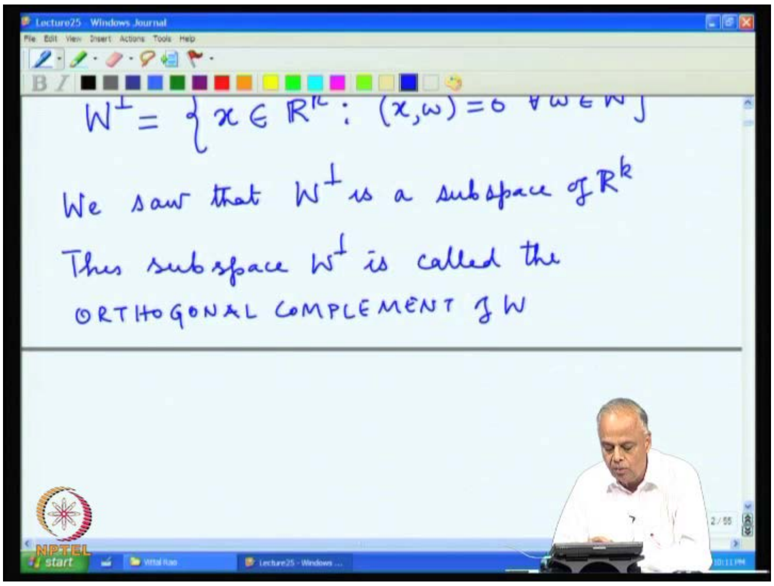
So, every vector x can be expanded in terms of the orthonormal set as the linear combination orthonormal basis, and the coefficients are easily obtained as the inner product of the vector x with phi j. The coefficients are simply the inner product of x look phi j. This was one of the important things that, we got out of this idea of orthonormal basis.

(Refer Slide Time: 02:30)



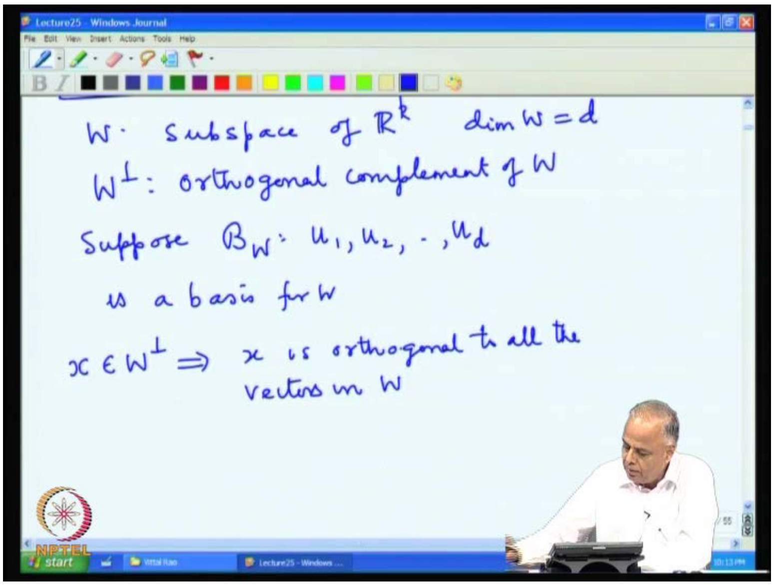
Then, we introduce the notion of the orthogonal complement of a set. In particular, we were interest an orthogonal complement of subspaces, so let say W is a subspace of R k. W is the subspace of R k then, we look at all the vectors which are orthogonal to every vector in W, so that set we call as W perp. W perp is the collection is a, collection of vectors what vectors, so all those vectors in R k, which are orthogonal to all the vectors in W. So, orthogonality is given by the fact that, the inner product is 0. So, mathematically, we can write in symbols as W lambda at all those vectors, it is the all those vectors in R k, this is this part such that, which are orthogonal to all the vectors. So, x comma omega must be 0 orthogonal to all the vectors, so should be for every omega in omega. So, W lambda if the set of all the vectors in R k, which are orthogonal to all the vectors in W perp.

(Refer Slide Time: 04:24)



We saw that, W perp is a subspace of R k. This subspace, W perp is called the orthogonal complement of W. So, every subspace as something called is orthogonal complements.

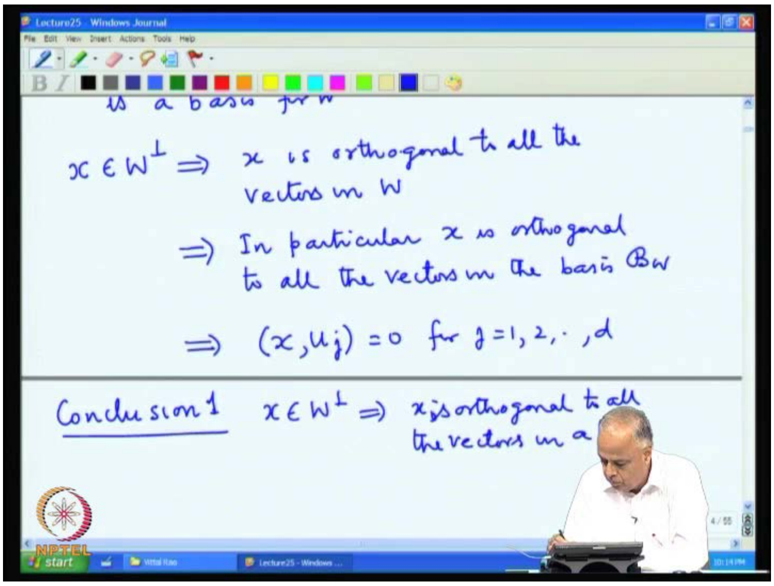
(Refer Slide Time: 05:08)



We will make one simple observation. So, again we start with the subspace W of R k and we denote by W perp, the orthogonal complement of W. So, we have a subspace and it is orthogonal complement. Suppose, we take a basis for W, suppose B w equal to u 1, u 2, u d is a basis for W, so let say dimension of W is d.

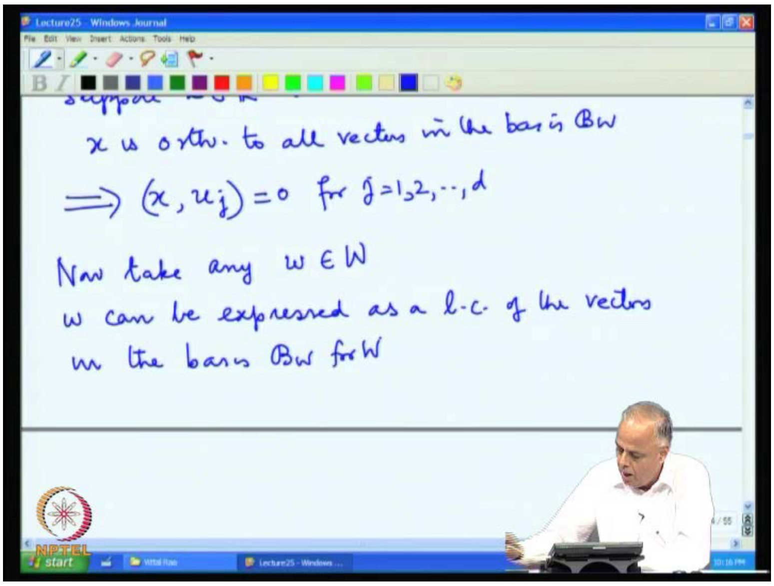
So, we have a subspace of dimension d, we call the w and W perp is the orthogonal complement of w and we are looking at a basis for w. It will consist of d linearly independent vectors let that, basis b u 1, u 2, u d. Now, suppose I have a vector x in W perp, this means x is orthogonal to all the vectors in W. That is the definition of W perp, W perp is the collection of all those vectors, which are orthogonal to all the vectors in W. So, x is orthogonal to all the vectors in W, this basis vectors u 1, u 2, u d are all in w, and therefore x must be orthogonal to these vectors.

(Refer Slide Time: 07:05)



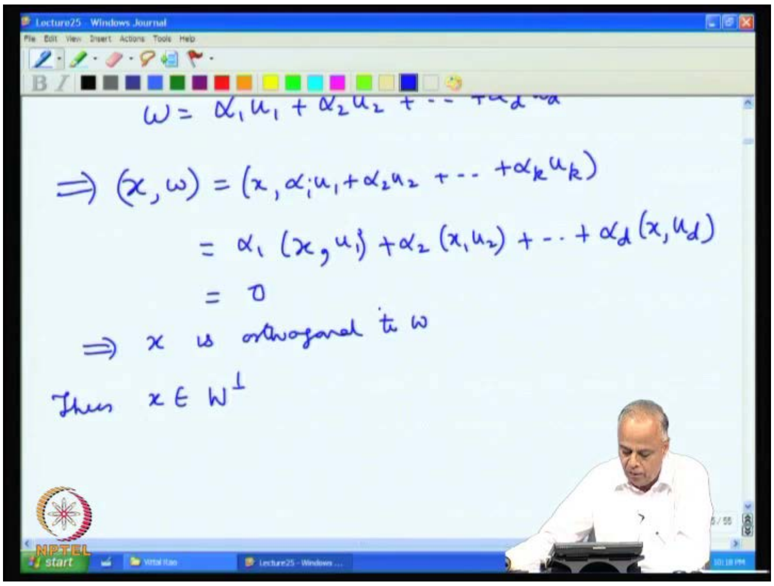
So, implies in particular x is orthogonal to all the vectors in the basis B w. And therefore, what is meant by orthogonality? The inner product x comma these vectors u 1 u 2 u k, so the inner product u j must be equal to 0 for j equal to 1, 2, d. So, what these mean says that, if a vector is in W perp, it must be orthogonal to every one of the basis vectors. So, from it up call it conclusion 1, x belongs to W perp implies x is orthogonal to all the vectors in a basis for W.

(Refer Slide Time: 08:26)



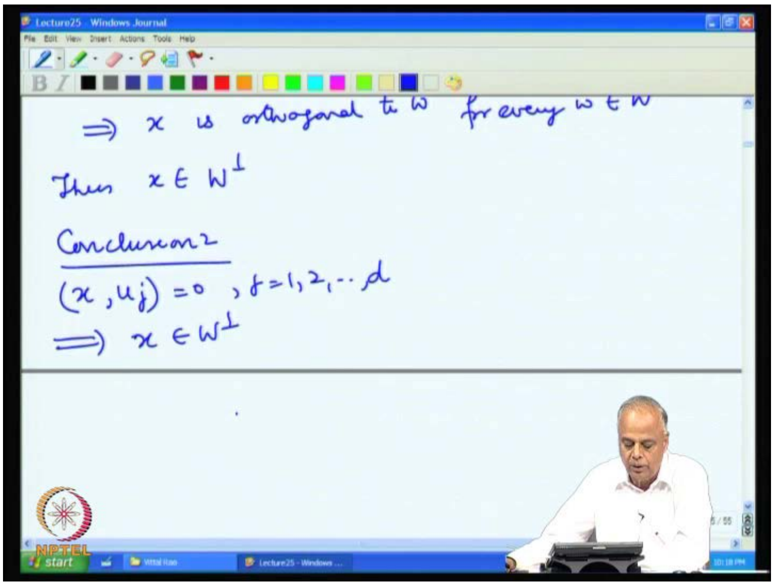
Conversely, suppose x belongs to R k is such that, x is orthogonal to all vectors in the basis B w. What does that mean? This means, x u j is equal to 0 for j equal to 1 2 up to d. Now, take any vector w in W. Now, since B w is a basis, we can expand w in terms of these B w vectors. So, w can be expressed as a linear combination of the vectors in the basis B w in the basis B w for W.

(Refer Slide Time: 09:52)



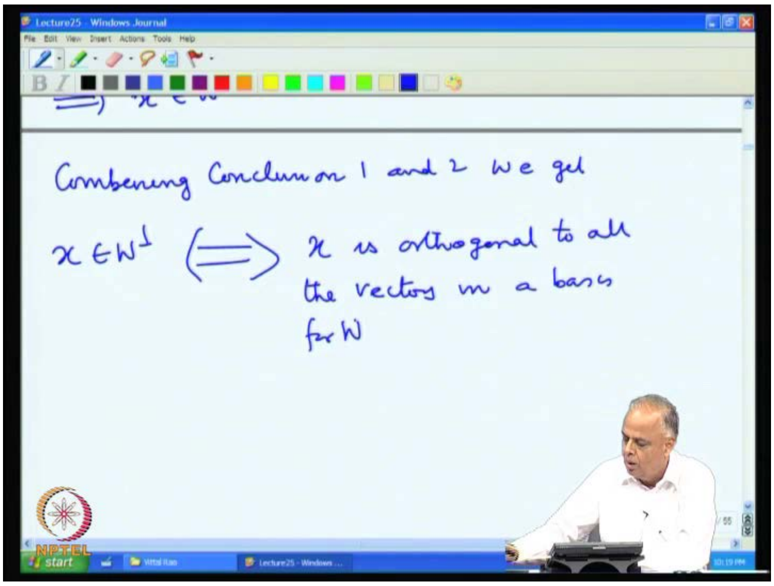
And therefore, we have x equal to from alpha 1 u 1 plus alpha 2 u 2 plus alpha d u d. Consequently, w equal to this. Consequently, This implies, if you now take the dot product of the inner product of x with w, I get x comma alpha 1 u 1 plus alpha 2 u 2 plus alpha k u k, but the dot product is distributive and constant come out of the dot product, this is equal to alpha 1, x 1 x comma u 1, alpha 1 x comma u 1 plus alpha 2 x comma u 2 and so on, alpha d x comma u d. But now, we are given recall, that we had assume that, x is orthogonal to all the basis vectors. And therefore, x comma u j is equal to 0 for every j, so this becomes 0. Which means, x is orthogonal to w, so what it says is that the moment the vector, if something is orthogonal to all the basis vectors then, it is automatically orthogonal to all the vectors, thus x belongs to W perp. This is true for any w for any w we can do this, and therefore x is orthogonal to w for every w in W.

(Refer Slide Time: 11:48)



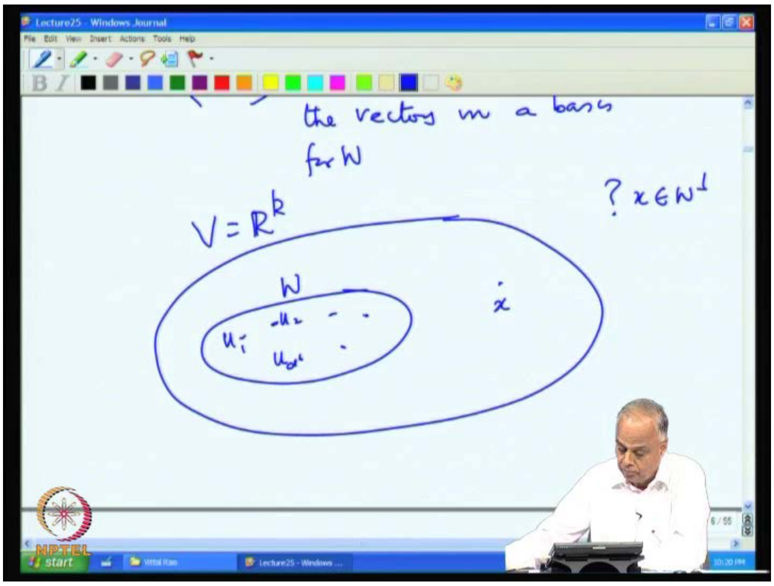
So therefore, conclusion 2 is, if we start with the fact that x is orthogonal to all the basis vectors then, that implies x is in W perp. Comparing, this conclusion in conclusion 1, we had that if x is in W perp, it must be orthogonal to all the basis vectors, and now we have say that if it conclusion orthogonal to all the basis vectors it must be in W perp.

(Refer Slide Time: 12:25)



So, combining conclusion 1 and 2, we get x belongs to W perp, if and only if x is orthogonal to all the vectors in a basis for W.

(Refer Slide Time: 13:02)

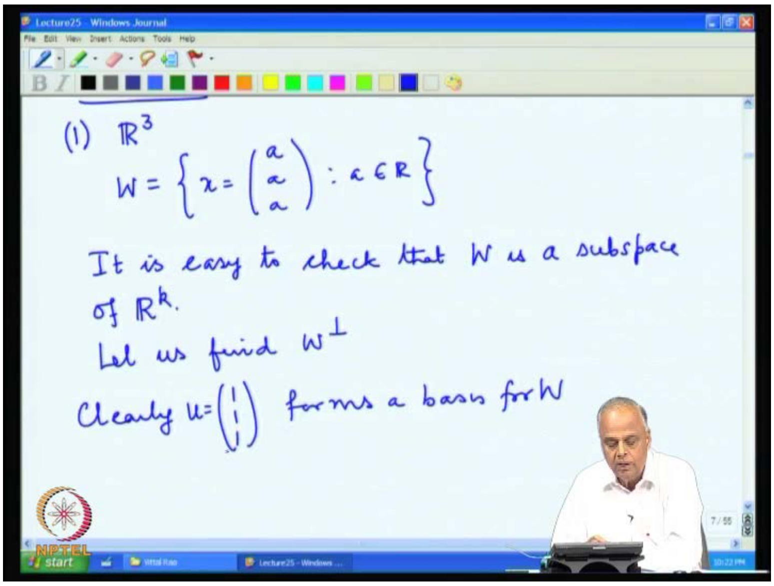


What means, what it means is the following, we have this vector space V and we have the subspace W. And we are going to pick a vector x in R k with the vector space V, we are dealing with R k. So, we are going to pick a vector in R k and we want to check, whether x belongs to W perp, so the question we are asking is, does x belong to W perp.

Now, basically in order to check that x belongs to W perp, we have to check whether x is orthogonal to every one of these vectors in this W.

However, what we have now shown is, you done have to go on checking with every vector. You choose this (()) d samples, namely the basis vectors, and just check whether x is orthogonal to u 1; x is orthogonal to u 2 and x is orthogonal to u d. And therefore, what it says (()) go on checking, whether x is orthogonal to all the vectors is enough you check x is orthogonal to all the basis vectors then automatically you can conclude, whether it is x is orthogonal to all the vectors or not, and thus in order to check whether vector x is in W perp or not. We have to check only whether x is orthogonal to all the vectors.

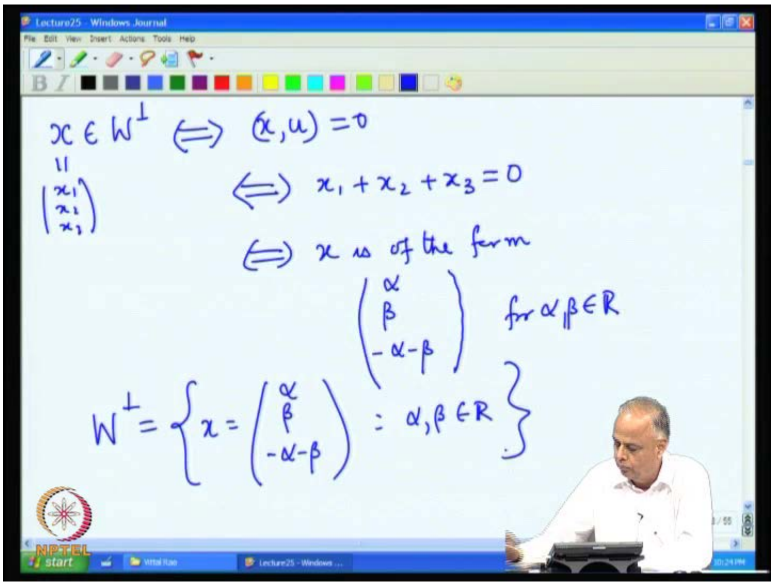
(Refer Slide Time: 14:36)



Now, let us look at some examples, this idea we will use in the examples to find the W perp. Let us look at, R 3 and consider the subspace of all those vectors, which are the form a a a: a is in R. In other words it consist of all those vectors everyone of this component is equal to the other. So, this is along the line 1 1 1, so now consider this space and that it is easy to check that it is a subspace. We know how to check whether it is a subspace, we have to see it is non empty; we have to check whether it is to close under addition, whether it is close to under scalar multiplication all these are easy things, we have done this before, so it is easy to check that so W is in it a subspace of R k.

So, let us find W perp the orthogonal complement of w, the first of all in order to check whether something is W perp or not. We have to check whether everything is orthogonal to a basis vector, so in order to do that we will first find a basis for w. So, clearly 1 1 1 this vector u alone forms a basis for W why is this so, because this vector belongs to W and every vector in W is a times u.

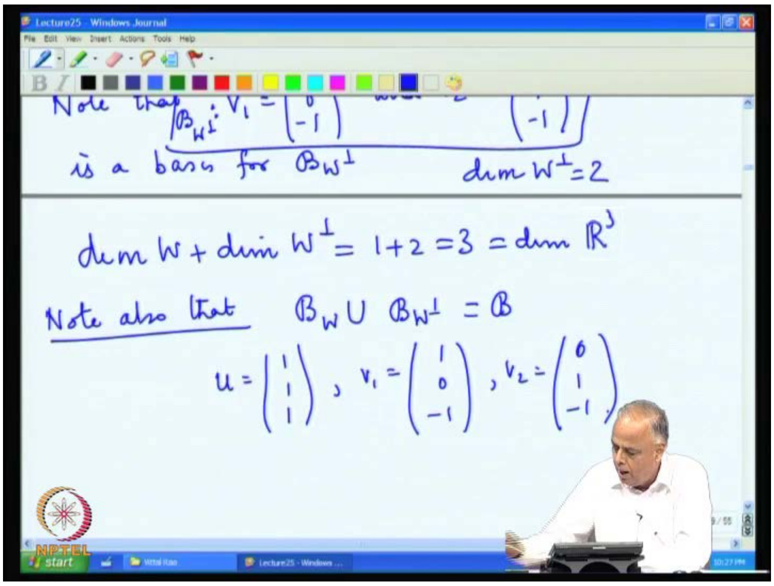
(Refer Slide Time: 16:42)



So Therefore, it is expands w and hence it forms a basis. So, of the B w the basis for u B w consist of only one vector namely u equal to 1 1 1. Now Therefore, a vector x will belong to W perp if and only if x is orthogonal to the basis vector and there is only one basis vector, and therefore we want x to the orthogonal to the basis vector. Suppose x is x 1 x 2 x 3 then, this is x comma u the dot product between x and u is x 1 into 1 plus x 2 into 1 plus x 3 into 1 that must be a 0.

So, the only requirement for a vector to get qualify to be in W perp is that the sum of each components is 0. And therefore, the vector x is of the form alpha beta x 1 and x 2 can be anything, but the moment, you will make give any values for x 1 and x 2 to make the total sum is 0, we have to make the third component as minus alpha minus beta for alpha beta in R. And therefore, we get W perp to be the collection of all vectors of the form alpha beta minus alpha minus beta for alpha and beta belong to R. So, the original space w which was all a a a now has this orthogonal component which complement which consist of vectors is third component. This is the negative of the sum of the first two components.

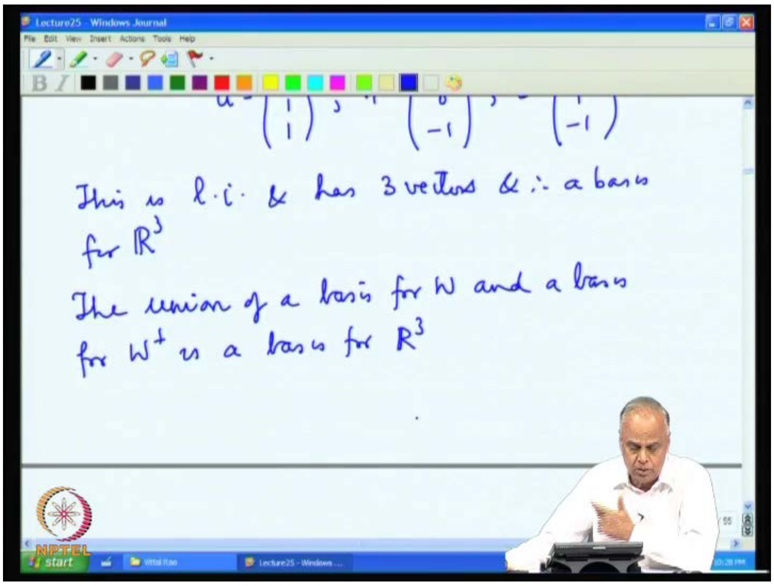
(Refer Slide Time: 18:40)



Note, that v 1 equal to 1 0 minus 1 and v 2 is equal to 0 1 minus 1 is a basis for let us call this a B w perp is a basis for B w perp. Because these 2 vectors are in W perp, and any other any vector alpha beta minus alpha minus beta in W perp can be written as alpha time v 1 plus beta time v 2; therefore, the span and there obviously linearly independent; and therefore, the they are form a independent set in W perp and span W perp, and therefore they form a basis for W perp. Now, notice the w had as basis consisting of the single vector, and therefore the dimension of W was 1 and the basis consist for W perp consist of two vectors; and therefore, dimension of W perp is 2 and in notice that the dimension of W plus dimension of W perp is equal to 1 plus 2 is 3, which is the dimension of the original vector space in these all the subspaces live, they are subspaces of R 3 so in this case k is 3.

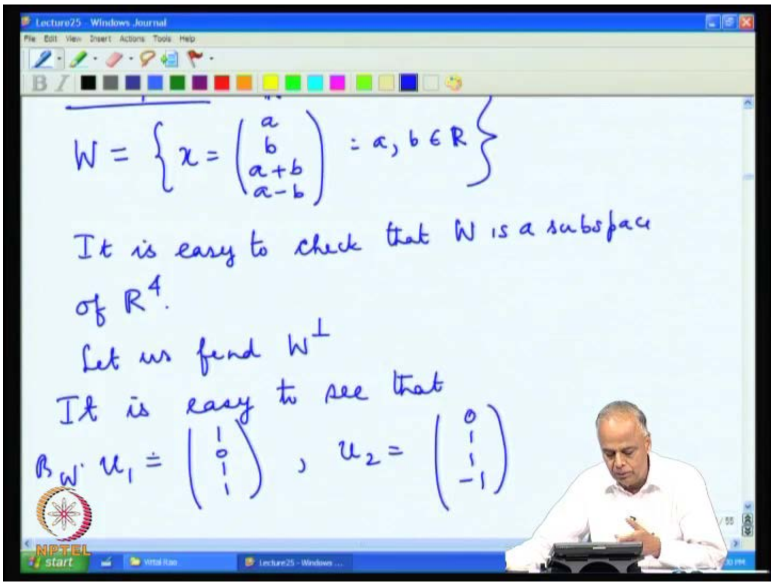
So, the dimension of w plus dimension of W perp tense out to be the dimension of this whole space. Note also, that we have this basis for w and we had the basis for W perp and if you put them together we get a set B, which consist of this vector u which was 1 1 1 and v 1 which consisted 1 0 minus 1 and v 2 which was 0 1 minus 1.

(Refer Slide Time: 21:04)



And this B is this is linearly independent and has 3 vectors, and we know that in R 3 any linearly independent set having 3 vectors is a basis, and therefore a basis for R 3. So, this example we observe lot of things are happening, namely the dimension of w and dimension of W perp added together to give the dimension of the whole space, and the basis of w and the basis of W perp added together to give now a basis for R 3. So, the union of a basis for W and a basis for W perp is the basis for R 3; that is in this example, we will see the more general version that this is in general true is litter while later.

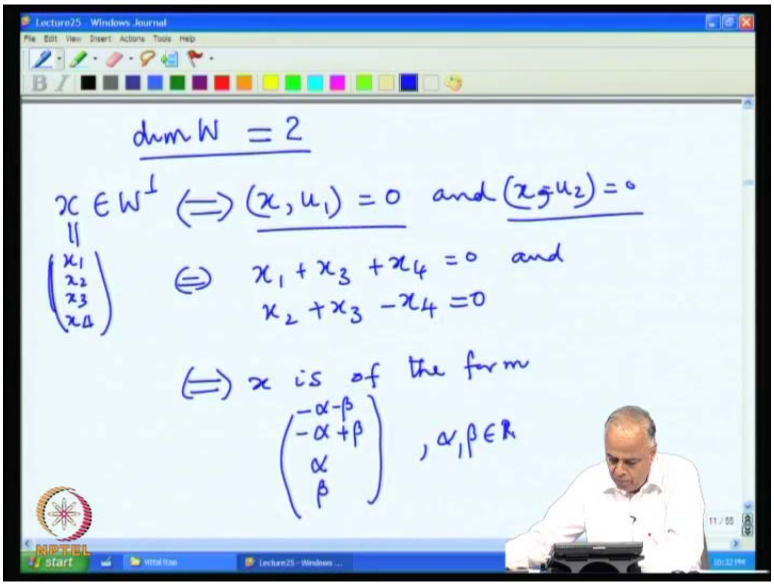
(Refer Slide Time: 22:12)



Let us now, look at another example of this construction of this orthogonal complement. Let us again, now consider R 4 straight bigger dimensional space than before, and now consider this space subspace w, which consist of all those vectors of the form a b, a plus b a minus b; where a b are in R. Now once again, it is easy to check that w is a subspace of R 4. So, w is a subspace of R 4, and therefore we will find let us find the orthogonal complement W perp. Now, again as before in order to find the orthogonal complement we first found a basis for w and then looked at all those vectors which are orthogonal to each one of these basis vectors. So, to this vector we will now go and find a basis for w, now from the structure.

It is easy now to see if I take a equal to 1 and b equal to 0 I get this vector 1 0 1 1, and if I take a equal to 0 and b equal to 1 I get this vector, and this forms a base this vectors are in w they are linearly independent, and any vector w is a times u 1 of the form a times u 1 plus u times u 2, and therefore they are linearly independent vectors in w which form w, and therefore they form a basis, so this is a basis for W perp w . So, what we observe from this about dimension of w.

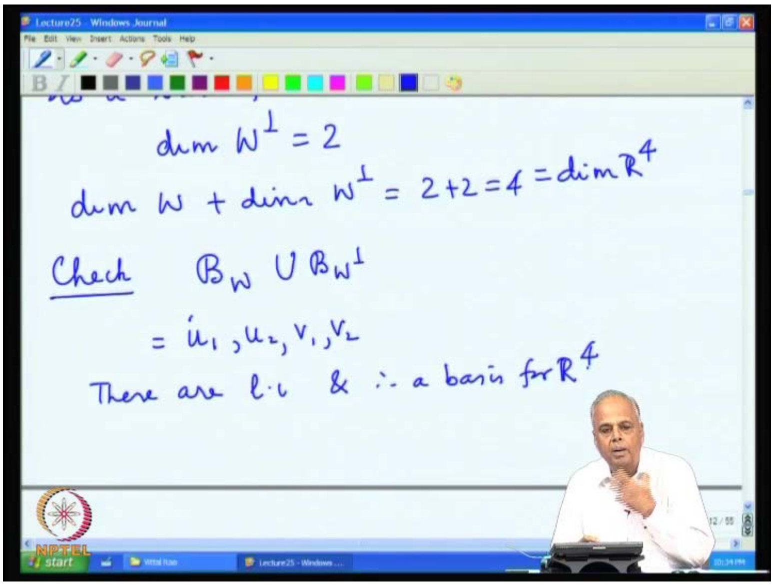
(Refer Slide Time: 24:24)



Since, there are two vectors in this basis the dimensional w is in it 2. Now, let us find W perp x belongs to W perp, we have seen that a vector belongs to the orthogonal complement, if and only if it is orthogonal to every vector in a basis for W and therefore, x must be orthogonal to u 1, and x must be orthogonal to u 2. Now, suppose x is x 1 x 2 x

3 x 4. Now, we are looking in the space R 4, so x will have 4 components. Now, what does the first condition say x must be orthogonal to u 1. So, the dot product of x with u 1 must be 0 u 1 is this vector. So, x 1 plus 0 times x 2 plus 1 times x 3 plus 1 times x 4 is 0, so we get x 1 plus x 3 plus x 4 equal to 0. That is this fact then, this fact tells us that x 2 plus x 3 minus x 4 is equal to 0. Now, that says that there are two equations that before component should satisfy, and they are going to be two floating variables and for x is of the form. We can choose x 3 and x 4 arbitrarily then, x 1 has to be minus x 3 minus x four and x 2 has to be x minus x 3 plus x 4, so alpha and beta can be anything.

(Refer Slide Time: 26:30)

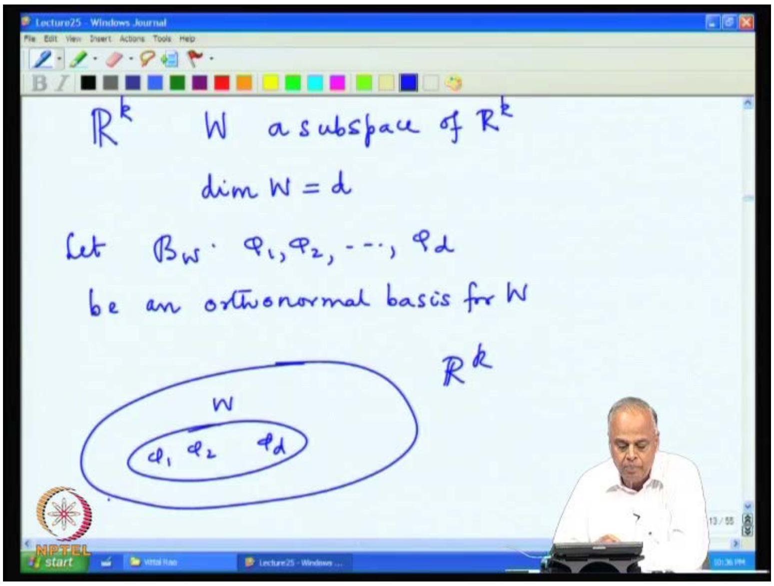


And therefore, we get W perp consist of all the vectors of the form minus alpha minus beta minus alpha plus beta alpha beta where alpha and beta R. Now, clearly again taking alpha as 1 and beta is 0, we get this vector and alpha is 0 and beta is 1, we get this vector and that will be a basis for W perp, so is a basis for W perp. And therefore so, in the basis for W perp consist of two vectors, we also observe the W perp as dimension 2 and it turns out u 1, in this case dimension of W plus dimension of W perp is 2 plus 2 which is 4 this is the dimension of this space R 4.

So, this dimension plus dimension W perp in both these examples added up to the dimension of the whole space, again check which is to check if you put the basis of w union basis of W perp, we get these vectors u 1, u 2, v 1, v 2 where u 1 u 2 v 1 v 2 are as we observed before, now these are linearly independent check that these are linearly independent, and these are four vectors which are linearly independent in R 4, and therefore a basis for R 4.

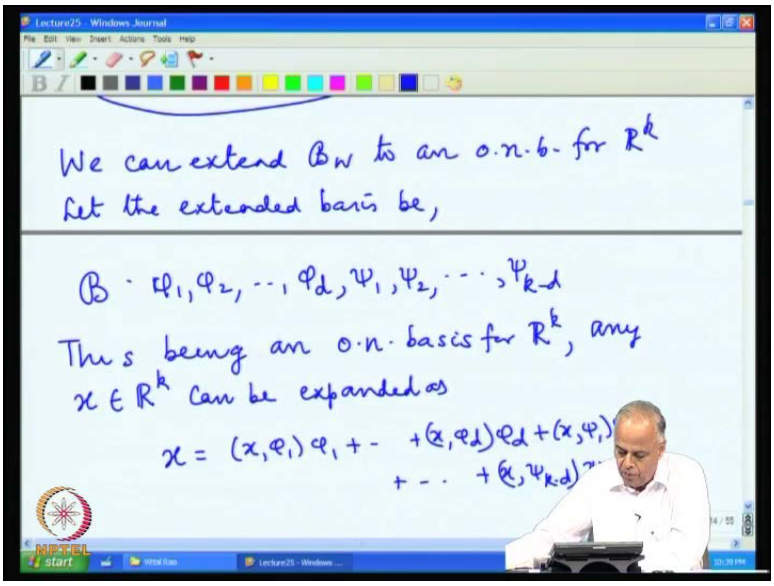
So, basis for W and a basis for W perp put together gave as a basis for R 4, and that it turns essentially gives as the fact that W the dimension of w plus dimension of W perp is indeed is equal to the dimension of the whole space. Now, let us perceive this further and analyse thesetwo2 orthogonal complement all these analysis will be used eventually to study our questions about system of equations diagonallisation etcetera.

(Refer Slide Time: 29:01)



So now, let us get back to R k and let say W a subspace of R k, let them dimension of w be equal to d, and therefore we will have an orthonormal basis, if you take any orthonormal basis for w, it will have a exactly d orthonormal vectors. So, let B w be phi 1 phi 2 phi d and be an orthonormal basis for W. Now, we have seen that every r orthonormal set is also linearly independent, and therefore phi 1 phi 2 phi d is in linearly independent set and it is an orthonormal set. It is an orthonormal set sitting in the big space R k, so here is R k and here is W in this phi 1 phi 2 phi d are sitting. Now, since phi 1 phi d phi 2 phi d is an orthonormal set we have seen that any orthonormal set in R k can be extended to a orthonormal basis in R k.

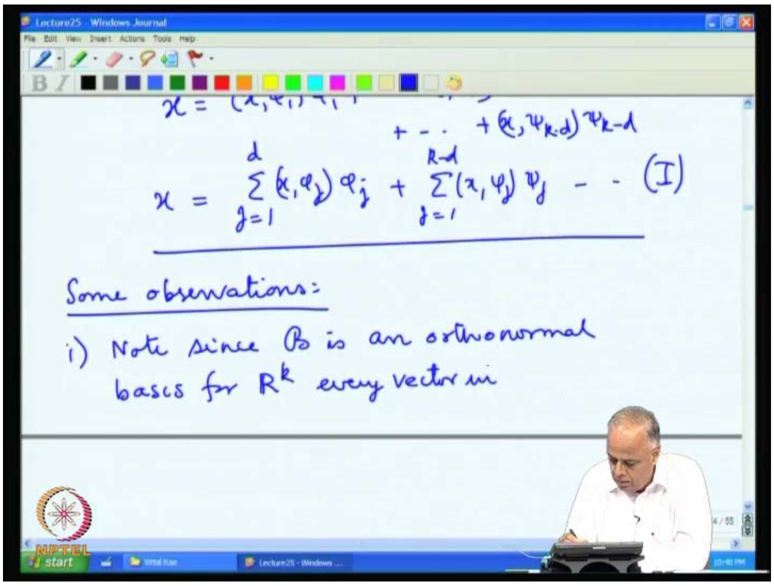
(Refer Slide Time: 30:35)



So, we can extend B w to an orthonormal basis for R k let how many vectors we will have to append the dimension of R k is k. So, we would have to pick some vectors how many of them k minus d call them psi 1 psi 2 psi k minus d, and these are going to be orthonormal there outside. So, let the extension the extended basis be, B we will have the entire phi 1 is that it what it on the extension to this. We will add this append this psi 1 psi 2 psi k minus d.

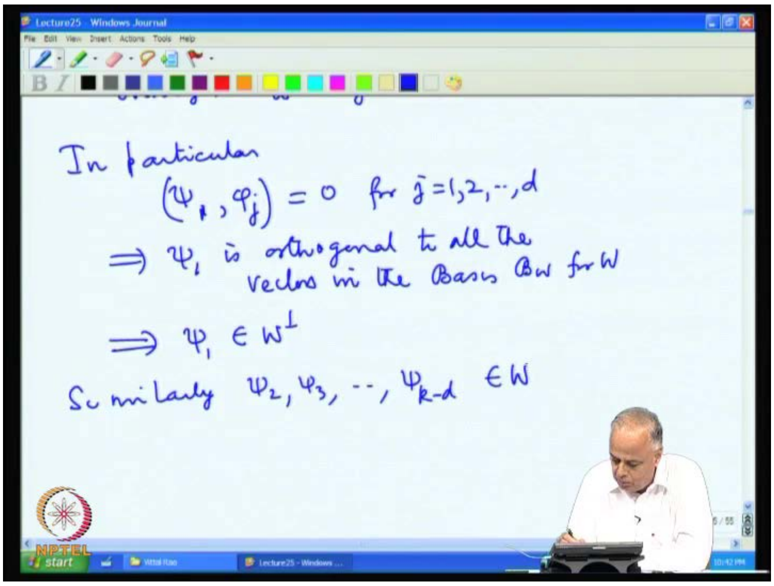
So, this is now a basis for R k, and it is an orthonormal basis. So, we have an orthonormal basis for so this being an orthonormal basis for R k, we have seen that any vector can be expended in a Fourier expansion or as a linear combination in terms of the orthonormal basis vectors, where the coefficients in the linear combination is simply the inner product of the vector with that corresponding basis vector. So, let us some this up, this being an orthonormal basis for R k any x in R k can be expanded as x equal to x comma phi 1 phi 1 plus etcetera x comma phi d phi d plus x comma psi 1 psi 1 plus x comma psi k minus d psi k minus d.

(Refer Slide Time: 33:05)



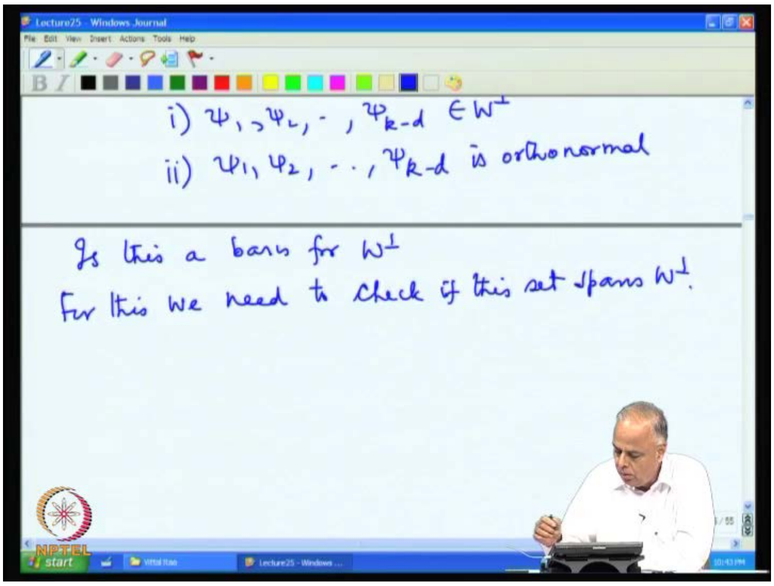
We will write this, a simply j equal to 1 to d x phi j phi j plus j equal to 1 to k minus d x psi j psi j is call this 1. Now, we are going to observe in facts from this examination, this expansion now for this we make some certain observations. Now, some observations that we make are the about this vector psi j. Now, since this is an orthonormal basis this is an orthonormal basis.

(Refer Slide Time: 34:30)



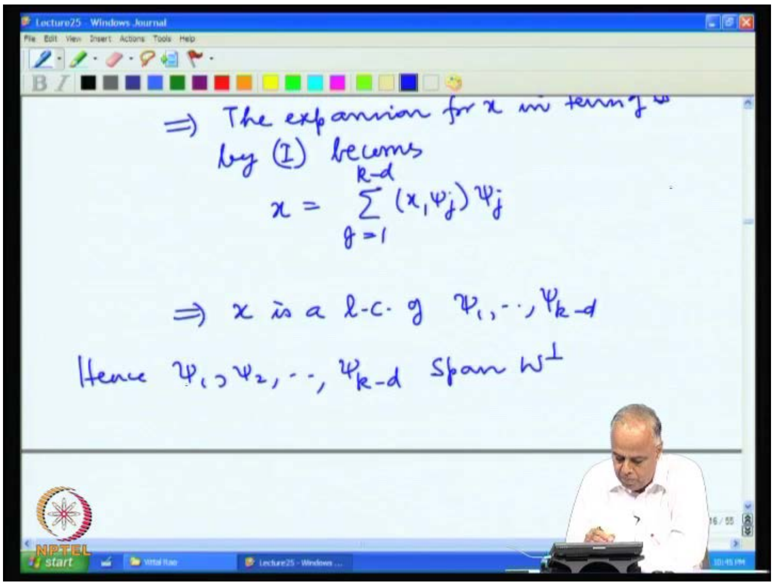
So, every vector in this is orthogonal to any other vector, and therefore psi 1 will be orthogonal to phi 1 psi will be orthogonal to phi 2 and psi 1 is orthogonal to phi d. So, note since, B is an orthonormal basis for R k, every vector in it if orthogonal to every other vector in it. And therefore, in particular psi 1 will be orthogonal to phi 1 phi 2 phi 3 phi j equal to 0 for j equal to 1 to d, so psi 1 is orthogonal to all the vectors in the phi 1 phi 2 phi j group, but these phi 1 phi 2 phi j form a basis for w, this means psi is orthogonal to all the vectors in the basis B w for w, and just a 2 minutes back we saw that if a vector is orthogonal to all the vectors in a basis then, it must belong to the orthogonal complements, so that says, psi 1 belongs to W perp. Similarly, psi 2 is orthogonal to all the vectors phi j, and therefore psi 2 belongs to W perp and so on. Similarly, psi 2 psi 3 psi k minus d all these belong to W perp.

(Refer Slide Time: 36:08)



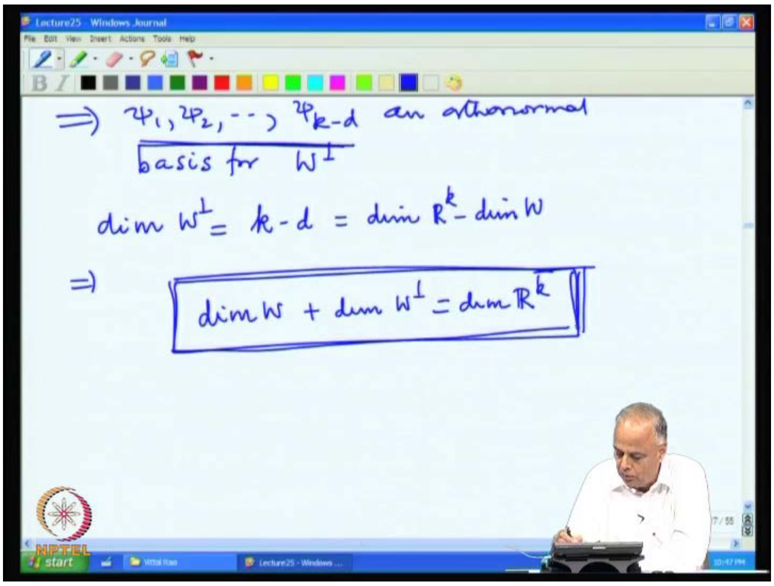
And therefore, we have the first observation psi 1 psi 2 psi k minus d are all in W perp and second since the whole set is orthonormal since, the whole set B is orthonormal even this part must be orthonormal and therefore, we get psi 1 psi 2 psi k minus d is orthonormal. So, we have an orthonormal set in W perp, we would like to know whether it is an orthonormal basis for w is this basis for W perp, for this we have to check whether every vector which is in W perp is a linear combination of this, because it is already orthonormal and hence linearly independent. Therefore, the only requirement to be check for this to form a basis is whether it spans the space. So, for this we need to check if this set spans W perp.

(Refer Slide Time: 37:37)



So, let check that let x be in W perp, if since x is in W perp x phi j, since x is in W perp x phi j must be equal to 0 for j equal to 1 to d. Why is this, because some vector is in W perp if and only if it is orthogonal to all the vectors in the basis for w? Now phi 1 phi 2 phi d forms a basis for w, and therefore x must be orthogonal to phi 1 phi 2 phi d; and that means, the expansion for x in terms of B, remember we had this equation 1 which said that any vector can be expanded of this form. Now, in particular if I have taken the x for which all the x phi j is 0, the only thing I will get is the remaining part so, by 1 by 1 becomes x is equal to summation j equal to 1 to k minus d x psi j psi j, which means x is a linear combination of this psi 1 psi 2, x is a linear combination of psi 1 psi 2 psi k minus d. Hence, psi 1 psi 2 psi k minus d span W perp, so we have seen, but they are in W perp they are orthonormal and now, they span W perp.

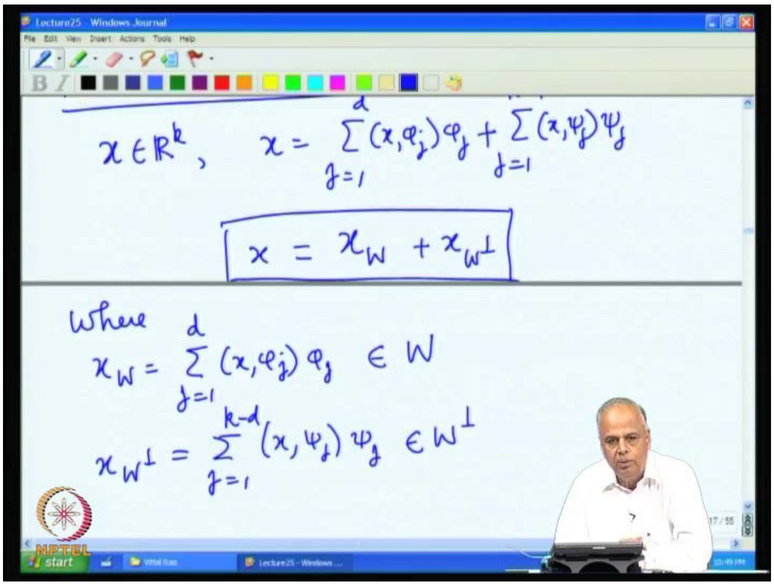
(Refer Slide Time: 39:31)



And hence, they form a basis an orthonormal basis for W perp so, what it says is that if we now start with a basis for w and orthonormal basis for w. Extended to an orthonormal basis for the whole space, the part that is coming in the extension; namely in this case this will automatically be a basis for the orthogonal complement. Now, we get some simply facts from this, what do you get from this as the dimension of W perp. This is a basis for W perp it has k minus d vectors.

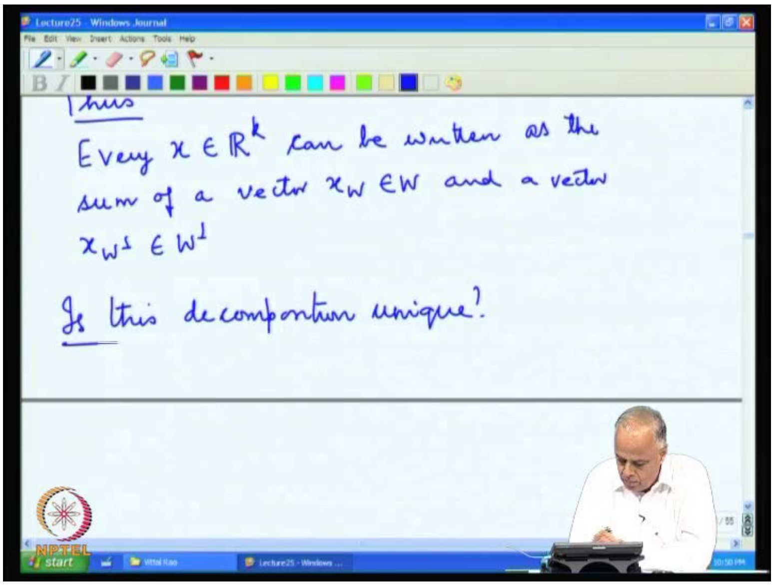
The dimension is precisely the number of vectors in a basis, so dimension of W perp is k minus d, which is dimension of R k in d was the dimension of w, which implies dimension of w plus dimension of W perp is equal to dimension of R k. So, this is a very important fact namely that, if you have any subspace w of R k then, the dimension of w and the dimension of is orthogonal complement add up to the total dimension of the whole space. This is very useful important fact, which we will use shortly.

(Refer Slide Time: 41:26)



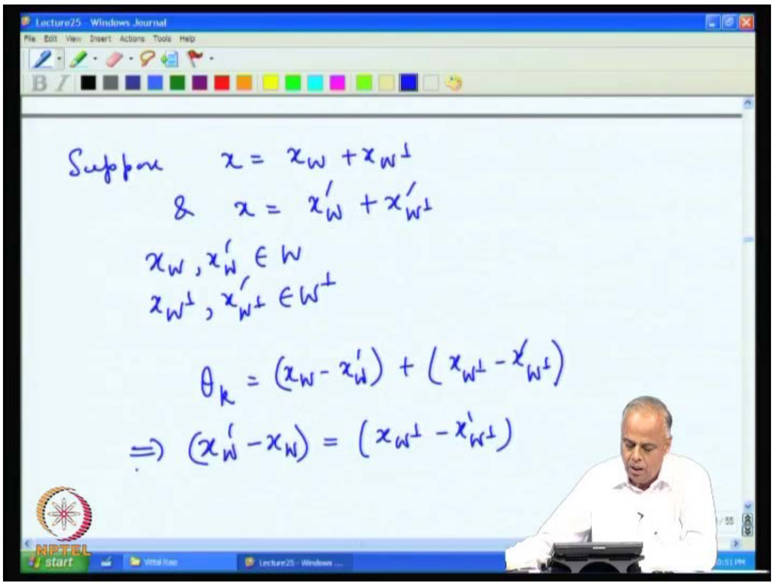
Then let us look at, (( )) the equation 1 which says that x is in R k then x can be expanded as sum equal to j equal to 1 to d x phi j phi j, where this phi 1 phi 2 phi d is an orthonormal basis for w plus j equal to 1 to k minus d x psi j psi j, where this psi 1 psi 2 psi k minus d is an orthonormal basis for W perp. We will call this as x w plus x w perp, where x w is the first sum x w is equal to summation j equal to 1 to d x phi j phi j, and x W perp is summation equal to 1 to k minus d x psi j psi j. Now, observe the since this phi 1 phi 2 phi d are all in w, the x w is a linear combination of the phi 1 phi 2 phi d and hence belongs to w. Similarly, psi 1 psi 2 psi k minus d are in W perp, and therefore this belongs to W perp any linear combination also be W perp. So, what this says is the vector x has been decompose, add this sum of two vectors the first vector in w and the second vector is W perp.

(Refer Slide Time: 43:20)



So, thus the conclusion is every x in R k can be written or decomposed as the sum of a vector x w belong to w, and a vector x W perp belong into W perp. Now, this show that we can decompose it, and to two parts any vector can be decomposed into two parts; one in w and other one in W perp. Now or there many ways of doing this decomposition, if so would like to choose the correct one or the useful one or if there is only one way of doing it, then we denote of much choice,so this decomposition unique.

(Refer Slide Time: 44:34)

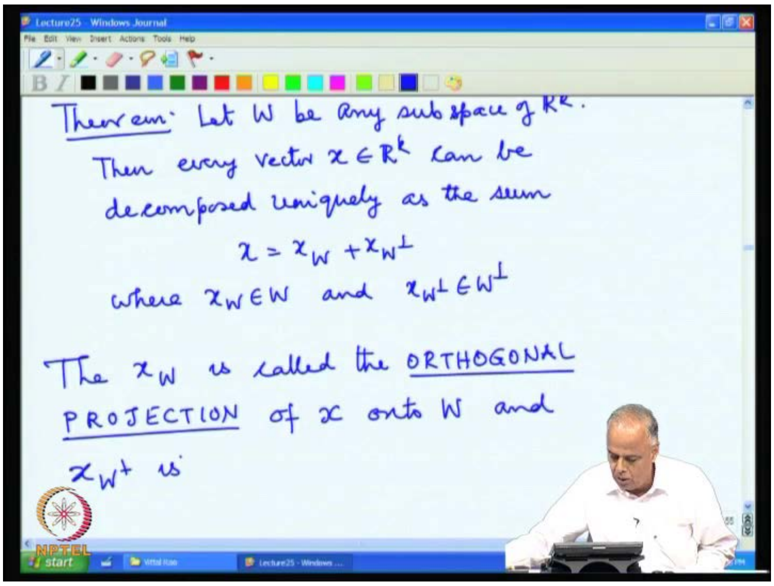


So, suppose we have two decomposition, suppose x equal to x w plus x w perp and x equal to x w prime plus x w prime, where x w x w prime are in w and x w perp and x w perp prime are in w perp. So, we have two such the composition say of a vector x into a w perp, and a w perp part. Then subtracting we get theta k is equal to x w minus x w prime plus x w perp minus x W perp prime or if you take 1 of these terms to the other side, we get x w prime minus x w is equal to x W perp minus x w perp prime. Now, what that says is this vector, and this vector is the same let us call this common vector as z.

Then this says, because z is equal to this, and x w prime is in w x w is in w this vector belongs to w. Similarly, this is in w perp and this is in w perp the difference of two w perp vector is in w perp and therefore, this is in w perp, because w and w perp are subspaces difference of two vectors in w will be w. Difference of two vectors in w perp will be in w; therefore, this say z is if a w vector it is also a w perp vector that says z belongs to w into section w perp.

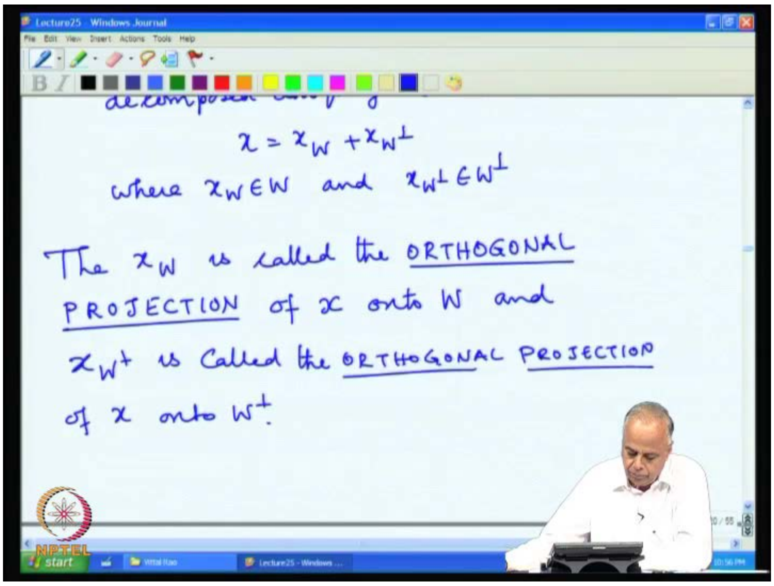
Now, whenever you have w and w perp the intersection can contain only the 0 vector, because if you now look at, z, z treat z as a vector in w perp w, and treat this z as a vector in W perp, we can do that, because that belongs to both w and w perp now, we have an inner product of vector in w and w perp, and that must be 0, but the inner product is the length squared, the length is 0 means z is equal to theta k. And that says x w must be equal to x w prime and x w perp must be equal to x w perp prime. So, not only we can decompose a vector x of the sum of a vector in w and vector in w perp that this decomposition is unique.

(Refer Slide Time: 47:26)



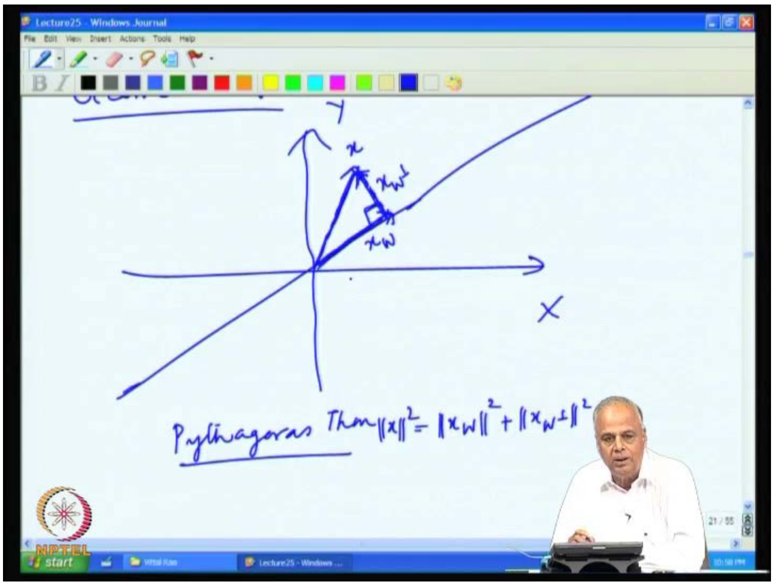
So, what do we get let w be any subspace of R k. Then every vector x in R k can be decomposed uniquely as the sum x is equal to x w plus x w perp, where x w belongs to w and x w perp belongs to w perp. There is a unique decomposition of every vector with respect to every subspace, this old decomposition depends on the subspace w with which you start, so moment you give me a subspace, there is a breaking of the vector into two parts one from w and one from w perp the x w is now, I consider the x w, because the whole decomposition is unique.

(Refer Slide Time: 49:31)



The x w is called the orthogonal projection of x onto W, and x w perp is called the orthogonal projection of x onto W perp, we see there if we know one orthogonal projection suppose, we know x w we can get x W perp as x minus x w, and an alternately, if we know the projection x w perp then we can get x w as x minus x w. So, that is we have the notion of the orthogonal projection of this.

(Refer Slide Time: 50:03)

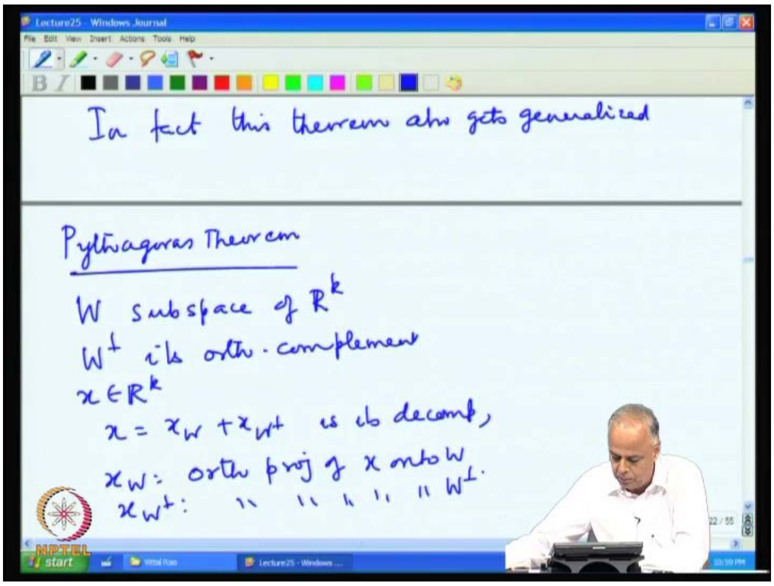


Let us look at what this says geometrically what this does is the generalization of the following fact that, we know into dimensions let us say 2 dimensions. Let us take this x and y axis suppose, what is as the notion of a subspace is the line passing through the origin. So, let us say this is the subspace, a line passing through the origin. Take any vector x, now draw the perpendicular from x to this. Now, this vector is what now, x w is and this vector is x W perp and what it says we know, but the resolution of force is that force x is equal to the velocity or force, whichever vector you want to use, if the sum of these two vectors, and this is what is now generalize in the set up of R k the line is replace by a subspace w, and this orthogonal dropping is the orthogonal projection, and their difference is the projection on to the other space.

So, this is the generalization of the simple projection idea, which we have been geometry. Now, in geometry when we have such a projection idea this angle being on orthogonal ninety degrees, we know by Pythagoras theorem, the length of the hypotenuse squared is equal to sum of the length of the squares of the other 2 sides.

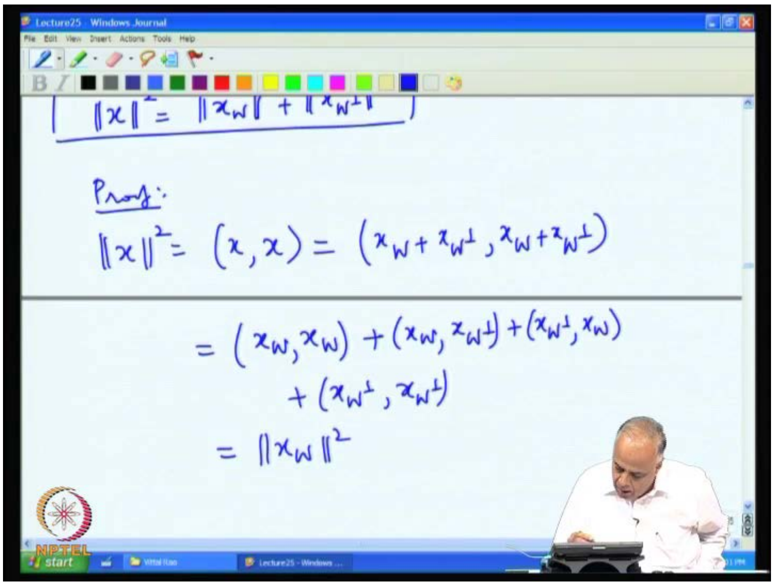
So, the Pythagoras theorem says in r 2, but the length of this vector x w squared plus the length of the vector x w perp squared is the length of hypotenuse squared. Now, since we have generalized the projection idea can be generalize this Pythagoras theorem also and in fact this is true.

(Refer Slide Time: 52:15)



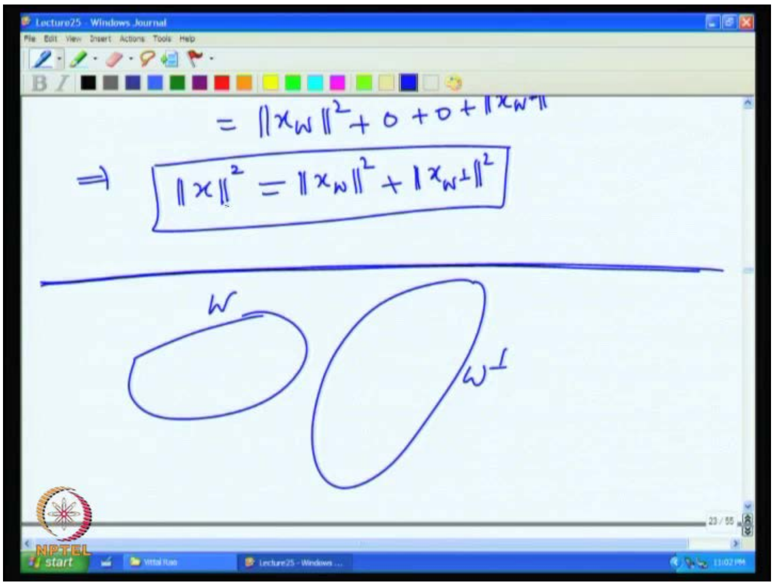
In fact this Pythagoras theorem, this theorem also gets generalized, so we have this Pythagoras theorem in the set up of R k is general version of the Pythagoras theorem. So, w subspace of R k, W perp its orthogonal complement x belongs to R k; x is equal to x w plus x w perp is it is decomposition, where x w is the orthogonal projection of x onto w and x w perp is the orthogonal projection of x onto W perp, this x belongs to w x w and x W perp belongs to W perp.

(Refer Slide Time: 53:26)



Now, let us look at the length of x squared, so and then length of x squared is equal to the length of w x w squared plus the length of x w perp squared. This is what the Pythagoras theorem is lets proof that, so very simple proof the length of any vector is given by the dot product of the vector with itself, but now we are given that x is x w plus x w perp. This will be writing like that, and the dot product is distributive, so we can take out the individual products this will be x w comma dot product x w plus x w dot product with x w perp plus x w perp dot product with x w plus x W perp dot product with x W perp the first term is the length of x W squared, because the dot product of any vector with itself is the length squared and the second term is 0, decause x W and x W perp are in orthogonal spaces and similarly, x W belongs to W and x W perp belongs to W perp so, that is 0and that gives me the length of x W perp squared.

(Refer Slide Time: 54:57)



And hence, we get the Pythagoras theorem, length of x squared is equal to length of x square is equal to length of w squared plus length of x w perp squared. So now, if look at this theorem that, any vector can be decomposed as x w plus x w perp. Now, we can add and also norm x squared is equal to norm of x w of norm of x w perp squared. So we have, what are all the things that, we have seen you have w and you have w perp, and everything can be decomposed in terms of these two. We can decompose a vector, we can decompose the basis into two parts one from w, and one from w perp, and we can decompose the vector and it is length into two parts. It is this sort of decompositions that, we will be using in our analysis. We shall look at an example, and see how would use it in your matrix context in the next lecture.