Advanced Matrix Theory and Linear Algebra for Engineers

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Lecture No. # 10

Linear Independence and Subspaces

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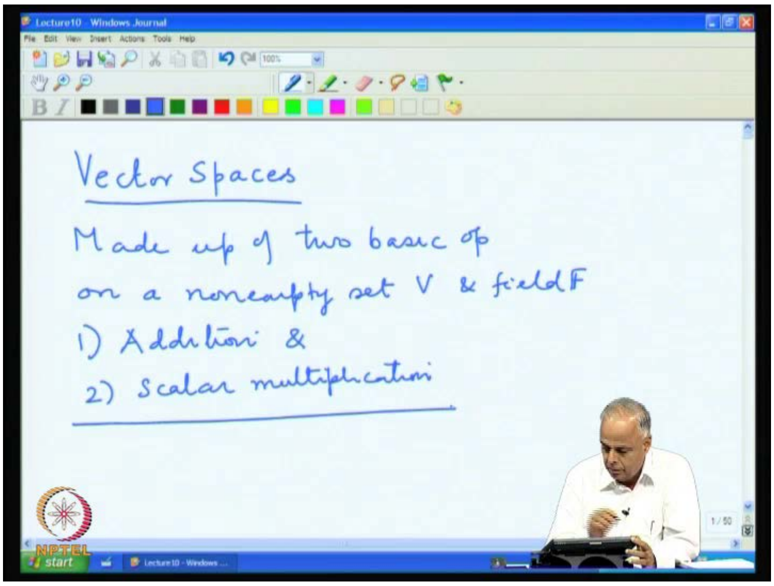

In the last lecture, we looked at the most important notion of vector spaces. Vector spaces are like universe in which we carry out linear algebraic calculations. We found that the vector space is made up of two basic operations on a non empty set V, and a field F, the two basic operations are addition and scalar multiplication. We had the various laws connected with addition and scalar multiplication.

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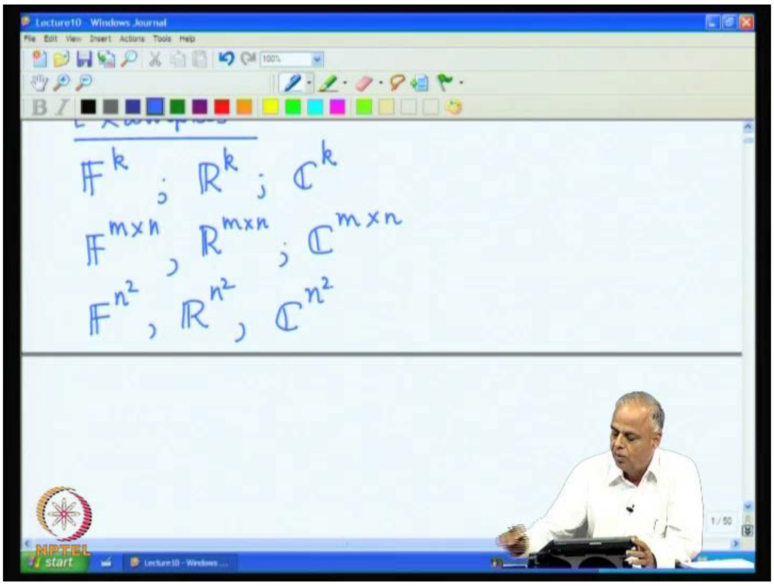

Then we looked at a number of examples of vectors spaces among them, where these standard models and examples of F k, the collection of k by 1 vectors over a field F in particular, if we take F equal to R we get the real vectors, and if we take F equal to the complex number field, we get the C k. Then we had the model of the m by n matrices of m cross n, and in particular if we take the field to be the real field we get the collection of all m by n matrices, the real matrices which form a vector space on analogously, if we take F equal to C we get the vector space of complex matrices. In particular, if we take m equal to n, we get the vector space of all square matrices of size n by n; similarly, when we take F equal to R, we get the collection of all real square matrices, when we get F equal to C we get the collection of all complex square matrices.

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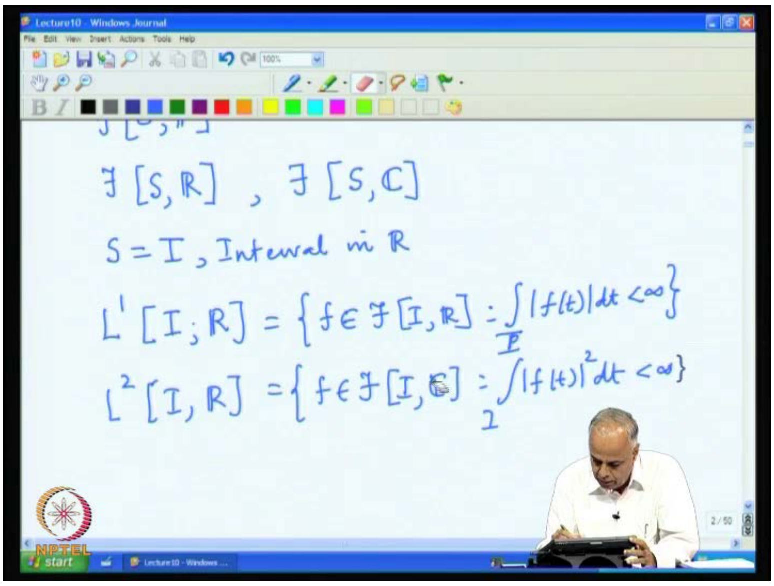

The next collection of examples we saw where functions. So, we will looked at a set S, and we will looked at a field F, and we will looked at all functions from S to F, and we are the standard point wise addition, and point wise scalar multiplication as the operations, and this was a vector space over F. In particular, if we take F equal to R, we get the collection of all real valued functions on S which forms a vector space over R, and if we take the F equal to C, we get the collection of all complex valued functions over S, and this forms a vector space over the field C.

In particular, if we take S to be an interval in R, then we looked at two special types of functions; one was L 1 I R, which was the collection of all functions which belong to the collection of all functions from I to R, such that their integral over I of mod f(t) dt is well defined and is finite. And similarly, we will looked at L 2 I R of all functions F from the interval I to C, such that the integral mod f t square d t is finite. They should read are we are looking at real valued functions; similarly, in place of R if we take C we will get complex valued functions in L 1, complex valued functions forming L 2.

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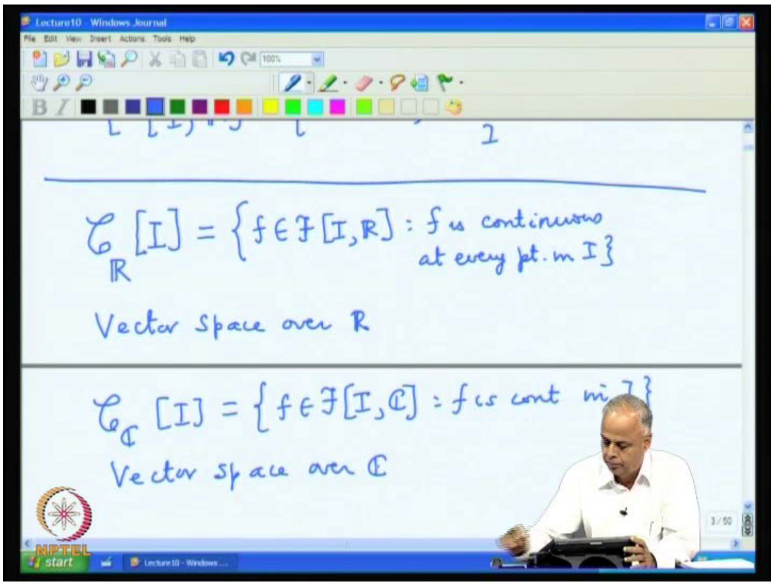

We shall now, look at more examples of functions and their vector spaces, again we look at the collection of all functions, which are defined and an interval, and which take real values and the C stands for continuous. So, the although functions which are in the collection of functions from I to R; such that, f if continuous at every point in I. Now, with usual addition laws for functions in scalar multiplication laws for functions, this is a vector space over R. Similarly, if we take the collection of all complex valued functions their functions from I, but the take complex values and f is continuous in I, then this forms a vector space over C. We shall look at one or two more examples, which are useful in analysis.

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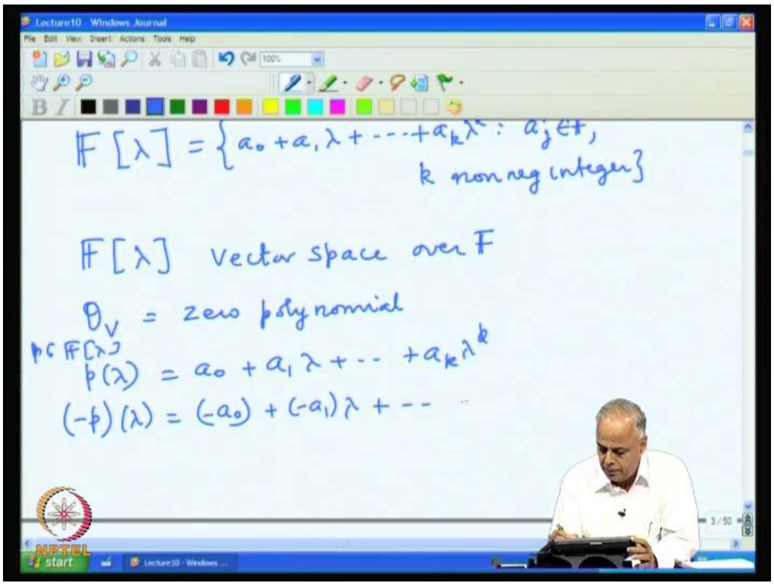

Let us look at the next example, which we denote by F a lambda this is the collection of all algebraic expressions of the form a 0 plus a 1 lambda plus a k lambda k, and as we vary this coefficients over f and vary k over all non negative integers. So, collection of all these expressions which we get by varying the a 0, a 1, a 2, a k's over f, and varying k over the non negative integers; such expressions are called polynomials in lambda, and we add the polynomials as normal functions are added, and we multiply a polynomial by a scalar as normal functions are multiplied by a scalar.

Then this F of lambda the collection of all polynomials forms a vector space over F and the 0 vector, in this vector space is the 0 polynomial and if we have a polynomial just like in functions. If we have a polynomial a 0 plus a 1 lambda plus a k lambda power k t is in f lambda, then the minus p is the polynomial, define as usual which is minus a 0 plus minus a 1 into lambda plus etcetera minus a k into lambda to the power k.

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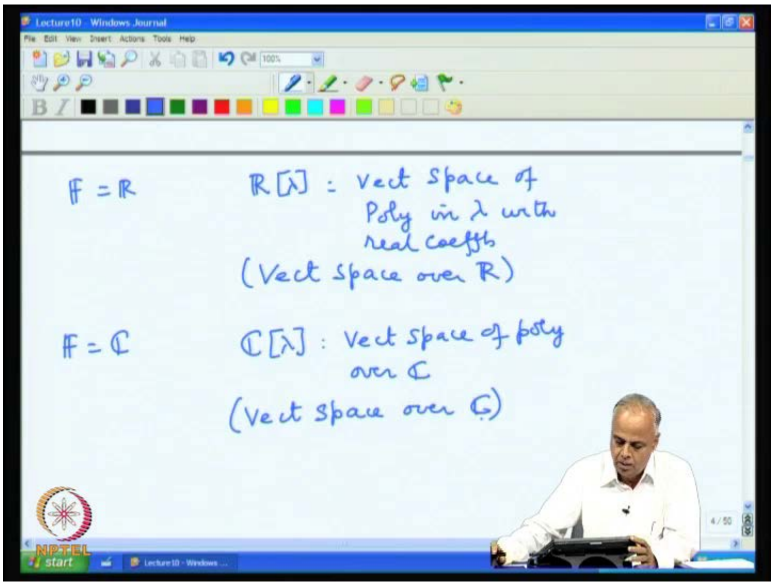

In particular, if we take F equal to R we get R lambda polynomials with the vector space of polynomials in lambda with real coefficients and this is a vector space over R. The scalars have to be taken as R and if we take F equal to C, these are 2 important fields which will always be looking at then we get C lambda. The vector space of polynomials over C when we say polynomials over C we mean the coefficients are C and this is a vector space over c, the next example is a special version of polynomials.

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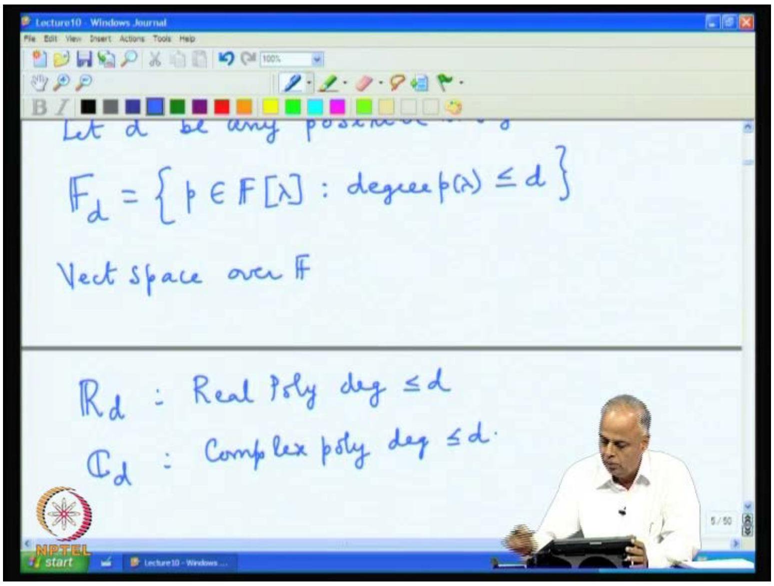

So, let d be any positive integer for example, we can take d equal to 4 or 5 or whatever positive integer we need. So, let d be a positive integer and let F d denote polynomials first. So, their all in F lambda, but they are special polynomials their degree of the polynomial p lambda must be less than or equal to d. So, take the collection of all polynomials of degree below certain threshold level say d, then these collection of all such polynomials form a vector space.

The usual laws of addition and scalar multiplication over F again, the repeat if we take F equal to R we get the polynomials, we will call simply I will call it real polynomials degree less than or equal to d and we get C d, if we take F equal to C. The polynomials with complex coefficients will simply write complex polynomials with degree less than or equal to d.

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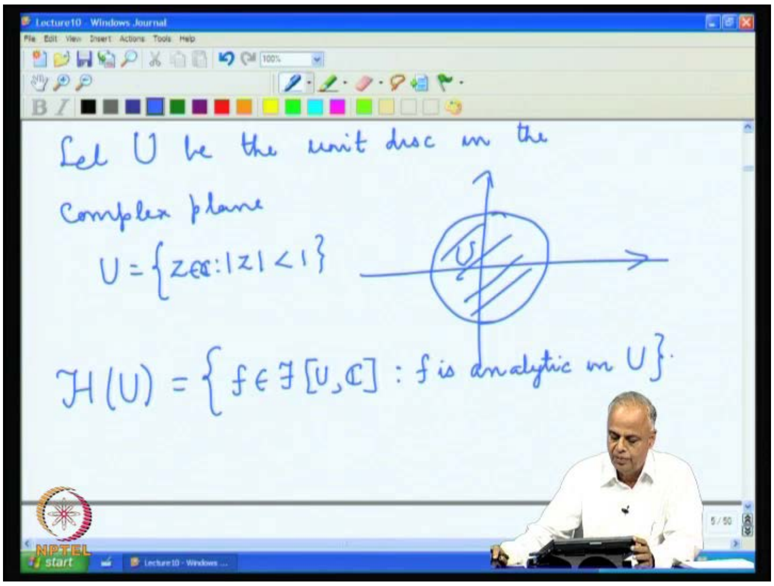

We shall conclude our discussions of these examples with 1 more important example, we will be coming across many more examples little later, we will take 1 example from complex analysis and 1 example from Fourier transforms. Now, let U be the unit disk in the complex plane. What do you mean by the unit disk? U is the collection of all the complex number z, such that mod z is less than 1. So, these are all complex numbers with means you just take the units circle in the complex plane, and look at the interior that is our unit disk.

Take the circle of unit radius about the origin and look at the interior of that circle and that is called the unit disk then the H U, we shall look at all functions which are from U to C these are functions from U to C, but we are not going to look at all functions from U to C. You are going to only look at functions U to C, such that f is analytic in U or whole a in u, f is analytic in... Then since these are functions we will know how to add function, we know to multiply scalar and function.

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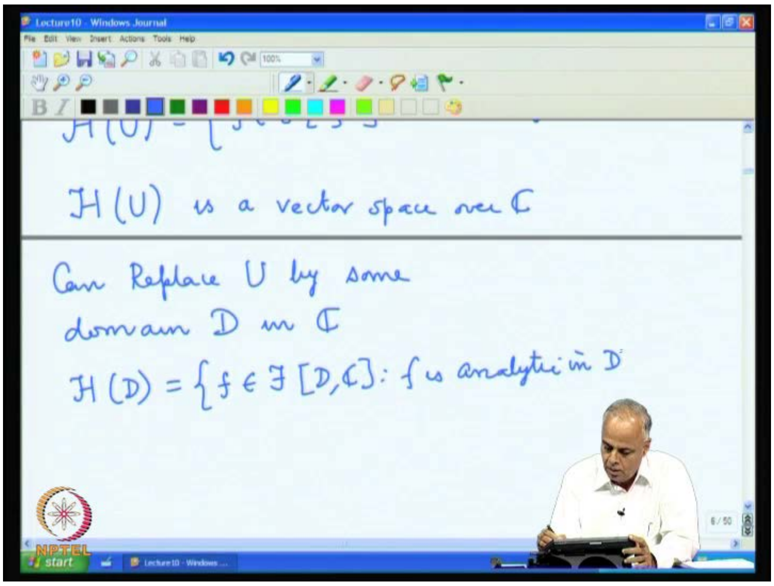

And With these laws H of U the sum of 2 analytic functions is an analytic function, if you multiply an analytic function, there are complex constant there is again analytic function. So, H U is a vector space over C. Now, we can replace U by some domain D in the complex plane C, and then we get H D the collection of all analytic functions, which are functions from D to C. Since, that f is analytic in D like this it is very useful in complex analysis 1 final example as we said on Fourier transforms.

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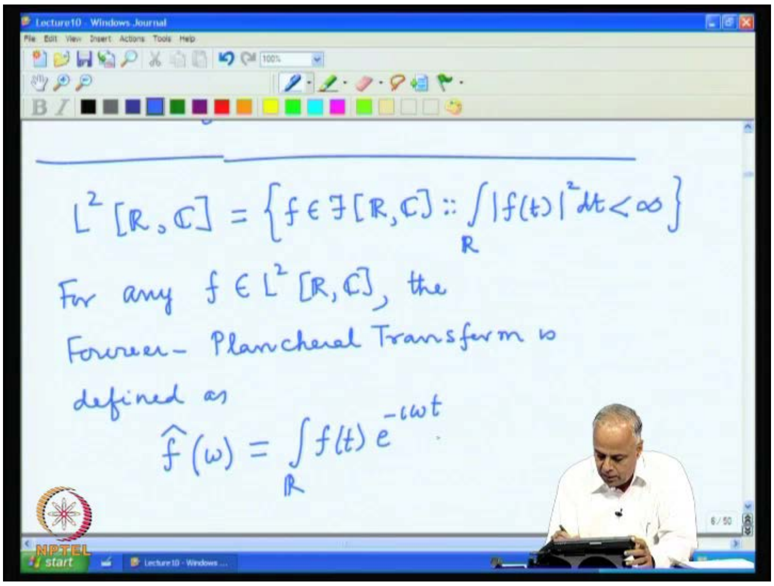

Let us look at L 2 R, C. What do we mean recall these means, we are looking at all functions from R to C that means, they are define for all values of the real number t. So, f of t we have a function of t where t can vary from minus infinity to infinity over the real numbers and the f of t. The value of the function at the point t is a complex number. So, it is complex valued function of a real variable and we are not going to look at all such functions these L 2 stands for those functions for which, if I take the mod f t square d t, but is integrable one has to be technical.

Here technical it has to be integrable and that mod f t square which is labag integrable, the integral is well defined and the integral is less than infinity, this we will call us L 2 R, C. Now, look at all these functions for any f in L 2 R, C. The Fourier transform actually should be called the Fourier Plancherel Transform is defined as we denote the Fourier Plancherel transform by f hat, which a function of omega defined as integral over R which is integral between minus infinity to infinity f of t e to the minus i omega t dt. Now, technically again we will not get into the details of the definition of the Fourier Plancherel transform, the integral has to be interpreted as a integral as a limit in the mean. Now, once we have this Fourier transform well defined.

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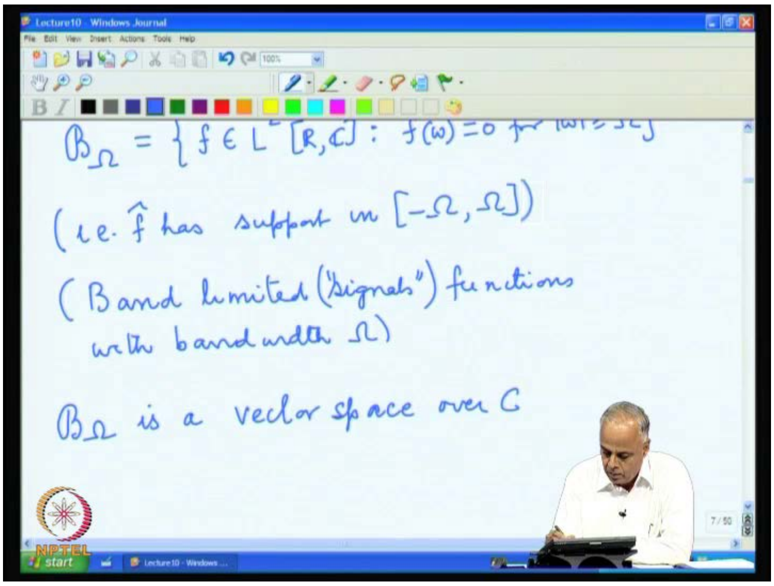

We look at a positive number omega. So, let omega be a positive a real number and look at B sub omega, which all those functions in L 2 R, C. So, we are looking at not all the functions in L 2 R, C. We are looking at all those functions in L 2 R, C for which the Fourier transform is 0 for mod omega greater than or equal to omega; here omega is again a real number, these function we take it as defined for all omega belonging to R.

So, we are looking at all those functions in L 2 R for which the Fourier transform f 0 beyond a certain stage, we say that the Fourier transform has support in the interval minus omega omega, that is f has f hat has support in the interval minus omega omega outside this, it is 0 such functions are called band limited functions. In the context of signal processing these are called band limited signals with omega as the band width.

So, these are band limited signals or band limited functions with bandwidth omega, and since, these are functions we know how to add functions, we know how to multiply a function by a scalar. And we see that these form a vector space B, omega is a vector space over C; these class of functions band limited functions come in handy in signal processing, and in Shannon's sampling theorem. Now, we have seen a number of examples are vector spaces, we shall be looking at more examples as we go on, but for the moment we shall have these examples in our bank, and try to use them whenever we need.

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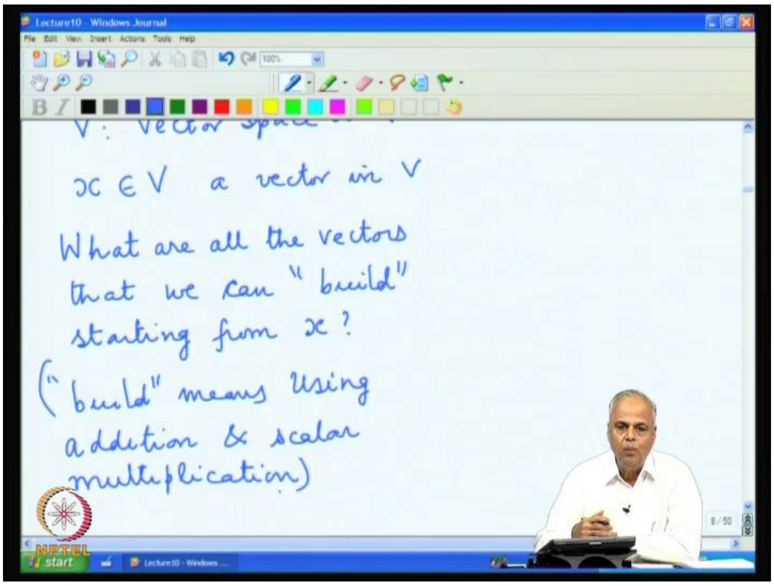

We now introduce another important concept, in the first important concept in vector spaces known as Linear Combination. What do we mean by Linear Combination? Let us consider a field F, F is any field and V is a vector space over F. So, we have vector space over F, let us look at a vector X in V. So, X is a vector in V. So, we are a vector in a vector a fixed vector X in the vector space V, and now we see what are all the vectors, what are all the vectors that we can build that we can build starting from X.

Now, this question has to be explain further, what do we mean by build? We have to start from X do something and then produce another vector; that is a vector, which we says build from X. Now, what can I to do X, the only 2 things that I note to do in a vector space R addition, and scalar multiplication. So, build really means using the 2 operations in the vector space namely - addition and scalar multiplication.

So, start with a vector X in V, and see what are all the things, such we can build using that X, and think that you are allowed to build are the only thing that you can do in that universe; in that universe of that vector space the only things that we known to do are the addition, and the scalar multiplication. These are the algebraic things that we have introduced, and these are the only things that we can do.

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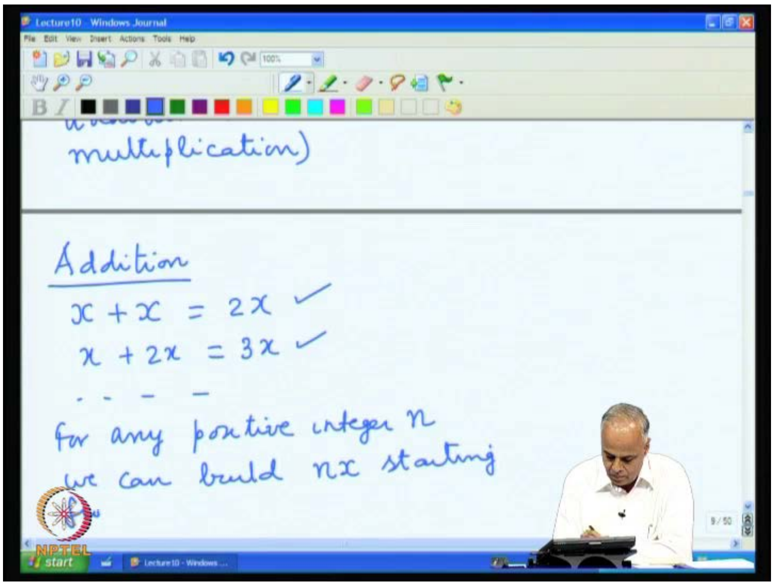

Now, let us look at addition. What can we do starting from x? You know only x, we can use only x. So, we can use x any number of times. So, you can use x and x. So, I will get 2 x. So, 2 x is a vector which I can build using x. Now, I can add x to 2 x, and I will denote this 3 x, and so I can build 3 x; continuing this way I will say for any positive integer n, we can build n x starting from x.

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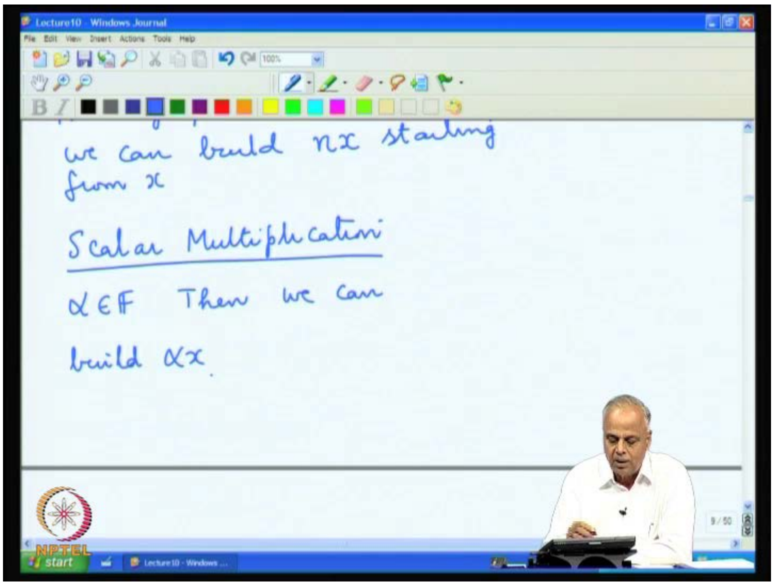

So, that is the first building process, we shall now look at what we can do using scalar multiplication. Now, we see we already have 1 type of scalar multiplication generated by addition namely n x. Now, instead of n scalar multiplication says we could have chosen any alpha in the field. So, let us look at alpha in F, then we can build alpha x, as x as alpha varies your F we get a large collection of vectors, and we see that this encompasses the n x which we have got already. So, addition can also be subsumed in this a scalar multiplication.

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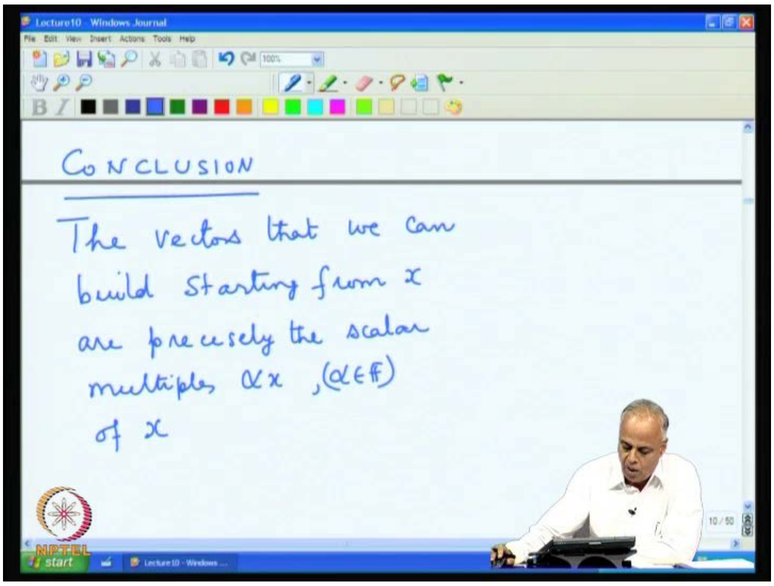

So, with conclusion is the vectors that we can build starting from x again using only the vector space operation of addition, and scalar multiplication are precisely, the scalar multiples in fact all the scalar multiples alpha x alpha belonging to F of x. So, starting from an x all that we can build are the scalar multiples of x. So, that is 1 step where we start from a small thing, and try to build as much information as possible then at next level.

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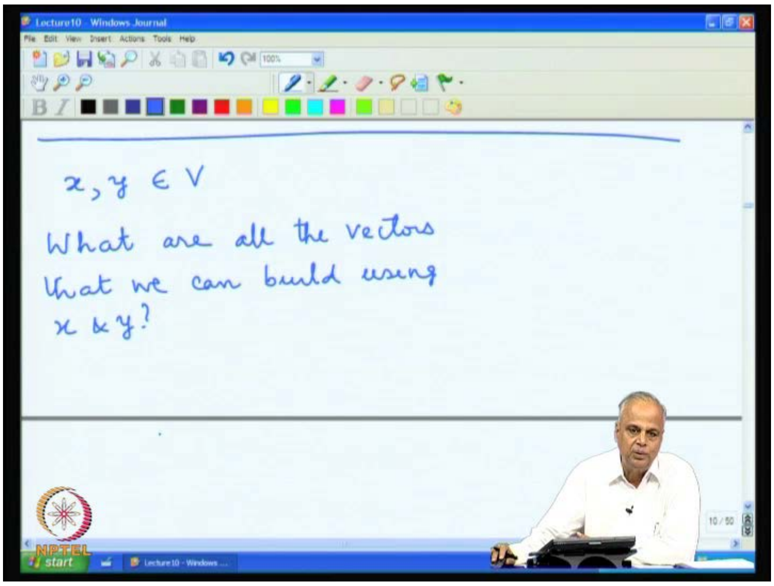

Let us start with 2 vectors x and y in v. Now, what are all the vectors that we can build using x, and y. Once again I repeat, when I say build the only 2 operation - the only thing that we can do this vectors are either add or do scalar multiplication.

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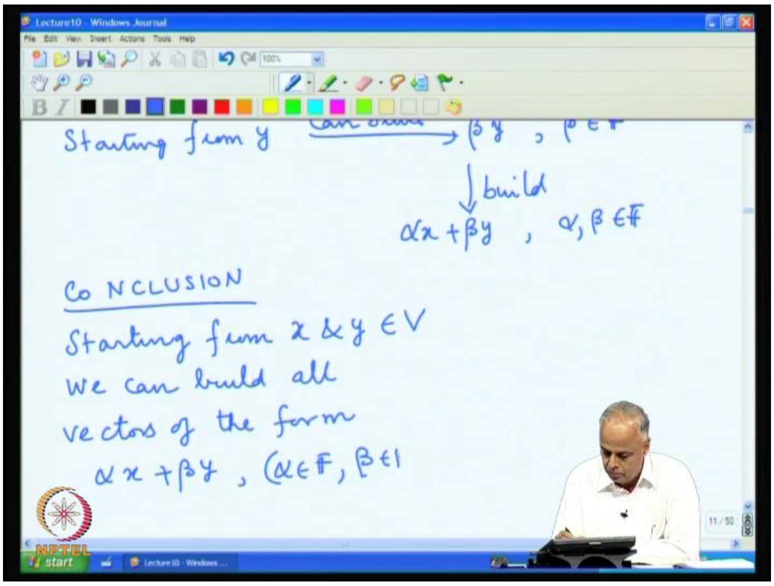

Now, let us look at the answer to this question starting from x, we have already seen that we can build alpha x their alpha there is over F. Similarly, starting from y we can build, so not all for x, we use some other notation beta y your beta can vary in F. Now, once where F this alpha x and the beta y's, we can add them and then from this we can build alpha x plus beta y where alpha, and beta belong to F. So, thus we see starting from 2 vectors x and y. So, the conclusion is, but starting from x and y in v starting with 2 vectors x and y in v. We can build all vectors of the form alpha x plus beta y where alpha belongs to F beta belongs to F.

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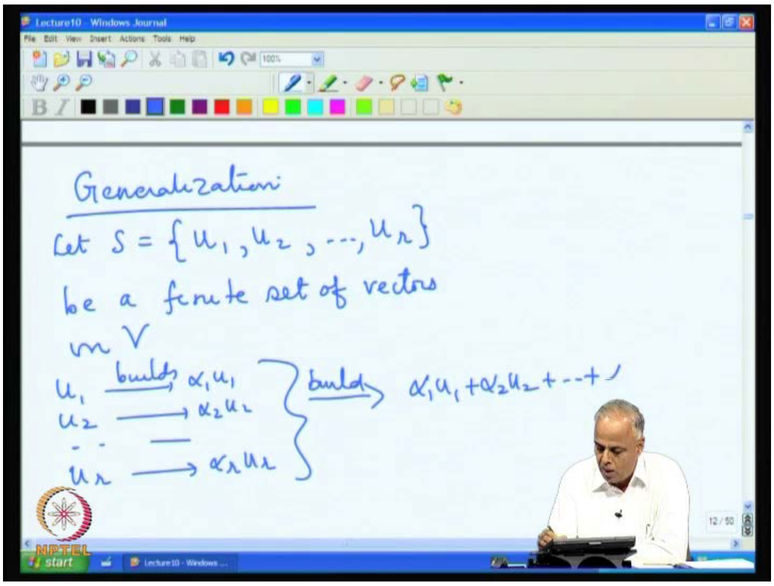

Now, extend this idea we see that if we have in general instead of 1 or 2, if we have a finite number of vectors. So, let S be u 1, u 2, u r be a finite set of vectors, in the vector space V then what are all the vectors that we can build using u 1, u 2, u r obviously you can build I for 1 u 1 from u 1, alpha 2 u 2 from u 2. So, u 1 builds alpha 1 u 1, u 2 builds alpha 2 u 2 and so on. u r builds alpha r u r. So, now these together build alpha 1 u 1 plus alpha 2 u 2 plus alpha r u r. So, the collection of all the vectors that we can builds starting from u 1, u 2, u r the vectors that we can build using the vector space operations are precisely, those vectors which are of the form alpha 1 u 1 plus alpha 2 u 2 plus alpha r u r.

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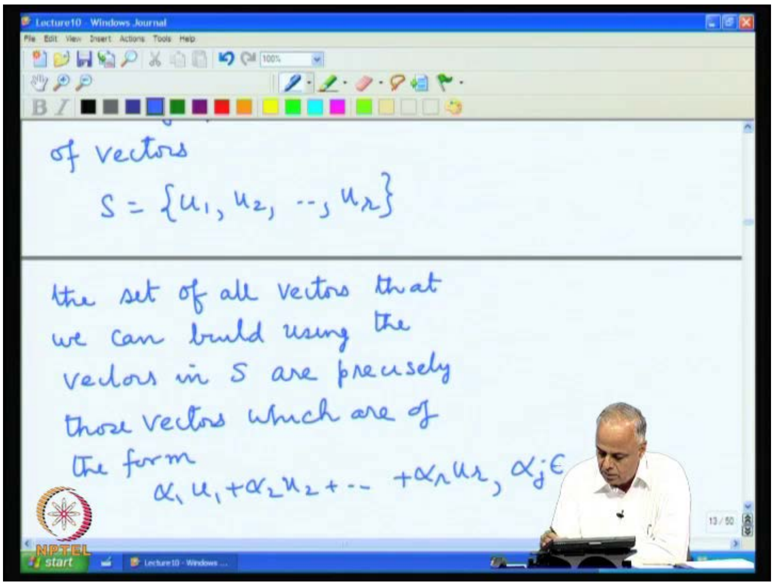

So, starting the conclusion of all our discussion, so far is that starting from a finite set of vectors S equal to u 1, u 2, u r; there are of course, a positive integer the set of all vectors that we can build using the vectors in S or we will call them S vectors, using the vectors in S or preciously those vectors, which are all of the form alpha 1 u 1 plus alpha 2 u 2 plus etcetera alpha r u r, where the alpha j's are all in F. So, we have a large collection of vectors, which we can pull starting from these S vectors. Now, we give a name for all these vectors or the type of vectors that we can build from u 1, and that is what is known as the linear combination of the vectors.

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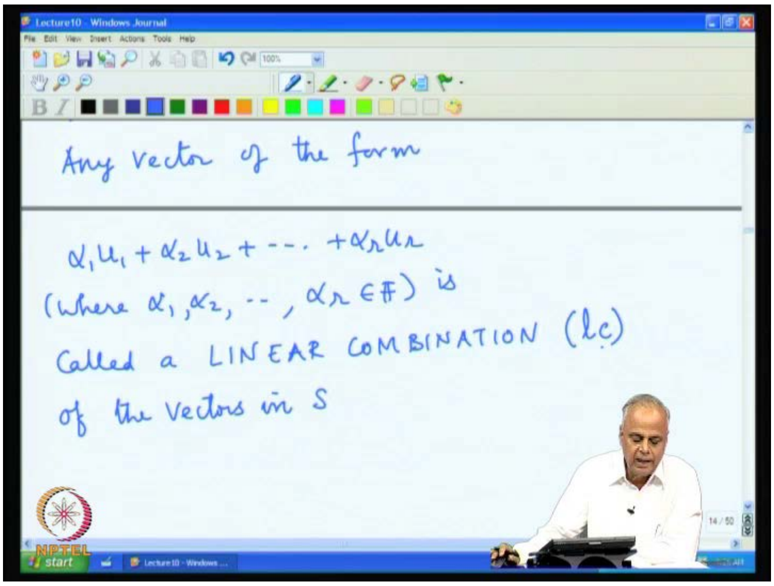

So, we have a definition let S equal to u 1, u 2, u r be a finite set of vectors in v, because we now have our universe as V vector space over a field F, this should be our universe all the work will be done in a vector space V over a field F. So, let us take a finite set of vectors u 1, u 2, u r in a vector space V any vector of the form, any vector of the form alpha 1 u 1 plus alpha 2 u 2 plus etcetera alpha r u r; where alpha 1, alpha 2, alpha r belong to F is called a linear combination is actually an engineering terms, it is a super position of the S vectors a linear combination of the vectors in S, and now on for linear combination we will just write l c as the short form.

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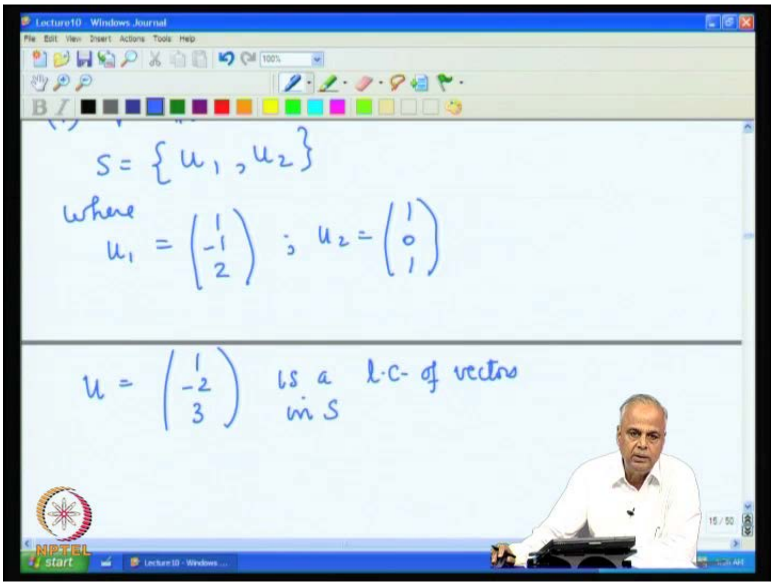

Now, let us look at some simple examples, let us take first example let us take the vector space V to be R 3, and let us take s to be a finite set. So, just 2 vectors where u 1 is the vector 1, minus 1, 2 and u 2 is the vector 1 0 1. Now, let us look at the vector u equal to 1, minus 2, 3. U is a linear combination of vectors in S, we claim this in order to claim this we must show that u can be written as the linear combination of u 1 and u 2 which mean, we must finds scalars alpha 1 and alpha 2 such that, u can be written as alpha 1 u 1 plus alpha 2 u 2.

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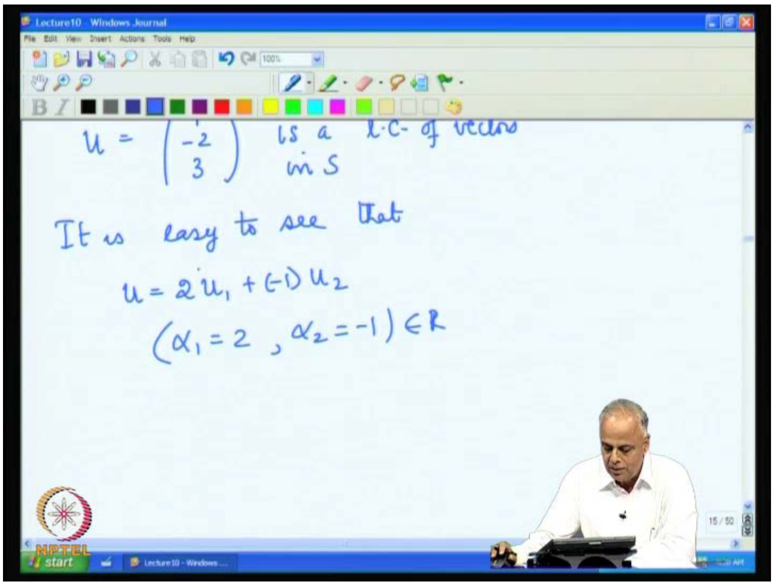

So, we will simply say it is easy to see not very difficult to check that u is equal to 2 times u 1 plus minus 1 times u 2; we see for example, 2 times u 1, 2 times u 1 plus minus 1 times u 2 gives as 2 minus 1, which is 1 minus 2 0 which is minus 2, and 2 into 2 four minus 1 is three. Therefore, it is easy to see that u is equal to 2 u 1 plus minus 1. So, in this case alpha 1 is 2 and alpha 2 is minus 1 and both are in R, see remember i vector space is R 3, and we must find scalars in the vector in the field F that is R, and does what we are found we are found alpha 1 and alpha 2 both real numbers.

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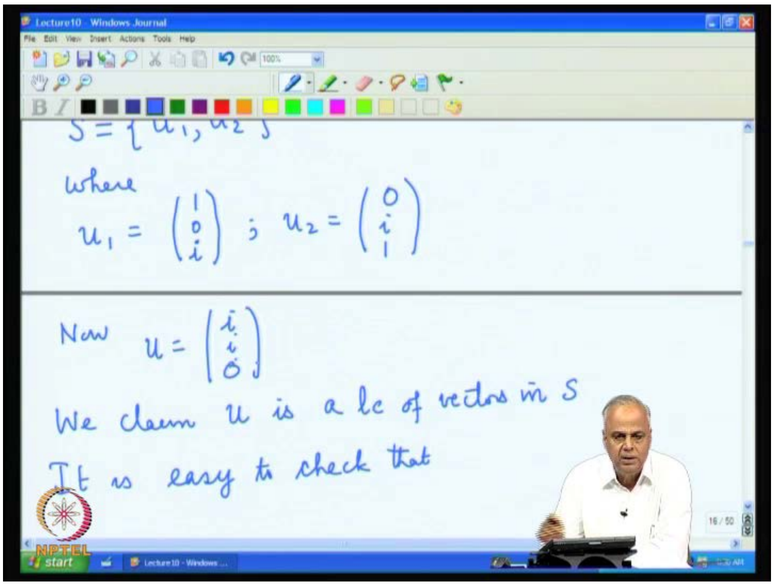

So, that is a first example let us look at our next example let us take the vector space to be the complex vector space. Now, c 3 and obviously of course, our field is this cases c. So, we have look at c 3 as the vector space over c, and now again we will look at a set s which consists of just say finite set. So, let us start again look at 2 vectors u 1 and u 2 where u 1 is 1 0 i. Now, the components can be (()) if can be complex because we are working over a complex space and u 2 is 0 i 1. So, we are a finite set vectors u 1 and u 2 where u 1 and u 2 are (( )).

Now, look at the vector u which is 0 i i is again a vector in c three, because we are allow to have complex vectors we claim u is a linear combination of vectors in S for this we have to find 2 scalars alpha 1 and alpha 2. Now, the scalars are allowed to be complex numbers, so we have to find 2 complex numbers alpha 1 and alpha 2 such that alpha 1 u 1 plus alpha 2 u 2 is equal to u, again it is easy to check that I am sorry, let me make a slight correction here this should be i, and this should be 0 this much easier the calculations become easier.

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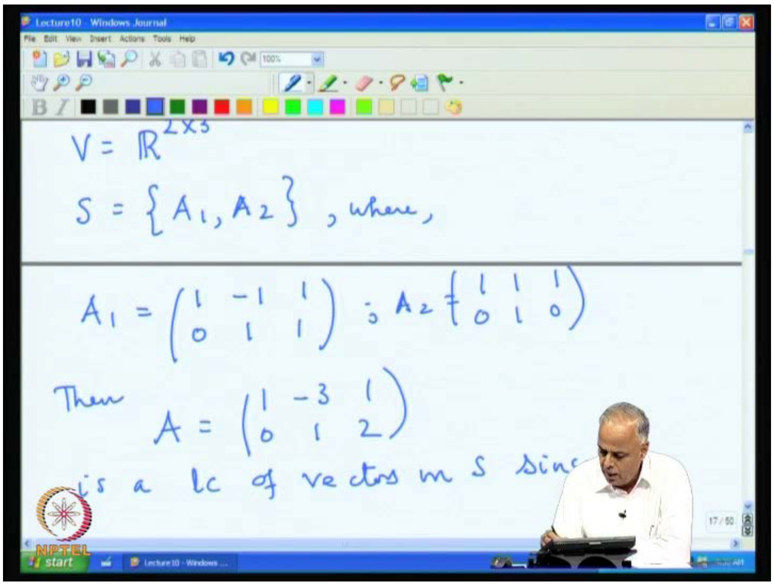

So, we have u is equal to i into u 1 plus u 2 or 1 into u 2 to be more precious i into u 1 plus 1 into u 2. Let us check this i into u 1 will give you i plus 0. So, that is i i into u 2 give 0 here, 1 into u 2 will give i. So, that is this i i into u 1 will give minus 1 here and if you had u 2 minus 1 plus 1 will give you this 0. So, u is equal to i u 1 plus i u 2. So, alpha 1 in this case is i alpha 2 is 1 both are in c therefore, u is a linear combination of vectors in S.

Let us look at another example let us take the vector space V to be the vector space of all 2 by 3 matrices again, we look at a finite set of vectors to matching things easy simple let us look at 2 vectors. Now, vectors are all matrices let us look at 2 vectors in this where A 1, if the matrix 1 minus 1 1 0 1 1 and A 2 if the matrix 1 1 1 0 1 0 then the matrix A, which is a vector in V, which is 1 minus 3 1 0 1 2 is a linear combination of vectors in S.

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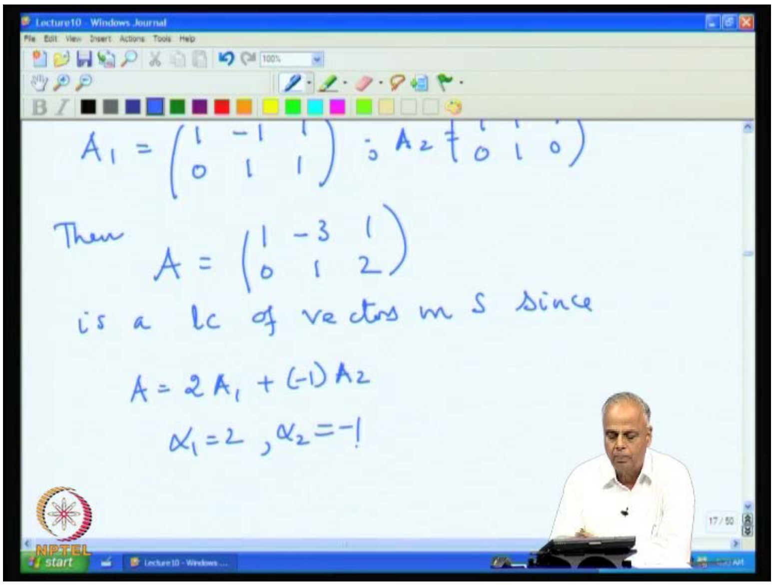

Since, again it is easy to check that A is equal to 2 times A 1 plus minus 1 times A 2. So, in this case alpha 1 is 2 alpha 2 is minus 1; and therefore, we are express A as a linear combination of the vectors in S.

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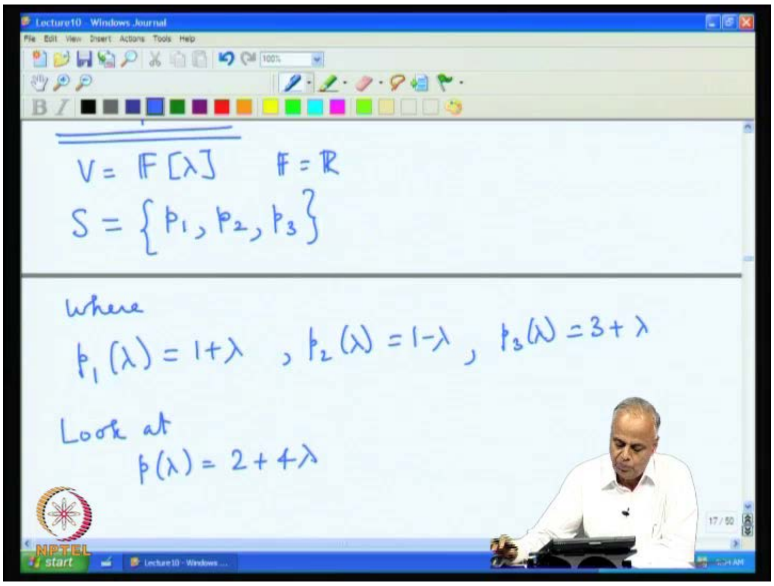

One final example, let us look at the vector space V to be vector space a polynomials in the variable lambda. Let us look at a finite set we should have vectors in V, but vectors in V are polynomials. So, we will say p 1, p 2, p 3 let us take a finite set of 3 vectors that is 3 polynomials, where p 1 is the polynomial p 1 lambda equal to 1 plus lambda p 2 is the polynomial p 2 lambda equal to 1 minus lambda p 3, the polynomial p 3 lambda equal to 3 plus lambda.

Now, let us look at p lambda which is 2 plus 4 lambda this again a polynomial it is coefficients R in R therefore, this is a polynomial. So, let us take F to be R to be much simpler. So, we have a polynomial over the realise, we have 3 polynomials; we have now looking at a fourth polynomial 2 plus 4 lambda.

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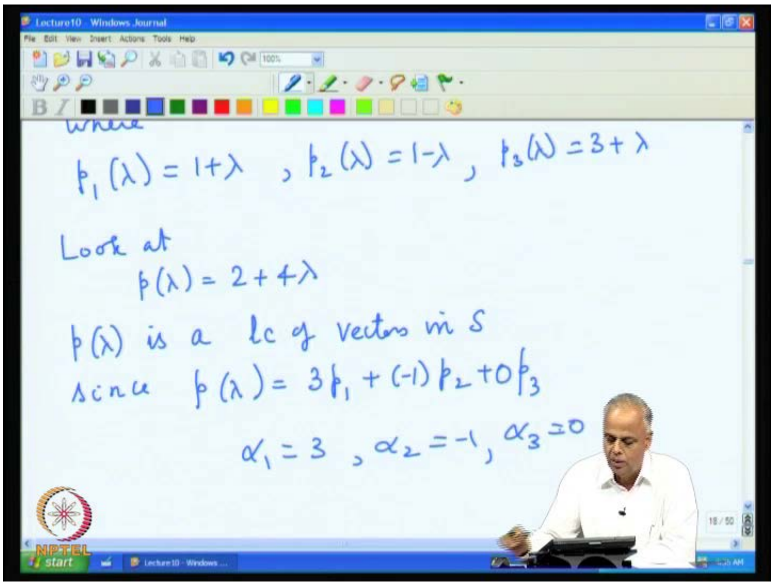

And p lambda is a linear combination of vectors in S; that is those polynomials in S. Since, again it is easy to check that p lambda is 3 p 1 plus minus 1 into p 2 plus p 3, again see for example, there 3 p 1 will give me 3 into 1 plus that is 3 minus p 1 will give me minus 1. So, 3 minus 1 will give me the 2, the 0 is not the I am sorry this is 0 times p 3 this would be 0 times p 3.

So, this is not contributing anything then look at the lambda terms 3 p 1 gives me 3 lambda, and minus 1 p 2 gives me another minus of minus lambda plus lambda. So, that gives me the 4 lambda, the third term has no contribution, so in this case where alpha 1 is 3 alpha 2 as minus 1 alpha 3 as 0. So, thus we have p lambda is the linear combination of the vectors in s.

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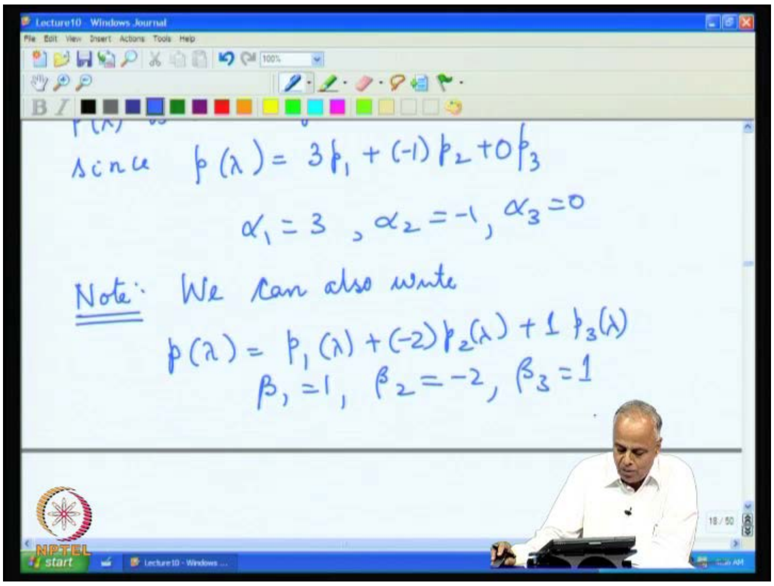

Note, we can also write p lambda the above p lambda namely this polynomial 2 plus 4 lambda as p 1 lambda plus minus 2 times p 2 lambda plus 1 times p 3 lambda; therefore, we see that the same polynomial p lambda is not only a linear combination of p 1, p 2, p 3 which can be written as this linear combination, it can also the written as this linear combination. So, we are now chosen beta 1 as 1 beta 2 as minus 2 beta 3 as 1. So, the same linear combination it may be possible to express as 2 different or more than 1 different ways as a linear combination of the vectors.

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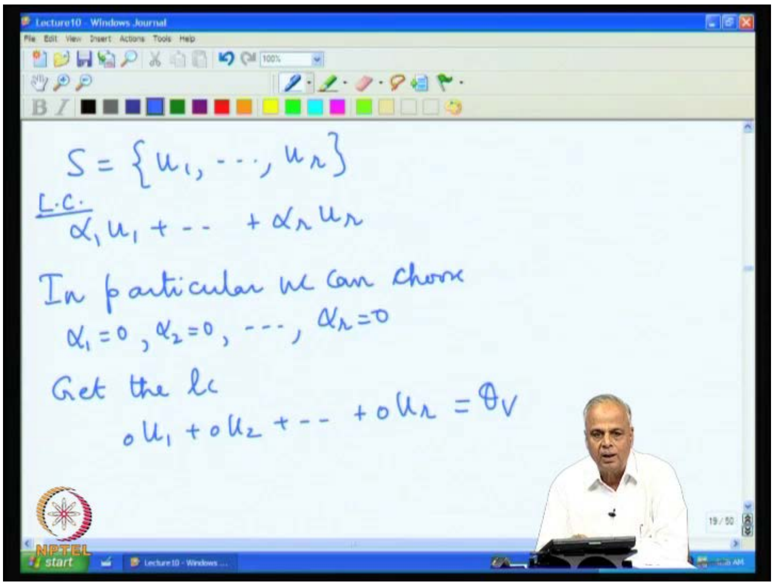

Let us pursue this idea a little bit to begin with, let us start with the vector space again this is our universe V is a vector space over F. Now, let us start with finite set of vectors u 1, u 2, u r and then look at linear combinations. What are linear combinations? These are vectors of this form alpha r, u r and where do I have to choose alpha 1, alpha 2, alpha r. I never to choose alpha 1, alpha 2, alpha r in the field F in particular, we can choose alpha 1 as 0 alpha 2 as 0 and so on.

Alpha r as 0; suppose, I choose all the alpha's to be 0 what do I get, I get the linear combination we get the linear combination 0 u 1 plus 0 u 2 plus 0 u r, which is a vector theta V, thus we say that whatever set we start with whatever finite state u 1, u 2, u r, we start within V. We can always get the 0 vector as a linear combination of these vectors. So, we can build the 0 vector from any finite set of vectors, so and therefore the 0 vector is not a much use in building process, because it can be anyway obtain from other vectors. Now, this particular linear combination that we have written in which all the coefficients alpha 1, alpha 2, alpha r as 0 is called the trivial linear combination.

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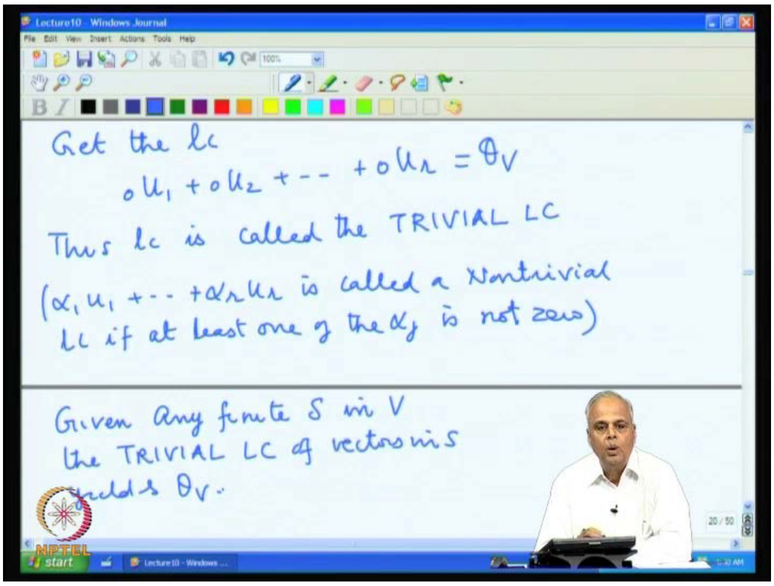

This linear combination is called the trivial linear combination that. What is the trivial linear combination? It is the linear combination which all the coefficients have 0 therefore, a linear combination we said to be non trivial linear combination give at least 1 of the coefficients is not 0. So, alpha 1 u 1 plus alpha r u r is called a non trivial linear combination, If at least 1 of the alpha j is not 0. Now, what we have observe is that given any, any finite set s in V the trivial linear combination of vectors in s yields. The 0 vector theta V; and therefore, we can always get the 0 vector and this form as a linear combination of any finite set of vectors.

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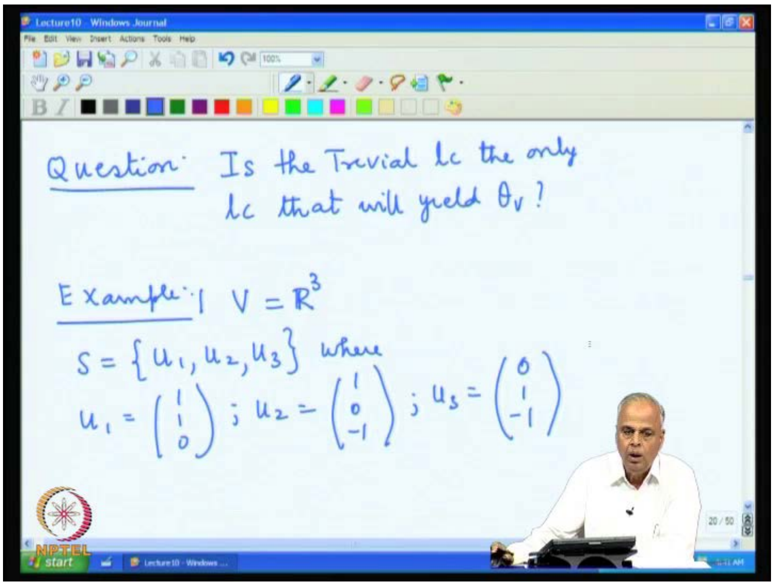

This raises the following question is the trivial linear combination, the only linear combination that will yield the 0 vector is the trivial linear combination. The only linear combination that will yield put 0 vector theta V, let us look at 1 or 2 simple examples before we look at answer this question let us take V equal to R 3, the worked space R 3 over F, let us take s to be u 1, u 2, u 3 a finite set of three vectors where, u 1 is 1, 1, 0 u 2 is 1 0 minus 1 u 3 is 0 1 minus 1. So, we are set of 3 vectors in R 3.

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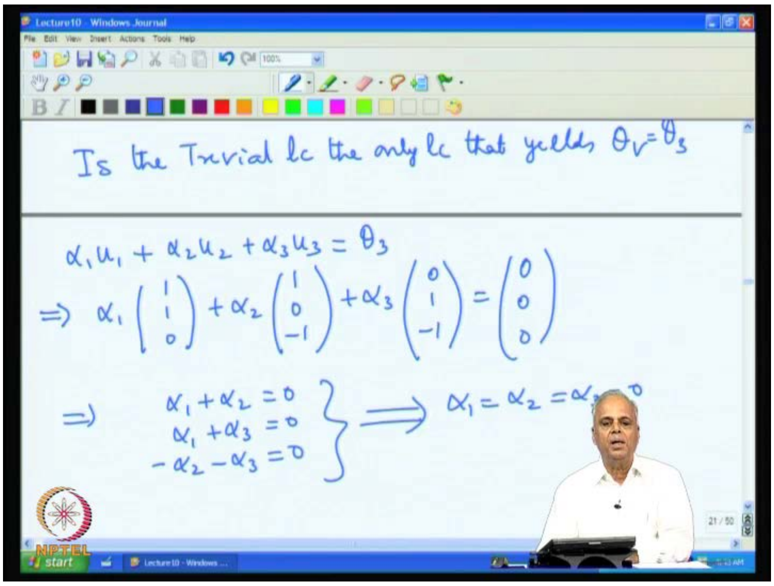

Now, the investigate the question is the trivial linear combination the only linear combination that yields theta V theta V in this cases theta 3 let us look, let us look at any linear combination alpha 1 u 1 plus alpha 2 u 2 plus alpha 3 u 3. If it has to be our aim is to find a linear combination, which gives the 0 vector, suppose I have a linear combination which use the 0 vector.

What does that say that says alpha 1 into 11 0 plus alpha 2 into 1 0 minus 1 plus alpha 3 into 0 1 minus 1 is 0 0 0, which implies alpha 1 plus alpha 2 comparing the first coefficients, comparing the second coefficients we get alpha 1 plus alpha 3 equal to 0. Comparing the third coefficients on both sides we get minus alpha 2 plus alpha 3, and the only thing that can satisfy all the these three equations is alpha 1 equal to alpha 2 equal to alpha 3. So, the only linear combination of these three vectors, but can yield the 0 vector is the trivial linear combination.

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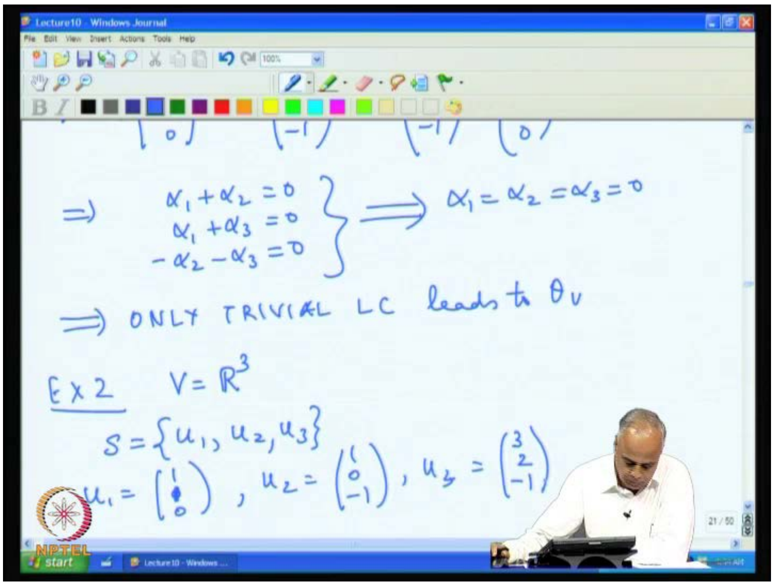

Therefore, only trivial L C leads to theta V, let us look at another example again let us take V equal to R 3 itself, but now let us take s to be u 1 u 2 u 3 again, but now let us take u 1 to be 1 0 1 will, I say 1 1 0 like before and u 2 to be 1 0 minus 1 and u 3 to be 3, 2, minus 1.

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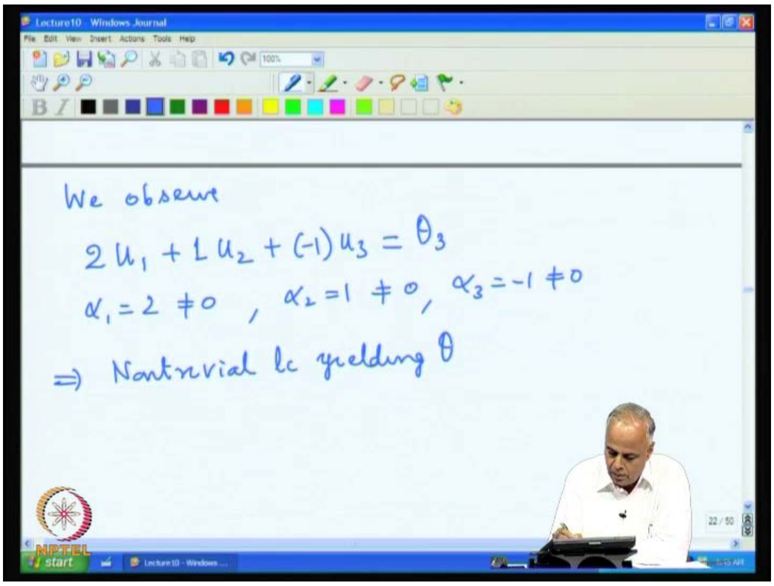

Now, we observe 2 u 1 plus 1 u 2 plus minus 1 u 3, if we look at these 2 u 1. Let us look at the first components 2 u 1 will give me 2 plus this 1 3; in these minus 3 that will be 0 and then second component 2 and minus 2 that will be 0. And similarly, third will be 0. So, this will be exactly the third. Now, we are a linear combination in with the coefficients are non 0. So, alpha 1 is 2 not equal to 0, alpha 2 equal to 1 not equal to 0 alpha 3 equal to minus 1.

So, we needed at least 1 coefficient not 0 in fact they were many coefficients not 0. So, this is a non trivial linear combination yielding theta V. Now, we are 2 examples in 1 of which only trivial linear combination yields 0 vector, in another example in which non trivial linear combinations that yield the 0 vector. So, there are sets for which only trivial will give 0 vector and there are such for which non trivial linear combinations, there are also give 0 vector we have to distinguish these 2 types of sets and therefore, we give the following definition.

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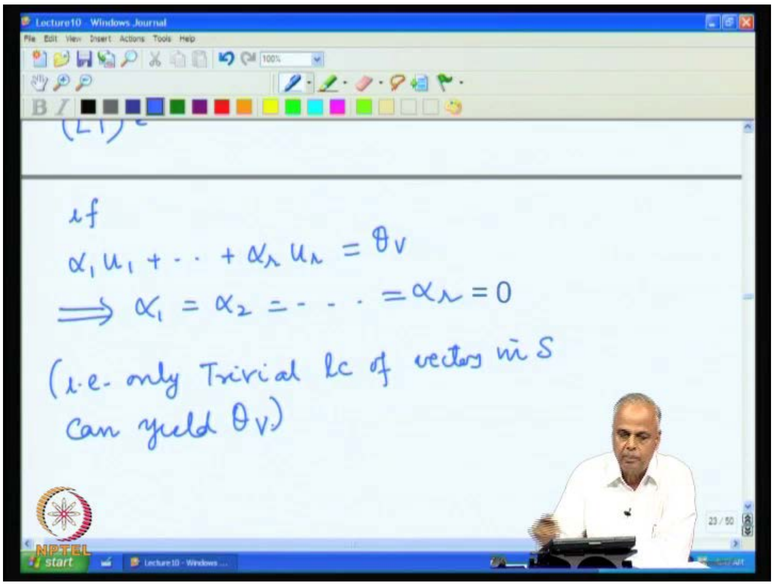

So, let S be a finite set of vectors in V. We say S is a Linearly Independent set of vectors linearly independent set from now what we will simply write L i are linearly independents set, so from now on L i means, linearly independent. So, we say S is a linearly independent set if the trivial linear combination the only 1 that will give 0 vector. What does that mean?

Alpha 1 u 1 plus alpha r u r, where S is that set, let us say S is the set u 1, u 2, u r, then we say it is linearly independent only trivial must give 0 vector. This means, if it is equal to 0 vector this must imply all the coefficients must be equal to alpha nothing else can give us be 0 vector, that is only trivial linear combination of vectors in S can yield the 0 vector.

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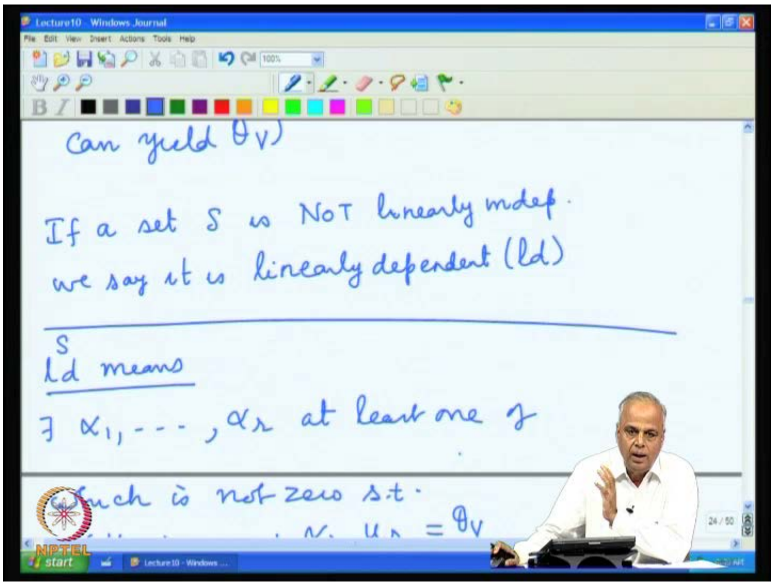

If a set s is not linearly independent, we say it is linearly dependent, we write l d for linear dependent, we will say it is linearly dependent. Notice linearly dependent means S is linearly dependent means, that it will not linearly independent, linearly independent means only trivial linear 0 vector, not linearly independent means, non trivial will also give 0 vector means, there exists alpha 1, alpha r at least 1 of which is not 0, such that alpha 1 u 1 plus alpha r u r equal to theta v. Now, this notion of linearly independent is a very important notion, and linearly dependent is the negation of linear independent.

The fact that it linearly independents in some sense means, it the vectors contain non redundant information; whereas, when we say linear dependents means the vectors contain a lot of redundant information. Now, when we want to keep matter in a compact information a compact manner we would like to remove all redundancy. So, we would like to keep only linearly, so independent information. So, these notion of linear independents is very important information, we will look at some examples of linear independents, and see how you tells in building up process in vector spaces.