

Integral Equations, Calculus of Variations and their Applications
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Lecture 56
Variational problems with moving boundaries-2

Hello friends I welcome you to my lecture on variational problems with moving boundaries.

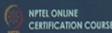
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Variational problem with moving boundaries

Let us find the shortest distance between a point (x_1, y_1) on the curve $y = g(x)$ and a point (x_2, y_2) on the curve $y = h(x)$ for which the functional

$$I\{y(x)\} = \int_{x_1}^{x_2} f(x, y, y') dx = \int_{x_1}^{x_2} \sqrt{1 + y'^2} dx,$$

is a minimum.



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So far we have considered variational problems where the boundary points x_1, y_1 and x_2, y_2 are fixed and we want to extremize the functional integral from x_1, y_1 to x_2, y_2 of $f(x, y, y')$ that we have considered initially. Now we will be considering the case where either both the boundary points x_1, y_1, x_2, y_2 are fixed are moving or one of them is moving. So let us take for example suppose we want to find the shortest distance between a point x_1, y_1 on a curve y equal to $g(x)$ and a point x_2, y_2 on another curve y equal to $h(x)$.

Then we would like to find the extremum value of the functional $I(y(x))$ equal to integral from x_1 to x_2 of $f(x, y, y')$ dx where $f(x, y, y')$ is the distance between the distance ds it is distance ds so that is under root $1 + dy$ by dx whole square into dx . So this $f(x, y, y')$ dx becomes under root $1 + y'$ square dx and the points x_1, y_1 and x_2, y_2 , either both of them are moving or one is fixed the other one is moving.

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At the free boundaries (x_1, y_1) and (x_2, y_2) we have the transversality conditions

$$\left[f(x, y, y') + (g' - y') \frac{\partial f}{\partial y'} \right]_{x=x_1} = 0 \quad \dots(1)$$

where (x_1, y_1) lies on the curve $y = g(x)$ and

$$\left[f(x, y, y') + (h' - y') \frac{\partial f}{\partial y'} \right]_{x=x_2} = 0 \quad \dots(2)$$

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So let us take a first problem where we will see that both the boundary points x_1, y_1 and x_2, y_2 are moving say and at the free boundaries x_1, y_1, x_2, y_2 , we know that we have the transversality conditions $f(x, y, y')$ plus g' minus y' into partial derivative of f with respect to y' at x equal to x_1 is 0, this is for the curve y equal to $g(x)$ and for the curve y equal to $h(x)$ similarly we have the other transversality condition $f(x, y, y')$ plus h' minus y' into $\frac{\partial f}{\partial y'}$ at x equal to x_2 equal to 0.

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where (x_2, y_2) lies on the curve $y = h(x)$.

The problem is therefore to find the shortest distance between the points (x_1, y_1) lying on $y = g(x)$ and (x_2, y_2) lying on $y = h(x)$.

Example: Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

Solution : We have

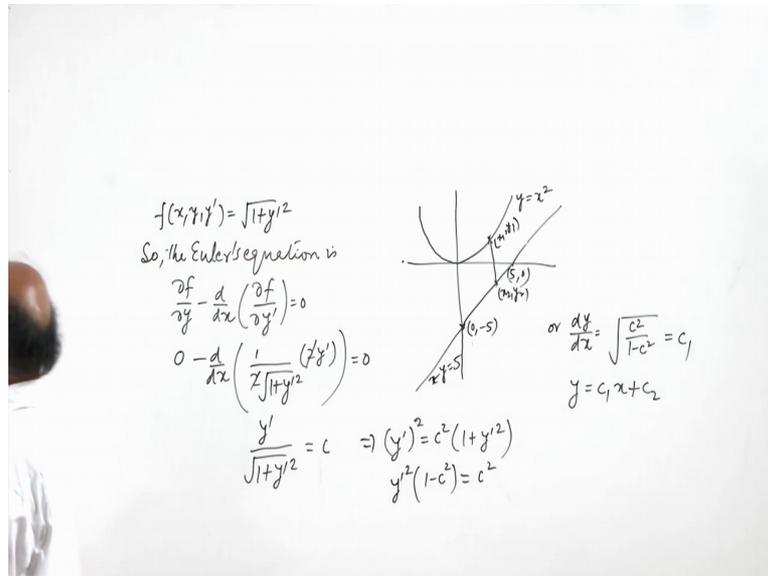
$$I(y(x)) = \int_{x_1}^{x_2} \sqrt{1 + y'^2} dx \quad \dots(3)$$

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Where x_2, y_2 lies on the curve y equal to $h(x)$. So the problem is therefore to find the shortest distance between the points x_1, y_1 lying on the curve y equal to $g(x)$ and x_2, y_2 lying on the

curve y equal to $h x$. For example suppose we have one curve y equal to $g x$ as y equal to x square and the other curve y equal to $h x$ as the straight line x minus y equal to 5.

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So let us see we can draw them, so this is y equal to x square y equal to x square and x minus y equal to 5, so when you put y equal to 0 we get x equal to 5 and when we put y x equal to 0 we get y equal to minus 5. So the straight line passes through 0, minus 5 here and 5, 0 here. So let us take one point x 1, y 1 here and another point x 2, y 2 here and we want to find the shortest distance between the curve y equal to x square and y equal to x minus y equal to 5.

Now you have seen that we have I have drawn the line joining x 1, y 1 to x 2, y 2 we can see that the shortest distance will among the extremal of this problem under root 1 plus y dash square actually is the shortest is the line. Say if you have here f x , y , y dash the extremal of this will will be a straight line this is equal to under root 1 plus y dash square. So if you differentiate so the Eulers Equation here is partial derivative of f with respect to y minus y d over dx , partial derivative of f with respect to y dash and what we get here is since f depends only on y dash so we have the partial derivative of f with respect to y 0 minus d over dx of partial derivative of f with respect to y dash. So that means 1 upon 2 under root 1 plus y dash square into 2 y dash equal to 0.

So we will get y dash upon under root 1 plus y dash square equal to some constant c when we integrate both sides. So or we can say this implies y dash square equal to c square 1 plus y dash square when we square both sides. So what we get is y dash square into 1 minus c square equal to c square or we can say dy by dx is equal to under root c square upon 1 minus

c square which we can write as some constant c 1, so y gives you c 1 x plus c 2 when we integrate.

So extremal of the functional integral x 1 to x 2 under root 1 plus y dash square dx is actually a straight line so we have but these points are moving x 1, y 1, x 2, y 2 so let us see the extremal for this problem but the extremal will be a straight line let us see what is that straight line.

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$g(x) = x^2, h(x) = x - 5$
 $f(x, y, y') = \sqrt{1 + y'^2}$

So the transversality conditions (1) and (2) imply

$$\left[(1 + y'^2)^{1/2} + (2x - y') \frac{y'}{\sqrt{1 + y'^2}} \right]_{x=x_1} = 0 \quad \dots (4)$$

and

$$\left[(1 + y'^2)^{1/2} + (1 - y') \frac{y'}{\sqrt{1 + y'^2}} \right]_{x=x_2} = 0 \quad \dots (5)$$

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$f(x, y, y') = \sqrt{1 + y'^2}$
 So, the Euler's equation is
 $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$
 $y = g(x) = x^2$
 $\left[f(x, y, y') + (g' - y') \frac{\partial f}{\partial y'} \right]_{x=x_1} = 0$
 $\left[f(x, y, y') + (h' - y') \frac{\partial f}{\partial y'} \right]_{x=x_2} = 0$

$y = x^2 = g(x) \quad h'(x) = 1$
 $\frac{d^2 y}{dx^2} = \sqrt{\frac{c^2}{1 - c^2}} = c_1$
 $y = c_1 x + c_2$

So with start with we start with the transversality condition the transversality condition here is what for the function for the curve y equal to g x y equal to g x is x square. So the transversality condition for the curve y equal to g x will be partial derivative of f with respect to y dash no this is f x, y, y dash, so f x, y, y dash plus the g dash derivative of d with respect

to $x - y'$ and partial derivative of f with respect to y' at $x = 1, y = 1$ equal to 0. This is the transversality condition for the curve $y = x$ on which $x = 1, y = 1$ lies.

So here $f(x, y, y')$ is given to be $1 + y'^2$ and this is $f(x, y, y')$ this is derivative of $g(x)$ with respect to x that is $g'(x) = 2x - y'$ and then this is partial derivative of f with respect to y' , so y' upon $\sqrt{1 + y'^2}$ at $x = 1, y = 1$ equal to 0 and similarly the transversality condition for the other curve $y = h(x)$.

So this is I can write $y = x - 5$. So the other curve is $y = h(x)$ where $h(x)$ is $x - 5$, so we have again $f(x, y, y')$ as $1 + y'^2$ and $g'(x)$ will be replaced by $h'(x)$ and $h'(x)$ is equal to 1. So the other transversality condition is $f(x, y, y') + h'(x) - y'$ into partial derivative of f with respect to y' at $x = 1, y = 1$ equal to 0 gives you this equation $1 + y'^2$ to the power half $h'(x)$ is equal to $1 - y'$, y' upon $\sqrt{1 + y'^2}$ to the power half is equal to 0, these are two are transversality condition.

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The general solution of the Euler's equation for the functional (3) is

$$y = c_1 x + c_2, \quad \dots(6)$$

c_1 and c_2 being arbitrary constants.

From (6), $y' = c_1$

Hence (4) $\Rightarrow 1 + 2x_1 c_1 = 0$

and from (5), we get $c_1 = -1$.

Thus $x_1 = 1/2$.



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$g(x) = x^2, h(x) = x - 5$
 $f(x, y, y') = \sqrt{1 + y'^2}$

So the transversality conditions (1) and (2) imply

$$\left[(1 + y'^2)^{1/2} + (2x - y') \frac{y'}{\sqrt{1 + y'^2}} \right]_{x=x_1} = 0 \quad \dots (4)$$

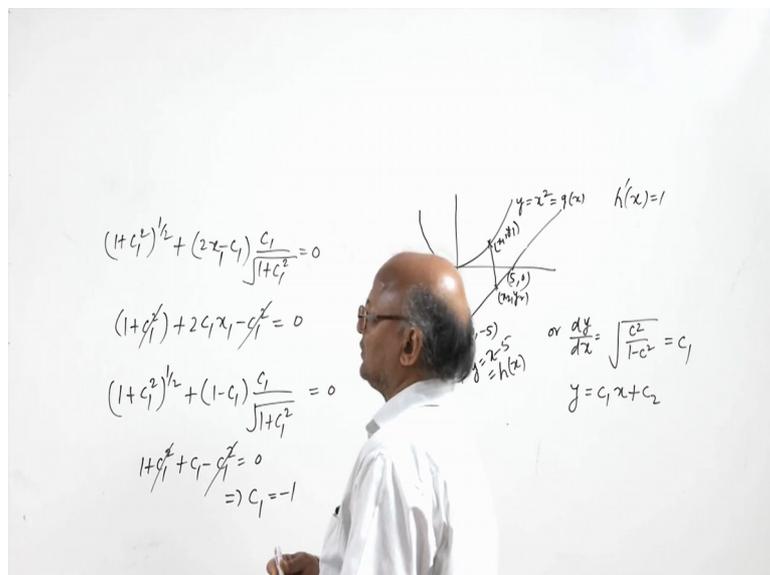
and

$$\left[(1 + y'^2)^{1/2} + (1 - y') \frac{y'}{\sqrt{1 + y'^2}} \right]_{x=x_2} = 0 \quad \dots (5)$$



The general solution of the Euler's Equation for the functional for the functional 3 that is this for the this is the general solution for this is this one y equal to $c_1 x$ plus c_2 the straight line y equal to $c_1 x$ plus c_2 where c_1, c_2 are arbitrary constants. Now from here we can see that y' is equal to c_1 so y' is equal to c_1 if you put y' equal to c_1 in equation 4 what we get is. Here if you put y' is equal to c_1 what we will get?

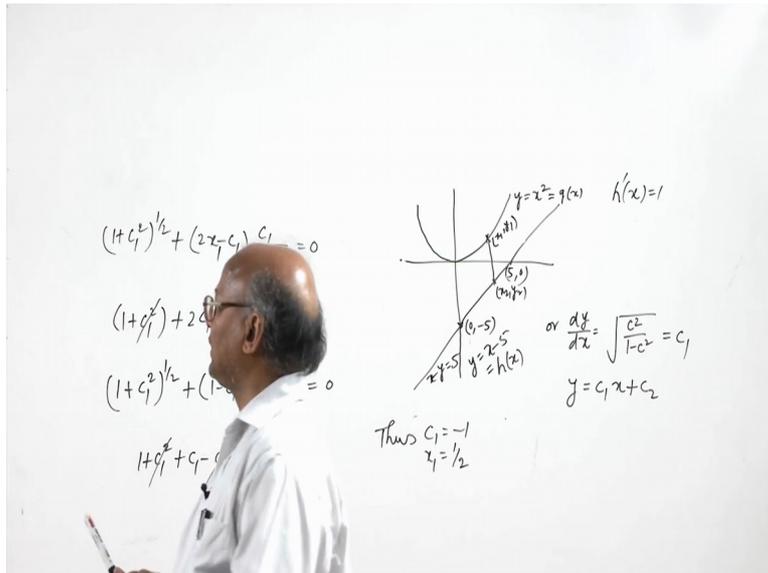
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$(1 + c_1^2)^{1/2} + (2x_1 - c_1) \frac{c_1}{\sqrt{1 + c_1^2}} = 0$
 $(1 + c_1^2) + 2c_1 x_1 - c_1^2 = 0$
 $(1 + c_1^2)^{1/2} + (1 - c_1) \frac{c_1}{\sqrt{1 + c_1^2}} = 0$
 $1 + c_1^2 + c_1 - c_1^2 = 0$
 $\Rightarrow c_1 = -1$

Graph showing $y = x^2 = g(x)$ and $h(x) = 1$. Points (x_1, y_1) and (x_2, y_2) are marked on the curve. The slope of the tangent at (x_1, y_1) is $y' = 2x_1 = c_1$.

or $\frac{dy}{dx} = \sqrt{\frac{c_2}{1 - c_2}} = c_1$
 $y = c_1 x + c_2$



$(1+c_1^2)^{1/2} + (2x-c_1) c_1 = 0$
 $(1+c_1^2)^{1/2} + 2c_1 = 0$
 $(1+c_1^2)^{1/2} + (1-c_1) = 0$
 $1+c_1^2+c_1 = 0$

So what we will get is $x = 1/2$. So we can multiply by $\sqrt{1+c_1^2}$ so we get $1+c_1^2+2c_1x-1+c_1^2 = 0$, so this cancels and we get $1+2c_1x = 0$ and from the equation 5 let us see the equation 5 from the equation 5 we have $1+y^2 = 1+c_1^2$ raise to the power half and then we get $1-y = 1-c_1$ and then we get $c_1 = 1$ upon $\sqrt{1-c_1^2} = 0$. So what we will get? $1+c_1^2 = 1-c_1^2$ when we take the LCM we get $1+c_1^2 = 1-c_1^2 = 0$.

So we get $c_1 = -1$ from the transversality condition number 5. Now so $c_1 = -1$ and thus $x = 1/2$ if you put $c_1 = -1$ here you get $x = 1/2$. So thus $c_1 = -1$ and $x = 1/2$.

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Since both the end points (x_1, y_1) and (x_2, y_2) lie on the extremal have $y = c_1 x + c_2$, we have

$$c_1 x_1 + c_2 = y_1 = (x_1)^2$$

and $c_1 x_2 + c_2 = y_2 = x_2 - 5$.

Using $c_1 = -1$ and $x_1 = 1/2$, we obtain

$$c_2 = 3/4 \text{ and } x_2 = 23/8.$$

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Now let us go to so since both the end points x_1, y_1, x_2, y_2 lie on the extremal y equal to $c_1 x$ plus c_2 they will satisfy this equation. So we get $c_1 x_1$ plus c_2 equal to y_1 and y_1 is equal to x_1 whole square and then we get $c_1 x_2$ plus c_2 equal to y_2 and y_2 is x_2 minus 5. Now let us use c_1 equal to minus 1 here and x_1 equal to half here, so then you can easily see that c_2 comes out to be 3 by 4 and x_2 comes out to be 23 by 8.

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Thus, the required extremal is

$$y = -x + \frac{3}{4}$$

and the shortest distance between the given parabola and the straight line is

$$\int_{1/2}^{23/8} \sqrt{1 + y'^2} dx = \int_{1/2}^{23/8} \sqrt{1 + c_1^2} dx = \frac{19\sqrt{2}}{8}.$$

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Since both the end points (x_1, y_1) and (x_2, y_2) lie on the extremal have $y = c_1 x + c_2$, we have

$$c_1 x_1 + c_2 = y_1 = (x_1)^2$$

and $c_1 x_2 + c_2 = y_2 = x_2 - 5$.

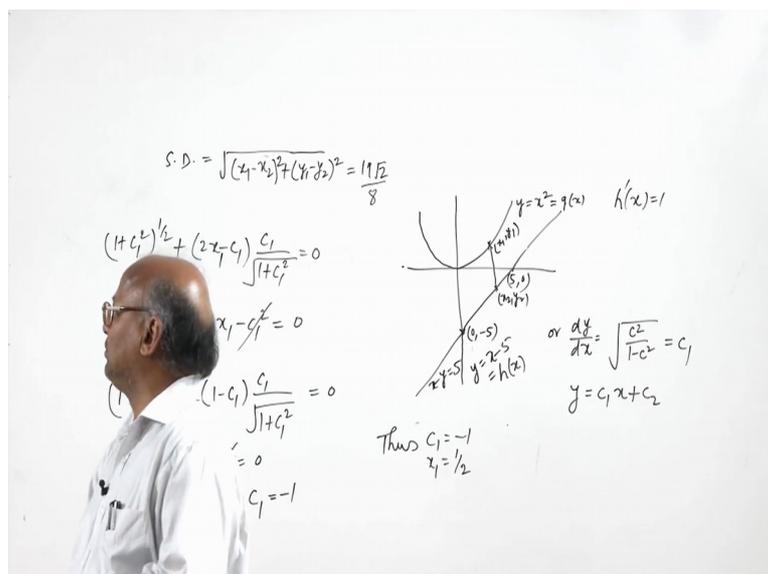
Using $c_1 = -1$ and $x_1 = 1/2$, we obtain

$$c_2 = 3/4 \text{ and } x_2 = 23/8.$$




And thus the required extremal line is y equal to $c_1 x$ plus c_2 so $c_1 x + c_2$ is $-1x + 3/4$ and c_2 is $3/4$, so we get y equal to $-x + 3/4$ and the shortest distance between the given parabola and the straight line is $\int_{x_1}^{x_2} \sqrt{1 + y'^2} dx$ where y' is c_1 , so $\int_{1/2}^{23/8} \sqrt{1 + (-1)^2} dx$ but c_1 is equal to -1 so we will be getting $\int_{1/2}^{23/8} \sqrt{2} dx$ that is $\sqrt{2} [x]_{1/2}^{23/8}$ so $\sqrt{2} (23/8 - 1/2)$ that is $19\sqrt{2}/8$. So this is how we can get the shortest distance between the given parabola and the straight line.

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$S.D. = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \frac{19\sqrt{2}}{8}$

$(1 + c_1^2)^{1/2} + (2x_1 - c_1) \frac{c_1}{\sqrt{1 + c_1^2}} = 0$

$x_1 - c_1^2 = 0$

$(1 - c_1) \frac{c_1}{\sqrt{1 + c_1^2}} = 0$

$c_1 = -1$

$y = x^2 = g(x)$ $h'(x) = 1$

$or \frac{dy}{dx} = \frac{2x}{1 - c_1^2} = c_1$

$y = c_1 x + c_2$

Thus $c_1 = -1$
 $x_1 = 1/2$

We can instead of this we can also use the formula under root $x_1 - x_2$ whole square $S.D.$ equal to because we know x_1, y_1 as well as x_2, y_2 and we get $19\sqrt{2}/8$. Now so here we we took a problem where both the points x_1, y_1, x_2, y_2 were moving.

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Example: Find the shortest distance between the point $A(1, 0)$ and the ellipse $4x^2 + 9y^2 = 36$.

Solution : We know that

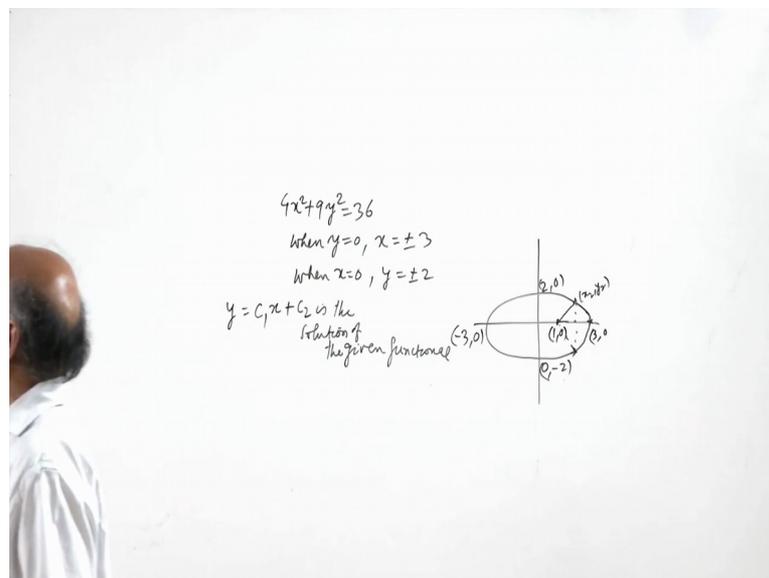
$$I(y(x)) = \int_{x_1}^{x_2} \sqrt{1 + y'^2} dx, \quad \dots(1)$$

where $4x^2 + 9y^2 = 36$ and $x_1 = 1$... (2)

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Now let us take a problem where one of the end points is fixed x_1, y_1 say for example is 1, 0 and x_2, y_2 lies on the ellipse and we want to find the shortest distance between the point A that is 1, 0 and the ellipse $4x^2 + 9y^2 = 36$.

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So let us draw the curves and plot the point. So if you so we have the ellipse $4x^2 + 9y^2 = 36$, you can see here that when y equal to 0, x is equal to plus minus 3 and when x equal to 0, y equal to plus minus 2. So we can draw the parabola sorry ellipse, so this is 3, 0 minus 3, 0, here is 0 minus 2, here is 2, 0 and 1, 0 point 1, 0 point let us say this is 1, 0 point.

So we have to find the shortest distance between the point $(1, 0)$ and the ellipse. Now we can see that ellipse is symmetric with respect to x as well as y axis. So if this is your point (x_1, y_1) , this is your point (x_2, y_2) then you can see that if the straight line and again we have that the extremal are the solution of the extremal the functional $\int y \, dx$ equal to x_1 to x_2 as we have seen in the previous problem under root $(y_1 + y_2) \sqrt{1 + y'^2} \, dx$ lies along the is the straight line.

So y equal to $c_1 x + c_2$, this is the solution of of the given functional. So if this point is (x_2, y_2) then you can get the corresponding y symmetry the other point also there actually so there will be two points (x_2, y_2) and its this point this also will have the same distance from $(1, 0)$. So what we will be doing is that when we solve this equation for the value of y we shall consider y to be positive, so by symmetry we can get the other points also. Now this is this is the equation $4x^2 + 9y^2 = 36$ and $x_1 = 1$, this is $x_1 = 1$.

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Now $y = g(x) = \frac{2}{3} \sqrt{9 - x^2}$.

The extremal of (1) is a straight line $y = c_1 x + c_2$, ... (3)

Since $(x_1, y_1) = (1, 0)$ lies on (3)

$\Rightarrow c_2 = -c_1$

Also $y_2 = c_1 x_2 + c_2 = c_1 (x_2 - 1)$

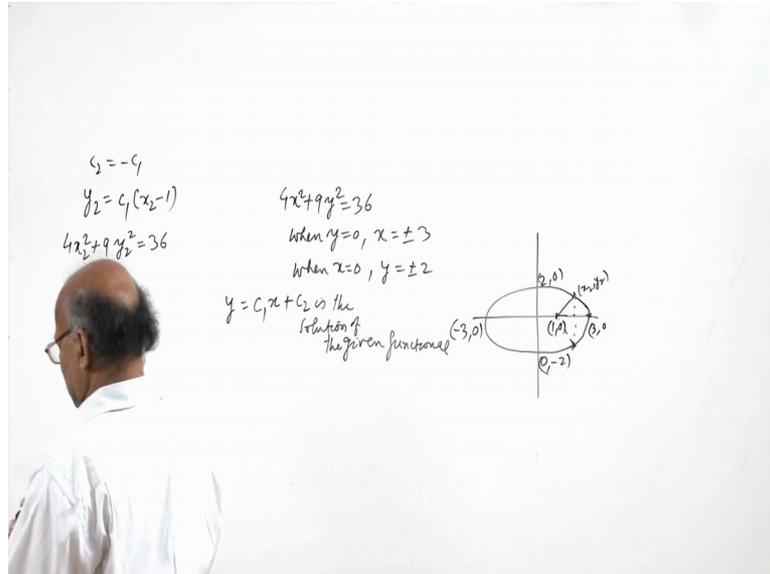
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Now what we do is we can solve the equation y equal to this is equation let us say $4x^2 + 9y^2 = 36$ can be written as $y = \frac{2}{3} \sqrt{9 - x^2}$. So while solving this equation as you can see we have taken the positive value of the square root. So we have taken this the point (x_2, y_2) here you can if you take negative value you get the point which is symmetric with respect to x axis.

Now extremal of the given functional is a straight line so y equal to $c_1 x + c_2$, (x_1, y_1) lies $(1, 0)$ which lies on this line so we get $c_2 = -c_1$ or (x_2, y_2) also lies (x_2, y_2) is the point on the curve (x_2, y_2) lies on the line $y = c_1 x + c_2$, so $y_2 = c_1 x_2 + c_2$.

to $c_1 x_2$ plus c_2 and when we put c_1 equal to c_2 equal to minus c_1 we get here c_1 times x_2 minus 1.

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So y_2 is equal to c_1 times x_2 minus 1, so this is the value of y_2 and we also have c_2 equal to minus c_1 .

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and hence (2) \Rightarrow

$$4x_2^2 + 9c_1^2(x_2 - 1)^2 = 36.$$

Now, the transversality condition at (x_2, y_2) is given by

$$\left[F + (g' - y')F_{y'} \right]_{x=x_2} = 0$$

$$\left[(1 + y'^2)^{1/2} + \left(\frac{-2x}{3\sqrt{9-x^2}} - y' \right) \frac{y'}{\sqrt{1+y'^2}} \right]_{x=x_2} = 0 \quad \dots (4)$$



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Now we go to the transversality condition before that x_2, y_2 point lies on these parabola. So $4x_2^2 + 9y_2^2$ is equal to 36. So putting the value of y_2 equal to c_1 times x_2 minus 1 we get this equation $4x_2^2 + 9c_1^2(x_2 - 1)^2 = 36$. Now let us write the transversality condition at the moving point x_2, y_2 is the moving

point so at x_2, y_2 which is F plus g dash minus y dash into the partial derivative of F with respect to y dash.

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and hence (2) \Rightarrow

$$4x_2^2 + 9c_1^2(x_2 - 1)^2 = 36.$$

Now, the transversality condition at (x_2, y_2) is given by

$$\left[F + (g' - y')F_y \right]_{x=x_2} = 0$$

$$\left[(1 + y'^2)^{1/2} + \left(\frac{-2x}{3\sqrt{9-x^2}} - y' \right) \frac{y'}{\sqrt{1+y'^2}} \right]_{x=x_2} = 0 \quad \dots (4)$$



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Now $y = g(x) = \frac{2}{3}\sqrt{9-x^2}$.

The extremal of (1) is a straight line

$$y = c_1x + c_2, \quad \dots (3)$$

Since $(x_1, y_1) = (1, 0)$ lies on (3)

$$\Rightarrow c_2 = -c_1$$

Also $y_2 = c_1x_2 + c_2 = c_1(x_2 - 1)$



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So here F_x by y dash is under root 1 plus y dash square. So we have 1 plus y dash square raise to power half and g_x is equal to this is $g_x = \frac{2}{3}$ under root 9 minus x square when we differentiate g with respect to x we get $-\frac{2x}{3}$ under root 9 minus x square minus y dash and the partial derivative of this F_x, y, y dash with respect to y dash is y dash over under root 1 plus y dash square at x equal to x_2 is 0 , this is the transversality condition at the moving point.

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Since (3) $\Rightarrow y' = c_1$, we have

$$(1+c_1^2)^{1/2} + \left(\frac{-2x_2}{3\sqrt{9-x_2^2}} - c_1 \right) \frac{c_1}{\sqrt{1+c_1^2}} = 0$$

(3) $\Rightarrow \quad 3\sqrt{9-x_2^2} = 2c_1x_2 \quad \dots(5)$

Since $y_2 = c_1(x_2 - 1)$, so $\frac{2}{3}\sqrt{9-x_2^2} = c_1(x_2 - 1)$

or $\sqrt{9-x_2^2} = \frac{3}{2}c_1(x_2 - 1) \quad \dots(6)$



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and hence (2) \Rightarrow

$$4x_2^2 + 9c_1^2(x_2 - 1)^2 = 36.$$

Now, the transversality condition at (x_2, y_2) is given by

$$\left[F + (g' - y')F_{y'} \right]_{x=x_2} = 0$$

$$\left[(1+y'^2)^{1/2} + \left(\frac{-2x}{3\sqrt{9-x^2}} - y' \right) \frac{y'}{\sqrt{1+y'^2}} \right]_{x=x_2} = 0 \quad \dots(4)$$



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Now we have y' equal to from this equation y' equal to c_1 x_2 plus c_2 y' dash equal to c_1 . So let us put that in this transversality condition we will get $1 + c_1$ square to the power half and here we will get $\frac{-2x_2}{3\sqrt{9-x_2^2}} - c_1$ this is c_1 this is c_1 upon $1 + c_1$ square under root, so we get this equation. So this equation we get this is what we get from the transversality condition.

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Since (3) $\Rightarrow y' = c_1$, we have

$$(1+c_1^2)^{1/2} + \left(\frac{-2x_2}{3\sqrt{9-x_2^2}} - c_1 \right) \frac{c_1}{\sqrt{1+c_1^2}} = 0$$

Hence $3\sqrt{9-x_2^2} = 2c_1x_2$... (5)

Since $y_2 = c_1(x_2 - 1)$, so $\frac{2}{3}\sqrt{9-x_2^2} = c_1(x_2 - 1)$

or $\sqrt{9-x_2^2} = \frac{3}{2}c_1(x_2 - 1)$... (6)

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Now when you simplify this equation what after simplification what you will get is 3 under root 9 minus x 2 square equal to 2c 1 x 2. Now what we have y 2 equal to c 1 into x 2 minus 1, so let us replace y 2 equal to 2 by 3 under root 9 minus x 2 here then this is equal to c 1 x 2 minus 1 or we can say 9 minus x 2 square equal to 3 by 2 into c 1 x 2 minus 1. So this equation and this equation be now combined together.

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From (4) and (5), we have

$$\frac{2}{3}c_1x_2 = \frac{3}{2}c_1(x_2 - 1) \quad \dots(7)$$

Now, $c_1 \neq 0$ because if $c_1 = 0$ then $c_2 = 0$ and so $y = c_1x + c_2 = 0$, i.e., x-axis then the shortest distance along this line from $(1,0)$ to the ellipse is 2.

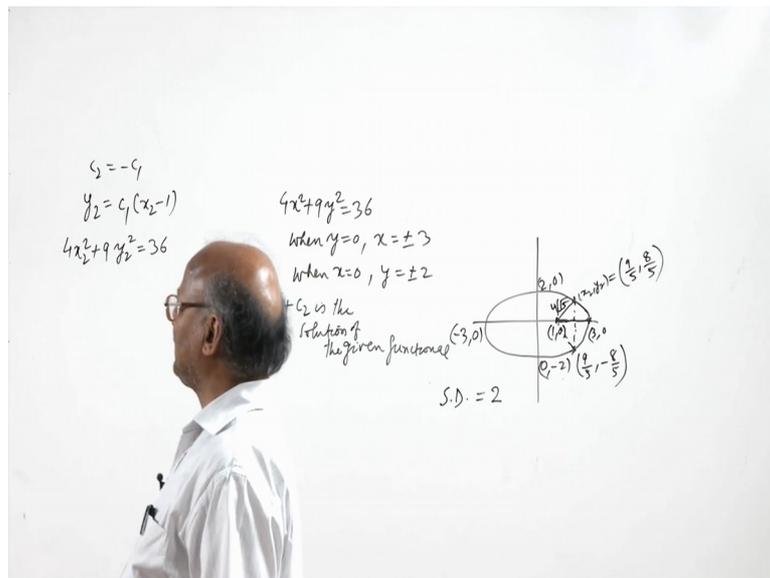
If we take $c_1 \neq 0$ then from (7) $x_2 = \frac{9}{5}$ and so

$$y_2 = \frac{2}{3}\sqrt{9-x_2^2} = \frac{8}{5}.$$

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And arrive at this equation 2 by 3 c 1 x 2 equal to 3 by 2 c 1 into x 2 minus 1. Now there arise two possibilities one is that c 1 is equal to 0 when c 1 is equal to 0 what will happen then c 2 is also equal to 0.

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So what will happen y equal to y equal to 0 we will get, y equal to 0 means x axis. Now x axis means the distance from 1, 0 to the ellipse will be along the line this that means this distance from 1, 0 to 3, 0 and the distance will be S. D. will be equal to 2, here so here the distance is 2 if you take c 1 equal to 0. But if you take c 1 non zero then what we will get from this equation we will get x 2 equal to 9 by 5 and from the relation y 2 equal to 2 by 3 under root 9 minus x 2 square y 2 comes out to be 8 by 5.

So we get x 2 equal to 9 by 5, so this is 9 by 5, 8 by 5 and so this distance of 9 by 5, 8 by 5 from 1, 0 is 4 by root 5 so this distance is 4 by root 5 and similarly by symmetry the other point here is 9 by 5 minus 8 by 5 so this.

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Then S. D =
$$\sqrt{\left(\frac{9}{5} - 1\right)^2 + \left(\frac{8}{5} - 0\right)^2}$$

$$= \frac{4}{\sqrt{5}}$$

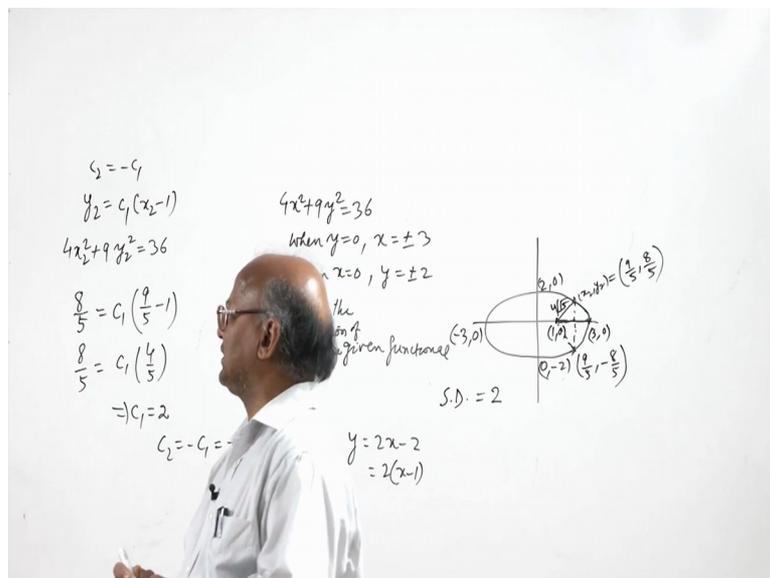
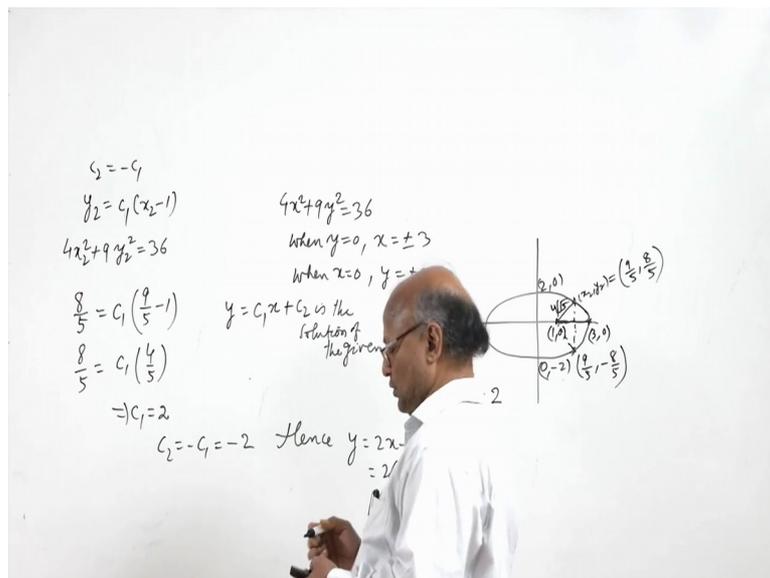
which is less than 2.

This S. D. is achieved along the line $y = 2(x - 1)$. Hence the extremal curve is $y = 2(x - 1)$.




Now you can see that 4 by root 5, 4 by root 5 is less than 2 we got when we took c 1 equal to 0 and when we took c 1 equal to 0 we got the line line y equal to c 1 x plus c 2 x as y equal to 0 which is the x axis and so the shortest distance from 1, 0 to the ellipse was the distance between 1, 0 and 3, 0 point which is 2 and when c 1 is not equal to 0 we have got S. D. equal to 4 by root 5 and clearly 4 by root 5 is less than 2. So this S. D. is achieved along the line y equal to two times x minus 1.

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So here actually when you calculate c 1 then it will come out to be 2, you can see from here y 2 equal to 8 by 5 y 2 equal to 8 by 5 c 1 times 9 by 5 minus 1. So this will give you 8 by 5 equal to c 1 times 4 by 5, so this gives c 1 equal to 2 and c 2 is equal to minus c 1 so it is minus 2. So y is equal to c 1 c plus c 2 that is y equal to 2 x minus 2 or two times x minus 1.

So the shortest distance is achieved along the line y equal to 2 times x minus 1 and thus we can say that the extremal curve is y equal to 2 times x minus 1 and the shortest distance is 4 by root 5 with this I would like to conclude my lecture thank you very much for your attention.