Advanced Matrix Theory and
Linear Algebra for Engineers
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Lecture No. # 17
Linear Transformations- Part 1

In the last lecture, we saw that in a finite dimensional space, we can use an ordered basis to convert or encoded digitize every vector in to a concrete vector in F n.

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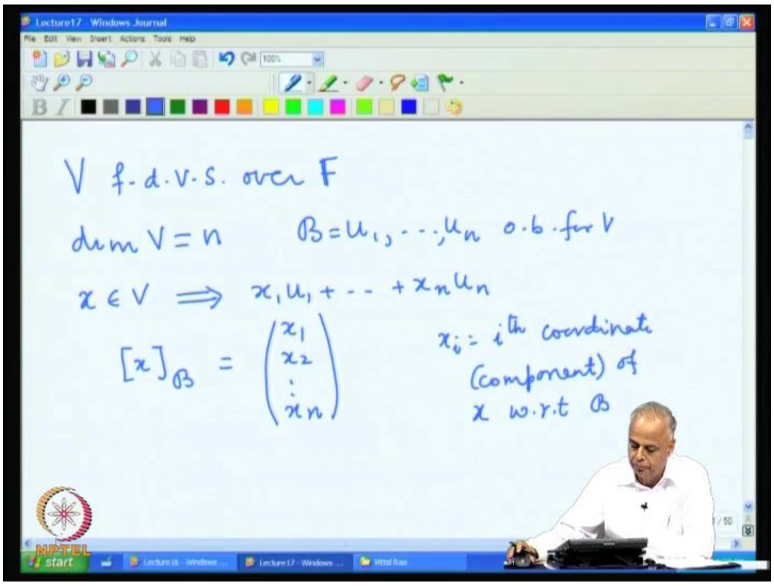

So, we have V a finite dimensional vector space over F, dimension of V is n, then x belongs to V can be digitize. For this, we need some digital equipment and that equipment is our basis.

So, B equal to u 1, u 2, u n and ordered basis for V, then every x in V has a unique representation as a linear combination of this basis vectors then we construct the vector in F n and call it as x B and that is this vector x 1 x 2 x n and x i is call the i th co ordinate or component i th component of x with respect to the ordered basis B.

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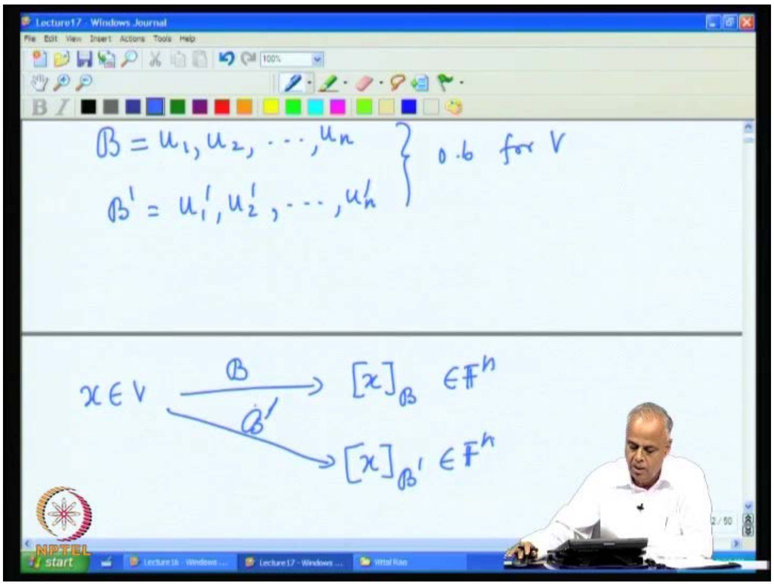

Now, we have seen suppose, we have two ordered basis B equal to u 1, u 2, u n and B prime is u 1 prime, u 2 prime and u n prime so both are ordered basis for V. Then, if we take a vector x in V using the ordered basis B, we can converted to a vector x B in F n. On the other hand, if you use B 1 or B prime; if you use B prime then we can convert the vector x in V to the vector x B prime in F n; if you now look at these two vectors x B and x B prime these are two different vectors in F n, but they are all both emerged from the same original vector x in V, and therefore we expect that since, they represent the same vector there must be some connection; there must be some relationship between x B and x B prime.

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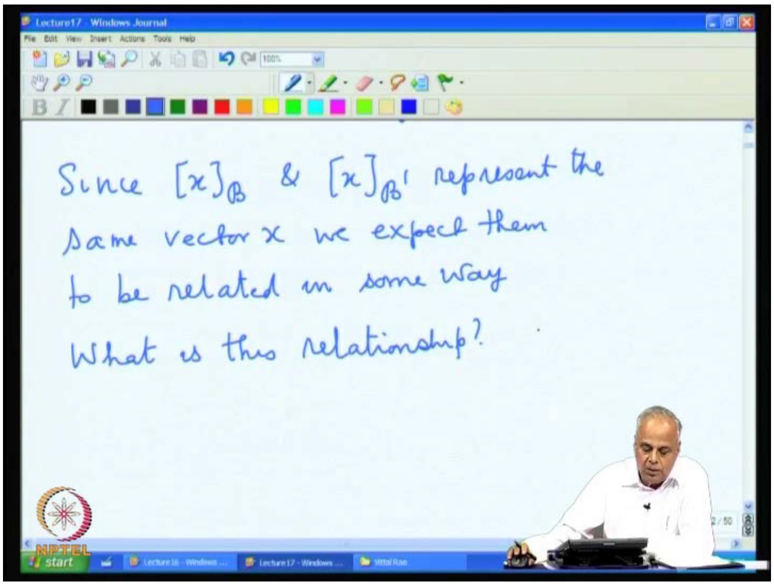

Since, x B and x B prime represent the same vector x, we expect them to be related, in some way. The algebra it is relation, because everything is algebra here, so what is this relationship between these two represent, so if you have the same vector represented in two different ordered basis, the two representations must some of the connected with each other, because they come from the same original vector. We are trying to explore and find out, what is this connection between this two? Obviously, the connection must come from the connection between the basis B and basis B prime, because these are the two fellows taking them in two vectors so, the connection between B and B prime will establish the connection between x B and x B prime. So, we must now look at the connection between x B and B prime.

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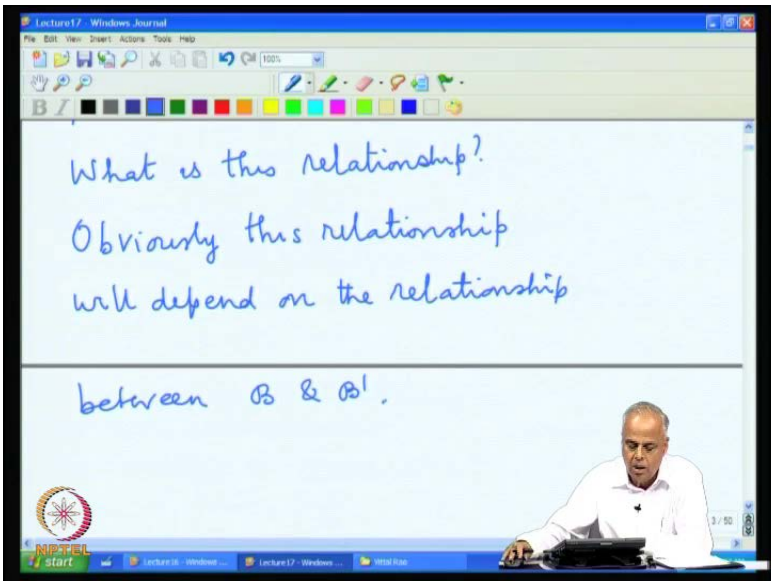

So, we write obviously, this connection; this relationship will depend on the relationship between between the two ordered basis B and B prime. Now therefore, we must ask, how do we find this relationship? How do we find this relationship.

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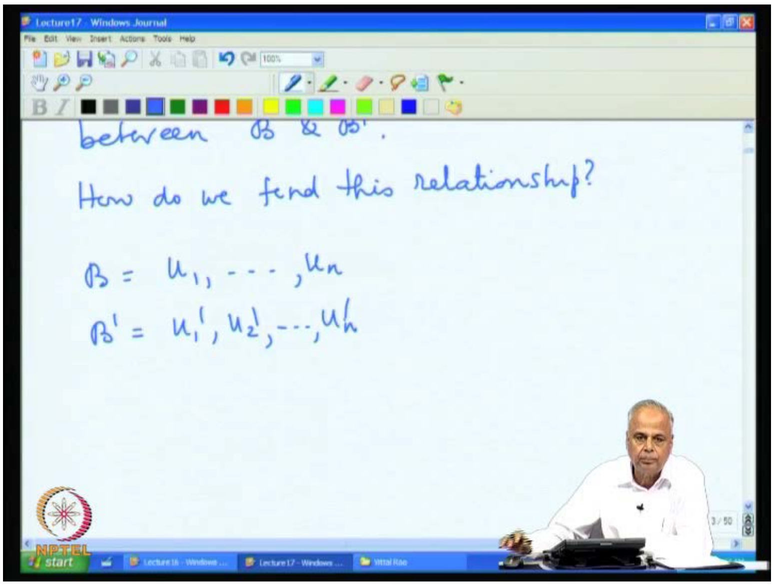

So, let us again look at this two basis B is u 1, u 2, u n. B prime is u 1 prime, u 2 prime, u n prime.

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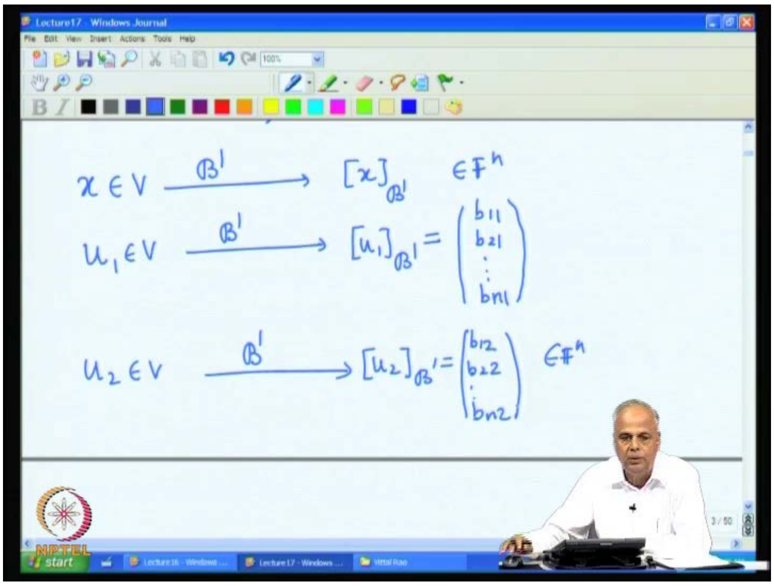

Now, we have seen that any vector x in V can be converted to a vector in F n, through the basis B prime has x B prime that connection is what will, that conversion is what we use denoted by x B prime? The digitize version of the encoded version in F n. Now, we now apply this transformation or this digitization using the basis B prime for the B vectors so, in other words I can take x to be u 1 u 2 certainly in B.

So, I can use B prime; I should get a vector in F n which is representing u 1 in the basis B prime since; this is the vector in F n. Let us, denoted by b 1 b 2 b n and in ordered to make sure that, we do not forget that, we are representing the first vector in the B basis we will put in additional index their one the second index denoting that we are looking at the representation of the first vector in the B basis. Similarly, if you now take u 2, that can be represented in the B prime basis as a vector u 2 B prime in F n and that vector again, we represent it as b 1 b 2 b n there be a second index 2 in order to ensure, that we are taking about the second base vector.

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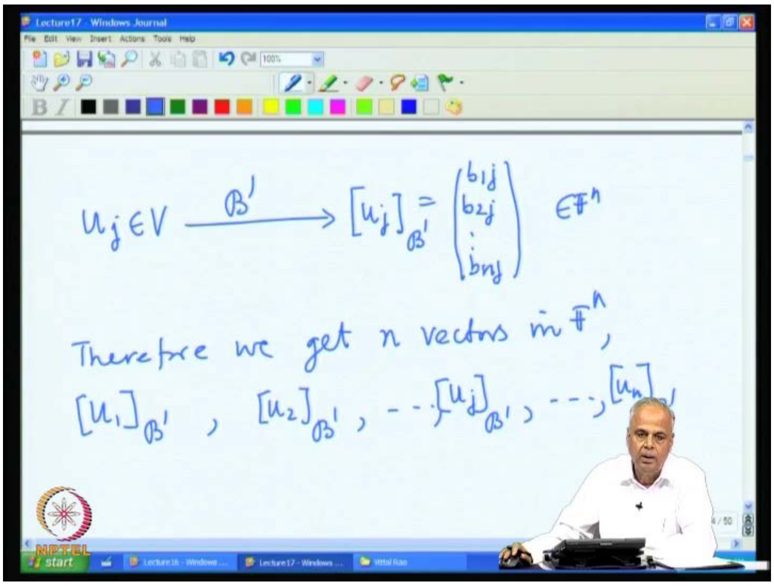

In general, if you take u j the j th vector in the B basis, you can use the B prime basis to represent it as u j B prime as a vector in F n and that vectors component are b 1 b 2 b n and the second index j says, that we are looking at the j th vector in the we are looking at the jth vector in the B basis.

So, what we have done is, we are looked at the basis B and each vector in the basis B is now converted in to an F n vector using the B prime basis. Now we have, how many vectors in F n? There is one vector in F n corresponding to u 1; there is one vector in F n corresponding to u 2 and there is one vector corresponding to u j for every j between 1 and n. Therefore, we get n vectors in F n, what are these vectors? The representation of u 1 in the B prime basis; the representation of u 2 using the B prime basis and so on; the representation of u j using B prime basis and so on; the representation using u n on the B prime basis.

So, thus we get n vectors in F n, we put them all together in the form of a matrix. The first column of which is the representation of u 1 in B prime basis; the second column of which is the representation of u 2 in B prime basis and so on.

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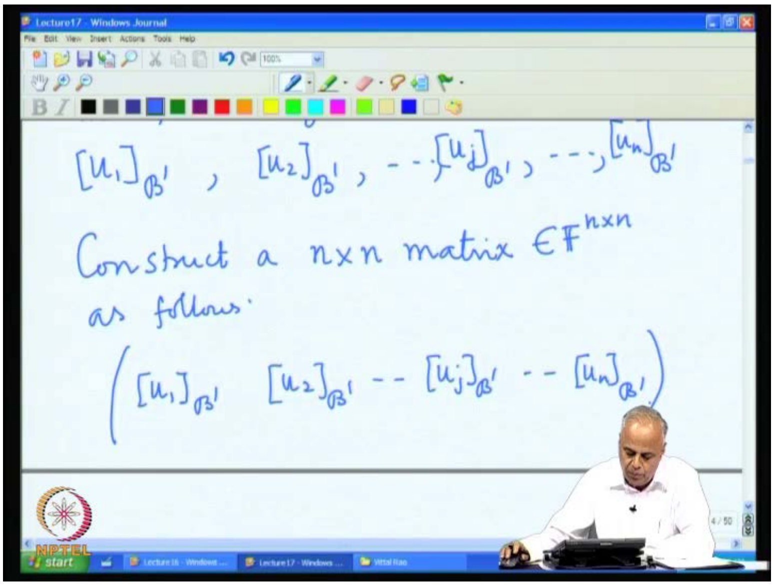

Now, we construct a matrix and it have n columns and each columns is in F n so, it have n rows, it will be an n by n matrix. Since, all the entries are from F, it will be an n by n matrix in F n cross n as follows. It is first column is u 1 B prime; it is second column is u 2 B prime; it is j th column is u j B prime and so on, is nth column is u n B prime.

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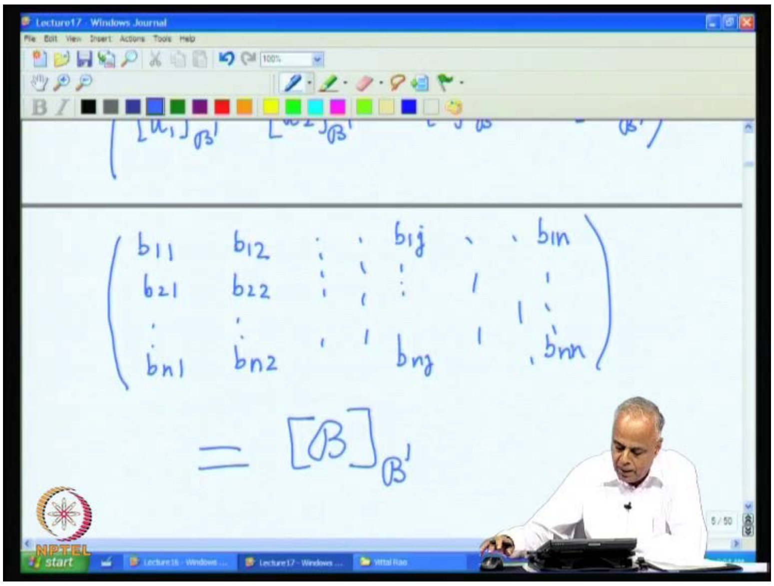

Now, if you look at the components we have already got, we got the u ones components as b 1 b 2 b n; we put the second index is 1 to say that, we are looking at the first vector. Then, we got the components of u 2 as b 1 b 2 b n and we use the second index 2 to say that, we are looking at the vector u 2 and so on and so far and when you come to u j we have b 1 b 2 b n and the second index j and so on.

And finally b 1 b 2 b n and the second index n. So, this is the n by n matrix that we construct. These are all the components of the B basis vector, in terms of the B prime basis vector. Let us call this matrix as; it is B basis vectors in terms of the B prime basis vectors. You will use this notation B sub B prime so, it is representation of the B basis in terms of the B prime basis.

How does this help? At least, this gives you a compact digitize version of the relationship of the B vectors in terms of the B prime vectors. This is what we are looking for? We were saying that the two representations of a given vector in terms of the B and B prime basis will obviously depend on the relationship between the B and B prime basis. Now we are captured in some form the relationship between B and B prime basis.

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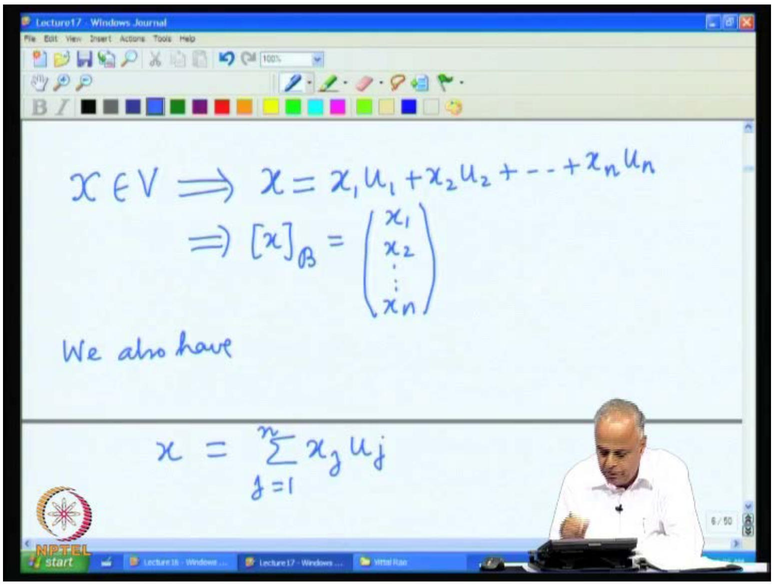

Now, let us take any vector x in B so, consider a vector x in V. We know, that any vector x in V as a unique representation in terms of the basis vectors. Let us represent x in terms of the B basis, it is x 1 u 1 plus x 2 u 2 plus x n u n. That is, the representation of x in terms of the B basis, this means when we digitize or we finding corresponding vector x, the coded version in F n that has to be formed out of the components so, it will be x 1 x 2 x n. On the other hand, we also have x if, we write this in simple to save time or to compact notation, we will write it as x j u j.

Now we have seen that, the u j is precisely having this representation in the B prime basis, that means u j is b 1 j u 1 prime b 2 j u 2 prime b n j u j prime.

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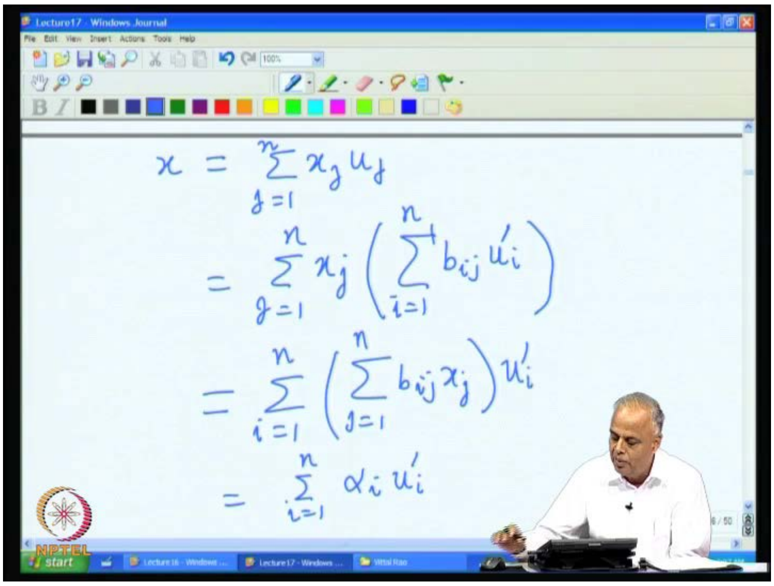

So this will tell me, that is equal to summation j equal to 1 to n x j u j can be written as i equal to 1 to n b i j u i prime, because now we are representing in the B prime basis. The B basis vectors in terms of the B prime basis. There is a double summation and everything is finite sum so, you can rearrange the summation and write it as i equal to 1 to n summation j equal to 1 to n b i j x j into u i prime. So, I can write this as i equal to 1 to n some alpha i times u i prime. So that says, x can be express as linear combination of the B prime basis vector with the co efficient of alpha i.

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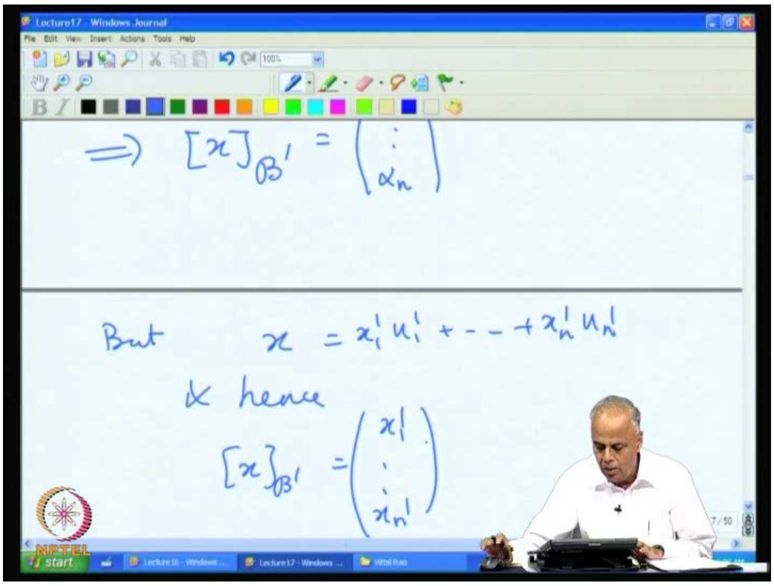

So therefore, the representation of x in the B prime basis must be alpha 1 alpha 2 alpha n. But we know, we have represented it as x 1 prime u 1 prime plus x n prime u n prime I am sorry x as we can rep usual use the notation x is equal to this and hence, x B prime must be x 1 prime x n prime. On the one hand, we have x B prime as alpha 1 alpha 2 alpha n and the other hand we have x B prime as x 1 prime x 2 prime x n prime.

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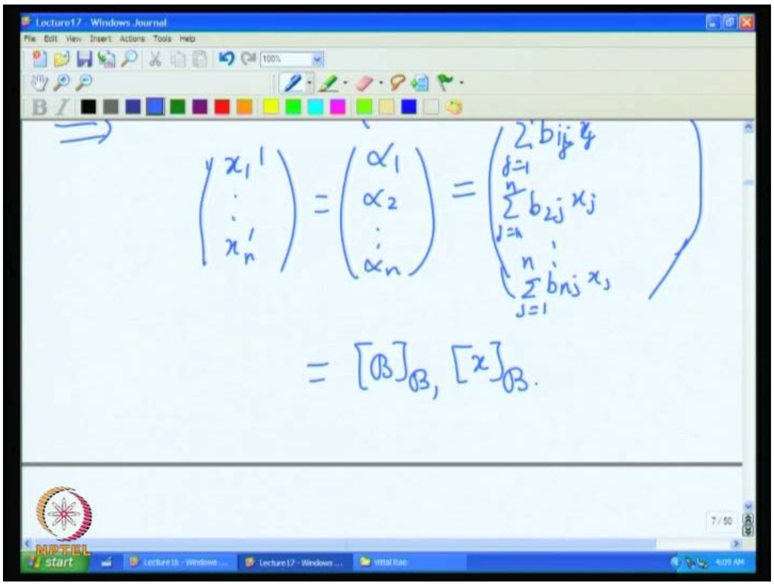

And that says, that the vector alpha 1 or the vector is, let us put this side the vector x 1 x 2 x n prime must be the same as the vector alpha 1 alpha 2 alpha n. So, comparing this representation here and comparing with this representation here, we get this x 1 prime x 2 prime x n prime must be alpha 1 alpha 2 alpha 3, but now, if you look at alpha 1 alpha 2 alpha 3 if you look at alpha 1 alpha 2 alpha 3, we we call this entire sum as alpha i and therefore, we get this is the same as summation; if you look at this it will be t b 1 1. Let it as put it this way b 1 i x b 1 j x j j equal to 1 to n b 2 j x j j equal to 1 to n and so on. Summation j equal to 1 to n b n j x j with with is simply the product of the matrix B B 1 and the x B.

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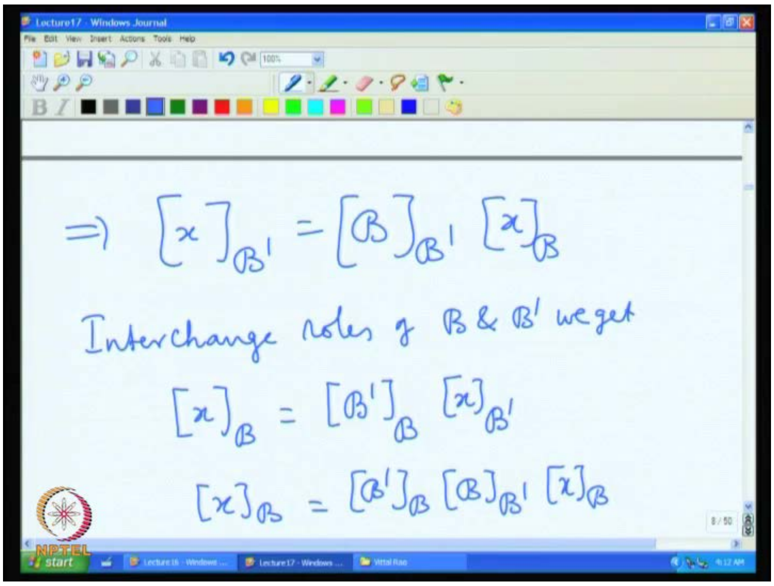

So thus, we get x B prime is B B prime times x B, this gives the connection between the representation of this vector x in terms of the basis B prime and the representation of the same vector x in terms of the basis B and the connection is through this matrix, which the compact form of representing the relationship between the basis B in terms of the basis B prime. So, thus we get x B prime is equal to B B prime, x B which is the connection between x B prime and x B which we were looking for. Similarly, if interchange if the role of B and B prime interchange roles of B and B prime we get, x B will be now, we have to represent the B prime basis in times of the B basis and here B x B prime.

So, we can knowing the representation of x in terms of the basis B, you can get the representation of x in terms of the basis B prime by the connection between B and B prime and knowing the representation of x, in terms of the basis B prime we can get the representation of x in terms of the basis B by connecting B prime with the B basis. Combining these two results, if you now subs if substitute, for x B prime from here we get this is B B prime in to x B.

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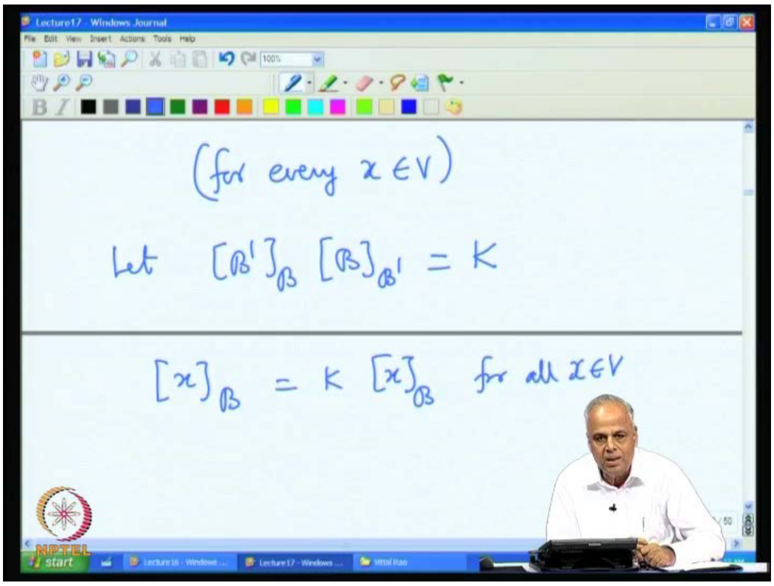

So therefore, what we get is x B is equal to B prime B in to B B prime in to x B and this is true for every x belong to V. Now, we leave it as an exercise to see that, take let us make it as simpler, call this matrix let B prime B remember this is a n by n matrix times B B prime. Which is also a n by n matrix call this matrix as K then, we have x B is equal to K x B, for all x in B. If you take any x in V and look at this representation in the B basis, it is the same as K times it is representation in the B basis and we expect therefore K identity.

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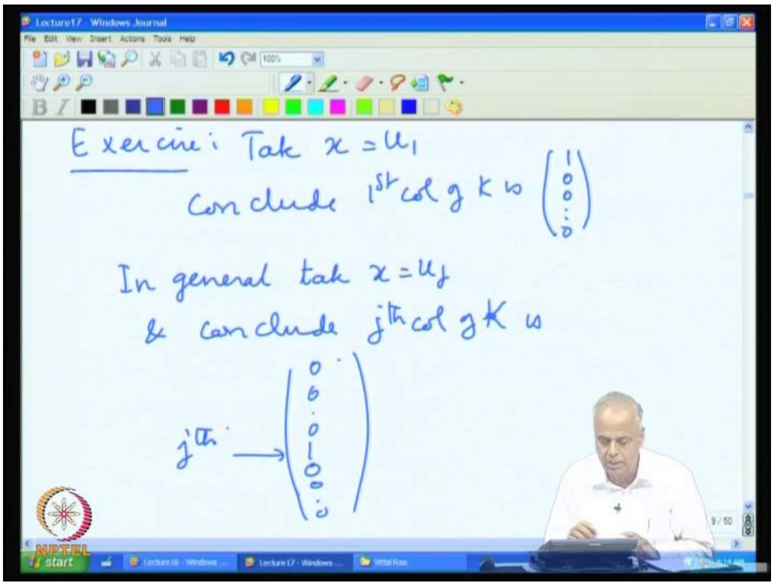

So exercise, take x equal to u 1 conclude first column of K is 1 0 0 0 0 and in general, take x equal to u j and conclude j th column of K is the mate the column 0 0 extra until, you come to the j th states and all are 0. This is the j th entry; the jth entry is for the simply means the columns of K or the same as the column of the identity matrix.

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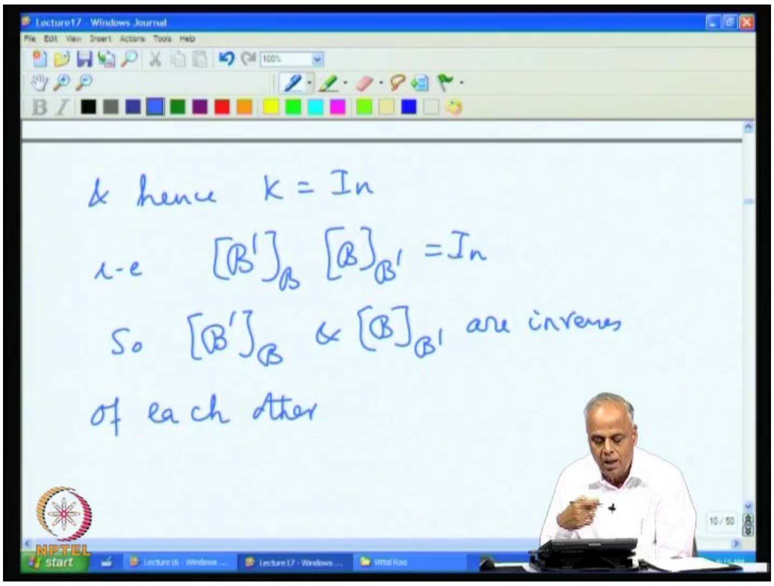

And hence, K equal to I n that is, B prime B in to B B prime is equal to identity. So, the representation of B prime in terms of B and the representation of B in terms of B prime are inverses of each other. To get, the representation converted from B language to the B prime language, you represent the B basis vectors in terms of the B prime basis vectors. to rec convert the information in the B prime language to the B language represent the B prime basis vectors in terms of the B basis vectors.

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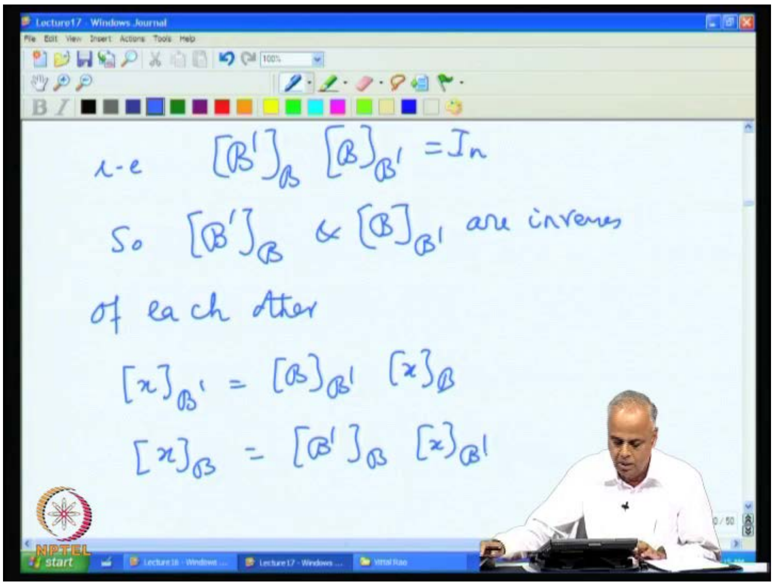

Once, we have this remember x B prime is B B prime in to x B and x B is B prime B in to x B prime.

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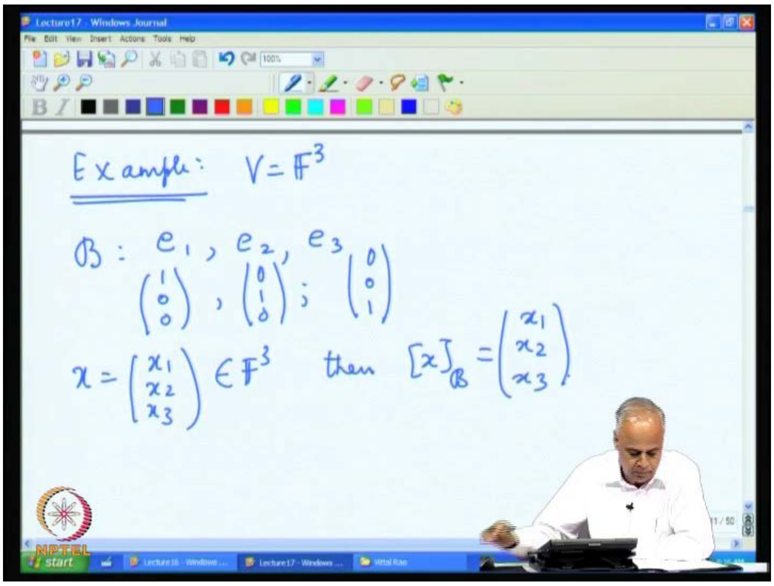

Let us look at a simple example; Let us look at the space V equal to F 3; let us first take a basis B which is the usual ordered basis be always start with, which is 1 0 0, 0 1 0 and 0

0 0 1 and then, if you take any vector x x 1 x 2 x 3 in F 3 then, it is representation in the B basis is just x 1 x 2 x 3.

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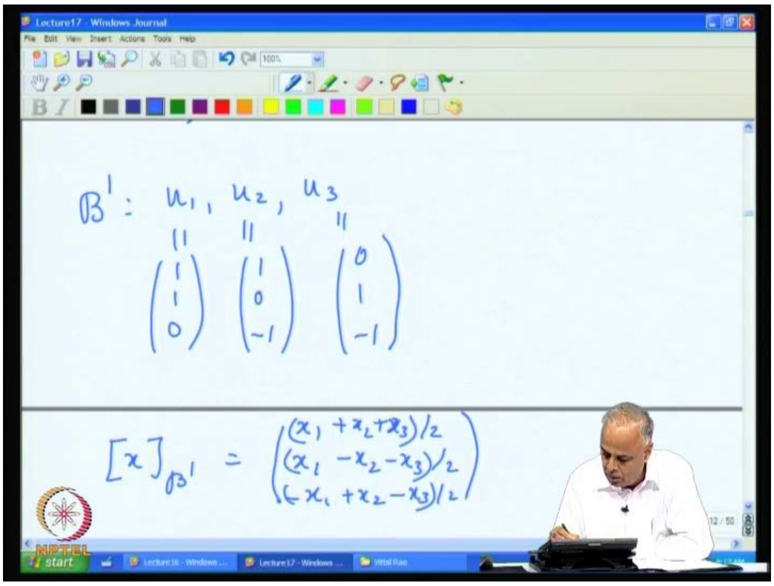

Let us now, will take a new basis B prime which is u 1, u 2, u 3 where the u 1 is 1 1 0 u 2 is 1 0 minus 1 u 3 is 0 1 minus 1 then, we have seen that the representation of x in the B prime basis is x 1 plus x 2 plus x 3 by 2 x 1 minus x 2 minus x 3 by 2 and minus x 1 plus x 2 minus x 3 by 2 this is, we found out in the last lecture.

So, we have the two representations now, how are they connected to look at the connection? We must represent V in terms of B prime. Let us look at the connection between the basis B and basis B prime.

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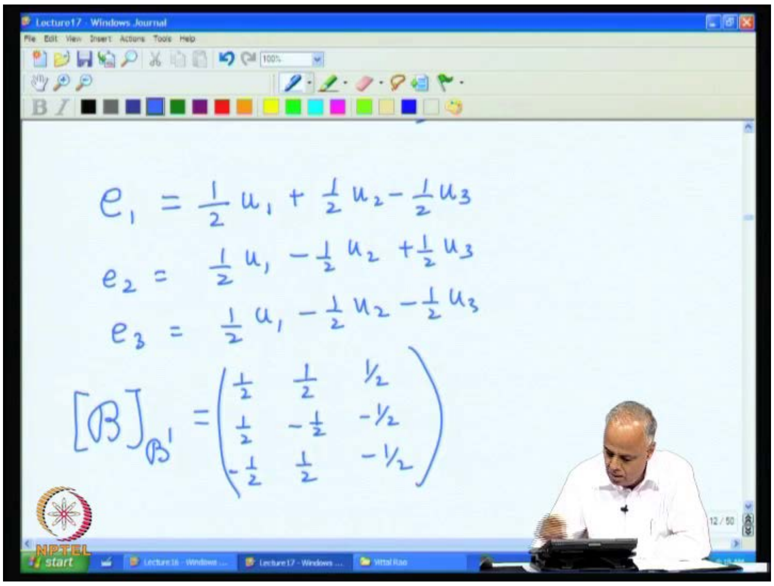

Now for that, we look at the first vector in the B basis and try to represent it in terms of the B prime basis, it turns out. We can see easily that half u 1 plus half u 2 minus half u 3 is e 1 similarly, e 2 is half u 1 minus half u 2 plus half u 3 and e 3 is half u 1 minus half u 2 minus half u 3. Now, how do construct the matrix B in the B prime basis? For that, I take the first basis vector in the B basis, represented in terms B prime and put those components as the first column. The components now are, 1 half half and minus half similarly, we look at the second basis vector e 2 it is representation in the B prime basis and put the components as the second column and finally, we look at the third basis vector and look at it is component and put it as the third column, we have half minus half and minus half.

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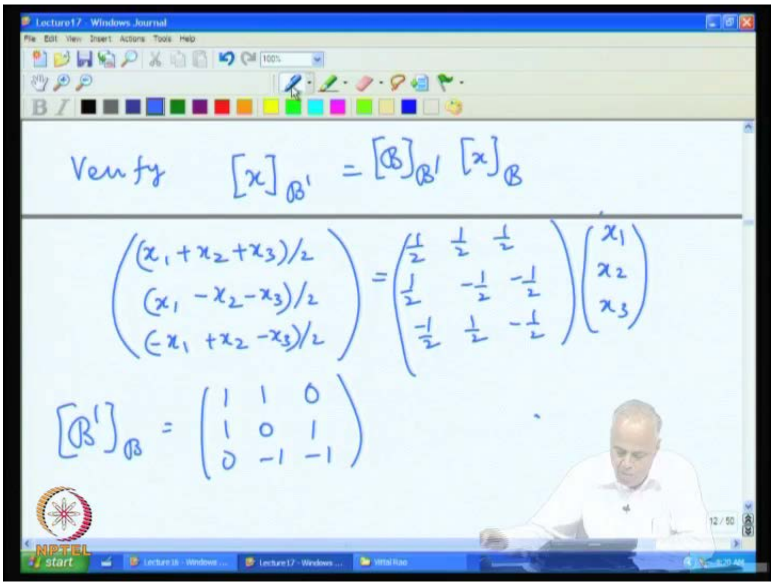

Now it is verify, we want to construct x B prime from the known information of x B, we must have B B prime. What is that mean? In this case, remember this was x 1 plus x 2 plus x 3 by 2 x 1 minus x 2 minus x 3 by 2 minus x 1 plus x 2 minus x 3 by 2 this must be equal to the B matrix into x B is just one x 2 x 3. It is now easy to verify that this wholes. Just, it is now a matrix multiplication verification and this is precisely, what you mean and this case, what is B prime in B? That is very easy, it is just 1 1 0 1 0 minus 1 0 1 minus 1, because we want to represent the B ee u 1 u 2 u 3 we want to represent u 1 u 2 u 3 in terms of e 1 e 2 e 3, which is a very simply straight forward competition.

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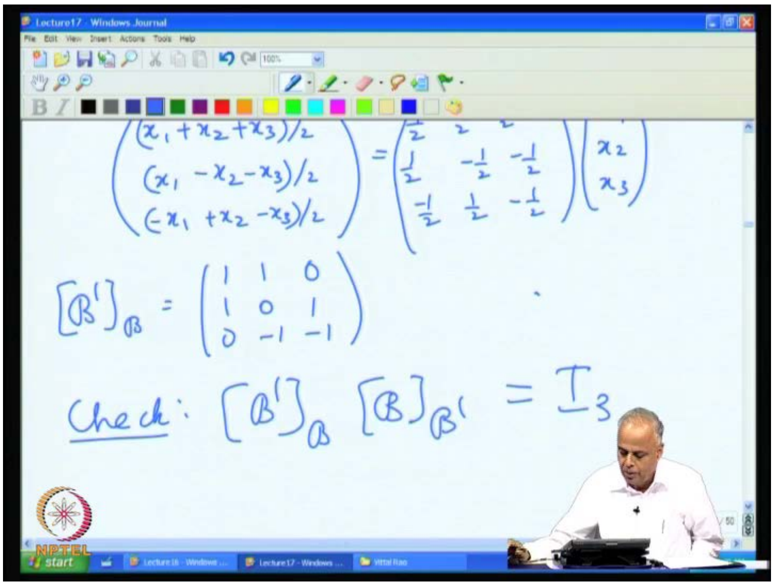

And therefore, we get B prime B as this, now all we have to do this check that, this B prime B we have got times B B prime that we have above gives as the identity matrix, this is to check this it just a matrix multiplication and we can verify and then, this is true. Thus, in order to look at the connection between two representations coming out of two different basis, we must always look at the connection between the two basis. If you want to convert from the B language to the B prime language, you should convert the basis vectors in B language in to the basis vectors in the B prime language. Similarly, if you want to convert from B prime language to B prime language, you must convert the basis vectors in B prime to the B language and that is why? That is how; we construct these matrix for the transformation. So therefore, we have the connection between these two represents representations neatly contained in a simple matrix, which stores this information about the connection between the two ordered basis.

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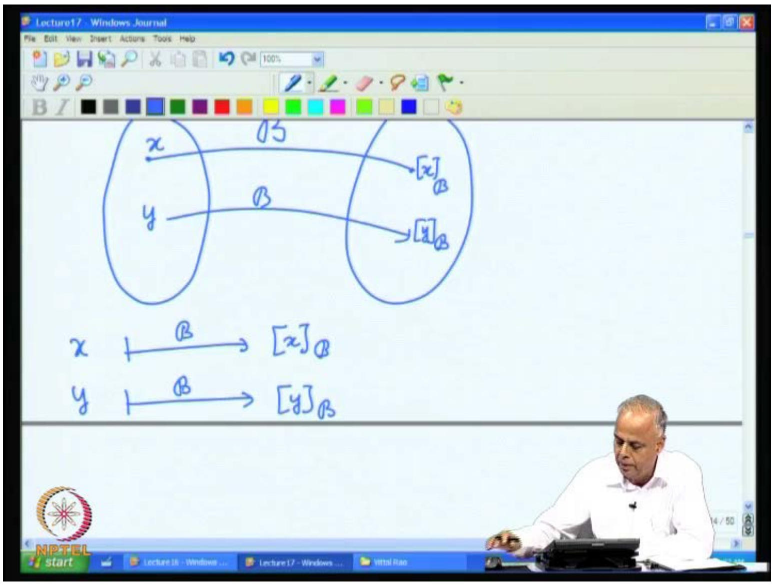

Now let us, have a further look; a closer look at the representation x B. We seems to be a very useful representation, because it helps us to convert any vector and the vector space V in to a concrete digitized form as x B. So, what we have is? Here is the vector space V whose dimension is n and here is F n so, V is the vector space is dimension V is n and we have F n so, if you choose in a ordered basis B and V for V and take a vector x it gets, converted to a vector x B through a basis B.

The moment have an ordered basis B for V, the basis the any vector in V gets converted to a vector in x x B in F n. Suppose, we take two vectors then, why will get converted to y B? So, we have this conversion of every vector in v into a vector in F n or you look at it x has been encode as a x B, y has been encoded as x B so, x B has been encoded as x B and the encoding operators is the basis ordered basis B and y has been encoded as y B. So, we have these two vectors.

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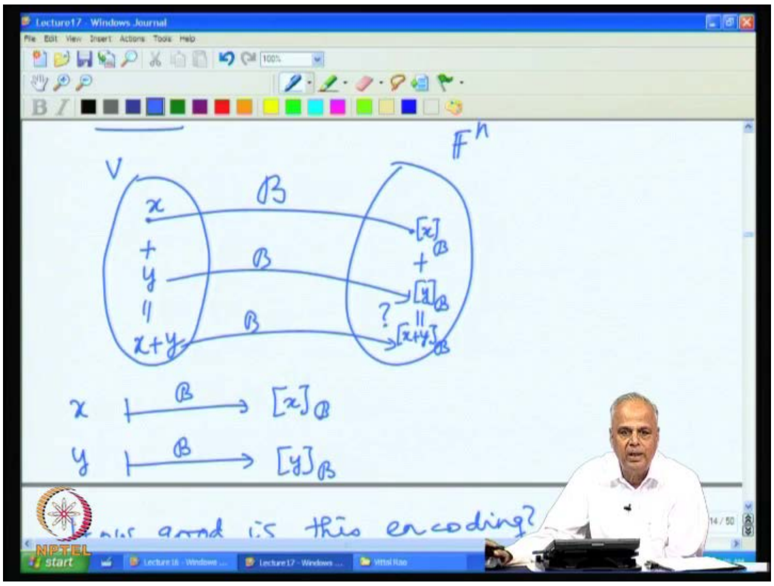

Now, how good is this encoding? How good is this encoding Now, we shall investigate impact, we do not you a know, what you mean by good? What is mean by good? But, let us look at this question carefully now, we have encoded the vector x in to x B; we have encoded the vector y in to y B. Now, in the vector space V this vector x and this vector is y can be added to get a vector x plus 1. In the vector space V, the vector x and the vector y can be added to get a vector x plus y.

Now since, x plus y vector in V that can be encoded as x plus y B in the space F n. Now we have done the addition on the V side suppose, we do the addition after encoding x. We did the vector x and the vector y and then, added and then encoded. Instead, we encode the x; we encode the y and then add, will I get the same? That is the question. In other words, if you add and encode or encode and add, you will get the same answer. If so it is good; if you get the same answer it is good, because then we know that the digitized version preserve additions.

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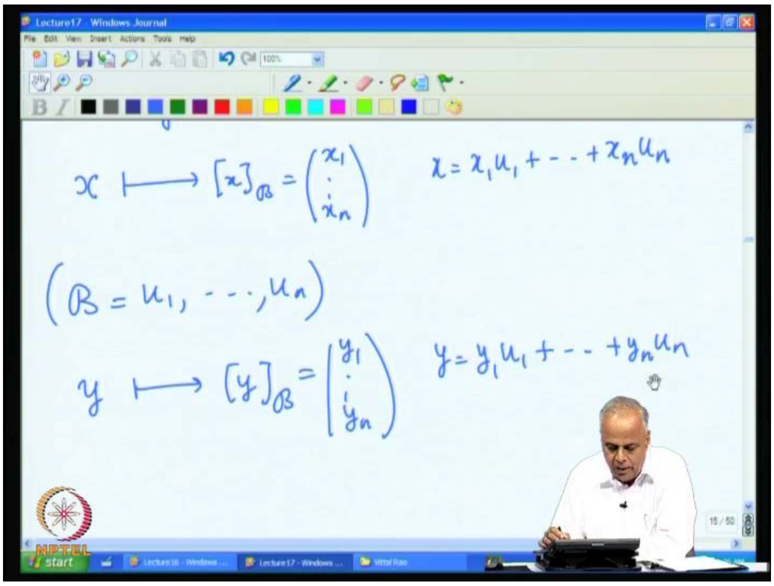

if you can add this side you can add the digitized version also. Let us look at this, suppose x gets coded as x B x 1 x 2 x n this means, x is equal to x 1 u 1 plus x n u n where u 1 u 2 u n is the ordered basis B. So, let us we collect B will always denote by u 1, u 2, u n. So, B is ordered basis for v x is x goes to x B means x is equal to x 1 u 1 x 2 u 2. Similarly, y goes to y B or y is coded as y B which is y 1 y 2 y n this means y equal to y 1 u 1 plus etcetera y n u n. Now, if x is now we are doing the addition in the V side, this is the vector in V. This is the vector in V.

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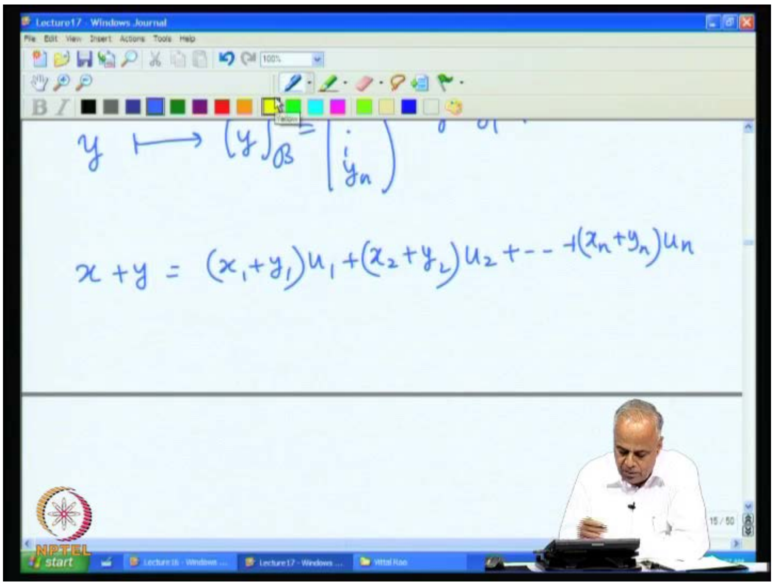

So we add, what do we get? We get x plus y is the x 1 plus y 1 in to u 1 plus x 2 plus y 2 in to u 2 and so on plus x n plus y n into u n that means, we have represented the vector x plus y as the linear combination of u 1 u 2 u n, the moment will represent as the linear combination the coefficients are one that are go that is going to determine in the digitization.

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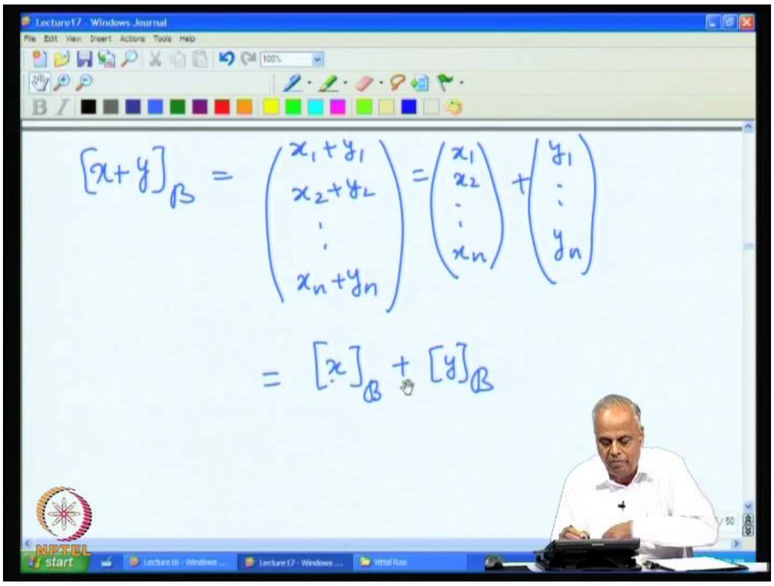

So x plus y, it is representation must be x 1 plus y 1 x 2 plus y 2 and so on x n plus y n. But, this is x 1 x 2 x n plus y 1 y 2 y n by the addition role in F n but, that is the same as x B the first one is x B and second is y B. So therefore, what we get is, if you add and then represent it is the same as represent and then add. So, add and represent or represent individually and then add Notice, this plus refers to plus in V; this ref plus refers to plus in F n.

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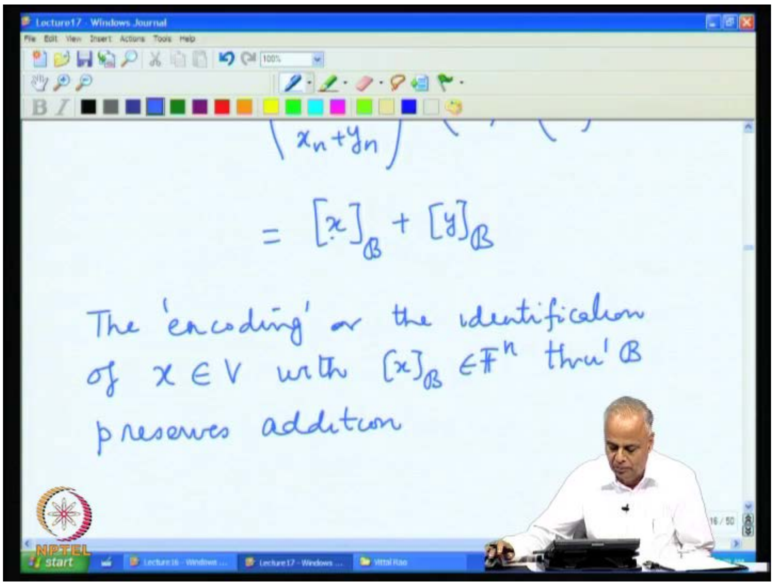

So we have therefore, that the encoding or digitization or the identification of x in V with x B in F n through the ordered basis B preserves addition. That is, you can add and identify or identify and adder these two are computing operations.

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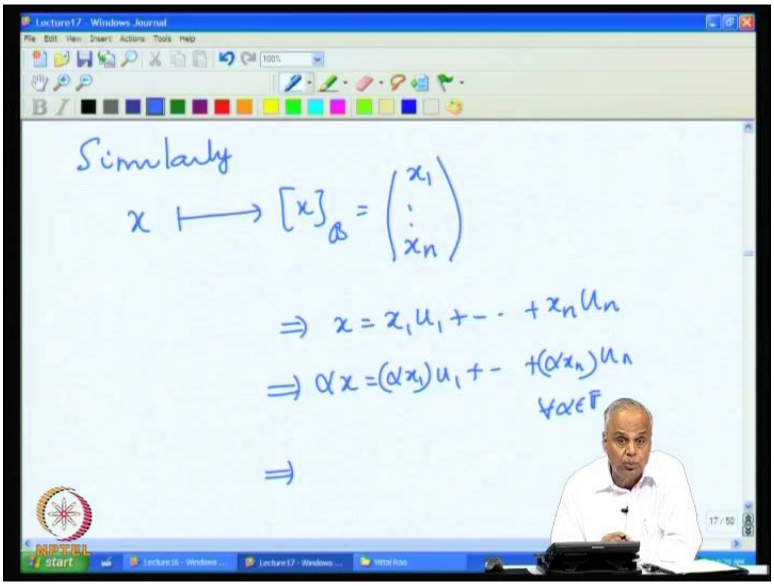

Similarly, if you identify x with x B which is x 1 x 2 x n then, that means x is x 1 u 1 plus x n u n; that means for any alpha, alpha x will be alpha x 1 u 1 plus alpha x n u n for every alpha in F n. Which means alpha x the scalar multiple of x with the scalar alpha has is linear combination representation in terms of u 1 u 2 u n.

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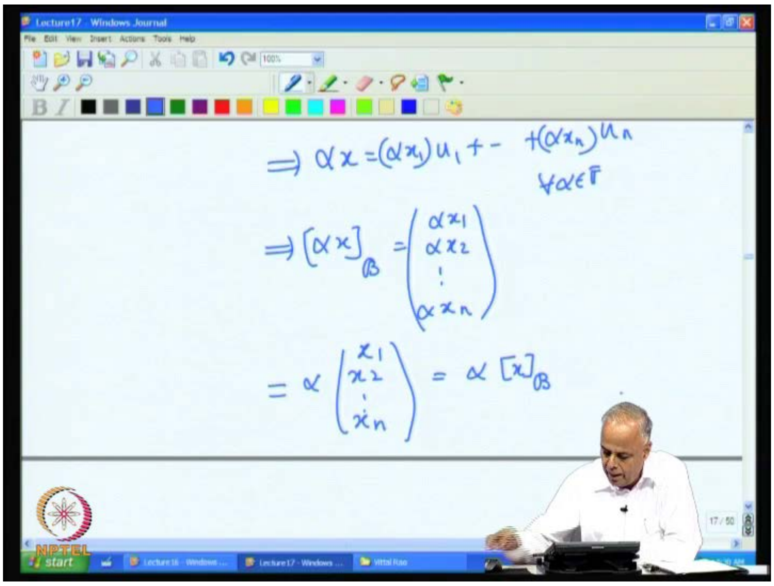

Now, the coefficients will be the once it be used to get representation and hence, alpha x B will be alpha x 1 alpha x 2 alpha x n. That is, indeed equal to alpha times x 1 x 2 x n which is equal to alpha x B.

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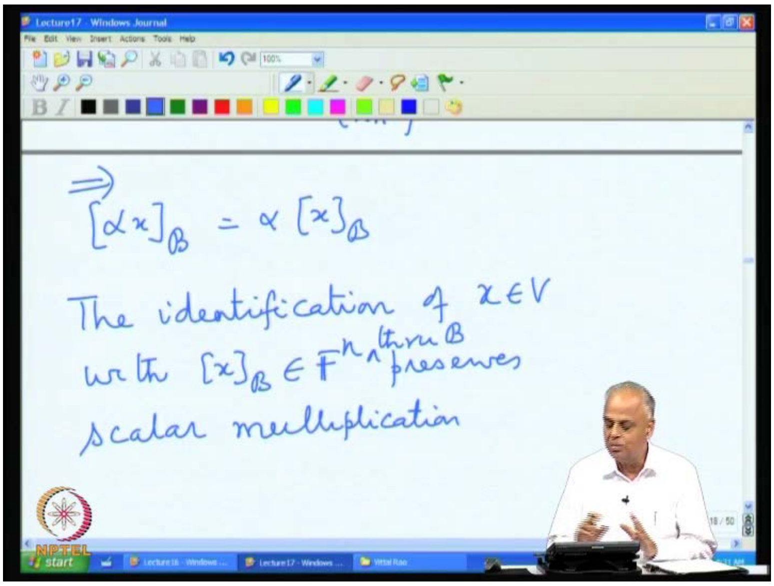

Which means therefore, we get alpha x it is representation is alpha times the representation of B. In other words, what is say this scalar multiply in V and then, get the representation, it is the same as first get the representation and then scalar multiply in F n and both are going to yield same results. So that means, the identification of x in V with x B in F n preserves scalar (( )) x B in f n of course,,, through the basis B preserves scalar multiplication. Thus we have seen, that this identification preserves addition in scalar multiplication; the addition in scalar multiplications of the (( )) algebraic operations in the vector space and these are preserved means; it preserves the algebra it preserves the algebraic operations.

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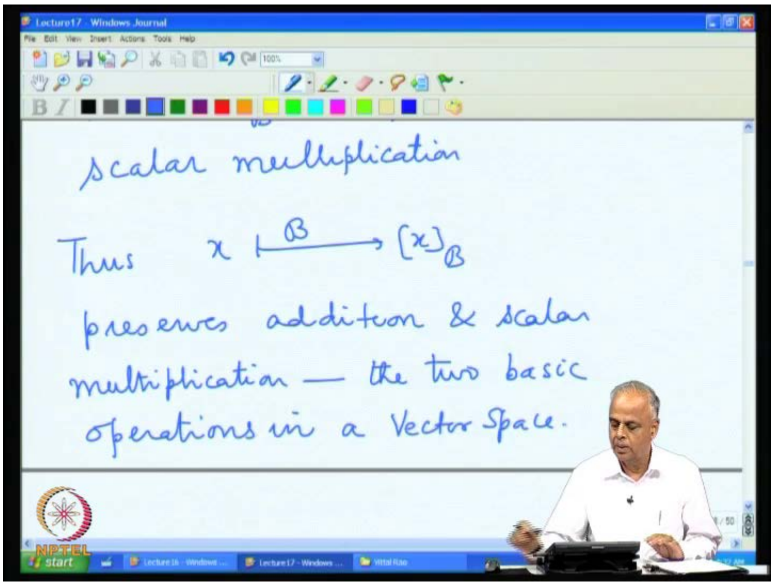

So, thus x to x B through the basis B preserves the addition and scalar multiplication, the two basic operations in a vector space. This suggests or this leads us to the notion of linear transformations.

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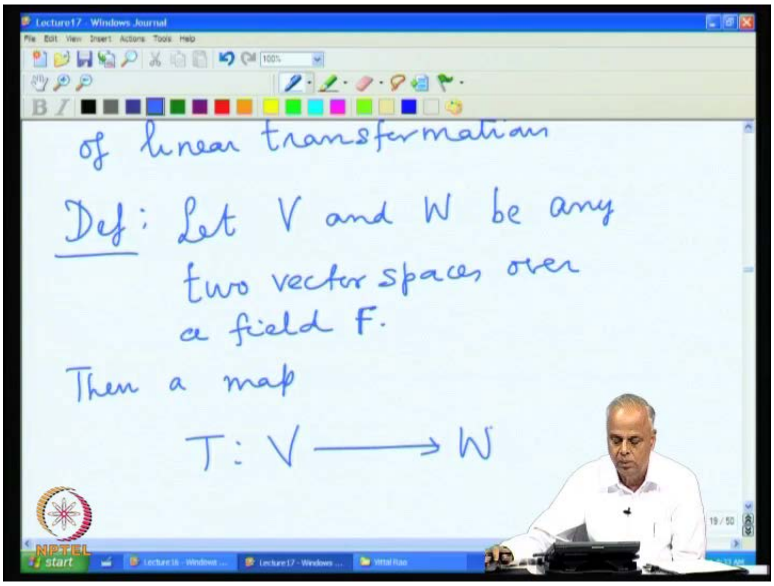

this lead us to the notion of linear transformations linear transformations Let us, give a definition let V and W be any two vector spaces over a field F. That V and w be any two vector spaces both the same field F then, a map a function T from V to W.

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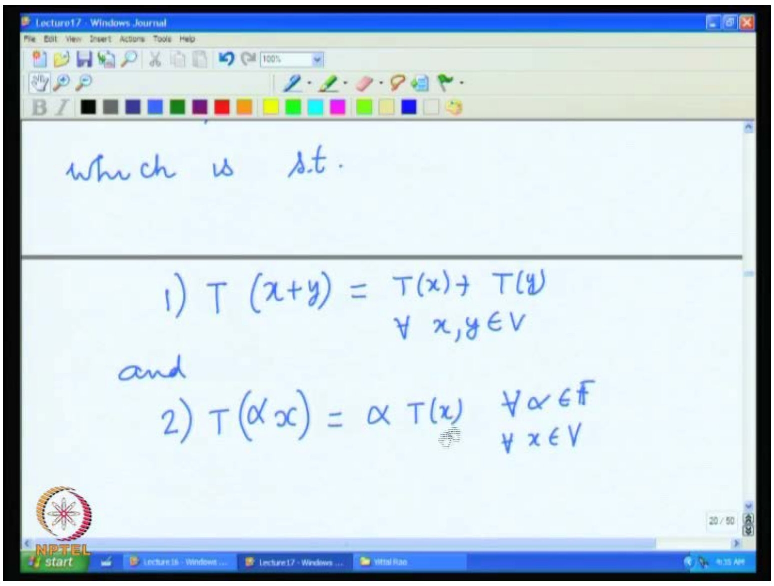

Which is such that, it preserves addition in scalar multiplication? What are that mean? It preserves addition means, if you add two vectors in V and then transform. It should be the same as transform individually and then, add for every x y is in V. Notice that, the plus here refers to the plus in B, because x is in y V y is in V therefore this plus is addition of vectors in V. The plus here is the addition in W, because T x is in w and T y is in W and therefore this is the W addition.

This is what you mean by preserves addition and it preserves scalar multiplication means take a vector x multiplied by a scalar in V and then take the transformation. It is should be the same as transform the vector x and then multiply by the scalar for all alpha in F and all x in V. Once again, notice that the scalar multiplication is in V and this scalar multiplication is in W, because this vector is in x and we are multiplying by alpha.

So, this is the scalar multiplication in V. This vector T x is in W, because T takes vectors from V two in W so, this is a vector in W and we are multiplying by a scalar, a vector in W so, this is the scalar multiplication in W. such a, if you have a transformation from V two W, which is such that it preserves addition and scalar multiplication.

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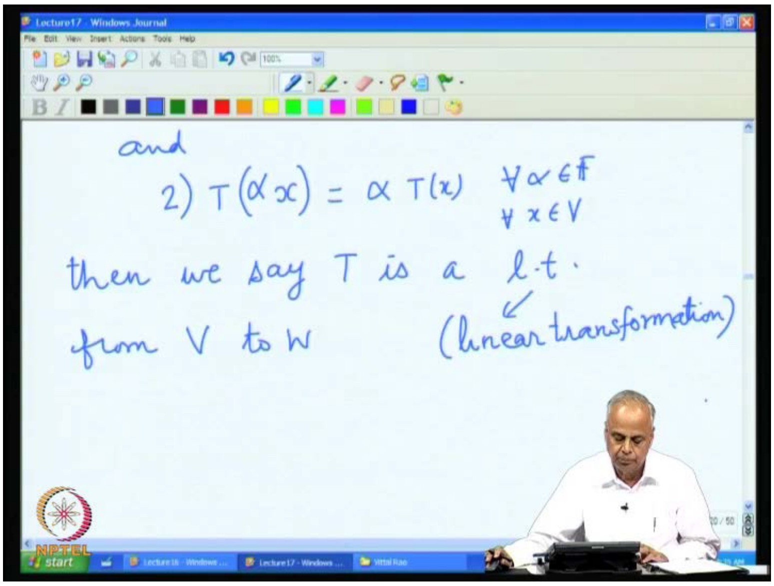

Then, we say T is a linear transformation from V to W. Now, at least for the l t stands for linear transformation. It is a linear transformation from V to W.

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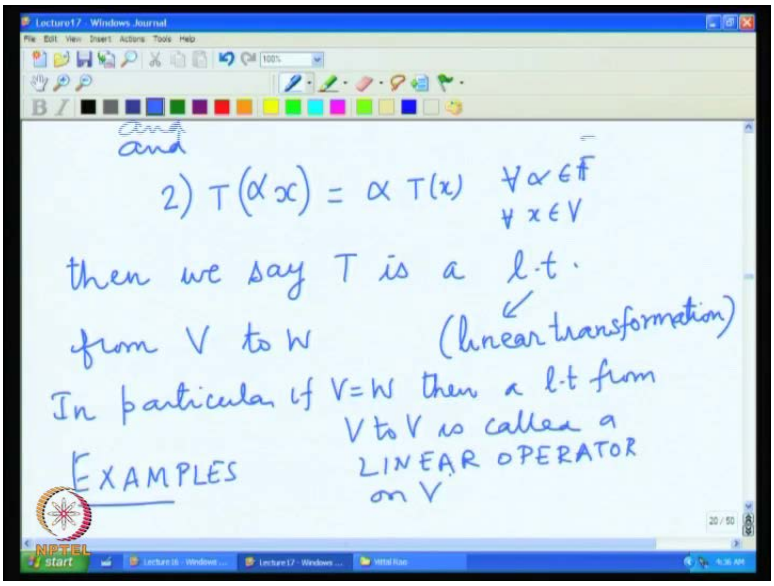

Let us looks at some examples; let us make a one remark, we have take any two vector spaces V and W in particular, if I take W to be also same as V then we get a transformation from V to V which preserves addition and scalar multiplication. Then in other words, you are encoding V vectors as V vectors without destroying the addition and scalar multiplication. In other words, in particular if V equal to W then a linear transformation from V to V is called a linear operator on V.

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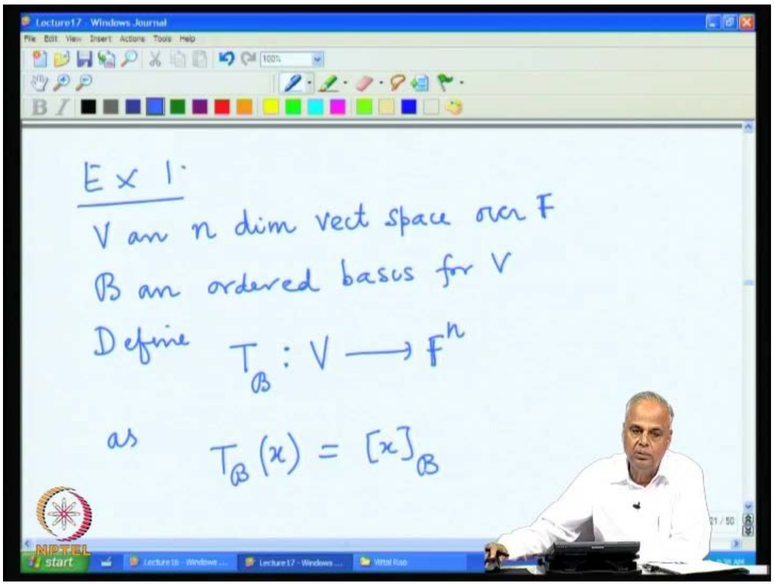

Let us, look at some examples of linear transformations and linear operations. Obviously the whole definition was motivated our identification of a vector in a n dimensional space through an ordered basis with a vector in F n. Let us take that the first example so, V an n dimensional vector space over a field F then, B an ordered basis for V. Now we define, a transformation T which of course, depends on ordered basis B so, we will write T B which take V vectors to F n vectors as, how is it define it takes the vector x in V to it is basis representation T B equal x equal to x B and we have seen, that this identification preserves scal addition and scalar multiplication this, what motivated as the definition of linear transformations.

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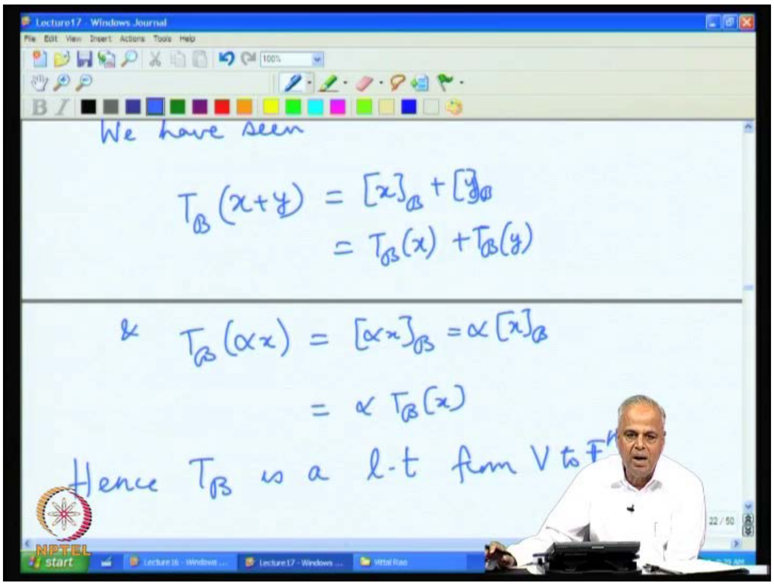

We have seen, if you add vectors and then, identify it is the same as x B plus y B and which is the same as T B x plus T B y and T B alpha x is, which is alpha x B we saw the this the same as identify then multiply which is equal to alpha T B x so, T B preserves addition sacral multiplication. Hence, T B is a linear transformation from V to F n so, thus every ordered basis on an n dimensional vector space produces a linear transformation from that vector space to F n.

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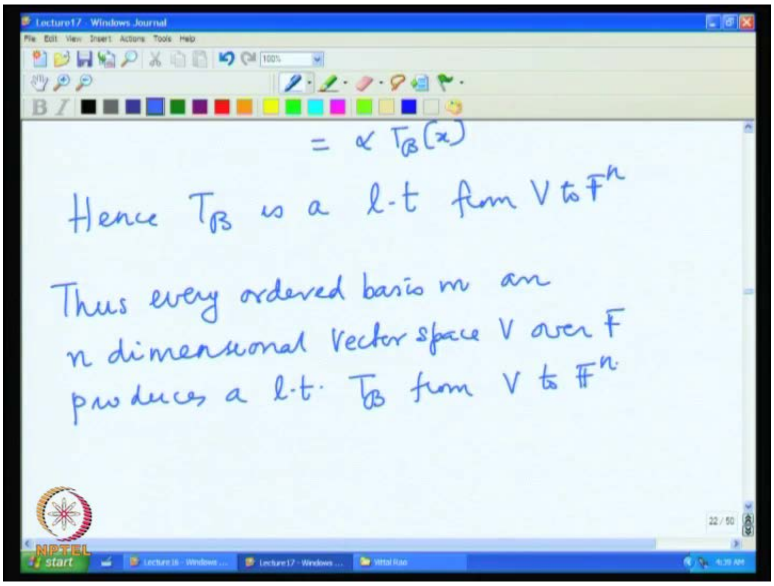

So, thus every ordered basis in an n dimensional vector space V over F produces a linear transformation T B from V to F n. So thats, our first example which actually motide motivated our definition for linear transformation.

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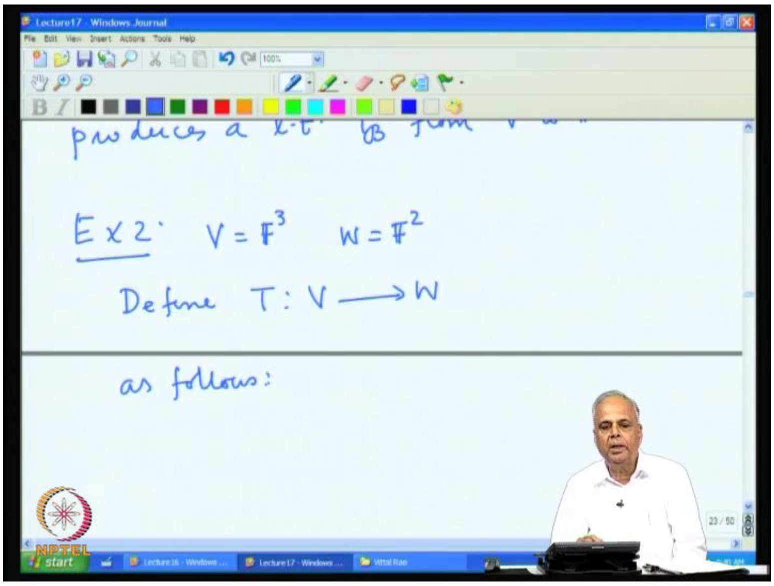

Let us looks at simpler examples now, take V to be F 3 and W to be equal to F 2. Define a transformation T from V to W as follows. Now, what should T do? T should take vectors from V and convert them to vectors in W; vectors in V are in F 3 so they have three components x 1, x 2, x 3 and then it should convert them to two component vectors.

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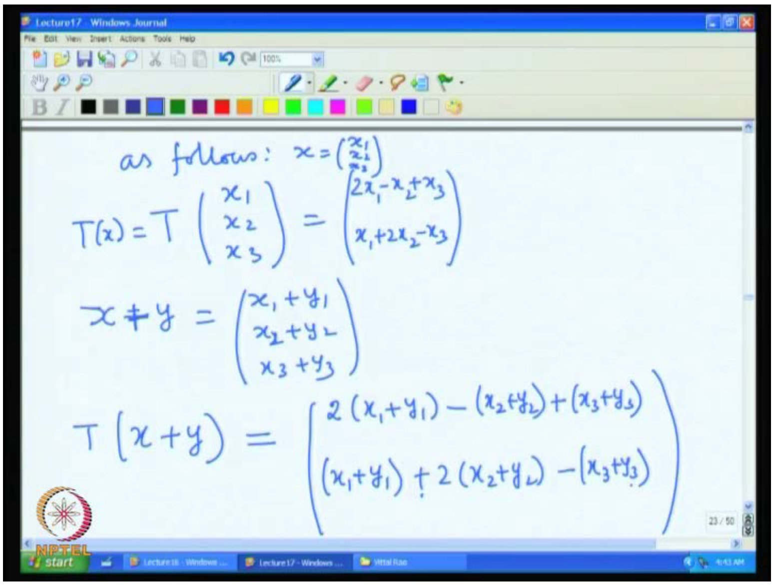

So, T must take a three component vector and convert it in to a two component vector. So, let us say 2 x 1 minus x 2 plus x 3 and x 1 plus 2 x 2 minus x 3, this is the transformation the vector x if, the vector x is x 1 x 2 x 3 it is now converted this is T x and it is converted into a two component vector.

Now, what is x plus y? x plus y is the vector x 1 plus y 1 x 2 plus y 2 x 3 plus y 3. It takes a x to be x 1 x 2 x 3 y to be y 1 y 2 y 3. Now, how does T transform x plus y? Let us observe this carefully, the way T transforms is, it is take a three component vector and converts it in to a two component vector. How are these two components form? The first component that is to be form is made up of all the three components of the original vector. It take two parts of the first component, subtract it is from one part of the second component and adds one part of the third component.

If you want to do the same thing, we have to take two times the first component of x plus y. What is the first component of x plus y x 1 plus y 1? From that, we must subtract the second component of x 2 and then add the third component of x 3. Then the second component of the transform vector is form by adding twice the second component of the original vector to the first component and then subtracting the third component. Doing the same thing, we get twice the first then the x 1 so; the first component is what should we take? The first component of x plus y plus twice the second component plus twice x 2 plus y 2 plus minus the third component of the vector you have transfer. This is what the transformation of x plus y.

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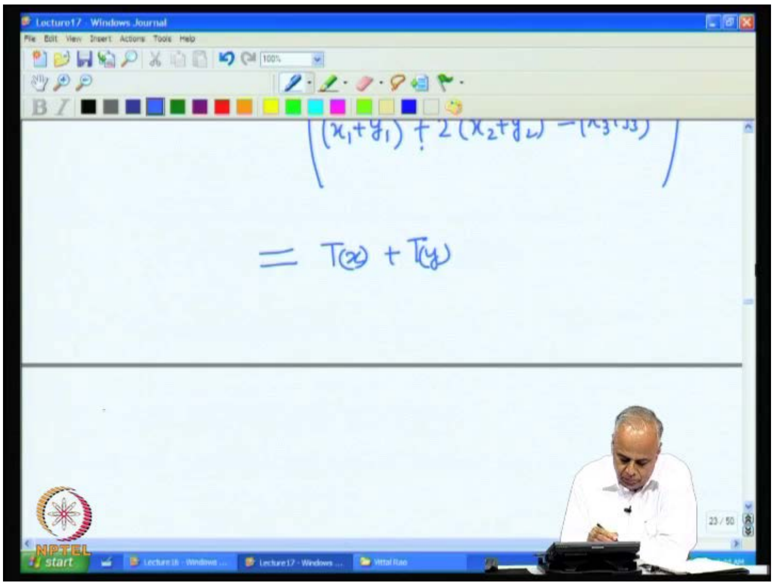

Now it check, now it is the simple manipulation to see or simplification to see that this is the same as T x plus T y. We know, what should be T x? T x should be obtained from here and T y will be obtained from here, replacing the x is by the y is if we add the two we get precisely this.

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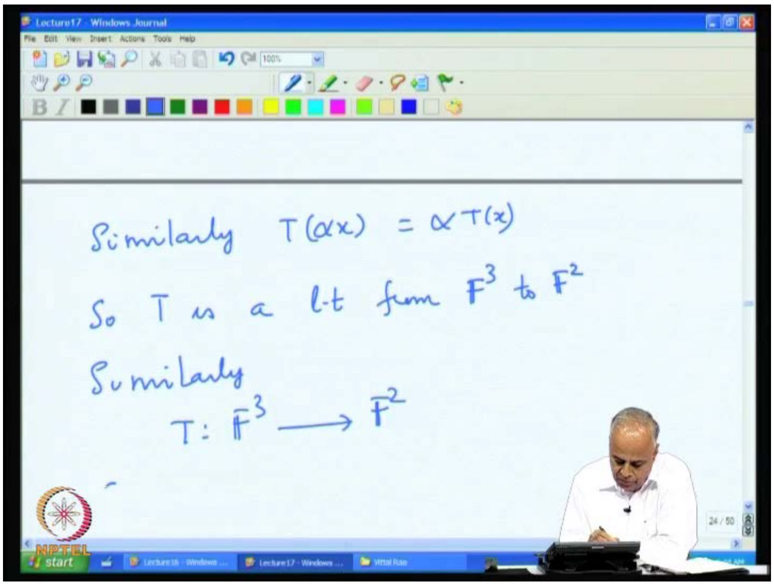

Similarly, similarly, T of alpha x is equal to alpha T x. T is a linear transformation from F 3 to F 2. Now we are chosen the values 2 minus 1 plus 1 here 1 2 minus 1 here, we could chosen any a b c and another alpha beta gamma, it would as still work.

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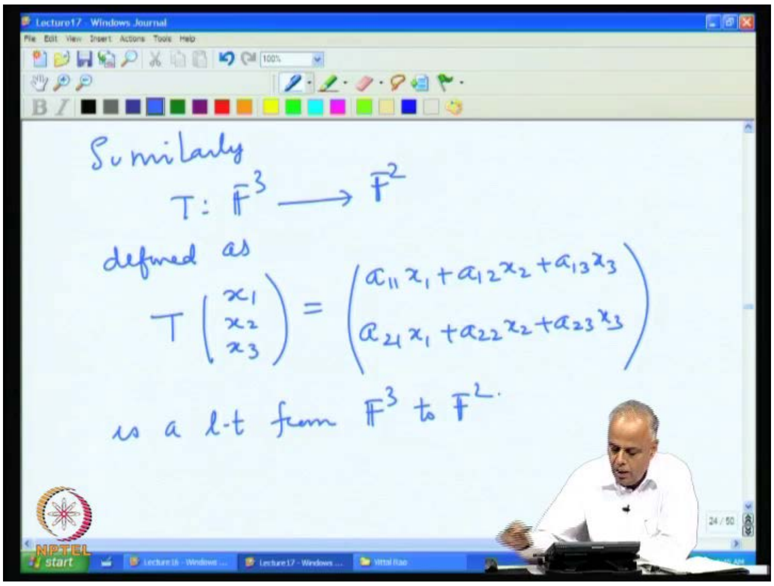

It is easy to check similarly, T from F 3 to F 2 defined as T of x 1 x 2 x 3 equal to a 11 x 1 plus a 1 2 x 2 plus a 1 3 x 3 so, certain proportion x n; certain proportion of x 2; certain proportion of x 3. Similarly, a 2 1 x 1 plus a 2 2 x 2 plus a 2 3 x 3 is a linear transformation from F 3 to F 2.

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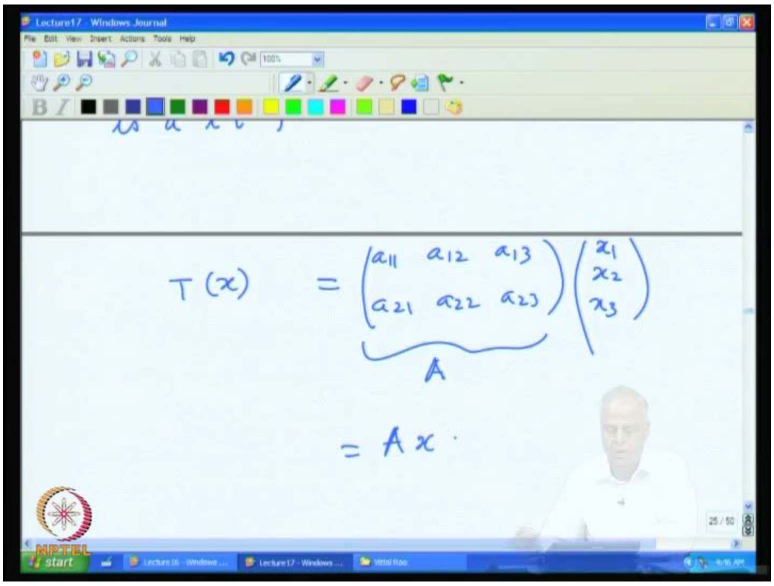

Let us observe this linear transformation, let it carefully T x 1 x 2 x 3 is T x and the right hand side can be written as a 1 1 a 1 2 a 1 3 a 2 1 a 2 2 a 2 3 times x 1 x 2 x 3. So, if you call this as matrix A then, this is equal to A x so, therefore if, you chosen arbitrary a 1 a 2 a a 1 a 2 a 3 extra we get a matrix A which is a 2 by 3 matrix and that automatically produces the a linear transformation on this space F 3 to F 2.

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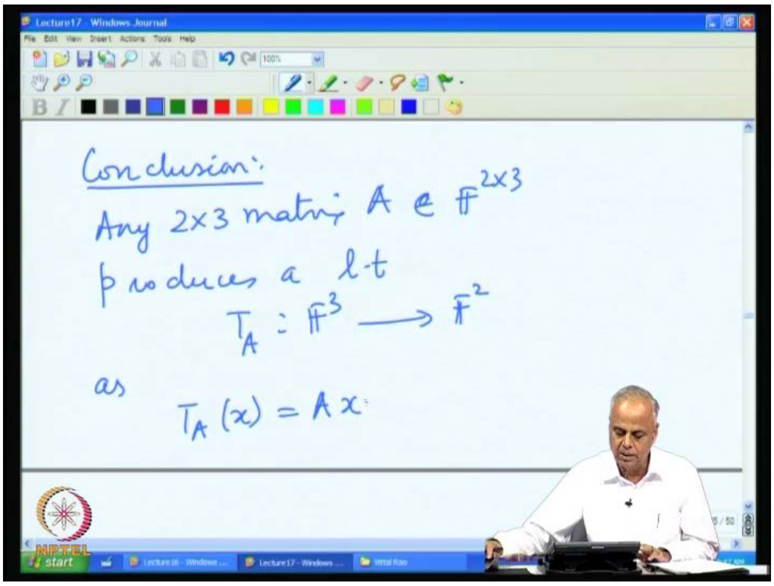

The conclusion is any 2 by 3 matrix A o belonging to F 2 by 3 produces, a linear transformation T from F 3 to F 2. Since, it depends on a will write it as a T A as x is equal to A times x.

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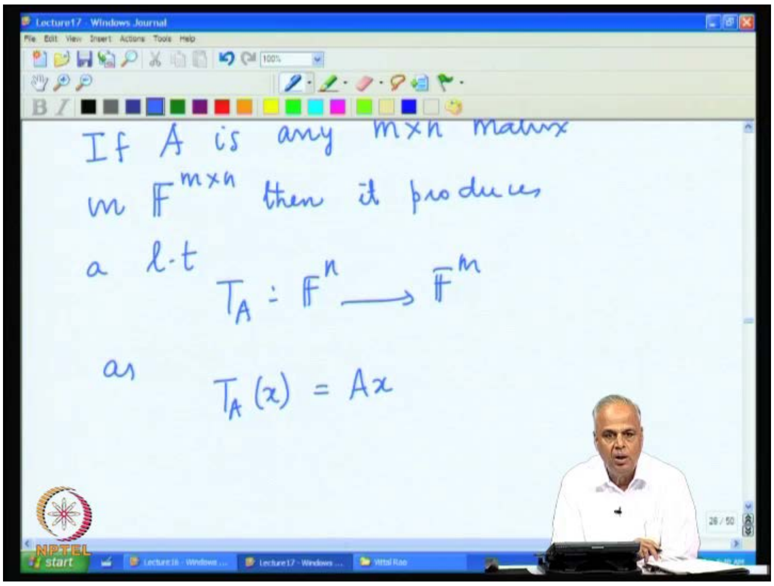

And this can be generalized as follows, written in this form is very easy to verify the T A x plus y is a times x plus a times y, because matrix multiplication a distributive and T A alpha x is alpha times A x, because scalars can be pulled out of matrix multiplication. The generalization is that, if A is any m by n matrix in F m cross n then, it produces a linear transformation T A from F n to F m as T A of x equal to A x. Now remember that, when we have talking about linear systems of equations, it is this art of matrix is that we were interested in. Therefore, this order of matrix is or connected with linear transformations.

The study of this structure of linear transformations will automatically give us a lot of information about this structure of the matrix A and hence, the structure of this solutions of the system A x is equal to B. We shall begin from on next lecture the study of this structure of such linear transformations on vector spaces.