Numerical Linear Algebra
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Lecture - 19
Sign Integer Representation

Hello friends, I welcome you to my lecture on signed integer representation. Ah in integer representation computers we know that computers used user fix number of digits to represent an integer.

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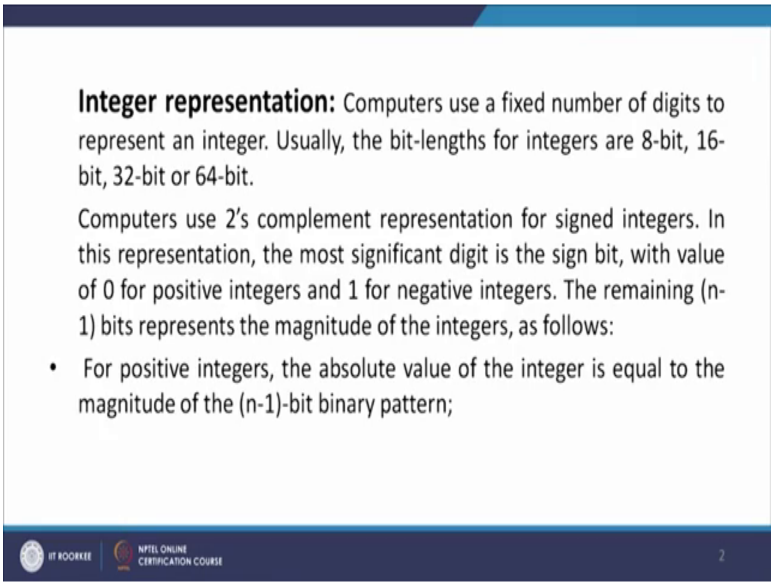

Usually the bit lengths are for integers 8-bit, 16-bit, 32 bits or 64 bit. Computers used 2s complement representation for signed integers. In this representation the most significant digit is the sign bit. And the sign bit has value 0 for positive integers, and the value one for negative integers. The remaining n minus 1 bits represents the magnitude of the integers ah. And the, they represent the magnitude of the integers as follows, for positive integers the absolute value of the integer is equal to the magnitude of the n minus 1-bit binary pattern.

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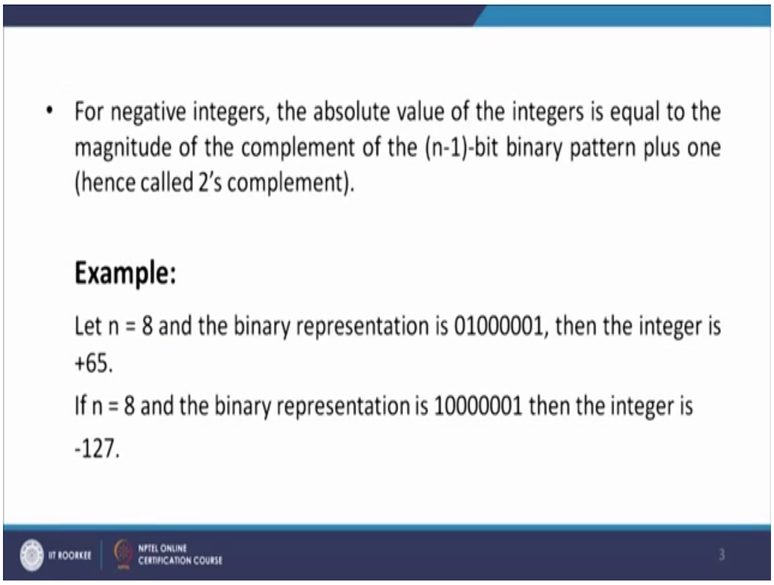

And for negative integers the absolute value of the integers is equal to the magnitude of the complement of the n minus 1-bit binary pattern plus 1 hence, called 2s compliment. For example, let us take n equal to 8 and the binary representation is 0 1.

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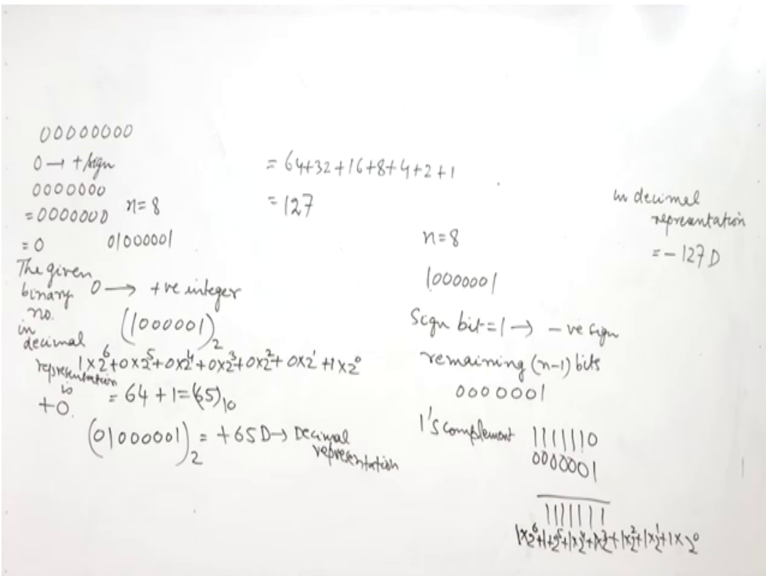

So, it is 8-bit representation, there are 8 bits. Now you can see the first integer is this sign bit. It denotes the sign bit. So, 0 means the 0 means that it is positive integer. And then the remaining n minus 1 bits, the remaining n minus 1 bits are 0, 1, 0, 0, 0, 0, 1. In decimal representation, this is equal to this is binary representation. And decimal representation is equal to 1 into 2 to the power 0, 0 into 2 to the power 1 0 into 2 to the power 2, 0 into 2 to the power 3, 0 into 2 to the power 4, 0 into 2 to the power 5, plus 1 into 2 to the power 6. Now 1 into 2 power 6 is 64, and then 0 0 0 0 0 1, we have 65 ok, 65 in the decimal representation ok.

So, the given number the given number in binary representation is equal 2 plus 65 in decimal representation D represents decimal representation. So, then decimal representation it is plus 65. Similarly, we can take another example. So, n is equal to 8, and then we take the binary representation as this ok. So, then the sign bit is one, sign bit is equal to 1 implies that the sign is negative. Remaining n minus 1 bits are 0 0 0 0 0 0 1 ok. Then we take one's compliment, in the ones complement, we take the complement of this n minus 1 a binary bits.

So, for 0 we write 1 for 0 we write 1, in here also 1, here 1 and how many? Ah 5 6 so, 5 6, and then for this one we write 0 ok, because there are in 2 bits either 0 or 1. So, for the complement of 0, we write one for the comple complement of one we write 0. And in the ones complement be 8 plus 1 ok. So, plus 1 means 0 plus 1, 1 then when you add the 2 integers, you write to the left of g one you write 0's. So, 0 0 0 0 0 0 so, edition means b b 8 0 and 1 0 and 1 is 1 1 plus 0 is 1 1 plus 0 is 1 1 plus 0 is 1 1 1 1.

So, we have 7 1's, 7 1's means, this is now binary number. So, in by so, it when you convert it to decimal representation, this is equal to this gives you ah 1 into 2 to the power 0, 1 into 2 to the power 1, 1 into 2 to the power 2, 1 into 2 to the power 3, 1 into 2 to the power 4 and so on, 1 into 2 to the power 5, 1 into 2 to the power 6, oh plus 1 into 2 to the power 7.

And this is how much these equal to ? Ah 64 now this is equal this is not 1 into 2 power 7 will go up to 1 to the 1 into 2 to the power 6, because there are 7 digits. So, this is equal to 64 plus 32 plus 16 plus 8 plus 4 plus 2 plus 1 ok. So, 4 plus 2, 6, 6, 12, 12, 8, 20, 24, 26, 27 and then we have to 6 2 8 3 11 and 1 12 ok. So, 127 and for the sign bit we are from sign bit we are getting negative sign. So, the given one number ok, the given binary number in decimal representation is minus 1270 D means decimal representation.

So, these minus 127, and similarly if you suppose that n is equal to 8 again and the binary representation is 0 0 8 0's. Then in binary representation if we have this then the first 0 means positive sign remaining 7 0's we have ok. So, absolute value of the n minus

1-bit binary representation is the same. So, this is ok, and this is 0 into 2 to the power 0 into 2 to the power 0 plus 0 into 2 to the power 1 plus 0 into 2 to the power 2 and so on; so, this is 0.

So, we have the plus the given binary number in decimal representation is plus 0 ok.

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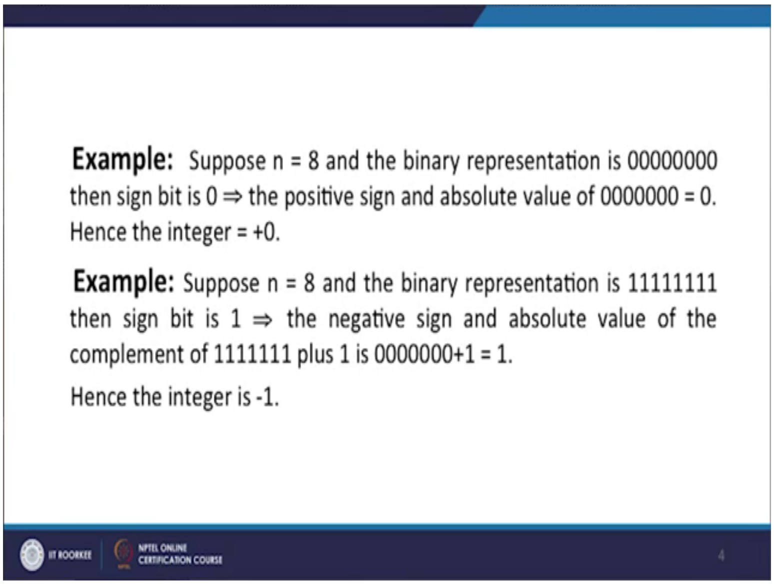

One more example we can take to clarify this suppose n is equal to 8.

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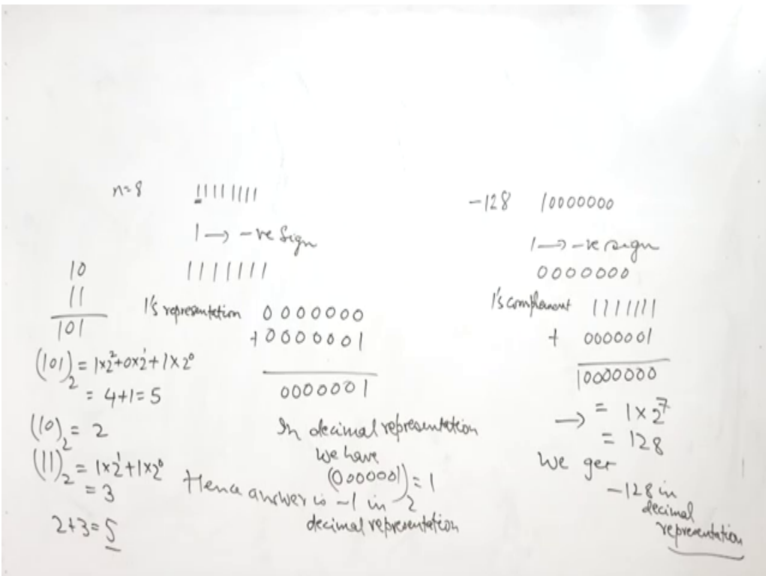

And the binary representation is 1 1 1 1 1 1 1 1 ok. So, there are 8 1's 8 1. Then the first o one this one first this this inte this one represent the sign bit. So, it is negative sign ok. Remaining ones let us consider remaining 7 ones ok.

So, then in 1's representation we take the complement of this n minus 1-bit number. So, 0 0 0 0 0 0 0, and then we add plus 1. So, we add 1 means so, this, and when you add them this is 0 0 0 0 0 0 0 and then ok. So, now this represents in decimal representation we we have 2 ok, this number 0 0 0 0 0 0 1 ok, these equal to 2 in decimal representation ok. So, answer is minus 2 in decimal representation. In some books, you will find it written as as minus 2 D D means, decimal representation it is not minus 2 it is sorry, it is 1 because this one billy multiplied by 2 to the power 0. So, it is one not so, this is minus 1 in decimal representation.

Ah now let us go to a here we have listed all the integers from minus 128 to plus 127, that is the range of the ah integer that can be represented here.

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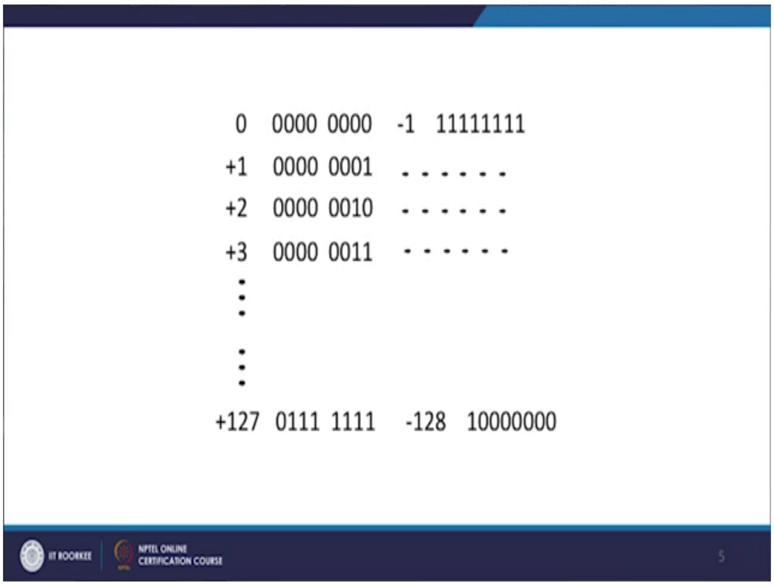

So, minus 128 to plus 127 that is the range of this sign integer representation 2 integer representation. Ah like we have done minus 127 we can do anyone else, say for example, minus 128 how let us see how we get this minus 128. This is represented by this one the first one, first one represents negative integer ok.

So, one represent minus sign, and then remaining n minus 1 bits ok, in a remaining n minus 1 bits, you take ones complement. Write 1 1 1 1 1 1 1 ok. So, 7 one's ok, 7 ones will represent and after taking complement you add one. So, 8 1, you put 0's here, now when you 8 1 in binary representation 1 plus 1 is 0 carry over one. How it happens? Because 1 plus 1 is 2 and 2 here is equal to in decimal 2 in binary 2 is same as 10 in decimal ah representation. So, 1 plus 1 means 0 carry over one ok, carry over one means 1 plus 1 again 0, then 1 plus 1 again 0, 1 plus 1 again 0, 1 plus 1 again 0, 1 plus 1 again 0, and 1 plus 1 again 0, carry over 1, and that one be discard.

So, we have this so, when we 8 1 to this ones complement 1 plus 1 is 0 then one carry over. So, 1 plus 1 0, then one can you over 1 plus 1 is 0 and so on when you reach here, 1 plus 1 is 0 1 carry over. So, we get this ok, and it is so, this is this is same as 0 into 2 to the power 0 plus 0 into 2 to power 1 plus 0 into 2 to the power 2 and so on, last at the last you get 1 into 2 to the power 7 ok. So, this is equal to 1 into 2 to the power 7 and 6 is equal to 128.

So, we get minus 128 in decimal representation. So, now inval let us as we have done the binary addition here, let us let me write the rules for the binary addition.

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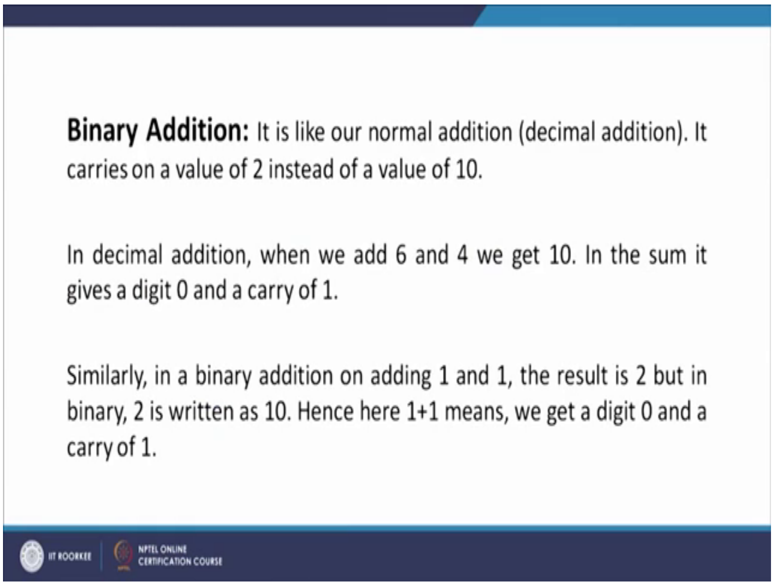

Ah this binary addition is like our normal addition, like the a decimal numbers decimal representation and numbers in decimal representation, when we add it is same as that here what we have is it carries on a value of 2, instead of a value of 10 in decimal addition when we add 6 and 4, we get 10 in the sum it will give you a 0 and carry of 1. Now similarly in a binary addition on adding 1 plus 1 1 and one the result is too, but in binary 2 is written as 10. And hence here 1 plus 1 means we get a 0 and carry of 1.

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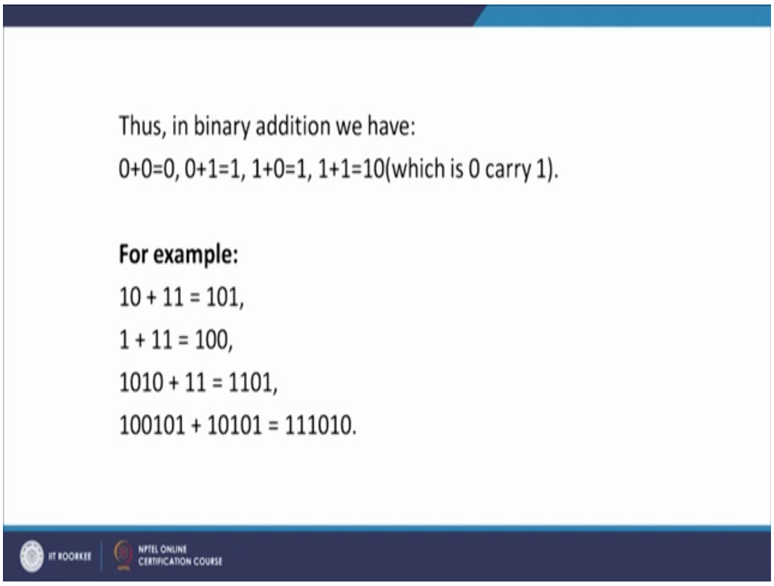

So, thus in binary addition we have 0 plus 0 equal to 0, 0 plus 1 equal to 1 1 plus 0 equal to 1, 1 plus 1 equal to 10 which is 0 carry 1. And for example, 10 plus 11 suppose we have 10 plus 11 then 0 plus 1 is 1 1 plus 1 is 2 1 plus 1 is 0 carryover 1.

So, this is 1 0 1 in binary representation in decimal representation 1 0 1 means 1 into 2 to the power 0 0 into 2 to the power 1, and then 1 into 2 to the power 2. So, this is 4 plus 1 5. And you can see this here also. 10 means in binary this 10 is in binary representation. And the this 10 is equal to in decimal representation in 0 into 2 to the power 0 1 into 2 to the power 1. So, this is equal to 2 and 11 number in binary representation will be equal to 1 into 2 to the power 0 plus 1 into 2 to the power 1, which is equal to 3, and you can see 2 plus 3 equal to 5 ok.

So, this 5 is same as your 1 0 1 ok. So, in binary representation when you add 10 and 11, you get 1 0 1 which is the, which is the destroyed binary equivalent of the decimal number 5.

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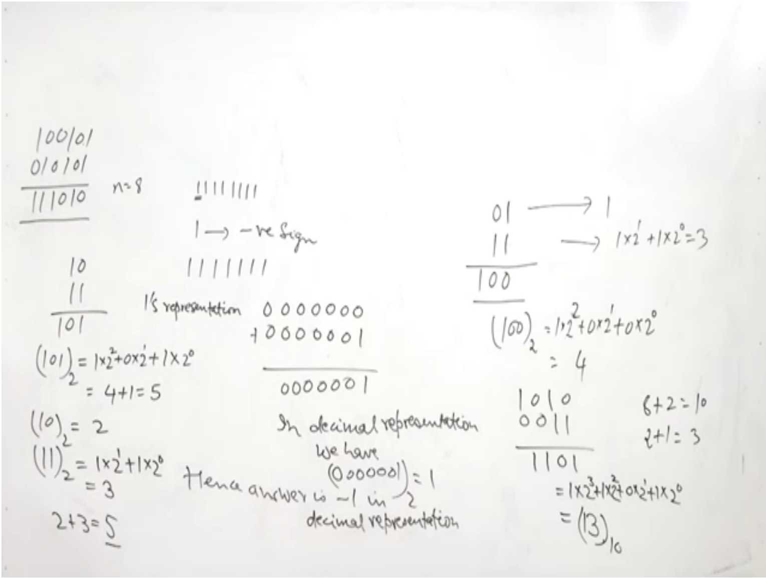

Now similarly if you add one and 1 1 when you add let us to the left of one we write 0, now 1 plus 1 is 1 1 plus 1 is 0 carry over 1 1, plus one is 0, and carry over one we get 100 and 100 in this the answer in the binary representation. If you convert into decimal representation it is 0 into 2 to the power 0, plus 0 into 2 to the power 1 plus 1 into 2 to the power 2.

So, this is 4 and so, this is a 100 is divided by one double 0 is the binary equivalent of 4. And you can see here also in this is binary representation, when you converted to decimal representation 1, means one 1 into 2 to the power 0 and 1 1 1 in 2 to the power 0 plus 1 into 2 to the power 1.

So, this in decimal representation is 1, while this in decimal representation is 3, 1 into 2 to the power 0 plus 1 into 2 to the power 1. So, which is 3, and you can you know that 1 and 3 together on the sum of one and 3 is 4. So, we get this similarly you can add 1 0 1 0 and 1 1 1 0 1 0 1 0 1 0 and 1 1. So, here again we put 0 0 to the left of 1 1, and then 1 0 plus 1 is 1 1 plus 1 is 0 carryover 1 carry over 1 plus 0 is 1, and then 1 plus 0 is 1. So, this is 1 1 0 1 and if you con if you converted to decimal equivalent what you get here 1 into 2 to the power 0 0 into 2 to the power 1 1 into 2 to the power 2 and 1 into 2 to the power 3, 8 plus 4 plus 1, that is 13 ok. In decimal representation, and here also you can see this is 0 into 2 to the power 0 1 into 2 to the power 1. So, we have 2 and then 0 into 2 to the power 2 plus 1 into 2 to the power 3.

So, 8 8 plus 2, and here we have 1 into 2 to the power 0. So, one and then 1 into 2 to the power 1 so, 2, this is 10 and this is 3 and 10 plus 3 is 13 ok, similarly you can add 1 0 0 1 0 1 and 1 0 1 0 1. So, 1 0 0 1 0 1, and then 1 0 1 0 1 1 0 1 0 1, now 1 plus 1 is 0 carry over 1 1 plus 0 is 1 1 plus 1 is 0 carryover 1 1 plus 0 is 1 0 plus 1 is 1, and 1 plus 0 is one so, we get 1 1 1 0 1 0.

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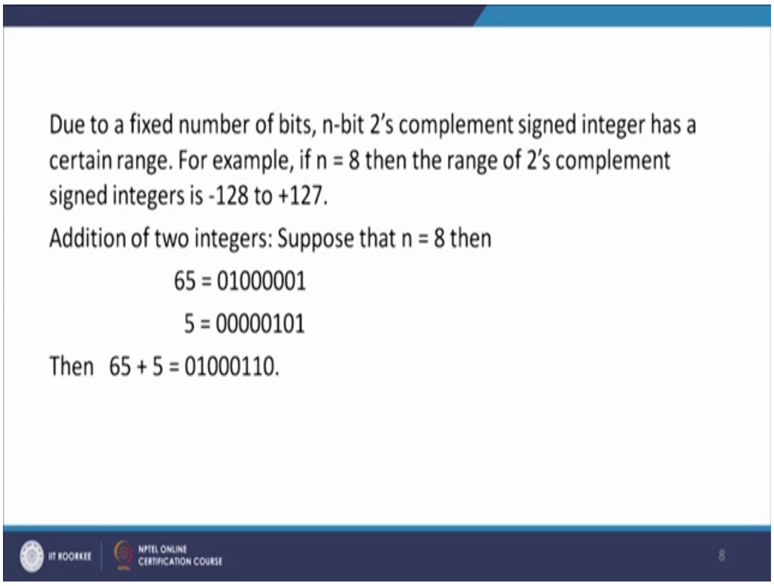

Now due to a fix number of digits, a due to a fix number of bits, and bit 2s complement signed integer has a certain range. For example, if n is equal to 8, then the range of 2s compliment signed integers is minus 128 to 1 plus 127 as have seen earlier. Now addition of 2 integers, suppose that n is equal to 8, then we want to add 2 numbers. So, you know that a 65 in decimal number 65 is equal to the binary number 0 1 0 0 0 0 0 1. And 5 is 0 0 0 0 0 1 0 1, and then when you add the binary representation then what you get is; so, let me let me write 65.

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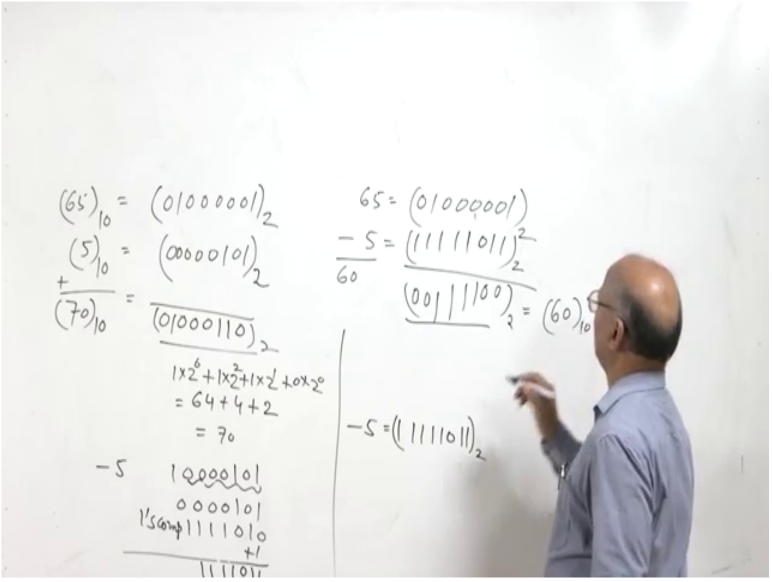

This is equal to 1, this is 2 to the power, this is 1 here than 0 0 0 0 0.

So, 1 2 3 4 5, and then we have 6 position here. So, we write here ok. So, this is the representation binary representation of this number oh, and then if you want to write 5 5 will be written as 1 0 1 ok. So, 65 plus 5 we know when we add we get 70 ok, in here what we will get when you add we will get 1 plus 1 0 1 carry over ok, then 0 plus 1 1 and then we have this, because this is this, this, this, this.

So, which represent 70, you can see because this is 0 into 2 to the power 0 then 1 into 2 to the power 1, then 1 into 2 to the power 2, and then this 0's and this is this one will be multiplied by 2 to the power 6, remaining are 0's ok. So, this is 64, plus 4 plus 2 which is 70 ok. So, this is the binary representation. Now when you subtract 2 integers ok Iit is treated as an addition of a positive and negative integer. Ah for example, 65 we have seen 65, 0 1 3 0 2 0 1 2 and when we have, but subtract 5 here ok. We get 60 in decimal representation n minus 5 is written as first minus 5 if you want to write minus 5, then minus means first bit must be equal to 1 ok. And then 5 you write in the absolute value representations of 5 you right as 0 0 0 0 1 0 1 ok.

So, these integers ok, you write the ones complement. So, 0 0 0 0 1 0 1 because write complement of this ones complement. So, 1 1 1 1 0 1 0 ok and then be add one to this. So, 0 plus 1 is one, then 1 plus 0 1 we have 0 we have 1, we have 1, we have 1, we have one ok, right? So, now to this we prefix this one ok. So, minus 5 will be written as in binary it will be written as 1, then 1 1 1 1 then 0 1 1 ok. So, 1 1 1 1 this, be a when we add these 2 integers 1 plus 1, 0 carryover 1, 1 plus 1 0 carry over once we get this, 0 plus 1 is 1, 0 plus 1, is 1 0 plus 1 is 1 1 plus 1 is 0, carry over 1 1 plus 1 is 0.

Now, the one that we get be discard that because these are 8 bits ok. And so, when you add ah this binary binary number to this binary number, the one that we get here we discard. That and this we know in binary, this in decimal these number the notes 60. When you add 2 negative integers, say minus 65 and minus 5 you get minus 70.

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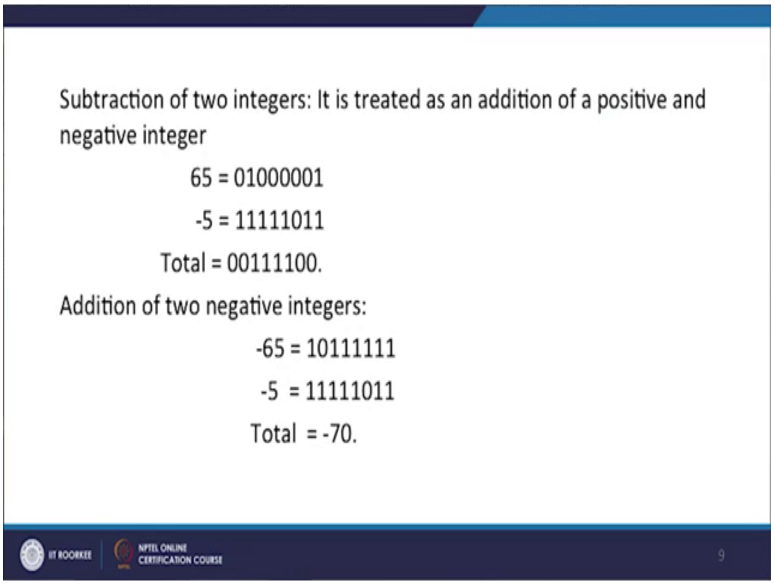

And we can see that minus 65, in which is in decimal is equal to 1 0 1 1 1 1 1 1 in binary. And this minus 5 is 1 1 1 1 0 1 1, when you add them, 1 plus 1 is 0 carry over 1 1 plus 1 is 0 carry over one, let us do this.

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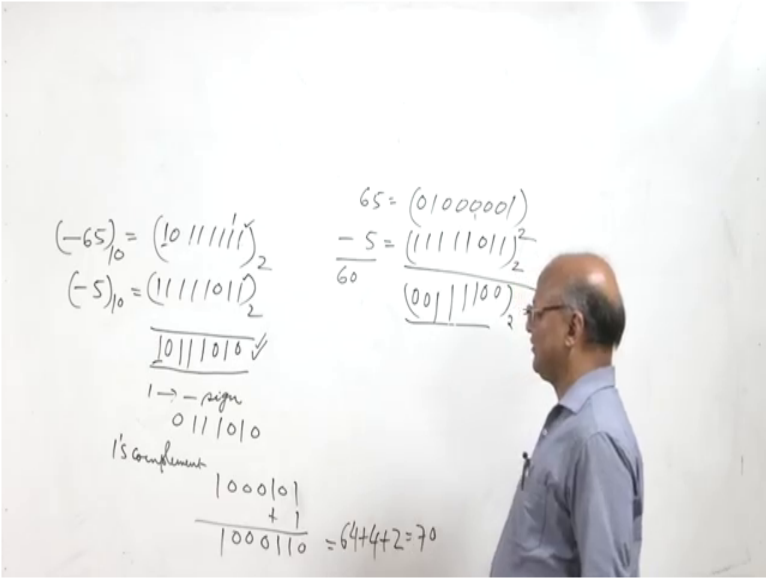

So, 1 0 minus 65, this is written as 1 0 1 0 double 1, double 1, double 1, and minus 5 is same as 1 1 1 1 1 0 1 1 so, let us 8 this ok. So, 1 plus 1 0 carry over 1, 1 plus 1 means 2 plus 1 3 3 means 1 1. So, we we have one here carryover 1 1 plus 1 0 carry over 1, 1 plus 1 2 plus 1 3 that is 1 1 in binary. So, we write one carry over 1 1 plus 1 2 plus 1 3 that is again a 1 1 in binary.

So, we have one here we have 1 plus 1 plus 1. So, we have one carry over 1 0 carryover 1 1 plus 1 means 2 2 plus 1 is 3 that is one carry over one. So, that one we leave and this is what we get when we 8. Now to the first integer is represents minus sign. So, this one, this one replace minus sign, remaining n minus 1 bits are 0 1 1 1 0 1 0 ok. To see that, this is nothing but minus 70, let us take the ones complement here. So, we get 1 0 0 0 1 0 0 sorry, 1 here ok. 1 0 0 0 1 0 1 and then we add 1. So, 1 plus 1 0 carry over 1. So, we get this, and then we have here, and then we get 0 0 0 1 ok. And this is equal to this binary number is equal to 0 into 2 to the power 0, and then we have the 1 into 2 to the power 1. So, we get 2 and then plus this is 1 into 2 to the power 2.

So, we get 4 and then we get here 1 into 2 to the power 6. So, 1 into 2 to the power 6 which is 64, this is 70 ok. 70 and then this minus sign represent minus 70. So, this number represents minus 70. So, this is how we get a minus 70, this one represents minus sign. And the remaining n minus 1 when we the complement of n minus 1 bits and then 8 1 we get 70. So, this number represents minus 70. So, with this I would like to conclude my lecture.

Thank you very much for your attention.