

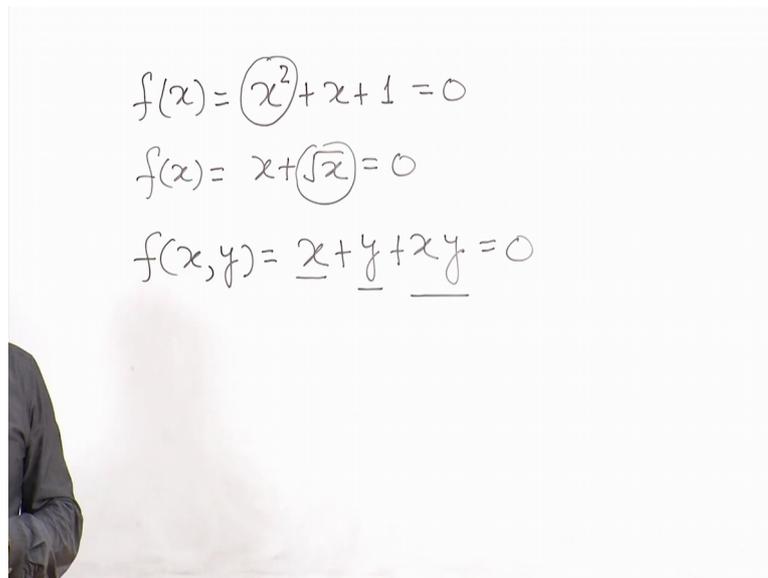
Numerical Methods
Dr. Sanjeev Kumar
Department of Mathematics
Indian Institute of Technology Roorkee
Lecture No 6
Introduction to Nonlinear Equations and Bisection Method

(Refer Slide Time: 0:47)

The slide features a blue header with the text "Nonlinear Equation". Below this, a light blue box contains the sub-section "Nonlinearity". The text in this box states: "One of the most frequent problem in engineering and science is to find the root(s) of a nonlinear equation." followed by the equation $f(x) = 0$. Below the equation, it explains: "Here, $f(x)$ is a nonlinear function in x . It means variables has an exponent either greater than 1 or less than 1, but never 1 or there can be product of variables in the equation." At the bottom of the slide, there are logos for "IIT ROORKEE" and "NPTEL ONLINE CERTIFICATION COURSE", and a small number "2" in the bottom right corner.

Hello everyone so today we are going to start a new unit in which we will learn few Numerical methods to solve nonlinear equations, so 1st of all what is a nonlinear equation? One of the most frequent problems in real life in engineering is to find the roots of unknown linear equation fx equal is to 0. The equation is nonlinear due to the unknown non-linearity of the function f . Here a function is said to be nonlinear if the variables or variable has an exponent either greater than 1 or less than 1, but never one or if you are having an equation having more than one variable you are having the cross product terms of the variable in the equation.

(Refer Slide Time: 1:25)



The image shows a whiteboard with three handwritten equations. The first equation is $f(x) = x^2 + x + 1 = 0$, where the x^2 term is circled. The second equation is $f(x) = x + \sqrt{x} = 0$, where the \sqrt{x} term is circled. The third equation is $f(x, y) = \underline{x} + \underline{y} + \underline{xy} = 0$, where each term is underlined.

If we take some example of nonlinear equation or nonlinear function here x square plus x plus 1 equals to 0, so here this equation is an nonlinear equation and it is because this particular term is having exponent as 2. Similarly if I am having some other functions let us say x plus root x and again it is a nonlinear equation because here the exponent is 1 by 2 of this particular x in this particular term, so in a nonlinear equation if any of the term is having exponent less than 1 or greater than 1 but not exactly 1 then the equation is called nonlinear equation.

If you are having an equation is in more than one variables let us say f is a function of x and y , where x and y are independent variables, so if I am having this function as xy equals to 0, so here you can see this term is linear, this term is linear but here we are having separately x and y having exponent one but with a dear product this particular function becomes nonlinear, so in next few lectures we will learn how to solve these nonlinear equations. Now what we mean by the solution of nonlinear equations?

(Refer Slide Time: 3:16)

Nonlinear Equation

Root

Given a nonlinear function $f(x)$, we seek a value of x for which

$$f(x) = 0$$

Such a solution value for x is called a root of the equation, and a zero of the function f . An example of a nonlinear equation in one variable is

$$f(x) = x^2 - 4 \sin(x) = 0$$

This equation has roots at 0.0 and near 1.9.

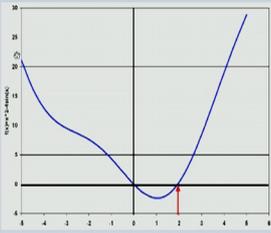
IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 3

Nonlinear Equation

Root: Graphical illustration

$$f(x) = x^2 - 4 \sin(x) = 0$$

This equation has roots at 0.0 and near 1.9.



IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 4

So given a nonlinear function f we seek of value of x for which f is 0. Such a solution value for x is called a root of the equation and a 0 of the function f . An example of a nonlinear equation in one variable if you take f equal to x square minus $4 \sin x$ equals to 0 and if I see the graft of this function, it can be clearly seen that the function is 0 when x is 0 and another point when x is near to 1.9, so hence this particular equation is having 2 roots 1 is 0 another one is just near to 1.9. So graphically we can say the solution of a nonlinear equation means we have 2 find the curve of nonlinear function and we need to see where this particular curve is touching or intersecting x axis.

(Refer Slide Time: 4:25)

Definitions

System of nonlinear equations
If we have more than one equations in more than one variables, then it is called a system of non-linear equations

$$x^2 + y^2 = 1$$
$$x^3 + x^2 + xy - 1 = 0$$

Transcendental Functions
A function which is not algebraic is a transcendental function. They are also referred to as Non-Algebraic Functions. For e.g., trigonometric functions, exponential functions, their inverse, and their combinations, etc.

$$f(x) = x^2 + \tan(x)$$

IT ROORKEE NITEL ONLINE CERTIFICATION COURSE 5

Like we had the system of linear equations in the previous unit in this unit also we can have system of non-linear equations. If we have more than one equation in more than one variables then it is called a system of non-linear equations. For example if you take these 2 equations that is 1st equation is x square plus y square is equal to 1 and the 2nd equation is x cube plus x square plus xy minus 1 equals to 0. So here we are having to equations in 2 unknown and both the equations are non-linear in x as well as in y.

So we will learnt in the last lecture this unit that how to solve such a system using some numerical technique. As you see in all these examples which I have given to you are basically algebraic functions. Algebraic function means they are the polynomials in x or in y. Apart from that you are having another type of nonlinear equation or nonlinear function those are called transcendental function, so a function which is not algebraic is a transcendental function. They are also referred to as non-algebraic function or example if you are having trigonometric term in your function, exponential term, logarithmic term or their combinations. For example if you see this equation fx equal to x square plus tan x. Here it is a trigonometric term in x and hence it is a transcendental function. So we will learn how to solve or how to find out zeros of such transcendental functions using numerical techniques.

(Refer Slide Time: 6:31)

Nonlinear Equations

Number of roots

An equation $f(x) = 0$ may have one or more than one root.

- an equation linear in x will have one root
- a quadratic equation will have two roots
- a cubic equation will have three roots and so on

The roots may further be distinct or repeated.

An example of a transcendental equation on the other hand would be $2 \sin(x) - 1 = 0$ which has a solution at $x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \pm 2n\pi$, where $n = 0, 1, 2, \dots$

Note that this equation has many roots corresponding to different values of n .

IT ROORKEE | HITEL ONLINE CERTIFICATION COURSE | 6

Now given a nonlinear equation we can have multiple roots. For example if you take a nonlinear equation fx equal is to 0. If it is a linear equation then it will be having one root but if it is a quadratic equation means it is a second degree polynomial in x , it will have 2 roots. If it is a cubic polynomial in x then the equation will be having 3 roots and so on. If the roots may further be distinct or repeated for example if you take a simple function x square minus 4 equals to 0.

So basically this quadratic polynomial is having 2 roots x equals 2 plus 2 and minus 2 and both the roots are distinct but or on the same time if you take another function let us say x square plus 4 minus $4x$ basically x minus 2 whole square then the roots are 2 and 2 and hence the 2 roots but both are same, so roots are repeated twice. In case of transcendental equation if we consider an example like twice of $\sin x$ minus 1 equals to 0 then if we find a solution of this it will be something like x equals to \sin inverse 1 by 2 and it is coming out π upon 6 plus minus twice $n \pi$ where n equals to 0, 1, 2. So you can see that this equation as many roots corresponding to different values of n .

(Refer Slide Time: 8:27)

Nonlinear Equations

There are two ways to solve an equation:

- 1 Analytical methods known as direct methods.
- 2 Numerical methods.

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 7

Nonlinear Equations

- Direct methods can solve the system in finite number of steps and gives an accurate solution.
- However, there are system of equations which are very time consuming when solving with direct methods or can not be solved by direct methods.
- So, to solve them, we resort to the numerical methods.
- Numerical methods provide a technique to find an approximate but accurate solutions of the system of equations.

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 8

So using numerical techniques we can find single root given in a particular interval or more than one roots. For this we are having 2 types of techniques as I told you one is analytical methods or direct methods like in case of linear equations and the other one is numerical methods. So direct method can solve system in finite number of steps and gives an accurate solution. However, there are systems of equations which are very time consuming when solving with direct method or there are number of equations which you cannot solve using the direct method hence we need to rely on the numerical methods to find out a solution of such equation those are not solvable using direct method, so numerical methods provides a technique to find an approximate but accurate solution to the system of equations.

(Refer Slide Time: 9:38)

Nonlinear Equations

Methods

We will discuss following methods to solve nonlinear equations

- 1 Bisection method
- 2 Secant Method/Regula Falsi method
- 3 Newton-Raphson Method
- 4 Fixed point method

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 9

So in the next few lectures we are going to discuss following numerical techniques like bisection method which we will cover in this lecture itself then in the next lecture we will take Secant method and regular Falsi method, then in the 3rd lecture of this unit we will go for Newton-Raphson method, in the 4th lecture of this unit we will discuss about fixed point method and in the last lecture of this unit will learn how can be solved system of non-linear equation using these techniques. So what we mean by numerical solution of a nonlinear equation.

(Refer Slide Time: 10:18)

Nonlinear Equations

Methods

In this lecture, we will talk about various iterative methods to find numerical solution of the equation

$$f(x) = 0 \quad (1)$$

- Numerical solution of (1) means a point x^* such that $f(x^*) \approx 0$.
- For this, we always assume that $f(x)$ is continuously differentiable real valued function.
- Also, it is assumed that roots of (1) are isolated.

Isolated roots: The root of (1) for which there is a neighbourhood which do not have any other root of (1).

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 10

So given a nonlinear equation $f(x) = 0$ numerical solution means a point we need to find out an approximate x^* such that $f(x^*) \approx 0$. For this we always

assume that $f(x)$ continuously differentiable and real valued function also it is assumed that root of this particular equation $f(x) = 0$ are isolated. What we mean by isolated root? The root of $f(x) = 0$ for which there is a neighborhood which do not have any other root of $f(x) = 0$ is said to be isolated root for example if you take $f(x) = x^2 - 4$ equals to 0 again, so here we are having 2 roots one is $x = -2$ another one is $x = 2$, so both the roots are isolated because in the neighborhood of 2 as well as neighborhood of -2 we do not have any other root.

(Refer Slide Time: 11:29)

Nonlinear Equations

To approximate the isolated roots, the main idea consists of the following steps:

- 1 **Initial guess:** Take a point $x_0 \in [a, b]$ as an approximation to the root of $f(x) = 0$
- 2 **Improving the value of the root:** If the initial guess x_0 is not in desired accuracy, then we should construct a method to improve the accuracy.

The methods of improving the initial guess are called iterative methods. A general form of an iterative method may be written as

$$x_{n+1} = T(x_n), \quad n = 0, 1, \dots$$

where, T is a real-valued function called an iteration function. In the process of iterating a solution, we obtain a sequence of numbers $\{x_n\}$ which are expected to converge to the root of $f(x) = 0$.



11

To find out such isolated roots the main idea consist of the following steps in a numerical technique first of all we need an initial guess that is take a point x_0 belong to a close interval a and b as an approximation to the root of $f(x) = 0$ and then we need to improve this initial root or initial solution by using an iterative equation that is $x_{n+1} = T(x_n)$, and equals to 0, 1, 2 and so on. Here T is a real valued function called an iteration function. In the process of iterating a solution we octane a sequence of numbers x_n which are expected to converge to the root of $f(x) = 0$. Like here we will start with x_0 then using this particular equation, iterative equation we will find out x_1 then using x_1 here we will find x_2 and so on.

(Refer Slide Time: 12:34)

Nonlinear Equations

Convergence
A sequence of iterates $\{x_n\}$ is said to converge with order $p \geq 1$ to a point x^* if

$$|x_{n+1} - x^*| \leq c|x_n - x^*|^p$$

for some constant $c > 0$.

Order/rate of the convergence
If $p = 1$, the sequence is said to converge linearly to x^* , if $p = 2$, the sequence is said to converge quadratically and so on.

IT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE | 12

Now in the previous I told you that we obtain a sequence of numbers x_n which are expected to converge to the root of $f(x) = 0$. Now what we mean by convergence of such an iterative scheme, a sequence of iterates x_n is set to converge with order p and p will be always greater than or equal to 1 to a point x^* if $|x_{n+1} - x^*| \leq c|x_n - x^*|^p$ that is the approximate in $n+1$ iteration minus x^* that is the approximate solution is less than or equal to c , $|x_n - x^*|^p$ for some constant $c > 0$. So if $p = 1$, the sequences said to converge linearly, if $p = 2$ we will say that the sequences converging quadratically and so on.

(Refer Slide Time: 13:35)

Bisection Methods

IT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE | 13

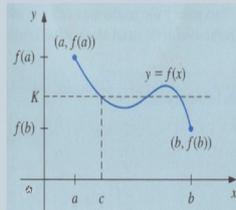
Nonlinear Equations

Bisection Method

This method is based on intermediate value theorem.

Theorem (Intermediate Value theorem (IVT)):

If $f \in C[a, b]$ and K is any number between $f(a)$ and $f(b)$, then there exists $c \in (a, b)$ such that $f(c) = K$.



NPTEL ONLINE
CERTIFICATION COURSE

14

So the 1st numerical technique which we are going to discuss is Bisection method. The method is based on intermediate Value Theorem, so what this theorem tells us that if f is continuous in a close interval a to b and K is any number between f_a and f_b then there exists c belongs to open interval ab such that f_c is equal to K . So if you see this graphics here we are having x on the horizontal axis fx on the vertical axis and this blue curve is showing the graph of fx between a and b . Now if I take a number K between f_a and f_b , so there will be some number c such that image of c under this function f equals to K .

(Refer Slide Time: 14:34)

Nonlinear Equations

Bisection Method

- Suppose that $f(x)$ is continuous on given interval $[a, b]$.
- The function f satisfies the property $f(a)f(b) < 0$ with $f(a) \neq 0$ and $f(b) \neq 0$.
- By intermediate value theorem, there exists a number say c such that $f(c) = 0$.



NPTEL ONLINE
CERTIFICATION COURSE

15

Nonlinear Equations

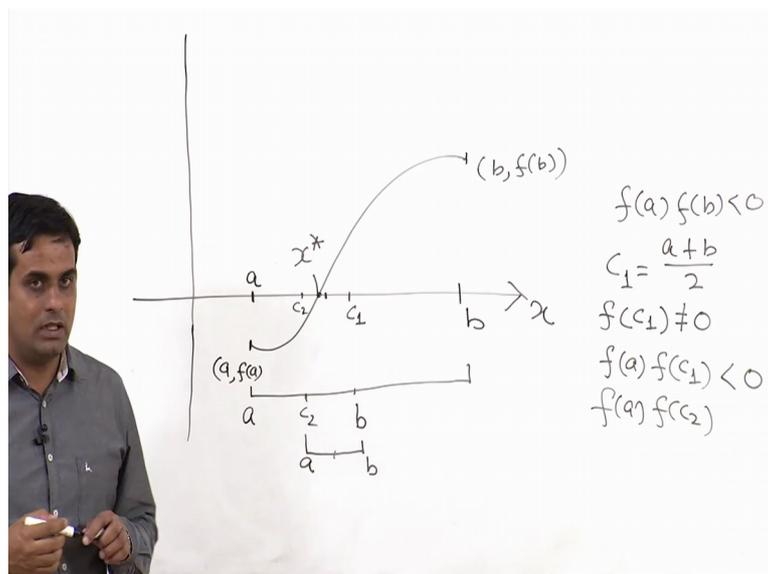
Bisection Method

- This procedure will also work if there exists more than one root in interval $[a, b]$.
- Here, we assume the root in $[a, b]$ is unique just for simplicity.
- This method calls for repeated bisection of subintervals of $[a, b]$ with locating the half containing p at each step.



It means if we are having a function $f(x)$ which is continuous on given interval ab the function f satisfies the property $f(a) \cdot f(b) < 0$ with $f(a) \neq 0$ and $f(b) \neq 0$ that a and b are not root of f then by intermediate value theorem, we can see there exists as c or a root between a and b such that $f(c) = 0$. This procedure will also work if there exist more than one root in the interval ab . Here we assume that root in ab is unique just for simplicity. This method calls for repeated bisection of subintervals of ab with locating the half containing p at each step.

(Refer Slide Time: 15:33)



So geometrically how this method will work? Let me explain here. Suppose we are having a function like this, so this is the point a , this is point b . Now as you can see this function is having a root at this particular point, so what I will see that this point is a and this point is b

fb. So what you can see fa is a negative value while fb is a positive value, so it means fa into fb means there is a root between a and b. Now what I will do? I will calculate c 1 which will be midpoint of a and b, so if I take the midpoint this interval a and b midpoint will be somewhere here, so let us say this is my c 1.

Now if I check f of c 1 and if it is equals to 0 I will say that c 1 is a root of the equation, if it is not 0 or I will check? I will check fa into fc 1. So product of these 2 if this product is negative the root will lie between a and c 1. If it is positive the root will lie in the right half interval that is c 1 to b, like in this example root is here, so what I will do? I will update this as a, and again this will become b, so what will happen in the next iteration where the length of the interval was b minus a in the 1st iteration it reduced to b minus a upon 2. Now again I will find out the midpoint of these 2, so let us say this one c 2, so c 2 will be somewhere here and again have the check fa into fc 2 which is coming positive.

Here it means root is in the right half interval that is in this interval, so what I will do? I will name it as a and this as b, so it means and then I will find out midpoint of this which will be somewhere here and continuing this process my method will converge to a root which is given here as b let us say or x star. So this is the geometric explanation of the bisection method in each iteration we reduce the half of our search intervals and then we will find out root by repeating this half and half and half like this.

(Refer Slide Time: 19:22)

The slide is titled "Nonlinear Equations" and contains a section for the "Bisection Method". It lists the following steps:

- The Bisection method consists of following steps:
- Step 1:** Given an initial interval $[a_0, b_0]$, set $n = 0$.
- Step 2:** Define $c_{n+1} = (a_n + b_n)/2$, the mid-point of interval $[a_n, b_n]$.
- Step 3:**
 - If $f(a_n)f(c_{n+1}) = 0$, then, $x^* = c_{n+1}$ is the root.
 - If $f(a_n)f(c_{n+1}) < 0$, then take $a_{n+1} = a_n, b_{n+1} = c_{n+1}$ and the root $x^* \in [a_{n+1}, b_{n+1}]$.
 - If $f(a_n)f(c_{n+1}) > 0$, then take $a_{n+1} = c_{n+1}, b_{n+1} = b_n$ and the root $x^* \in [a_{n+1}, b_{n+1}]$.

The slide footer includes the IIT ROORKEE logo, the NPTEL ONLINE CERTIFICATION COURSE logo, and the number 17.

So the algorithm for bisection method can be described in the following steps, the 1st step is given an initial interval a naught b naught set n equals to 0. In step 2 define c n plus 1 equals

to a n plus b_n upon 2 like you we are finding the midpoint of the original interval if f of a_n into f of c_n plus 1 equals to 0 then the root is c_n plus 1 if it is the product of f of a_n with f of c_n plus 1 is negative then we take root as means root will lie in the interval a_n plus 1 and b_n plus 1 where n plus 1 is a_n and b_n plus 1 is c_n plus 1. Similarly if this product is positive then take a_n plus 1 equals to c_n plus 1 and b_n plus 1 equals to b_n means the root will lie in the 1st half of the interval and so on.

(Refer Slide Time: 20:40)

Nonlinear Equations

Bisection Method

Step 4: If the root is not achieved in step 3, then, find the length of new reduced interval $[a_{n+1}, b_{n+1}]$. If the length of the interval $b_{n+1} - a_{n+1}$ is less than a recommended positive number ϵ , then take the mid-point of this interval ($x^* = (a_{n+1} + b_{n+1})/2$) as the approximate root of the equation $f(x) = 0$, otherwise go to step 2.

IIT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 18

Step 4 if the root is not achieve in step 3 then find the length of new reduce interval and plus 1 b_n plus 1. If the length of the interval b_n plus 1 minus a_n plus 1 less than a threshold epsilon then take the midpoint of this interval as the root otherwise go to step 2 for the next iterations.

(Refer Slide Time: 20:57)

Nonlinear Equations

Bisection Method: Graphically

19

Nonlinear Equations

Convergence and error in Bisection Method

Let $[a_0, b_0] = [a, b]$ be the initial interval with $f(a)f(b) < 0$. Define the approximate root as $x_n = (a_{n-1} + b_{n-1})/2$. Then, there exists a root $x^* \in [a, b]$ such that

$$|x_n - x^*| \leq \left(\frac{1}{2}\right)^n (b - a) \quad (2)$$

Moreover, to achieve the accuracy of $|x_n - x^*| \leq \epsilon$, it is sufficient to take

$$\frac{|b-a|}{2^n} \leq \epsilon \quad \text{i.e.} \quad n \geq \frac{\log(|b-a|) - \log(\epsilon)}{\log 2}$$

20

So this is the same which I have described on the board graphic really how bisection method works. Now if you talk about the convergence of this particular method let a naught b naught equals to a b be the initial interval with fa into fb less than 0 that is negative. Define the approximate root as x n which is the midpoint of a n minus 1 plus b n minus 1 upon 2 then there exist a root x star a b such that x and minus x star less than equals to 1 by 2 raise to the power n into b minus a where b a to b is the original interval.

Moreover to achieve the accuracy it is given that the accuracy should be of order of epsilon where epsilon is a small positive number. It is sufficient to take b minus a raise to power b minus a upon 2 raise to power n less than equal to epsilon because we need this number less than equals to epsilon for getting the desired accuracy in our numerical solution, so it is less

than equal to epsilon. It means if we take the logarithm I can write it n should be greater than equal to log b minus a minus log epsilon upon log 2. So this particular equation tells us about the number of iterations required to get a given accuracy epsilon.

(Refer Slide Time: 22:47)

Nonlinear Equations

Example

Consider the equation $x^3 + \cos x + 1 = 0$
 Let the length of the initial interval be $|b - a| = 1.0$. If the permissible absolute error is 0.125, i.e. $|x_n - x^*| \leq 0.125$,
 Then, the minimum number of iterations required to be carried out to achieve accuracy are

$$n \geq \frac{\log(1.0) - \log(0.125)}{\log 2} = 3$$

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 21

For example consider this function $x^3 + \cos x + 1$ and this equation equals to 0. Let the length of the initial interval is 1 that is $b - a$ is 1. If the permissible absolute error is 0.125 that is $|x_n - x^*| \leq 0.125$. Then minimum number of iterations required to be carried out using the earlier formula so it will become n greater than equals to log of $b - a$ minus log of epsilon upon log 2, so $\log 1$ minus $\log 0.125$ upon $\log 2$ and it is 3. So you need at least 3 iterations to get the accuracy of the order 0.125.

(Refer Slide Time: 23:36)

Nonlinear Equations

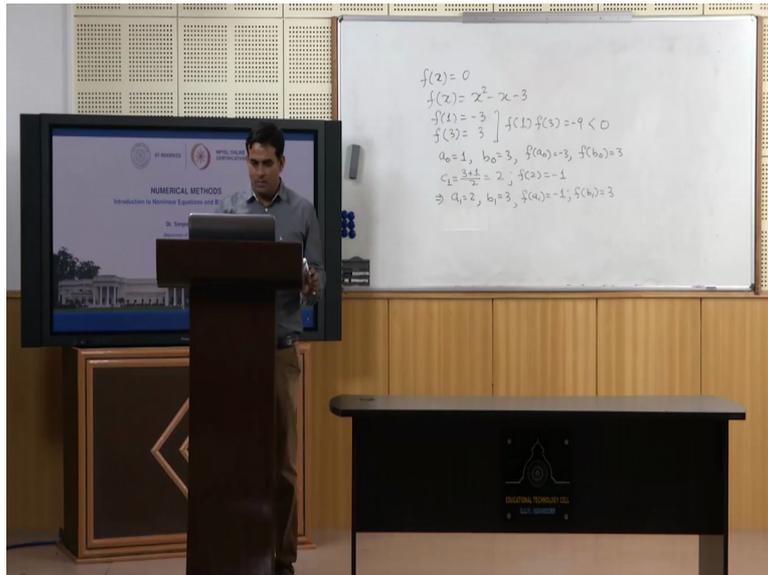
Example

Find the root of the equation $x^2 - x - 3 = 0$ using Bisection method correct up to 3 decimal places.

Solution

$f(x) = x^2 - x - 3$
 We see that $f(1) = -3$ and $f(3) = 3$. As, $f(1)f(3) < 0$, the initial interval will be $[1, 3]$ i.e. the root lies between 1 and 3.
 To find the root correct up to 3 decimal places, we fix the permissible absolute error as $0.001 (= 10^{-3})$.

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 22



Now we will take an example and we will see how we can solve this particular example using the bisection method, so example is an nonlinear equation and we will follow the steps of bisection method to get the solution, so let I need an nonlinear equation $f(x) = 0$ where $f(x)$ is given by $x^2 - x - 3$. Now I need to find out root of this, so if I check $f(1)$, $f(1)$ comes out to be $1 - 1 - 3$ as -3 and if I check $f(3)$, $f(3)$ will be negative if I check $f(3)$, $f(3)$ will become positive that is $3^2 - 3 - 3$ so it is positive.

As we can see that product of $f(1)$ into $f(3)$ is -9 which is a negative number it means the root of this particular equation $f(x) = 0$ where $f(x)$ is given by this function is lie between one and 3, so what I will do in the 1st iteration I will set a_0 equals to 1, b_0 as 3, $f(a_0)$ is -3 , $f(b_0)$ as 3. Now I will find c_1 , c_1 comes out to be the middle point of a_0 and b_0 so it will $\frac{3+1}{2}$ so which is 2, now I will check $f(2)$, $f(2)$ is $4 - 2 - 3$, so it is -1 , so check here if I take the product of $f(1)$ into $f(2)$ it is coming out a positive number hence the root lies between 2 to 3 not from 1 to 2. So it means my a_1 becomes 2, b_1 becomes 3, $f(a_1)$ is -1 , $f(b_1)$ is 3.

(Refer Slide Time: 26:58)

Nonlinear Equations

Example

N	a_n	b_n	x_n	$f(x_n)$	$ x_n - x_{n-1} $
0	1.0000	3.0000	2.0000	-1.0000	
1	2.0000	3.0000	2.5000	0.7500	0.5000
2	2.0000	2.5000	2.2500	-0.1875	0.2500
3	2.2500	2.5000	2.3750	0.2656	0.1250
⋮	⋮	⋮	⋮	⋮	⋮
11	2.3027	2.3037	2.3032	0.0016	0.0005

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 23

Nonlinear Equations

Example

At the 11th iteration, the absolute error is less than 0.001 i.e absolute permissible error and the root is accurate to 3rd decimal place. Thus, the root of the equation is 2.303.

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 24

So if I need to find out n accuracy up to of the order 10 raise to power minus 3 then the iteration will go like this in the 11th iterations I will get a n as 2.3027, b n as 2.3037, x n is 2.3032 and the value of fx at this particular number is 0.0016. If I check this x n minus x n minus 1 when the difference between 2 consecutive iterations it is coming out to be 0.0005 and hence in the 11 iteration the absolute error is less than 0.001 which is permissible error and the root is accurate to 3rd decimal place. Thus the root of the equation is 2.303. We are having some advantage when we are using the bisection method as well as some disadvantage.

(Refer Slide Time: 27:53)

Nonlinear Equations

Pros of Bisection method

- 1 This method is very easy to understand.
- 2 It always converges to a solution.
- 3 That's why it is often used as a starter for the more efficient methods.

Cons of Bisection method

- 1 This method is relatively slow to converge.
- 2 Choosing a guess close to the root may result in requiring many iterations to converge.

IT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 25

Handwritten notes on a whiteboard illustrating the bisection method for finding roots of the equation $f(x) = x^2 - x - 3 = 0$.

The notes show the function $f(x) = x^2 - x - 3$ and the interval $[1, 3]$ where $f(1) = -3$ and $f(3) = 3$. The product $f(1)f(3) = -9 < 0$ is noted. The midpoint $x_1 = \frac{3+1}{2} = 2$ is calculated, and $f(2) = -1$ is found. The new interval is $[2, 3]$ with $f(2) = -1$ and $f(3) = 3$.

Convergence formulas are written as $|x_{n+1} - x^*| \leq \frac{1}{2} |x_n - x^*|$ and $e_{n+1} \leq \left(\frac{1}{2}\right) e_n$, with $c = \frac{1}{2}$.

Advantage like this method is very easy to understand. It always converges to a solution. That is why it is often used as a starter for other more efficient numerical techniques. The disadvantages are this method is relatively slow to converge actually you can see if I am having $x_{n+1} - x^*$, so that is the error in n th iteration. Hence $e_{n+1} \leq \frac{1}{2} e_n$, so if I check with the convergence formula you can see if I take c equals to half, so this method is having exactly linear convergence and hence this method is quite a lot to converge, moreover if you choose a guess close to the root may result in requiring many iterations to converge, so in this lecture we have learn about bisection method. In the next lecture we will learn to other methods those are quite close to bisection method in the same category iteration updates in the same manner however they are having better convergence when compared to the bisection method. Thank you.