Advanced Matrix Theory and Linear Algebra for Engineers

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Lecture No. # 11

Linear Independent and Subspaces

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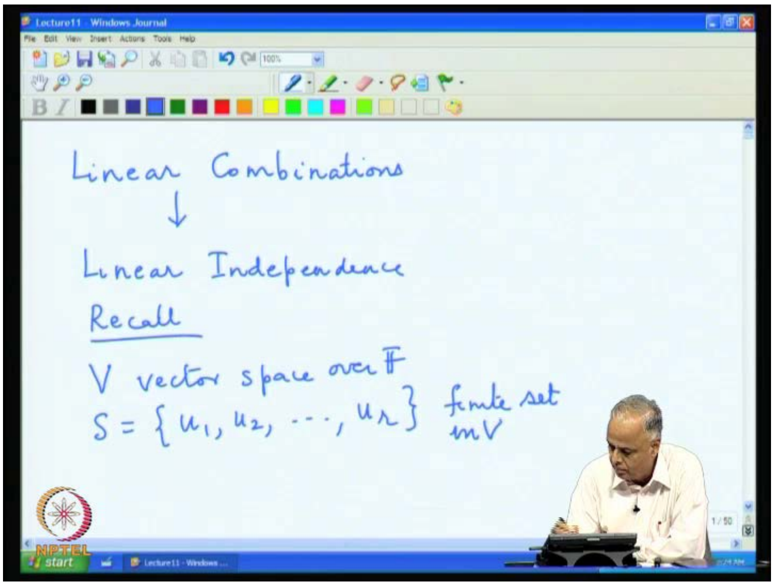

In the last lecture, we introduced two important notions. The first one was the notion of linear combinations as a consequence of this let us, to the notion of linear independence. Let us recall the definition of linear independence suppose, V is a vector space over F can be any field. Suppose V is a vector space over F and we have a finite set u 1 u 2 etcetera u r a finite set in V.

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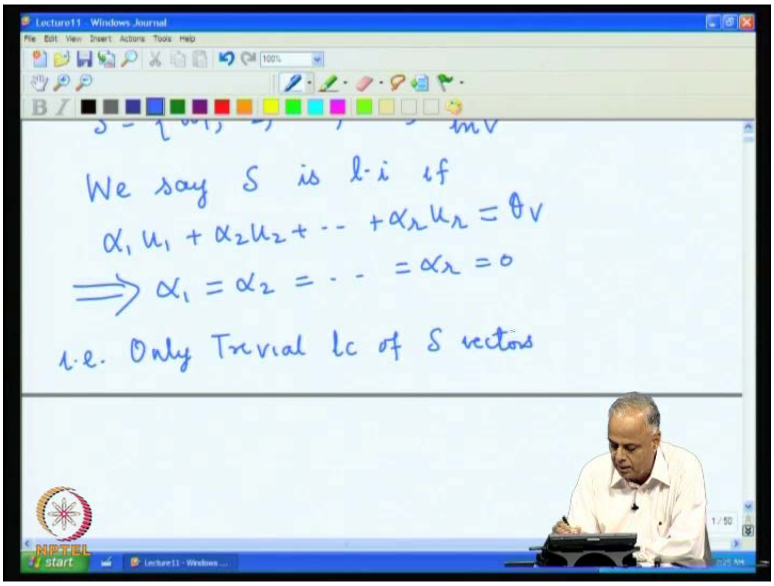

Then, we say S is linearly independent as I mentioned last time will write one I for linearly independent, then we say S is linearly independent. Whenever we have a linear combination of these vectors giving raise to the 0 vector, then all the coefficients must be equal to 0. This is the same thing as saying but, the only way to get the 0 vector as a linear combination of the vectors in S it is so called trivial linear combinations.

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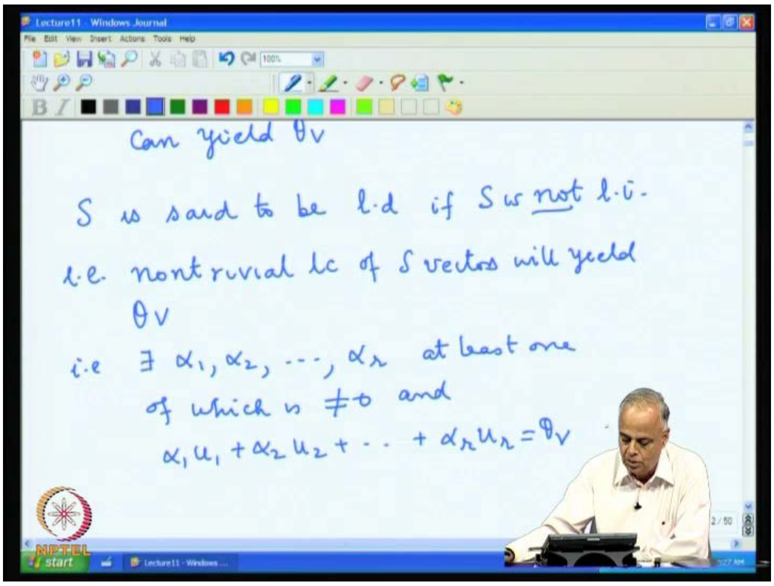

So, that is only trivial linear combination of S vectors or the vectors in S can yield the 0 vector. If the set is not linearly independent it is set to be linearly dependent. The S is set to be will write L d for linear dependent, then set to be linearly dependent if S is not linearly independent. What are that mean a set is linearly independent? If only trivial linear combination of give 0 vector, so not linearly independent means non trivial linear combinations should also be 0 vector, that is non trivial linear combinations of S vectors will yield the 0 vector. What are that mean we must have a linear combination in the all the coefficients are there at least one of them should be non 0, Because there exists alpha 1, alpha 2, alpha r at least one of which is not equal to 0 and the linear combination alpha 1 u 1 plus alpha 2 u 2 alpha r u r gives the 0 vector.

Thus when we say linear independence, whenever linear combination is 0 all coefficients must be 0. When we say linear dependence there can be a linear combination, which is the 0 vector without all the coefficients being 0.

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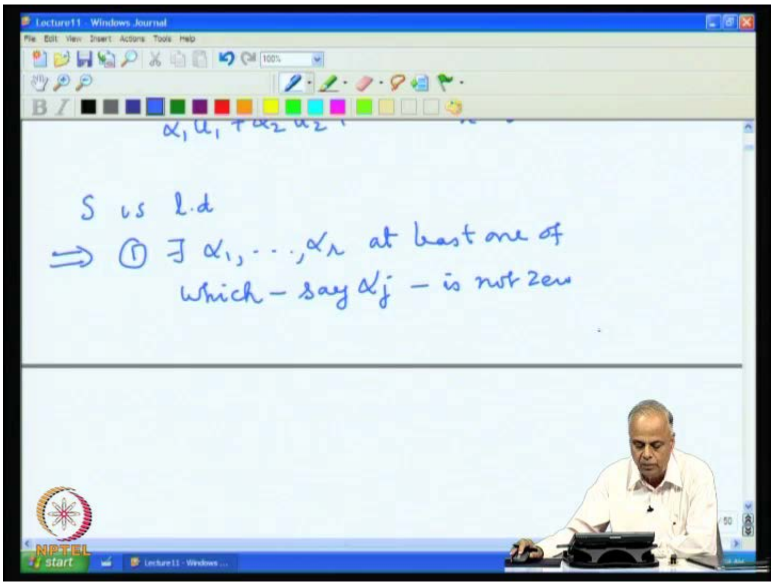

Now, let us look at one simple property of the linearly independent vectors suppose, S is linearly dependent. This implies one just does we observed above there exists alpha 1 alpha 2 alpha r at least one of which is not 0, then that is linear combination is 0. Let us say that alpha j is not 0, so there exists alpha 1 alpha 2 alpha r at least 1 of which say alpha j is not 0 so that will be first thing that will notice is linearly independent.

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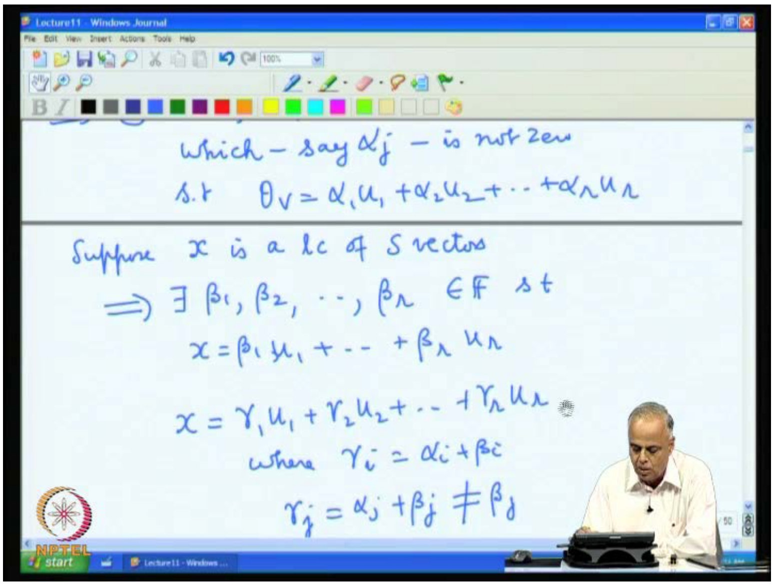

Suppose x is a linear combination of S vectors, what are that mean we must be able to express x, as the linear combination of the vectors in s. That means there exists beta 1 beta 2 beta r all these scalars are from the field F, such that x is beta 1 u 1 plus etcetera beta r u r. Now note give me say that there exists alpha 1 alpha 2 alpha r at least 1 of which is not 0. To such that the 0 vector can be obtained as this linear combination, this is what is meant by linearly it gives this is obtained by the notion of linear independence. Suppose we have a vector S which is a linear combination of S vectors, then we can expressed as beta 1 u 1 beta 2 u r u 2 beta r u r. If we add this representation and this representation the left hand sides add up to x, because theta V plus x is S and the right hand side add up to alpha 1 plus beta 1 into u 1 plus alpha 2 plus beta 2 into u 2 alpha r plus beta r into u r.

Let us, call this as gamma 1 u 1 plus gamma 2 u 2 plus gamma r u r where gamma i is alpha i plus beta i. Since we have assumed that this alpha j is not 0, then for assume that this alpha j is not 0. We will have gamma j equal to alpha j plus beta j since alpha j is not 0 this cannot be beta j. What is that mean we have 1 representation of x is a linear combination of the u 1 u 2 u r, we are now another representation of x is u 1 u 2 u r in these two representations thus beta j is different form gamma j and therefore, there are two distinct representations this means x as at least two representations as a linear combination of the S vectors.

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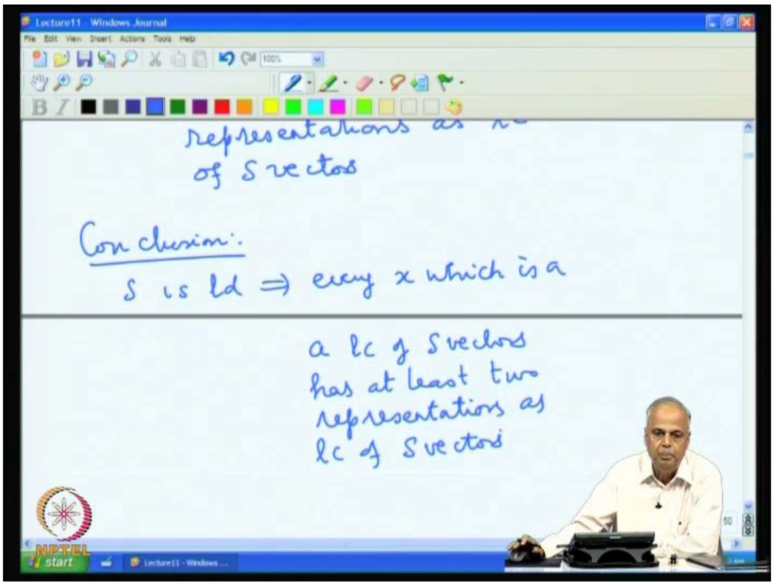

Then say x has at least two representations as linear combination of S vectors thus, whatever we have a linearly. What is this conclusion? Whenever we have S is linearly dependent means every x, which is a linear combination of S vectors has at least two representations as linear combination S vectors. The linear dependence brings in certain amount of vagueness, because thus lot of redundant information.

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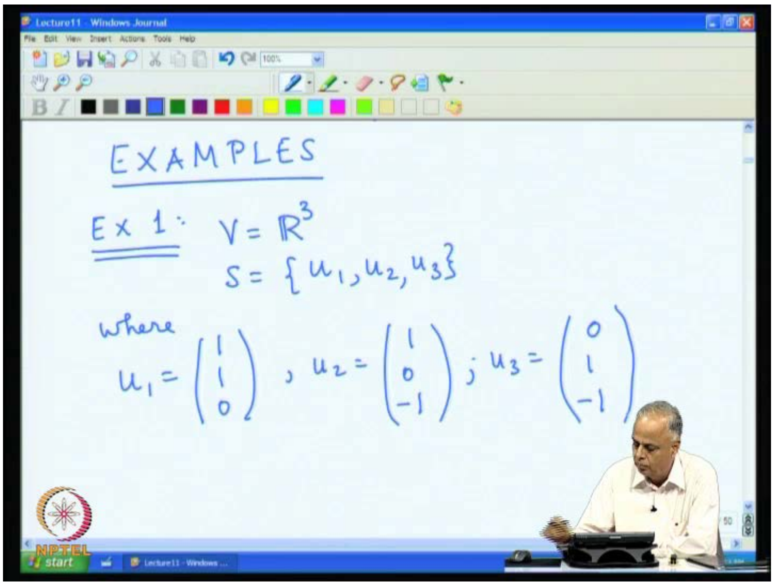

We shall, first look at some examples of this notion of linear dependence and linear independence first. Let us take V to be our standard R 3 and let us take the S to be the set of 3 vectors u 1 u 2 u 3, where u 1 is the vector 1 1 0 u 2 is the vector 1 0 minus 1 and u 3 is the vector 0 1 minus 1. Now, see whether this set is linearly independent for these we must check whether, if we linear combination vanishes does it imply all the coefficients are 0.

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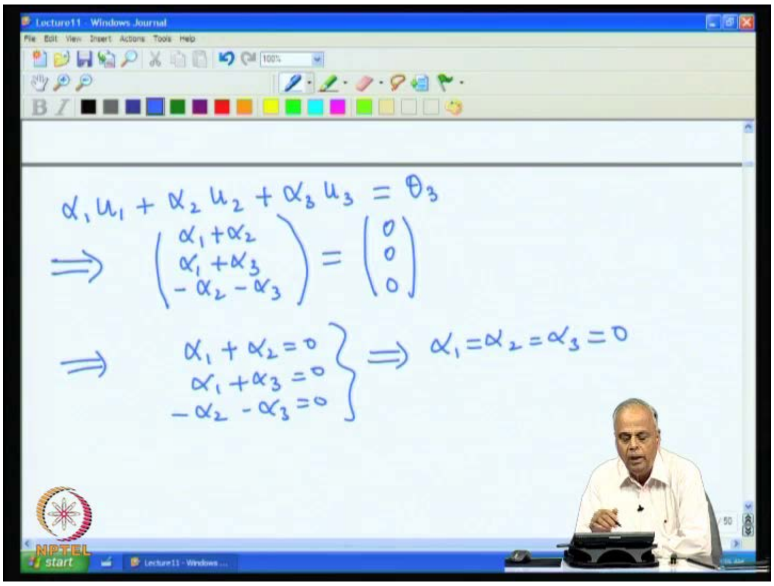

Let us, to start with a linear combination alpha 1 u 1 plus alpha 2 2 u 2 plus alpha 3 u 3 suppose, this is the 0 vector what are that imply? Let us find out, what alpha 1 u 1 alpha 2 u 2 alpha there u 3, so we have alpha 1 u 1 you give me the first entry of alpha 1 plus and alpha 2 here.

And then nothing here, the vector I will get on the left hand side is alpha 1 plus alpha 2 and a second component, I will get a alpha 1 0 and alpha 3. So I will get an alpha 1 plus alpha 3 similarly, I get the last as minus alpha 2 minus alpha 3. This is the left hand side is the right hand side is the 0 vector, that must be equal to theta 3, which is 0 0 0.

Now comparing both sides, we get alpha 1 plus alpha 2 is 0 alpha 1 plus alpha 3 is 0 minus alpha 2 minus alpha 3 is 0 and this automatically implies alpha 1 alpha 2 alpha 3 are all 0s. A linear combination can yield the 0 vector only, if all the coefficients are 0.

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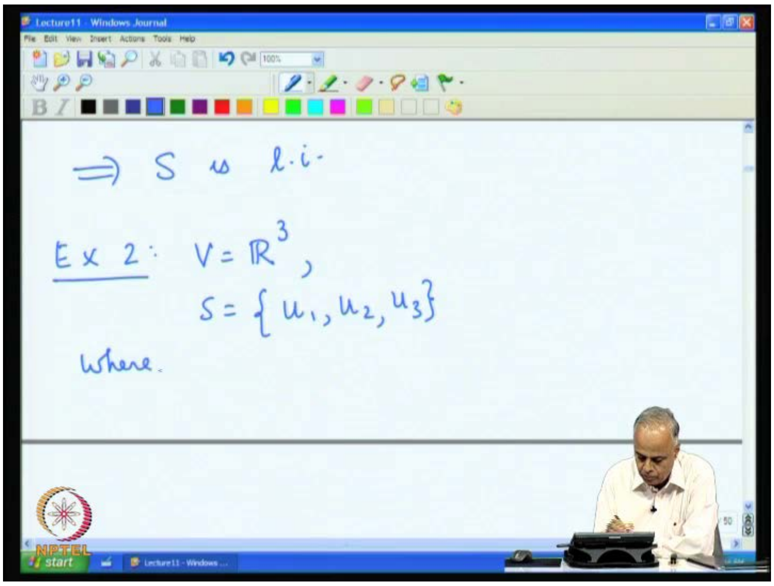

That is the set S is linearly independent. Just look at another example; again let us take V to be the vector space R 3, now let us takes again S to be a set with 3 vectors u 1 u 2 u 3.

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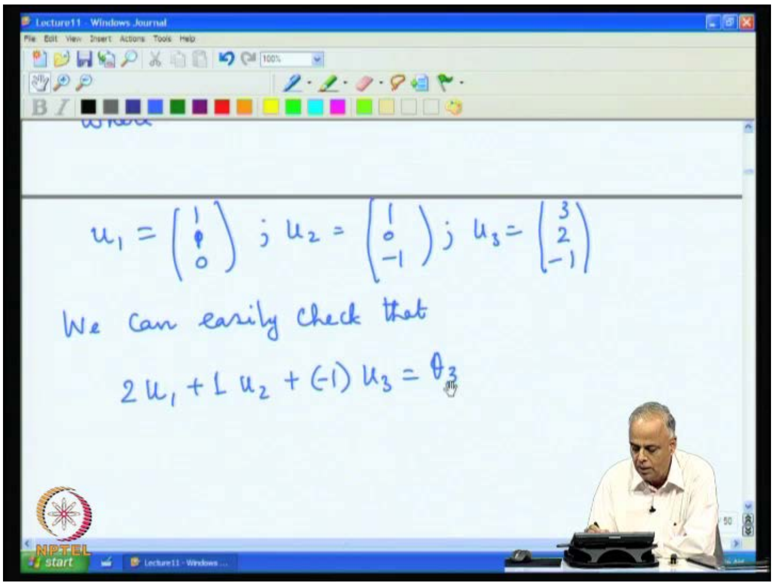

Where u 1 is as before u 2 is as before x, now take u 3 to be 3 2 minus 1. We see that we can easily check if, we are done within the last lecture. We can easily check that 2 u 1 plus 1 u 2 plus minus 1 u 3 gives as the 0 got and therefore, we have a linear combination in which we have non 0 coefficients and still that gives the 0 vector. Whenever such a non trivial linear combination is the 0 vector, we know the set is called linearly dependent.

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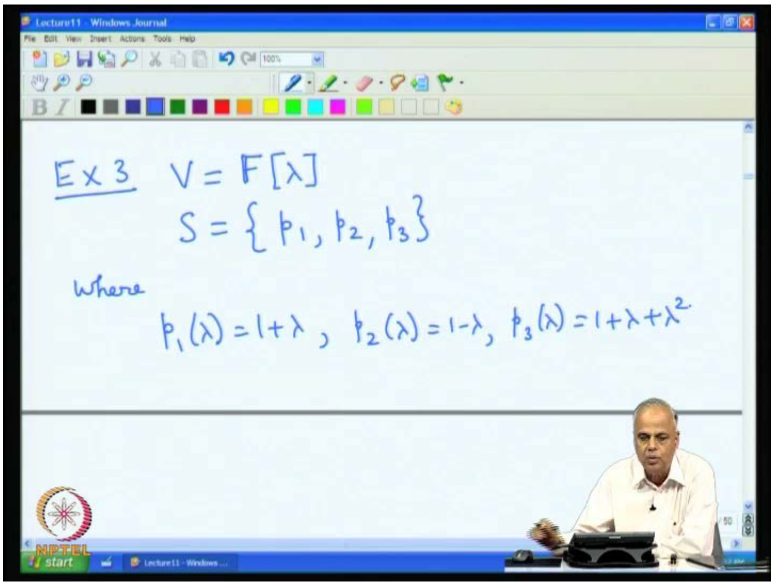

So, that is says S is linearly dependent look at another example, to take the vector space V to be the vector space are all polynomials in lambda with coefficients from the field F. let us take the set S to be consisting of the 3 polynomials p 1 p 2 p 3, where p 1 lambda is 1 plus lambda p 2 lambda is 1 minus lambda p 3 lambda is 1 plus lambda plus lambda square. We are the vector space of all the polynomials we are taken a set of 3 vectors, so we are finite set vectors these are the 3 polynomials vectors, we have a polynomials because this vector space consists of polynomials.

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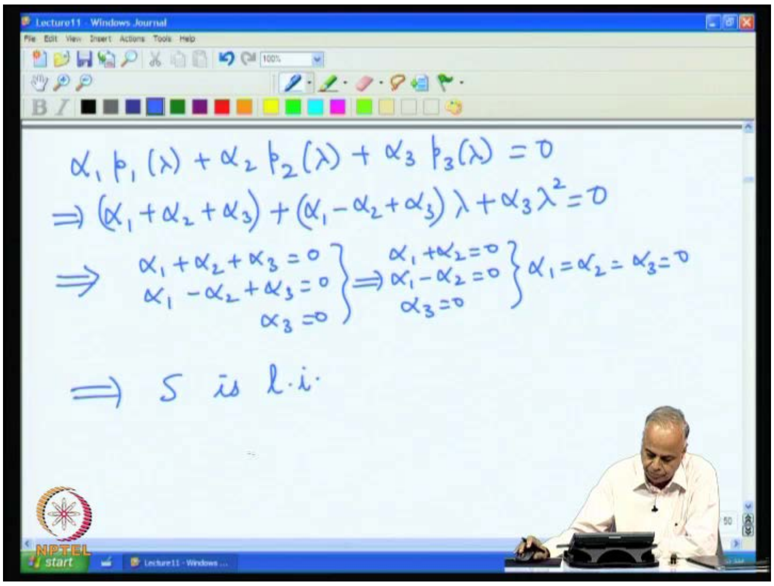

Now, suppose a linear combination alpha 1 p 1 lambda alpha 2 p 2 lambda alpha, there p 3 lambda gives as the 0 polynomial. What is that mean, if we substitute for p 1 lambda and p 2 lambda, let us collect the constant terms first alpha p 1 will give me an alpha 1 alpha 2 p 2 will give me an alpha alpha 2 alpha 3 p 3 will give me an alpha 3. The constant term will be alpha 1 plus alpha 2 plus alpha 3, now collect the lambda terms alpha 1 p 1 will give me an alpha 1 alpha 2 p 2 will give me an minus alpha 2 alpha 3 p 3 will give me an alpha 3 1. We have alpha 1 minus alpha 2 plus alpha 3 into lambda and now, if we collect the lambda square it comes only from p 3 so will get alpha 3 lambda square and that is 0.

There is the 0 polynomial a polynomial the 0 polynomial my all coefficients must be 0, that is say alpha 1 plus alpha 2 plus alpha 3 is 0 alpha 1 minus alpha 2 plus alpha 3 is 0 and alpha 3 is 0 now, if we use the fact that alpha 3 is 0 in the first equations we get, alpha 1 plus alpha 2 is 0 and alpha 1 minus alpha 2 is 0 now it is clear that the first equation imply alpha 1 is 0 alpha 2 is 0, then we get alpha 1 equal to alpha 2 equal to alpha 3 all of them 0. Whenever a linear combination of p 1 p 2 0 then all the coefficients must be 0, this means this set S is linearly independent.

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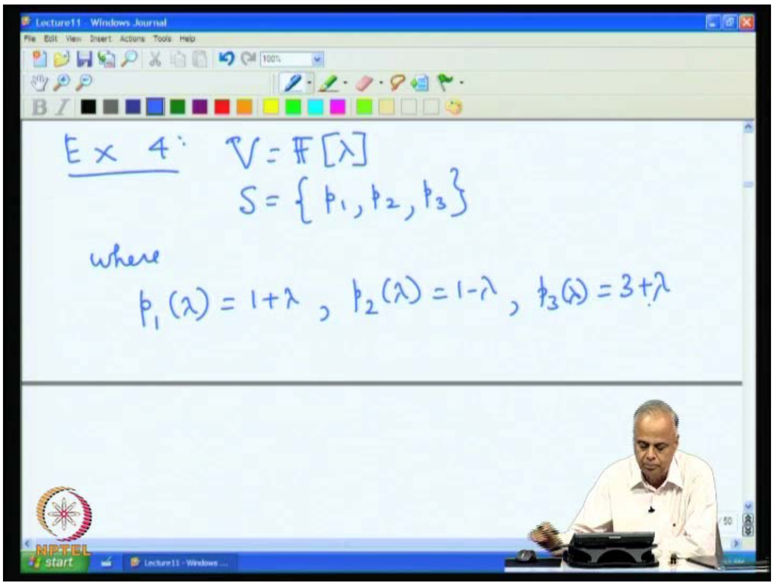

One final example, again take a vector space V to be the space of all polynomials, again take S to be p 1 p 2 p 3. Where p 1 lambda is 1 plus lambda p 2 lambda is 1 minus lambda and p 3 lambda is 3 plus lambda.

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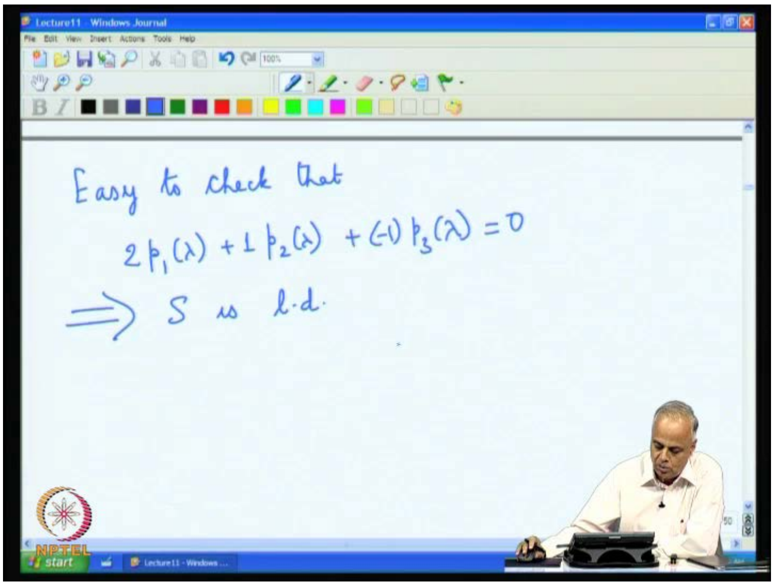

Again as we had done in the last lecture, it is easy to check that 2 p 1 lambda plus 1 p 2 lambda plus minus p 3 lambda is the 0 point. From this means we have nontrivial linear combinations, which give raise to the 0. Whenever this happens we say that the set is linearly dependent. Now we have seen some examples of what it means to say when a set of vectors a finite set of vector is a linearly independent in order to show that something is linearly independent. Assume that linear combination to this 0 and then, we try to conclude that all the coefficients must be 0 in order to prove something is linearly dependent, if we try to produce non 0 coefficients with which we can form a linear combination of the given vectors to produce the 0 vector.

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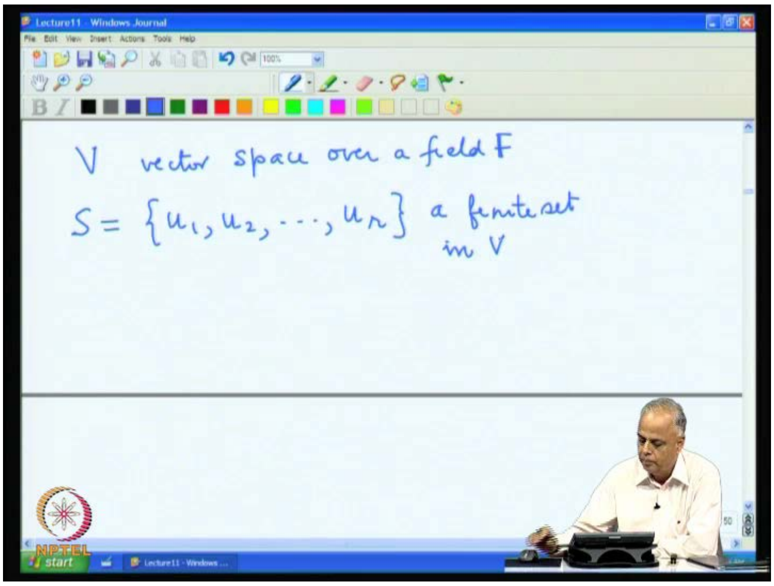

We, now move on to introduce a very important concept of subspace of a vector space it is begin with a vector space V over a field F. This will be always at the background because, as a set last time this is the universal which we work the universal, which we work in always a vector space over a field F. We get in t o the concrete situation by choosing V to be specific and F to be specific that. When we develop general theory, it will be for a general vector space V over a general field, so let us see take a vector space V over a field F and let us take a finite set u 1 u 2 u r a finite set in V.

Now we have seen in the last lecture, that when we try to build a newer vectors starting from a S vectors when we say build using the operation of addition and scalar multiplication preciously, we get a linear combinations of the S vectors.

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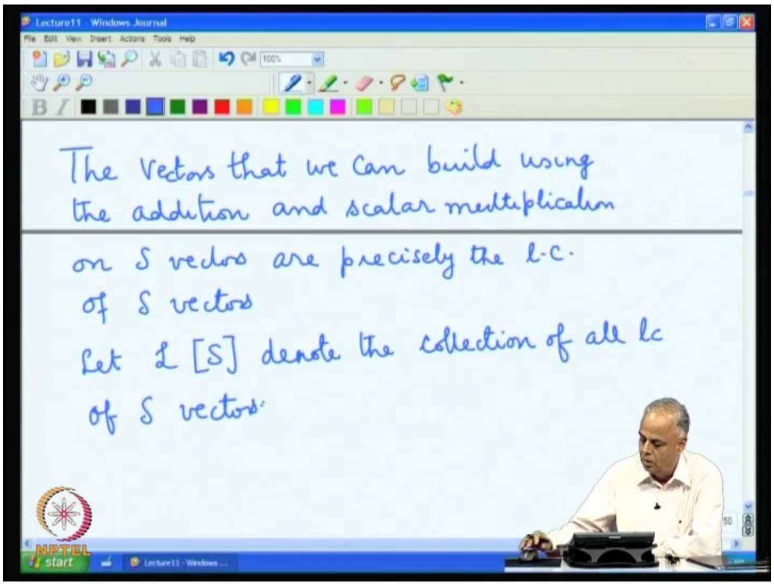

In this vectors, that we can build using the addition and scalar multiplication on the S vector are preciously the linear combinations of S vectors. What we do? We collect all those people, who we can will use this S vectors. Let L S denote the collection of all linear combinations of S vectors.

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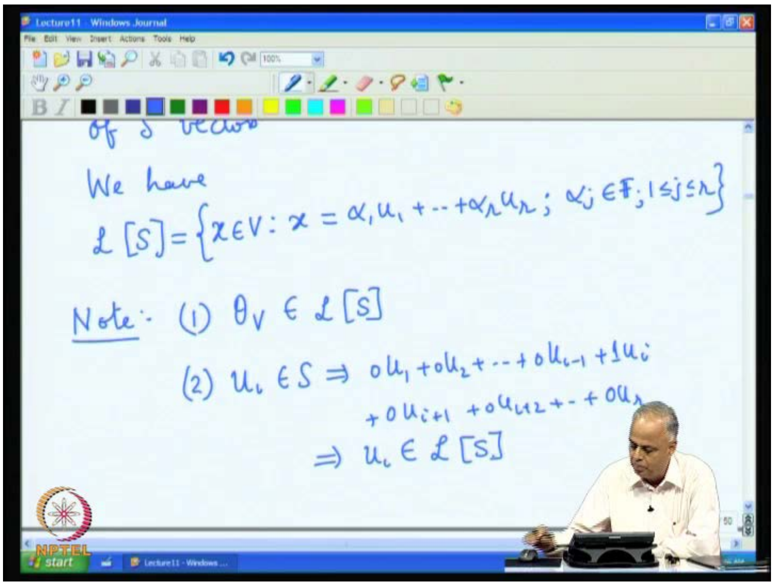

So, what do we have we have L S is equal to all those vectors in V. Such that that vectors, which can be expressed as linear combinations of u 1 u 2 u r, then there linear combinations all these coefficients must be in F. To collect all the linear combinations of the S vectors and these are follows similar, build using the S vectors. Note we had observed that the 0 vector can always be obtained at the trivial linear combination of any finite set of vector and therefore, the theta vector will always belonging to this collection linear combination of S vectors, secondly if we take u i in S then any 1 of the S vectors then we can write as 0 u 1 plus 0 u 2 plus etcetera plus 0 u i minus 1 plus 1 times u i plus 0 u i plus 1 plus 0 u i plus 2 plus etcetera plus 0 u r

Otherwise, a linear combinations in which all the coefficients except the i th coefficient as 0 and the i th coefficient is one and therefore, that is says u i is a linear combination of the S vectors and therefore, u i must be in that collection.

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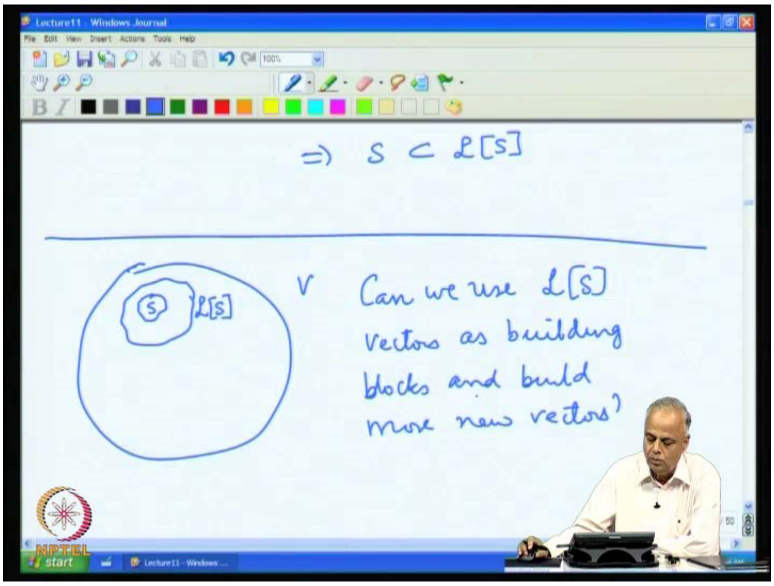

That means S is contain in L S, every vector of S is a part of this collection of a linear combination of S vectors and therefore, S must be a part of L S. These are two simple observations, now what we achieved first? We had this vector space V in the derived this small finite set S using that set, we have been a bigger set called L S. The big vector space inside that a small collection of sets S, where given using that is small collection of sets S. We have built a lot number of vectors and these are the linear combinations of the S vector. We have put that together, what we have got the L S? Suppose, I think of the L S vectors as building blocks, we started with S vectors is the building blocks and build L S it is like starting from the elementary particles in building a atoms.

Then we put atoms together, to get some molecules. Now we want to see whether, we can put the L S vectors to create anything new, so can we use L S vectors as building blocks and build more new vectors. To see, how far we can push starting from S, how much we can build? How build the structure? But we can build it.

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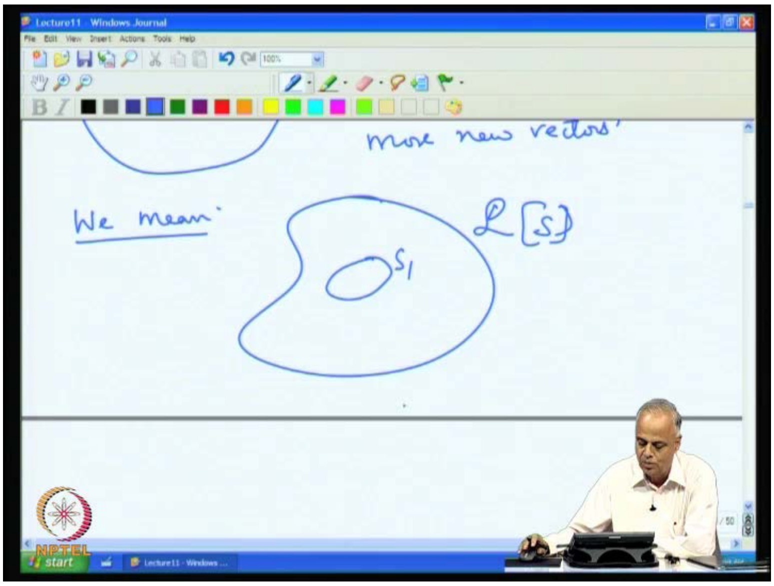

What we mean by this question, we mean now take here is your L S, which are build in this L S you pick a small finite subset.

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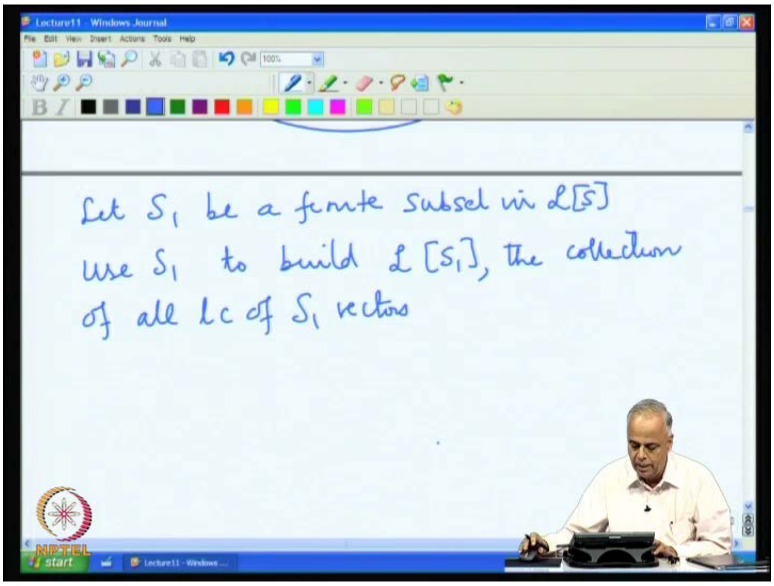

So, let S 1 be a finite subset in L S, now goes through the building constructions starting from S 1 to use S 1 to build. Then we start with S, what we build for L S? The collection of all linear combinations of S vectors, when we start with a S 1 we will build a L S 1, which is the collection of all linear combinations of S 1 vectors.

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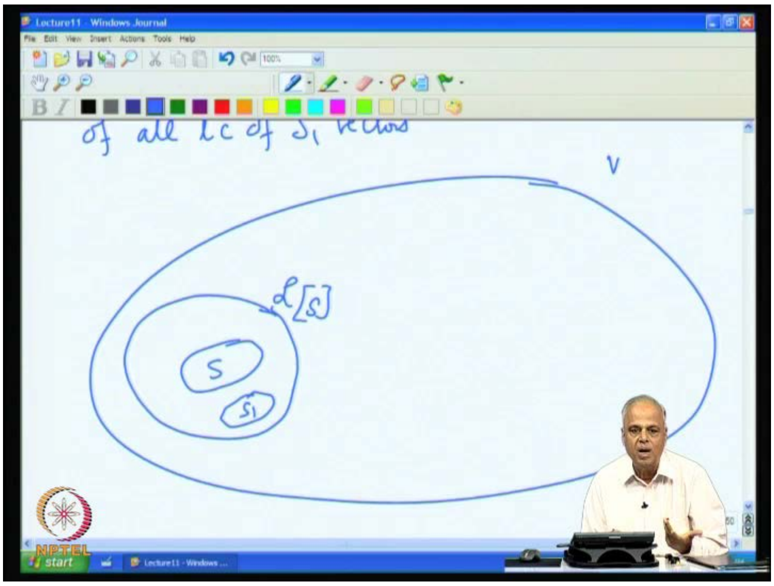

Again look at the picture, we had this u space V inside that u space V was the finite set S and starting from this finite set we construct the build L S. In this L S, again we picked up a small set S 1 we are constructing the L S 1. If by chance L S 1 take as outside L S, then we would have constructed more new things but, if L S 1 falls within L S then we would not now constructed anything new.

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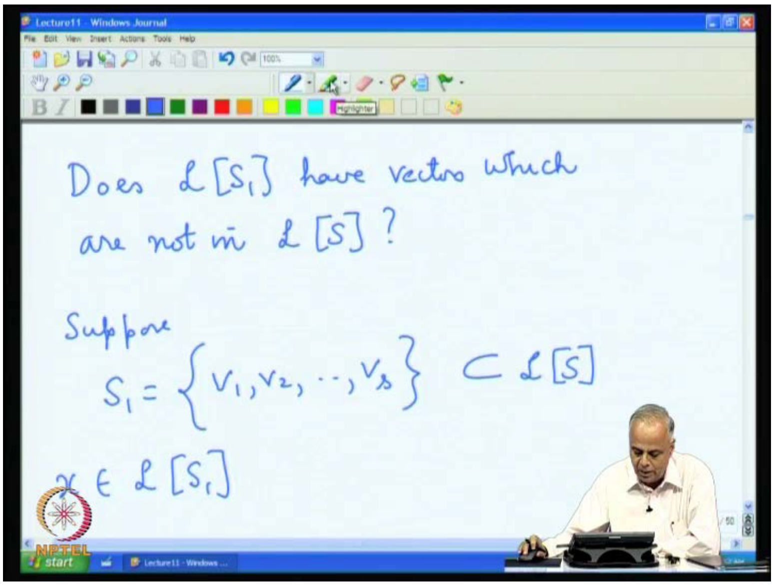

I want to investigate does L S have vectors, which are not in L S. If it is show then, we would have continued a building process with this various subset of L S. Let u look at answer to this question, suppose S 1 remember S 1 is a finite set sitting inside L S so S 1 is a finite set, to remember there was a finite sets. Then take and call it as V 1 V 2, some V S L S 1 is the finite set and it is sitting inside L S. No, we are trying to build things out of this suppose x belongs to the thing that we are build from S 1. What are we build from S 1 L S? Suppose, something is in this L S 1, what is that mean L S 1 the collection of all the vectors which are linear combinations of S 1 vectors, L S 1 will be collection of all linear combinations of S 1 vectors.

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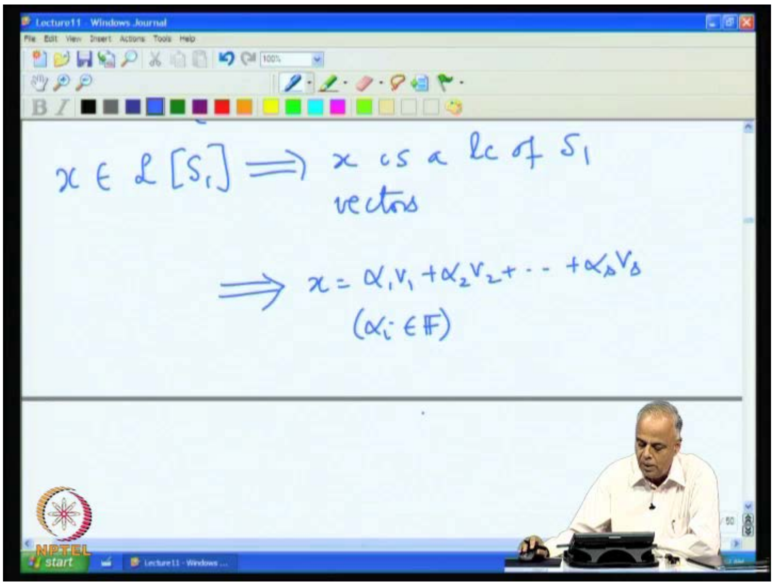

Therefore, any vector in L S 1 means x is linear combinations of S 1 vectors. What is that mean? That means we can write x as alpha 1 V 1 plus alpha 2 V 2 plus alpha S V S, where these alpha is are all in F. Because, S 1 consists of this vectors V 1 V 2 V S and x is a linear combinations of the S vectors but, now where this V 1 V 1 is is sitting inside L S but, L S is the straight of all linear combination of S vectors. The V 1 V 1 is sitting inside L S anything L S a linear combination of S vectors. So V 1 is a linear combination of S vectors similarly, V 2 is a linear combination of S vectors V S is a linear combination of S vectors.

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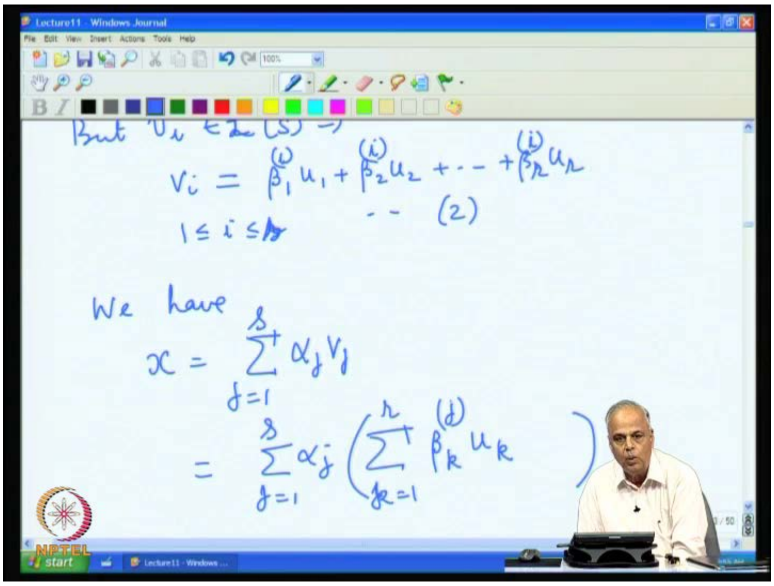

Let us keep this as 1 but, V i belongs to L S implies V i must be a linear combination, how do we write it? Beta 1 u 1 beta 2 u 2 plus beta r u r, the S vectors are u 1 u 2 u r and V i must be a linear combination of this. Since, we are talking with the i th vector we put a super script here to say that is the coefficient of the i th vector and we can do this for every one form the is from one to S. Because, there are S if we substitute all this in the equation 1 what do we get will get x equal to alpha 1 into a linear combination of the u 1s plus alpha 2 into linear combination of the u 1's and so on.

Therefore, if we combine the V 1 terms combined the V 2 terms we will get finally, only a linear combination of the S vectors. Let us do this in a slightly start a notation we have x is equal to sum believe the summation index j equal to 1 to r 1 to this. Because x now, we are the S 1 set so we have alpha j V j the equation that x is a linear combination of the V 1 V 2 V S we are written in the short form as j equal to 1 to j alpha j V j, now we observed that V j itself can be written as a linear combination of the write as j k equal to 1 to r. We want to look at the j th vector the j th vector will have a linear combination representation as V j 1 u 1 V j 2 u 2 V j u r.

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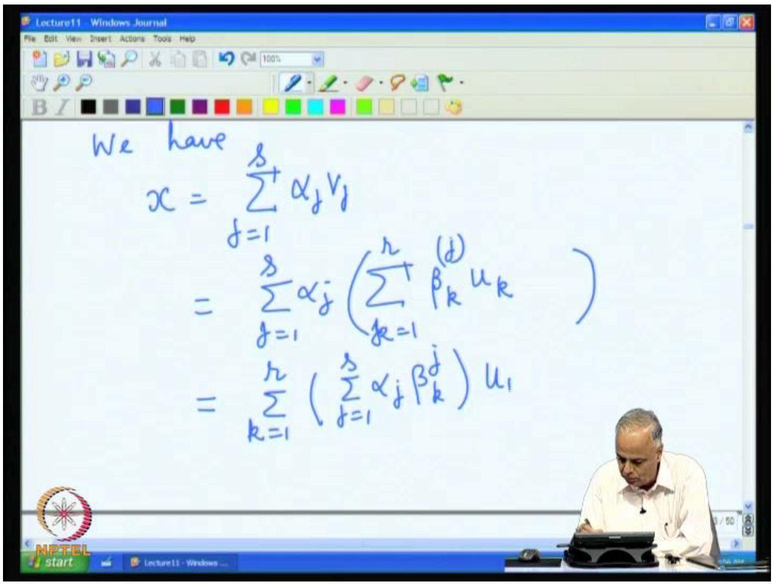

We can write this x is this form, by combining this are interchanging this summation notations, we get k equal to 1 to r summation j equal to 1 to S alpha j into u beta k j into u k.

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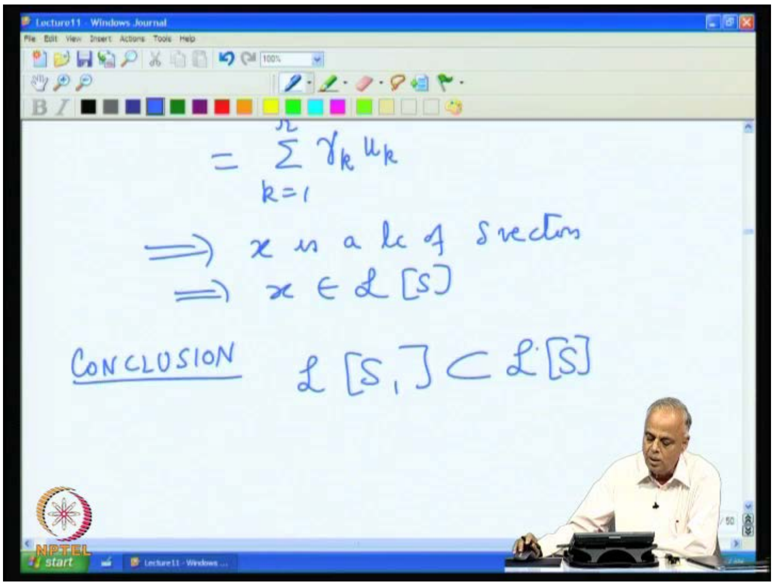

Which says that this is equal to we can write as some k equal to 1 to r, these whole thing in the inside summation, we can call it as some gamma k u k, which says x is a linear combination of S vectors, which means x belongs to L S. What will be in fact of all this, what we are shown is that we start with S. then, we build L S inside L S picks up small S 1 and we try to build L S 1 L S 1 does not go outside L S. Whatever vector x in L S 1 we choose that goes back to L S, any vector L x is L S 1 must be in L S. The conclusion is that L S 1 is contained L S.

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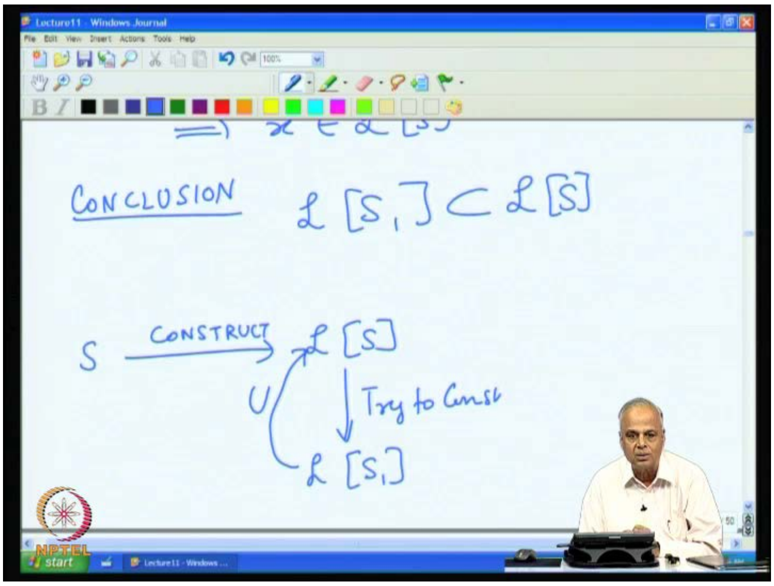

So what is that all mean we started with S we constructed L S, then from L S try to construct L S 1. How do we construct this? Choose a S 1, which is in L S and try to construct the linear combination of S 1 it pull u back to L S L S 1 is containing L S. In other words this construction process the building process saturates the L S level.

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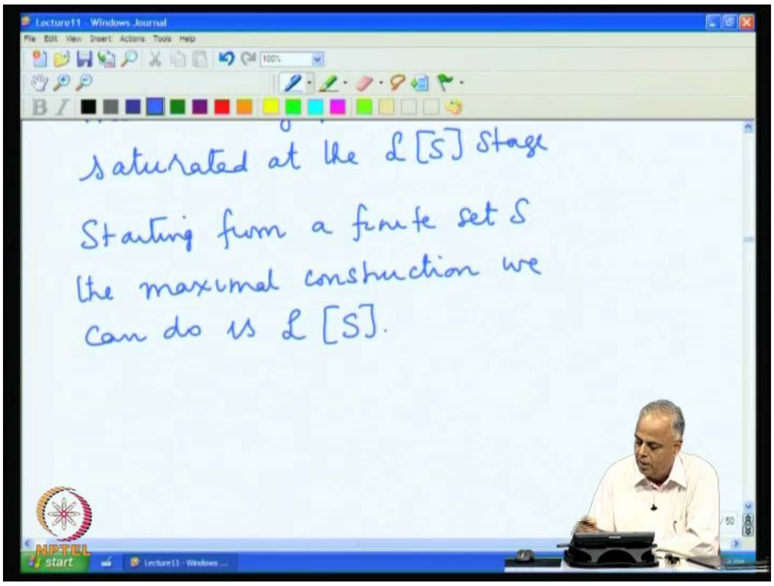

The building process has saturated at the L S stage, which means that starting from any finite set S in V, adjusting collection that we can build a out of this S is preciously L S. Then starting from S a finite set S, the maximal construction of building we can do then we do construction we use only the operation for addition and scalar multiplication that we should keep in mind because the vector space is nothing else we know we know that only the addition and scalar multiplication make the assumes at the vectors space so the maximal construction that we can do is L S there is a reason the construction was saturated.

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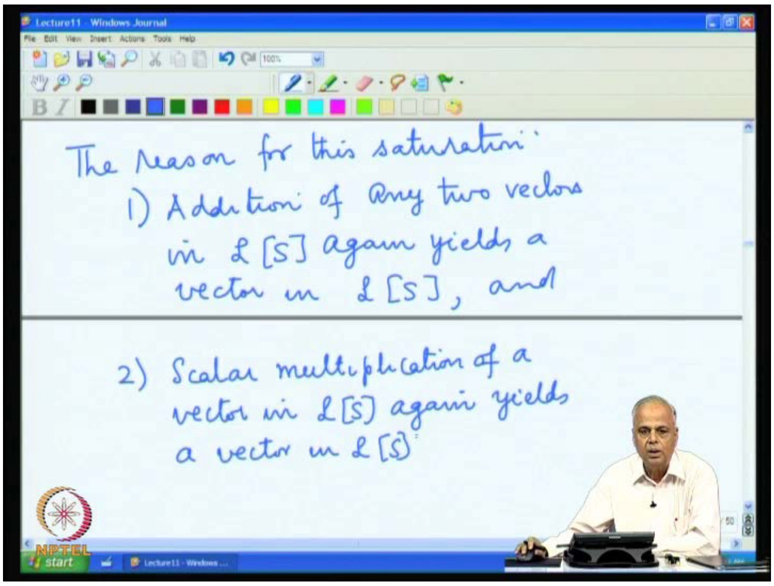

What make this saturation? What make this saturation happen? Now, if we try to construct the only things that we do is again, a repeat addition and scalar multiplication but, if we take the L S and if we take any two vectors in L S and we try to add them we have only add a vector back in L S. Because, linear combination plus linear combination is again linear combination similarly, if we take a vector in L S and scalar multiply again is going to be in L S so these two basic construction operations do not take a outside L S at all whatever construction you make and therefore, you are always in L S so the reason for this is the reason for this saturation a twofold in addition of any two vectors in L S again yields a vector in L S.

The second reason is scalar multiplication of a vector in L S again yields a vector in L S, there is a certain amount of solid wall created about L S, that you cannot outer the wall by using this basic operations of vector space addition and scalar multiplication, so we simply say L S is close with respect to addition and scalar multiplication you cannot open the door and get out by using only scalar multiplication and addition.

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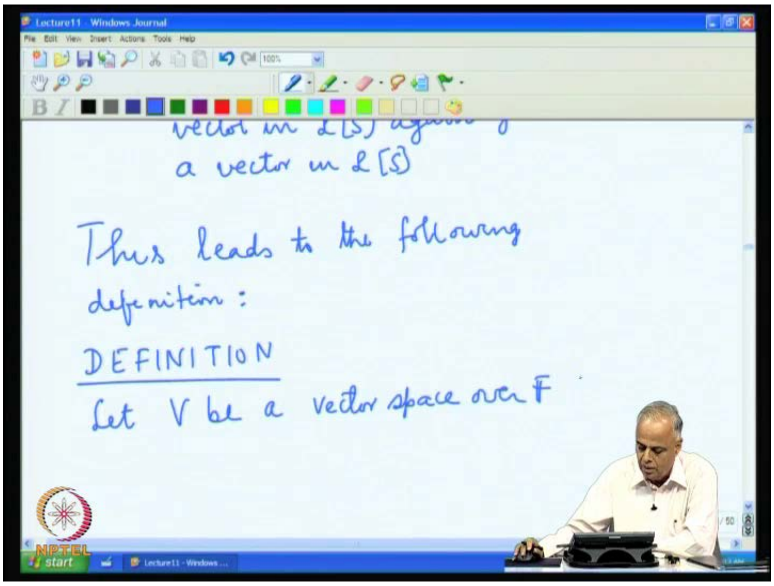

This leads to the following notion of sub spaces, this leads to the following definition any sub collection which is close in this sense with respect to this addition and scalar multiplication all. This is saturated as for as construction is concern you try to b e anything outer them you will be study or you will not be constructing anything new so we have this following definition let V be a vector space over a field F.

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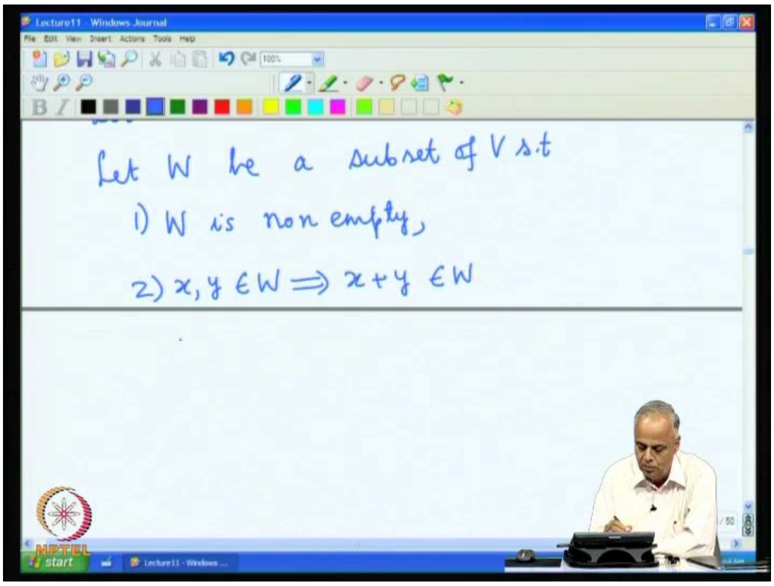

Now, we are going to replace L S by a general set let W be a subset of V. Such that 1 remember L S had lots of vectors, these 0 vector was in L S because, the 0 vector can be written as a linear combination of any set and we also add every S vector was already in L S L S contained vectors. Then it was non empty, we could like W to the non empty and the L S was close with respect to addition that in addition did in focus out of L S similarly, if we are in W if we take any vectors the sum must also being w.

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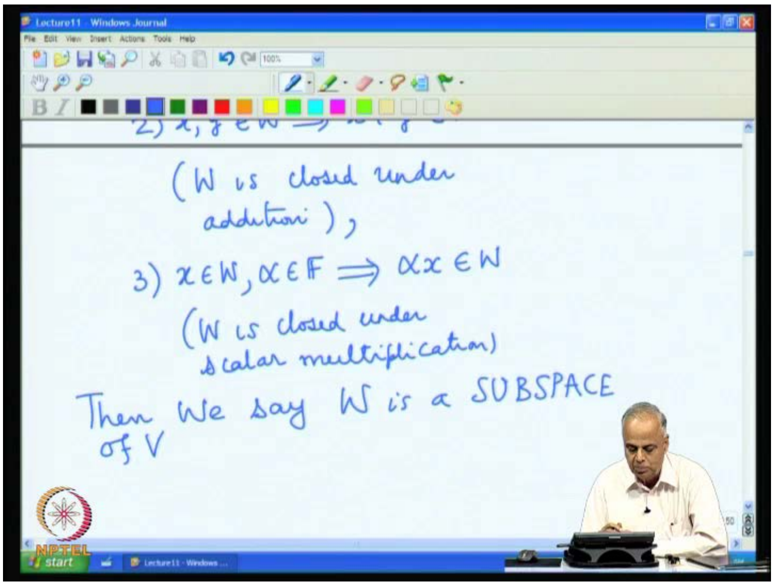

We say W is closed under addition and the third reason for if the other reason for the saturation namely closes with respect to scalar multiplication. That is we take any vector in W and if we take any scalar. Look at this scalar multiple that must be again in W. We say W is closed under scalar multiplication. If we start with a set in W which was these three properties namely that it is non empty. It is closed under addition and it is closed under scalar multiplication.

Then we say W is a sun space of V to make a sub space. First we do the non empty set the non empty set should be reach enough to contain all additions all sums that is equal to what is in that collection the sums should also already there and we should be reach enough to contain all scalar multiples that is if we take a vector there and multiplied by any scalar must be so it must be evaluate which collection of vector we make one or two simple observations about sub spaces.

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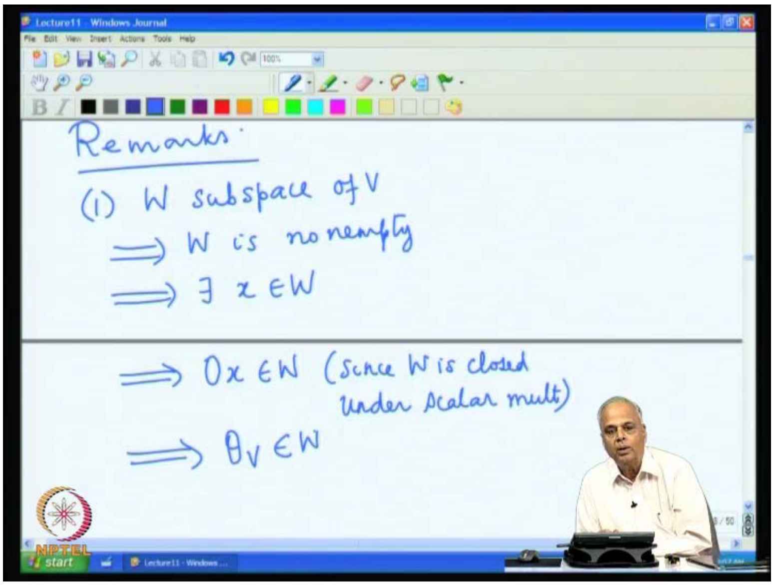

The first few remarks before we see some examples, we make some remarks the first thing we observe row is that suppose W is sub space of V, if W is subspace of V the first condition for the subspace is there is should be non empty what is it meant to say the W is non empty there was be at least some vector in W which says W is non empty which means there at least 1 vector x in W.

If there is a vector in W and if I multiplied by any scalar you should be in W because a sub space is closed under scalar multiplication so if I multiplied by the number 0 or the scalar 0 that must also being W since W is closed under scalar multiplication but, 0 x an exercise to verify that the 0 x is nothing but, this 0 vector belongs W therefore, so what we see here is that 0 vector whatever subspace we have 0 vector must belonging to it

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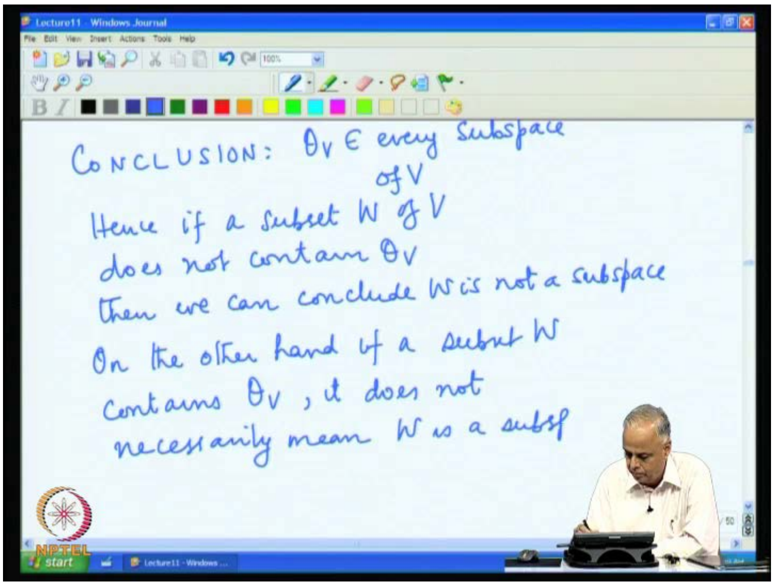

The conclusion is that theta V belongs to every subspace of V and therefore, hence if a subset W of V does not contain the 0 vector, if the subset W if we does not contain the 0 vector it automatically fails to be a subspace. Because, we have just seen in order to this subspace first followed every subspace must contain the 0 vector, then since it does not contain the 0 vector we can conclude W is not a subspace any subset, which does not contain the 0 vector fails to be a subspace on the other hand, just because some subset contain is 0 vector it does not make it as subspace. It is only a minimal requirement that is all on the other hand if a subset W contains the 0 vector theta V it does not necessarily mean W is a subspace.

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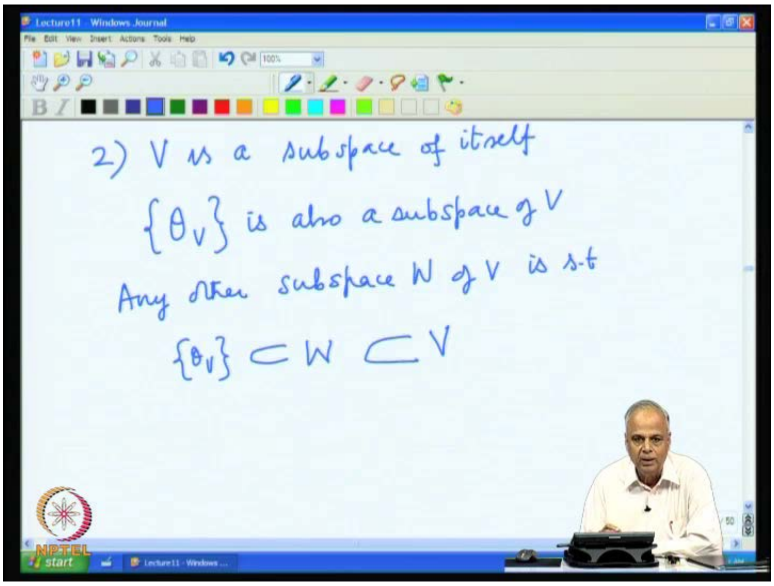

The second remark that we make to the following, we want to construct the subspace we want to look at a subspace. Now what is the thing that we have to make sure, first of all we must take some vectors because, we want a non empty set or we have to make sure that addition takes you back to the set. The scalar multiplication takes you back to the set, so it is say first way to do it is take everybody every vector in the set V we collect. We know about, whatever vector in V a choose there summing there this scalar multiplication, then it is the action of the vector space and therefore, V is a subspace of itself this V this is constructing by a very safe way, that is totally very thing and make sure that the entire, we do not get out of it on the other hand L S.

It is section, we already seen that the 0 vector must belongs to all these subspaces so just take that and construct a set W which confuse only of this 0 vector and that is also a subspace of V these are the towards subspaces every subspace must have at least the 0 vector so take only that and every must be a part of this so any other sub space any sub space W of V which is different from these two is such that it may certainly be inside V because its sub space and the since the theta V belongs to a every subspace it must be like this so these are the time to extremes of sub spaces being the largest subspace and theta V will be smallest subspace.

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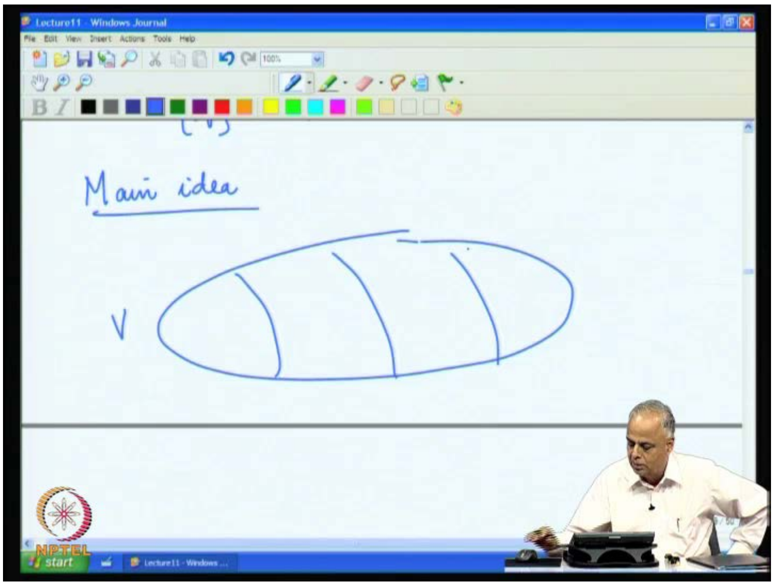

The main idea in analysing a problem in vector spaces is we take the full vector space V and try to chart into small subspaces.

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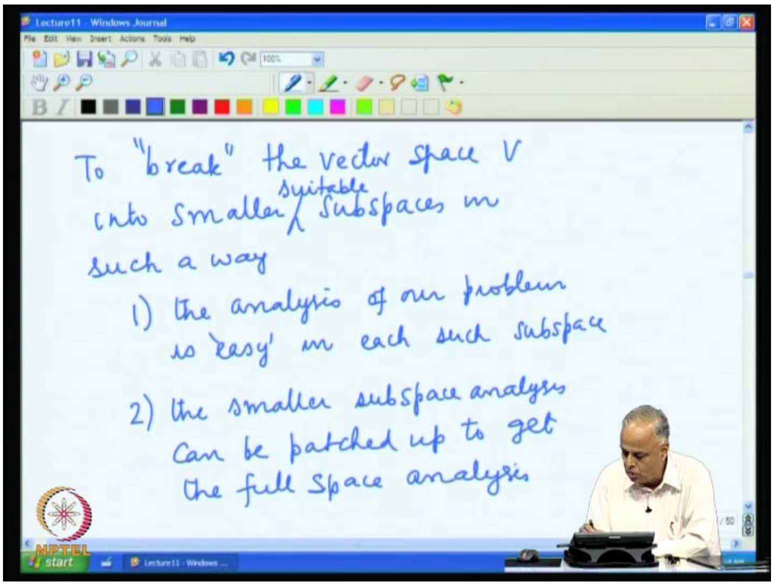

Our idea will be to break of decompose the vector space V into smaller subspaces in such, a way when we say smaller subspaces we do not take like, which are taken short chart in the vector space we may did in a suitable manner suitable subspaces. Such a way what we mean by suitable? That the analysis of our problem of our problem. It is easy each subspace we are broken in the subspace into smaller pieces.

Then, we analyse the problem and when we each one of this pieces we analyse the problem and the way we break should be such that, the smaller piece analysis becomes easy and number 2 from this easy analysis of the smaller pieces we must be able to patched up and get the full analysis the smaller the subspace analysis can be patched up to get the full space analysis most of your problems and analysis that is spaces will be an entire to find such breaking what are the mechanism of such breakings, how should be a break with reference to a particular problem? Basically most of our course will be devoted, what is are the breaking of vector space is possible? What is are the vector space breaking, we should look for builder the available and what conditions will there be available. If there available, how do we get them and that is that forms the core of vector space and linear algebra analysis.

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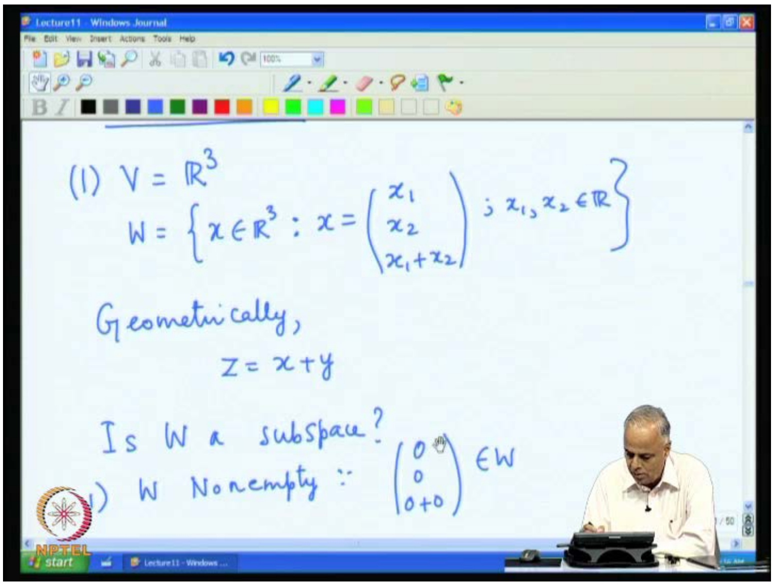

We shall, now begin looking at some examples or subspaces the first will start with some simple examples and move on to some very serious important subspaces that would be needed in our analysis. The first example, we take is always in R 3 is the word in which will the 3 dimensional well up R 3 we take V equal to R 3 and let us take W to be the set of all vectors in R 3, which are such that there are the form the first component on the anything.

The second component can be anything that the third component must be the sum of the first and second components of course, all components must be row so we have a set of all vectors for with the third entry the some of the first two entries geometrically speaking we can think of this let x be a coordinate system then it is the Z equal to x plus y it is a plane passing through the origin, what we are looking at is the plane passing through the origin but, let us smoothing algebra is W a subspace now for this we have to verify three things one is it non empty W is non empty. Because 0 0 0 plus 0 belongs to W look at this is of the form x 1 is 0 x 2 is 0 and x 1 plus x 2 is again 0 therefore, W is non empty the 0 vector.

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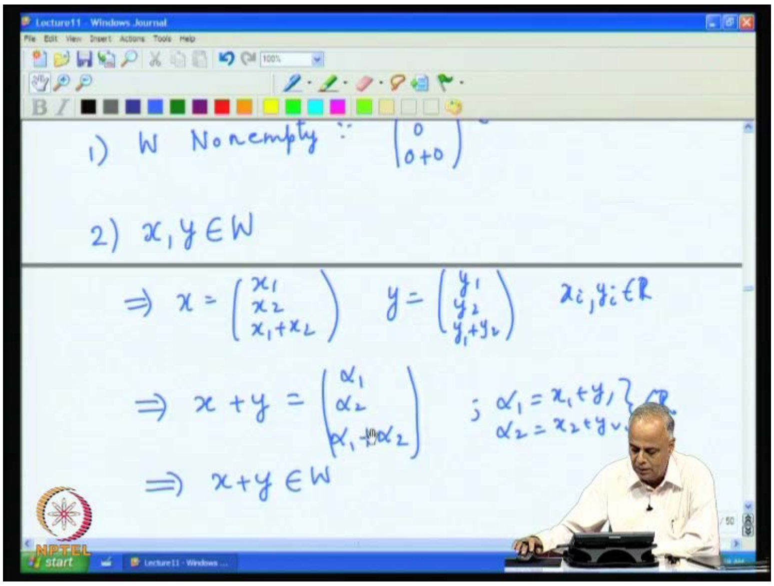

If we take, two vectors in W then next thing that we have to verified if that it is closed under addition, if we take two vectors in W a geometrically what is this mean these 2 vectors are in this plane. We take two vectors in the plane there some is also in the plane that is what we are going to verify algebraically, x must have this form x 1 x 2 x 1 plus x 2 y must have the form y 1 y 2 y 1 plus y 2 and of course, all these x is and y is belong to know why add this 2 vectors what we are get x 1 plus y 1 I will call it as alpha 1 x 2 plus y 2 I call it as alpha 2 and then I get x 1 plus y 1 which is alpha 1 plus x 1 plus y 2 which is alpha 2 where alpha 1 is x 1 plus y 1 alpha 2 is x 2 plus y 2 now since x 1 and x 2 are real numbers x 1 plus y 1 is real number x 2 plus y 2 is real number so these are all real numbers and this is again that without is the sum of the first 2 is says x plus y belongs to W.

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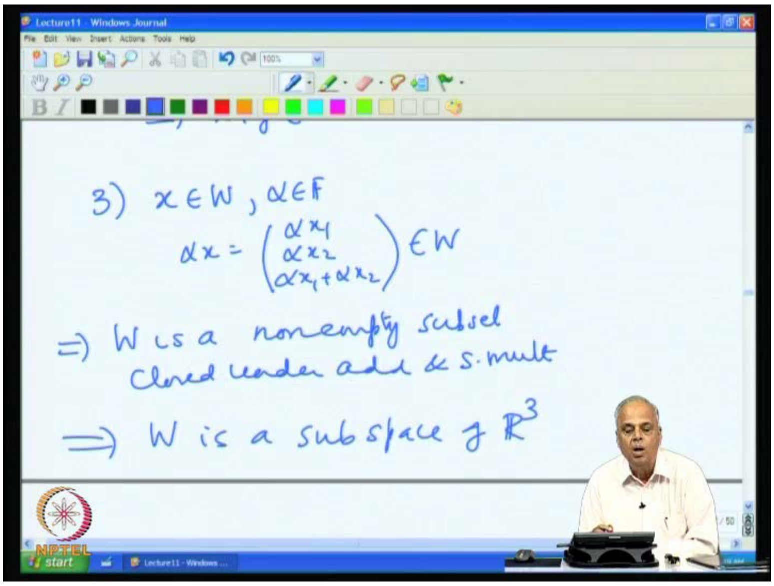

Analogously we see that, if x is in W alpha is in F alpha x will be the vector alpha x 1 alpha x 2 alpha x 1 plus alpha x 2 which is again in W and therefore, the first condition first this is W is non empty, the second one verified. Choose the W closed under addition and the third one is verified that W is closed under scalar multiplication. So, W is a non empty subset closed under addition and scalar multiplication which means W is a subspace of V which will more examples of subspaces in the next lecture.