Approximate Reasoning using Fuzzy Set Theory
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Lecture - 22
(N, T, I) – An Organic Relationship

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Hello and welcome to the last of the lectures in this week, wherein we have predominantly and exclusively discussed Fuzzy Implications, under this course titled Approximate Reasoning using Fuzzy Set Theory. A course offered over the NPTEL platform.

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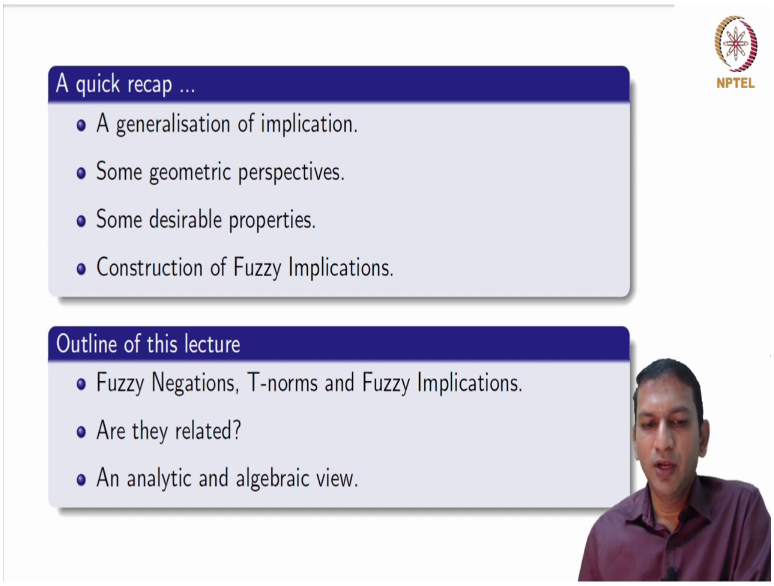

So far in this week we have seen a generalization of classical implication to the setting of fuzzy set theory. We have had some geometric perspectives of these operations in terms of the 3D plots. We have also listed down some desirable properties that we expect a fuzzy implication to process. Finally, we have discussed at length at least two different ways of constructing fuzzy implications. One, from other fuzzy logic connectives, and the second one from simple unary functions from [0, 1] to R^+.

In this lecture, let us look at the basic fuzzy logic connectives that we have dealt with so far that of T-norms, triangular norms, fuzzy implications and negations. An interesting question could be are they related in some way? Does there exist any relationship between them? Now, this is the question that we would like to explore in this lecture from an analytic and an algebraic perspective.

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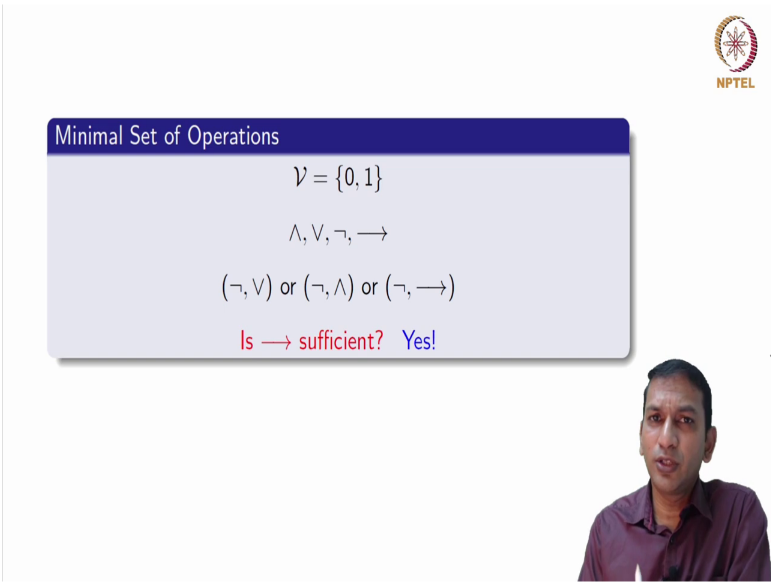

Towards this end, let us look at classical logic connectives and the relationships and the construction that we could have on them.

So, we know that in the setting of classical logic, the set of truth values is just restricted to 0 and 1, true or false. And these are the 4 operations that we have that of conjunction, disjunction, negation and implication. If one way to ask from these 4 operations can we extract a few of these operations as the minimal set of operations; that means, with a fewer than these 4 operations will we be able to obtain the other operations. In fact, it can be easily seen this is true. For example, if you take the negation and distinction, we could easily obtain the conjunction and the implication.

Also, let us look at it.

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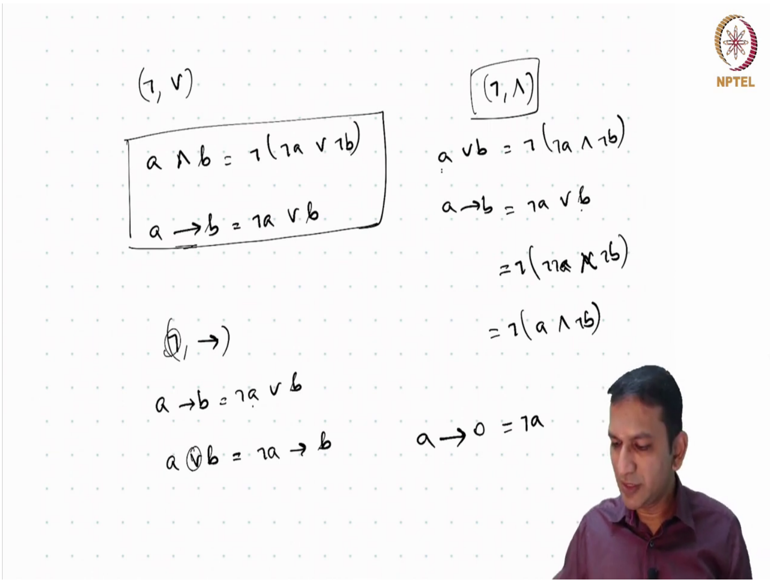

So, now we are considering the set of the pair of operations, negation, and disjunction, and what we need to define is the conjunction and implication from this. Clearly, because in the classical logic setting the negation is involutive. We know that disjunction and conjunction are related to one another by the duality. So, this could be written as negation a or negation b.

Now, when we have, when it comes to a implies b, we know from the previous lectures that one of the inspirations or motivations for us to come up with the S, N implication is the material implication which is given as follows, negation a or b. So, that means, from negation and disjunction we are able to obtain the other two operations.

What if we take negation and conjunction? Well, the properties due to the duality between conjunction and disjunction ensure that given a negation and conjunction, we can still obtain a disjunction and the implication. So, disjunction comes out because of duality.

And now we already know that once you have a disjunction a implies b can be written in terms of the disjunction. But we could also write it purely in terms of the negation and conjunction.

Note that what we need is negation a or b, we have a or b here. So, we need to put a negation here, so negation of negation negation a or negation and negation b. But we know that by the negation being involutive, this essentially is nothing, but negation a and negation b, negation of a and negation of b. Which means once again this forms this set forms a minimal set.

In fact, with negation if you take the implication, once again this forms a minimal set. For instance we know that a implies b can be written as negation a of b. So, using this formula, you could get a disjunction b as instead of negation take it as negation a implies b. So, the moment you have disjunction and a negation, we know how to obtain the corresponding conjunction.

Now, people are not happy with this. They went further and ask this question, is the implication operation alone sufficient? So, now, that we know that with a negation if you club any other binary operation that of conjunction, distinction, or implication, we actually could obtain the other two operations.

Let us start with just only the implication. So, it means that if we could somehow get negation from implication, then we are done. But then that is immediate; if you have implication a implies 0, if you take that, we know that this gives us the negation.

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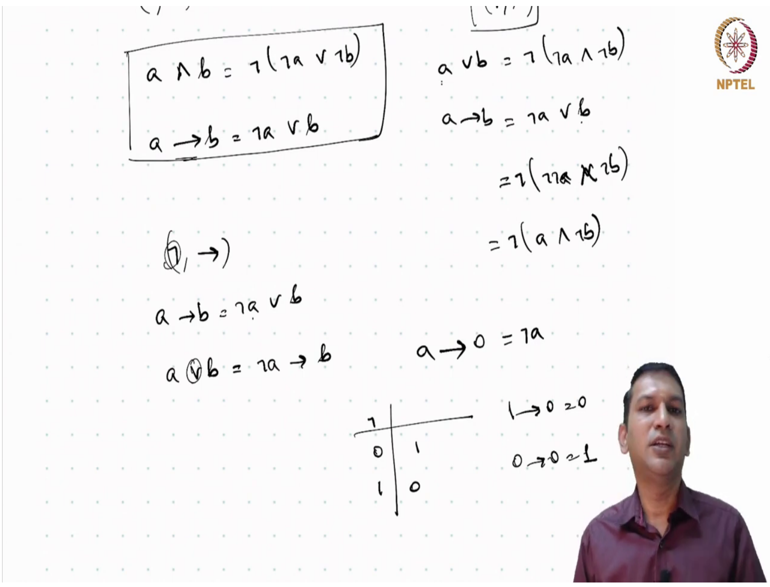

Remember, from the classical truth table what we have is for negation 0 is 1 and 1 is 0. And when you look at implication 1 implies 0 is 0 and 0 implies 0 is 1. This is something that we have seen so far when we have discussed the case of fuzzy implications also. That is how we have defined the natural negation.

So, which means that we can obtain the other binary logic operations just from implication, not just binary also the unary operation of negation. So, that is how important implication is in the classical logic setting. This, the last few minutes should have also shown you that there are relationships among these 4 classical logic operations.

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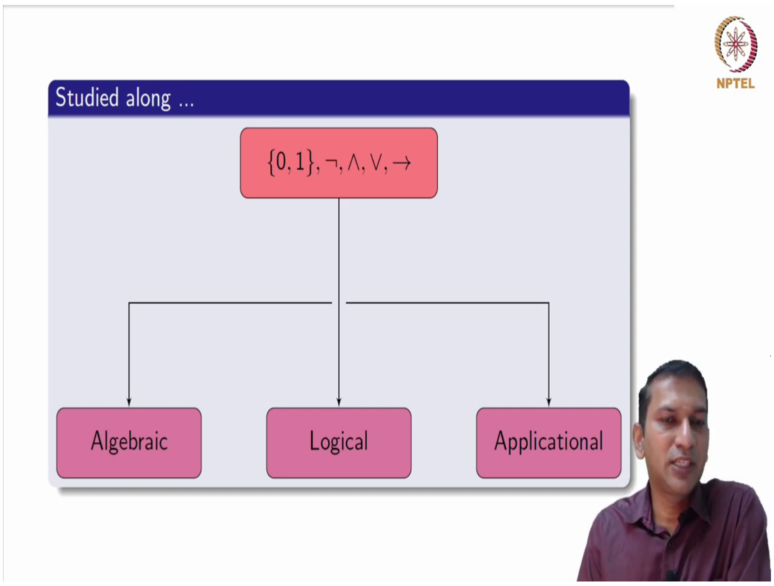

Well, let us move ahead. So, if you take the classical logic operations and the set of truth values 0 and 1, these have always been studied along different aspects and perspectives. They have discussed the logical aspects of this set with these operations. Of course, the algebraic aspects and also the application aspects, where they can be utilized. We know that logic is very much used in reasoning and they form the backbone of earlier expert systems and decision making systems.

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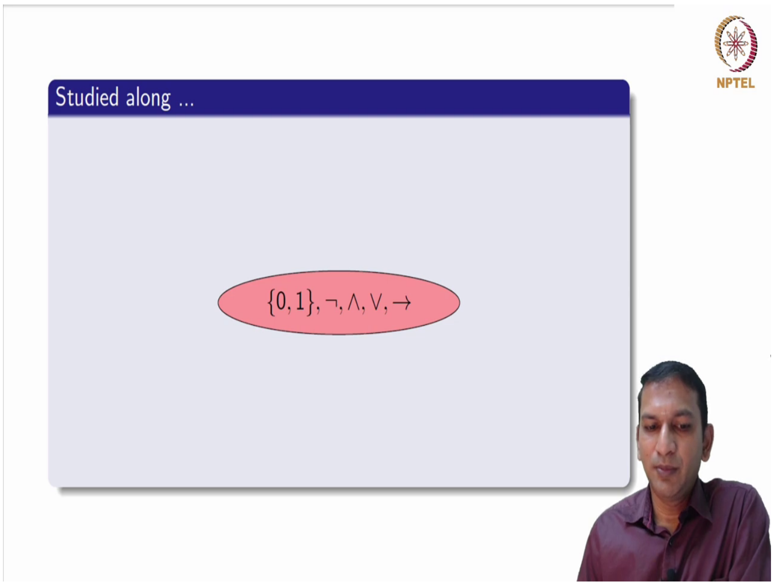

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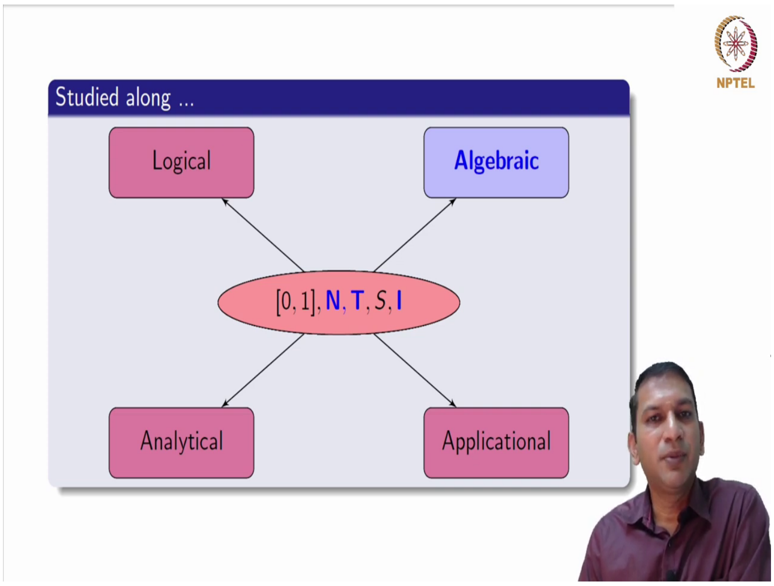

Now, when you move to the fuzzy logic connectives, let us look at how these connectives are studied.

Note that from the classical logic setting, we move to the fuzzy logic setting, but by first expanding the set of truth values, by defining a fuzzy negation, a generalization of a classical negation. And one particular generalization of conjunction, basic one is that of a t-norm, the distinction was generalized to a t-co-norm, and the implication to a fuzzy implication.

So, in this setting, how have these operations been studied? Of course, their logical aspects have been studied. The algebraic aspects have been studied. The application aspects have been studied. However, in this case we also have on the underlying set of [0, 1] a topology available for us, which meant that we could also study the analytical aspects of these operations. That is why we have discussed about different types of continuity on the set of t-norms.

Well, so far in this week, or the last 2 weeks, we have discussed lot of analytical properties of these operations. Going forward we will look at the application aspects, where they are useful in applications. The logical aspects have always been the underpinnings for us, even though we have not dealt with these in depth.

In today's lecture let us look at some algebraic aspects of these operations. So, specifically we will try to relate the negation, t-norm and the implication from an algebraic perspective.

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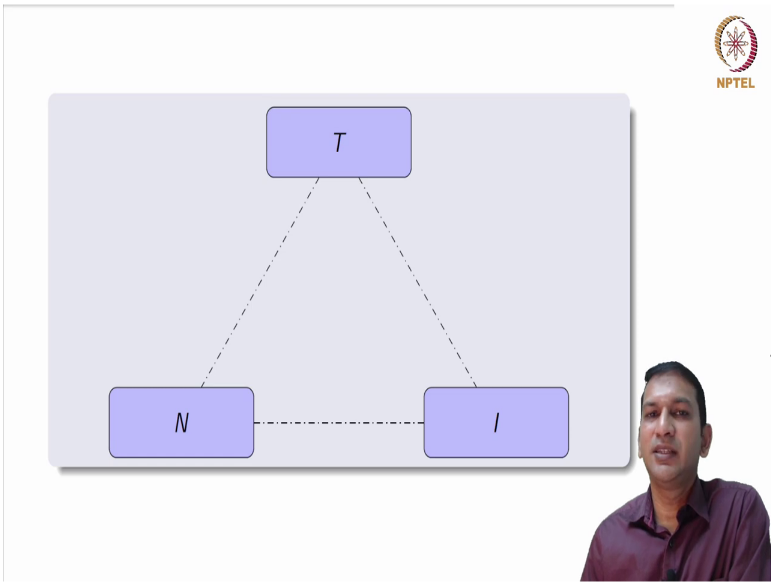

So, the question now is these are the 3 basic fuzzy logic connectives that we have seen, are there relationships between them, between T and N, T and I, and I and N? And we will see by the end of this lecture. Yes, there are.

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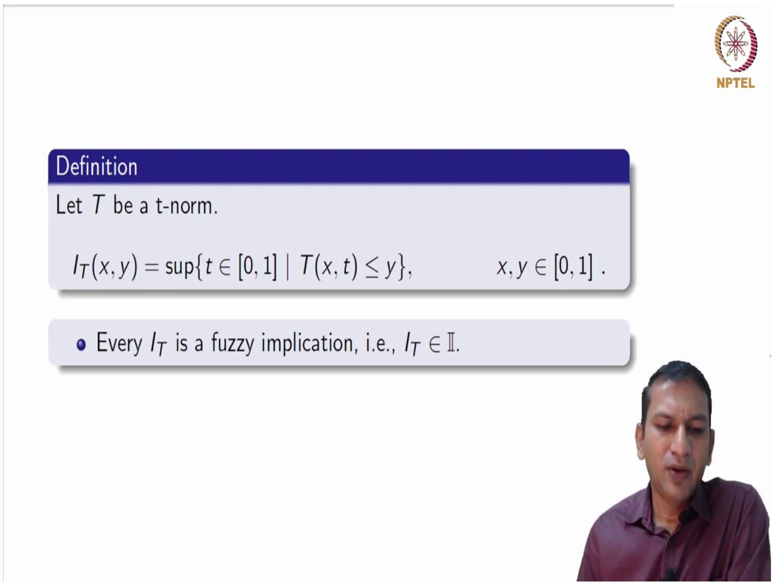

Let us begin by looking at it from an algebraic perspective. We have seen what an R-implication is. Given a t-norm if you define this function I_T, denoted as I_T, as supremum of all those T such that T(x,t) is less than or equal to y, we know that such a function is an is a fuzzy implication. This is what we call an R-implication.

Now, in the literature there is a very special kind of lattice called the residuated lattice.

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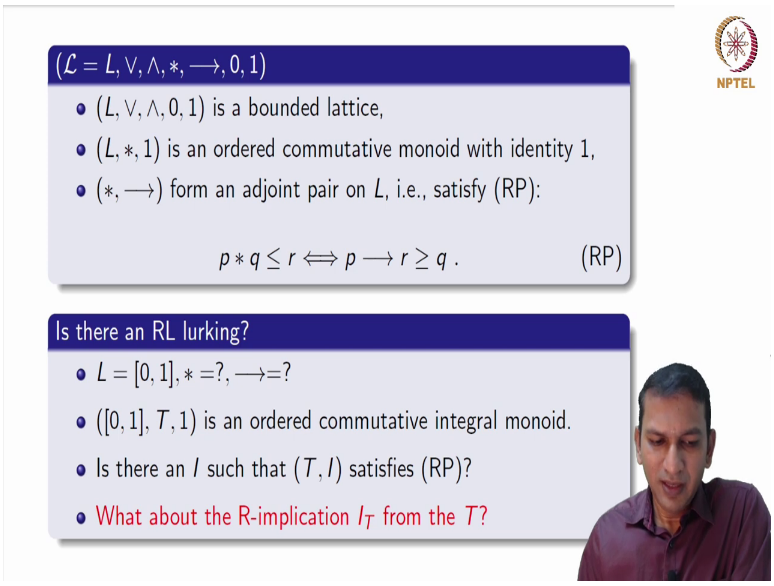

What is it? It consists of a set L and 4 binary operations that of joint and meet, the star operation, and the implication operation or the arrow operation as they call it in that setting, and two constants 0 and 1. These are elements coming from L.

We call it a residuated lattice if the following properties are true. Firstly, L with this joint and meet, and these two constants 0 and 1 should be a bounded lattice. We know what a bounded lattice is. So, this is what expect on this structure.

And L along with the binary operation star and the element special element 1, should be an ordered commutative monoid with identity 1, right. We understand what is commutativity, star is symmetric, we understand what is monoid, it is an associative operation binary operation and also has the has an identity.

It is an integral monoid, if the identity also happens to be the top element and here we have an order. So, order comes from the lattice underneath and this operation star should be compatible with the order; that means, it should respect the order, monotonic increasing with respect to the order that we have.

Finally, the star operation, the arrow operation, they should form an adjoint pair on L. What does it mean? They should satisfy the following property.

It is called the residuation property and that is why we have shortened it and given this the tag RP for this. What does it say? Whenever p star q is less than or equal to r, this should imply p implies r or p arrow r is greater than or equal to q. And conversely whenever p arrow r is greater than or equal to q, p star q should be less than or equal to r. Now, this is how the residuated lattice is defined.

Now, the interesting question is we also have lots of these operations and we can ask the question, is it possible to take these operations and make the residuated lattice out of them. Is there a residuated lattice working somewhere?

Now, for us the set of values that we all the domain for us is the 0, 1 interval. So, let us take L to be 0, 1. And let us ask and the moment we take 0, 1 to be the set we already have the usual ordering, the total ordering on that, which means the joint and meet are there and immediately the least element is 0 and the top element is 1 and so we have a bounded lattice. The question is how should the star and the arrow operation be defined.

We already know that if you consider a t-norm on the 0, 1 interval we know it is an ordered commutative integral monoid. That means, it is a monoid where the identity is also the top element, which is 1 here. So, now, the question is what should be considered for the arrow operation. That means, if you take a T does there exist an implication I, such that the pair T, I satisfies the residuated property.

Where do we go in search of this I? Question is why not look at R-implication. Can they be the ones that will help us to obtain a residuated lattice? Let us discuss this.

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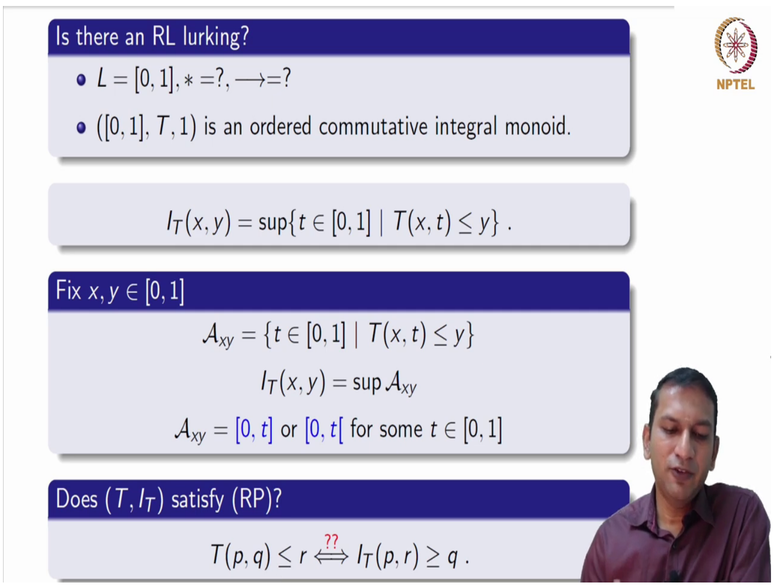

Now, recall the definition of an R-implication, supremum of all those T, such that T of x t is less than or equal to y. Now, if you are given an x and y let us fix them and look at this set A xy. This must be familiar from one of the previous lectures. This is nothing, but the set of all those T sets such that T of x t is less than or equal to y. Immediately, it is clear then I_T of x, y is nothing but supremum of A_xy.

Now, we have also seen due to the monotonicity of the t-norm T A xy is always an interval, in fact it will look like in the closed interval 0, t or closed 0 and open t. So, either this t may belong or may not belong which is essentially the supremum value of this interval and the I_T of x, y assumes that value.

So, now the question that we are going to ask is does the path T, I_T, where I_T is the R-implication satisfy RP? This essentially means we need to discuss this equation. So, instead of using the infix notation, we have just started to use the prefix notation. So, T for star and I_T for the arrow operation. So, this is what we want to ask T of p q less than or equal to r if and only I_T of p, r greater than or equal to q.

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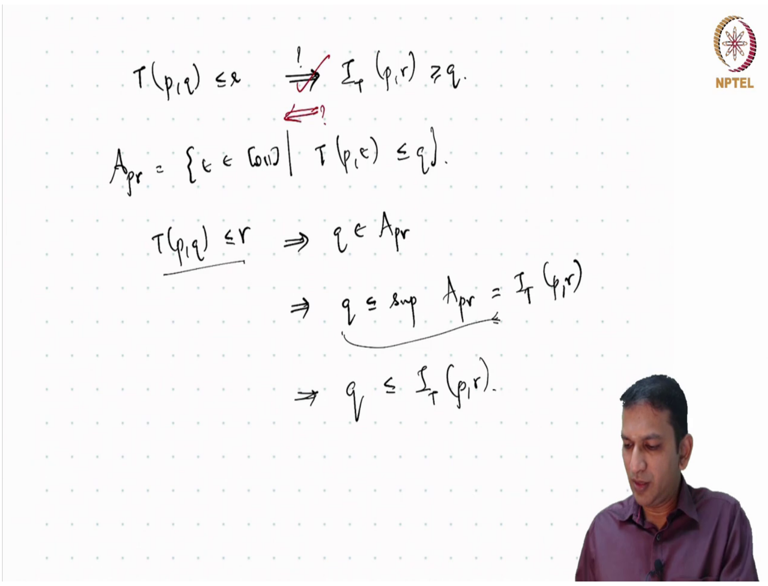

Now, let us look at this particular equation. T(p, q) less than equal to r. Does it first of all does it imply I_T of p, r is greater than or equal to q?

So, note that if you fix p and r, A_pr is nothing but set of all those t in 0, 1 such that T of p, t is less than or equal to q. Now, we are given T of p, q is less than or equal to r. This immediately implies that q belongs to A_pr. Now, what it means is q is less than or equal to supremum of A_pr which is actually equal to I_T of p, r.

So, from here we get that whenever T of p, q is less than or equal to r; q is in fact less than, implies q is less than I_T of p, r. So, this implication is in fact true.

Now, what about the reverse implication? Let us look at some examples here.

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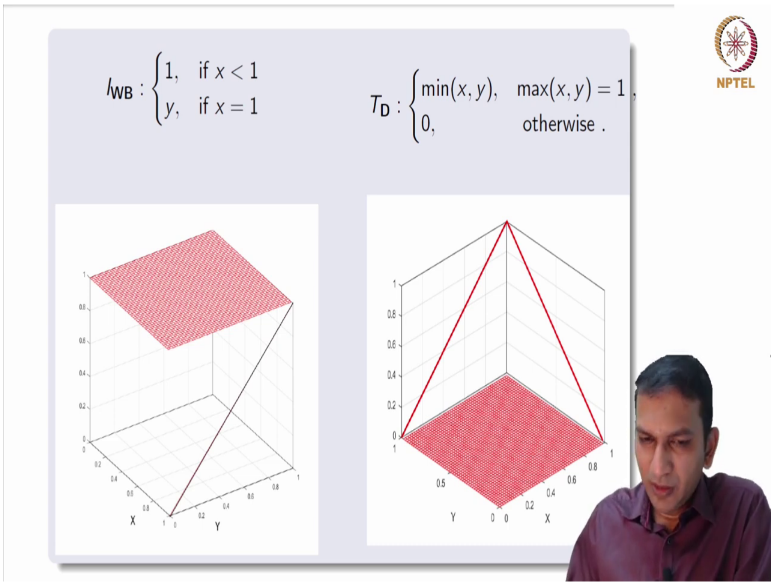

We know the weber implication is an R-implication which is obtained from the drastic t-norm T_D.

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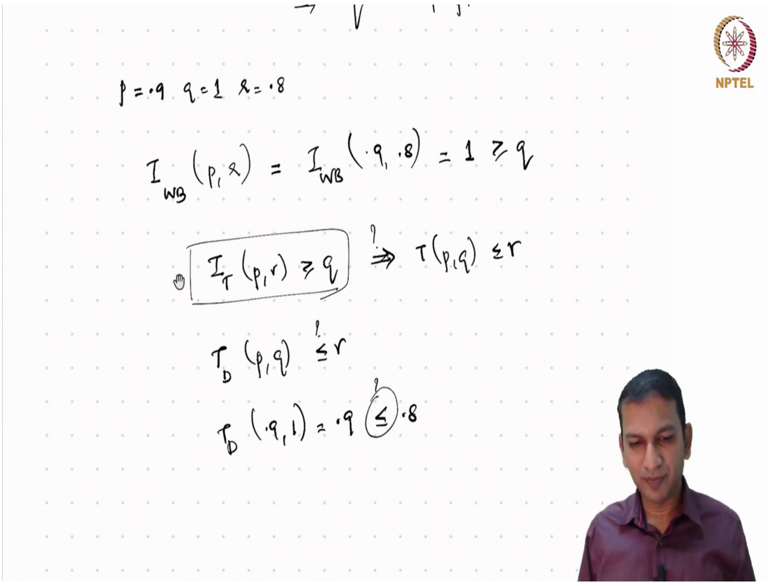

Now, towards helping us look at this formula and then let us consider some values for p, q and r. Let us take p to be 0.9, and q to be 1, and r to be 0.8. Now, let us look at what is I weber of p, r. This is I_WB of 0.9, 0.8.

Now, we know that only along when x is equal to 1 because of the neutrality property it takes the value y or everywhere else it is 1. Which means I_WB of 0.9, 0.8 is actually equal to 1 which is greater than or equal to q. So, that means, what we are looking at now is we want to see if this implication is in fact valid.

So, now, we have seen that for the weber implication the antecedent part so to say is true. So, now, we need to verify whether the consequent is true. Now, that means, T D of p, q is it less than or equal to r. This is the question we are asking.

Now, T_D of 0.9, 1 because q is 1 and p is 0.9, we know that for any t-norm 1 is the neutral element which means this is 0.9. Now, we are asking the question, is this less than or equal to r which is 0.8? We see that this is not true. That means, there exists t-norms and the corresponding R-implications I_T for which the converse or the reverse implication is not true.

Now, you might suspect maybe because it is the smallest t-norm, it is not even border continuous and perhaps that is why this is happening.

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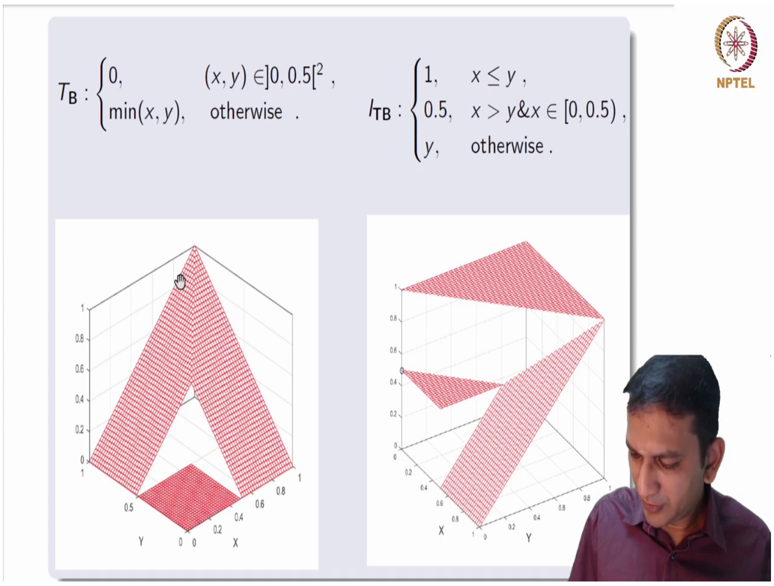

So, let us consider an example of a border continuous t-norm which is what we have seen earlier and the corresponding R-implication obtained from it.

Now, for this pair of t-norm and the corresponding R-implication, let us look at whether the converse of this implication is true.

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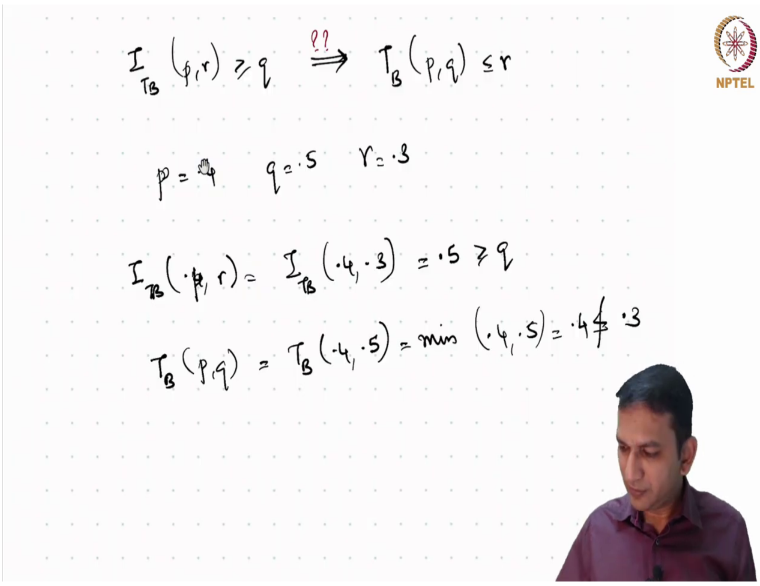

That means, we are still looking at I_TB of p, r if it is greater than or equal to q, does it imply; this is the question you are asking, does it imply that TB of p, q is less than or equal to r? Once again let us look at the values p is equal to 0.4, q is equal to 0.5 and r is equal to 0.3.

Now, I_TB of 0.4, p, r is equal I_TB of 0.4, 0.3. Let us go and look at the formula. So, now, here x falls from 0 to 0.5 because p is 0.4, and it is greater than y, greater than r which is 0.3 with. From the formula we then get that I_TB of 0.4, 0.3 is actually 0.5.

Now, this is greater than or equal to q, clearly. So, now, question is what about T B of p, q, is it less than or equal to r? This is TB of 0.4, 0.5. Let us look at what is this value. Remember, it is 0 on the open 0, 0.5 square and minimum rest of the places.

Now, what we have is 0.4, 0.5 which means it is not in this open square and hence this has to be minimum. So, minimum of 0.4, 0.5, this is actually equal to 0.4 and r is 0.3, clearly this is not less than or equal to 0.3. So, we see that even border continuity does not help us here.

So, then what will help us here in ensuring that T and I_T this pair does satisfy the residuated property?

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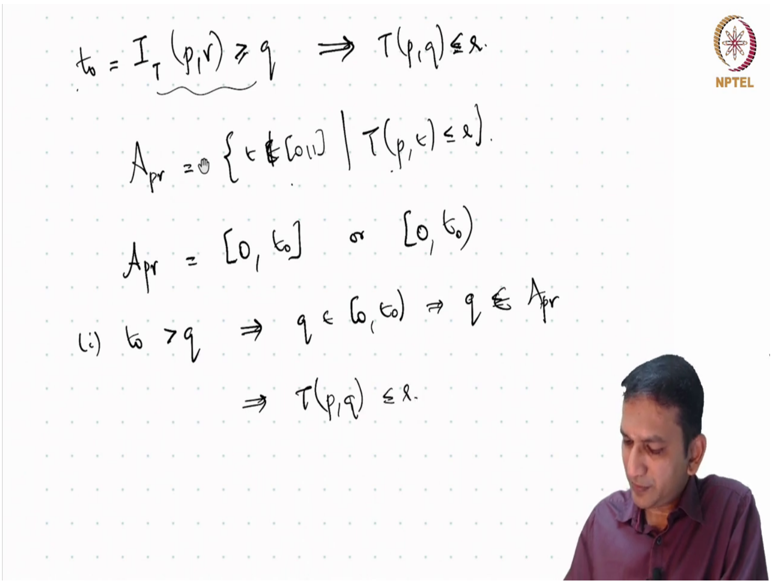

Now, let us look at what we have. This is what we need to prove, T of p, q is less than or equal to r. But let us for the moment assume that I_T of p r is actually equal to t_0, so that means, t_0 is greater than equal to q. Now, this is given to us. For some triple of p r, p, r and q we need to show the right answer, ok.

Now, for this fixed p, r let us write what is A of p, r. It is nothing but all those t, element of element of 0, 1 such that T of p, t is less than or equal to 1. Now, we know since I_T of p r is equal to t_0 either it is 0, t_0 closed or 0, t_0 open.

Now, there can be two cases. First case is t_0, remember t_0 is definitely greater than or equal to q, that is what is given to us. If t_0 is greater than q this implies q actually belongs to 0, t_0 strictly greater than this, which implies q is in fact it belongs to A_pr, which implies since it belongs to A_pr; A_pr consists of all those elements as the T of p q is less than equal to r, we see that T(p, q) in fact is less than or equal to r. So, in this case everything is clear.

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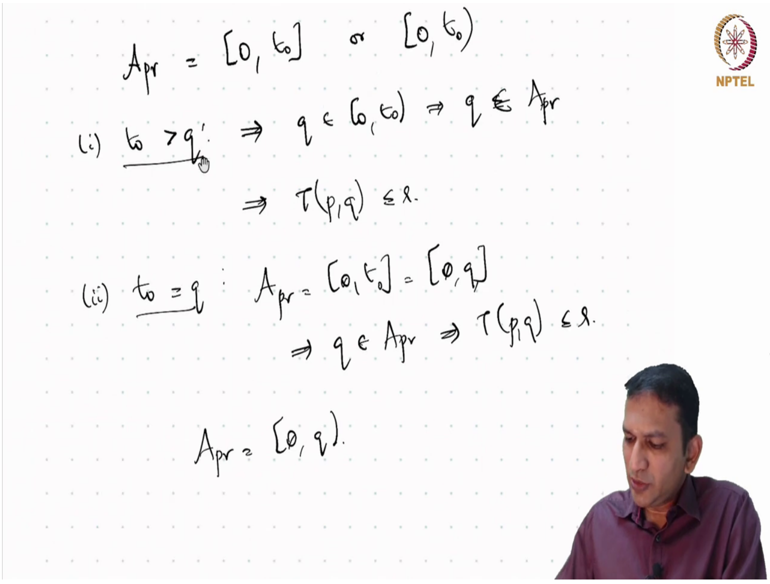

Now, what if t_0 is actually equal to q? This is the second case. Now, if A_pr is in fact, a 0, t_0 then it is actually 0, q. And we know that q belongs to A_pr implies T of p, q is less than or equal to r. So, once again it is done.

So, now, question is what if A_pr is in fact, 0, q open? This is the only case where we can fail to have T of p, q not being smaller than or equal to r.

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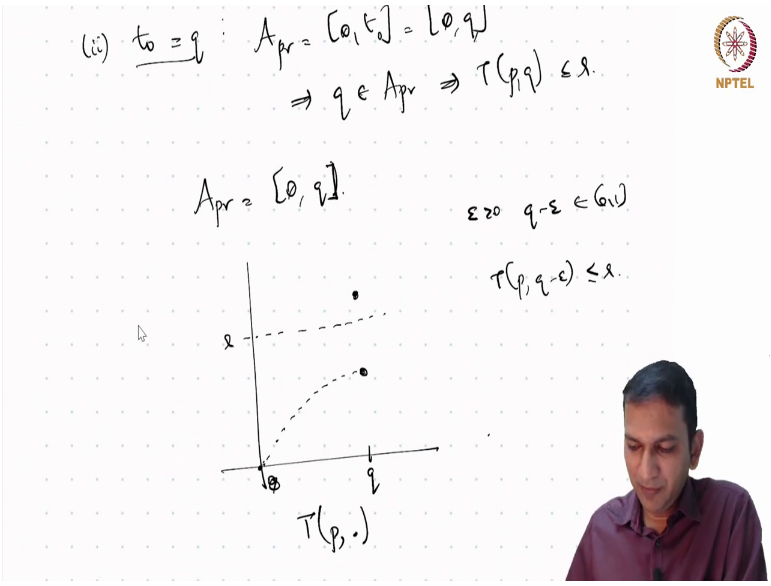

Let us look at what exactly happens from a geometric perspective. Consider a t-norm and we are fixing this p and then varying the second value. So, this is the q, at 0 it is 0, and let this value be r.

Now, since we know that the set A_pr is close 0 open q, we know that for any epsilon q minus epsilon actually belongs to A_pr, and that means, for any epsilon greater than 0 small ones, such that q minus epsilon is still in 0, 1. We see that T of p, q minus epsilon is in fact less than or equal to r, because that is how A_pr is defined.

Which means, if you go here, given this is r perhaps all the values are in fact smaller than r except at q. So, there seems to be a jump discontinuity here. So, at q remember this T of p, dot; that means, we are fixing the first variable to be p and we are only looking at the partial function as q varies. So, the second variable varies.

So, for every value of course, at 0 to 0, at q what we expect is the T of p, q should be less than or equal to r, but somehow it goes up. But for every other value it is smaller than r. Now, if this were not to happen what we want is that this r in fact, this value is in fact, less than r.

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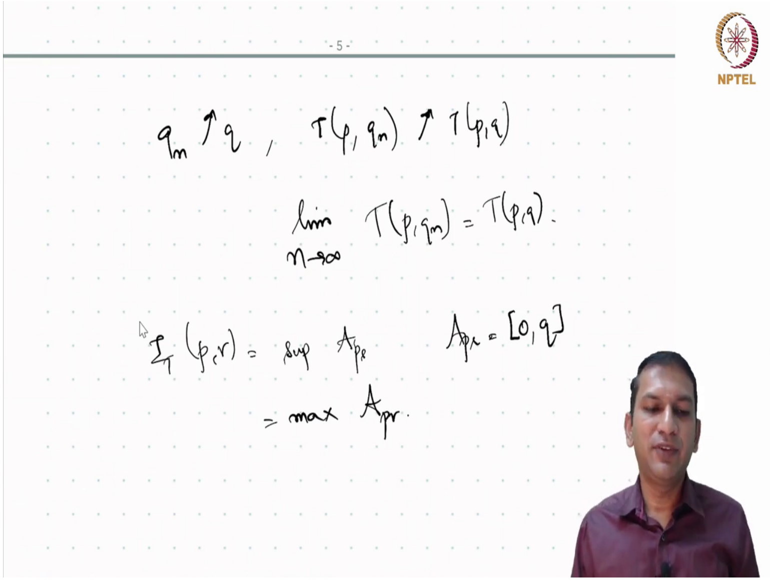

But what does that mean? It essentially means that this T has to be left continuous, that is for any increasing sequence q n going to q at this p, T p, q n should actually greater than T p, q.

So, at another way when ln n tends to infinity T of p, q n is actually T of p, q. So, if this property is held then it cannot jump here, it will smoothly converge to T of p, q. And this is essentially the concept of left continuity.

The moment we have the t-norm to satisfy left continuity, then we know that this kind of jump discontinuity cannot happen and this A_pr will always be a closed interval in which case in fact, when we may write I_T of p, r as supremum of A_pr.

In the case A_pr when the case T is left continuous we know that this will always look like this closed interval. And so you could in fact, replace supremum by maximum because that element the supremum supremum in fact, belongs to the set A_pr. So, the left continuity of T is in fact very important here.

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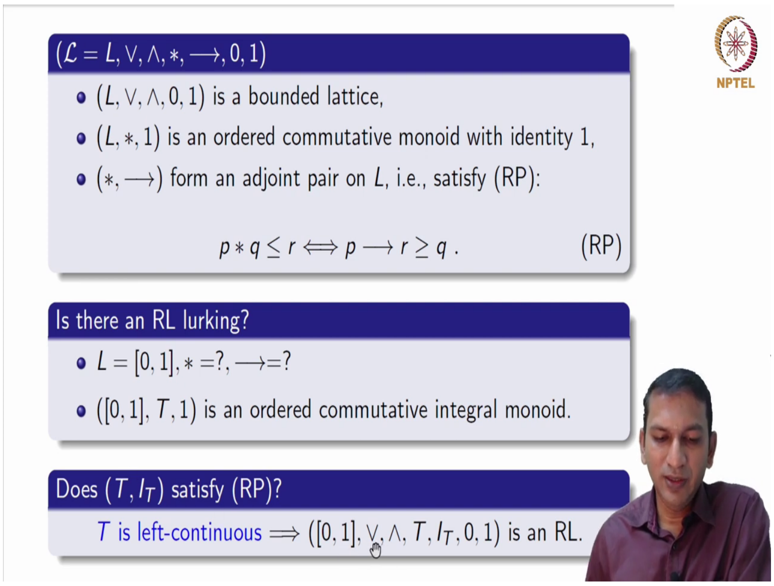

So, we ask the question does T, I_T this pair does it satisfy the residuated property? And now what we see is if T is left continuous then definitely it does satisfy the residual property. And we have the structure where [0, 1] with the usual order, and the left continuous T, and the corresponding R-implication obtained from it, it is a residuated lattice.

If you recall we had discussed this nomenclature when we discussed R-implication, we said that unless T is a left continuous t-norm we cannot call it a residuated implication of the residuum of T, even though the term R-implication itself comes from there.

So, typically, if you do not know anything about T, then I_T is only called an R-implication. But if you do know that T is left continuous t-norm, then I_T can be called the residual implication or the residuum of T.

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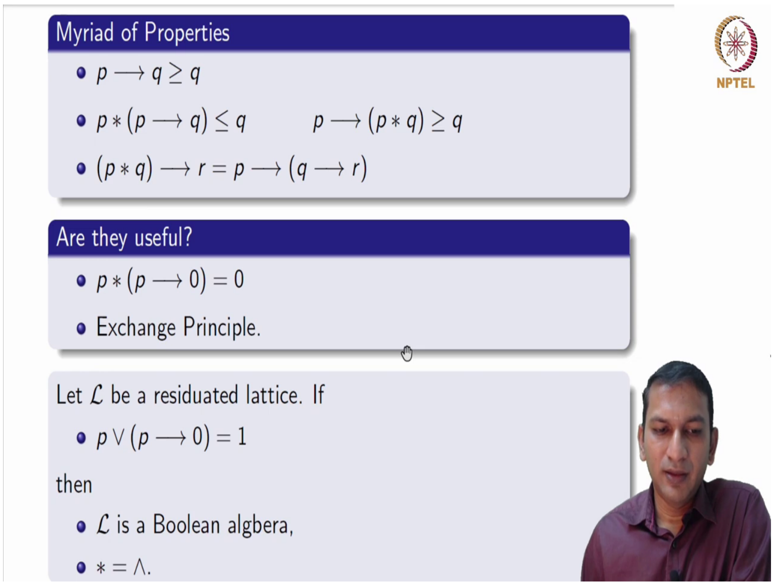

Why do we need a residuated lattice? This is something that we will see moving forward. It has a myriad of properties. For example, look at this, p implies q is always greater than equal to q. p star p implies q is less than or equal to q, and p implies p star q is greater than or equal to q. These are some of the properties that we will be making use of later on when we will discuss some desirable properties of fuzzy influence systems.

And most importantly what is called the law of importation, we will discuss this little later, which says that with respect to star and implication if you are looking at residuated lattice, p star q implies r is actually equal to p implies q implies r.

Now, how do these help? In many ways, but quickly let us look at them. If you put 0 here, we see that p star p implies 0 is actually equal to 0. Now, recall p implies 0 is nothing but a negation that you can obtain from p and star is some kind of conjunction, some kind, not necessarily the meet of the lattice. So, what you have is p star negation p is 0.

If you recall this is one of the properties that you need to be able to define a complemented lattice. So, this is something that is available to you. This also goes under the name of the law of contradiction in set theory from the logic. We will have another occasion to speak a lot more about this.

Now, look at the exchange principle. What is an what is the exchange principle?

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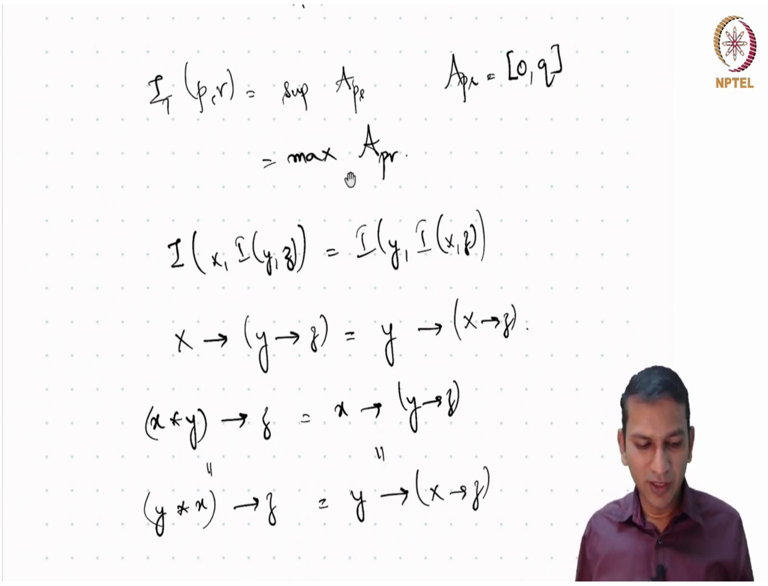

So, written in terms of arrow, so this is the exchange principle that as we known from an implication. In terms of arrow, we can write this as x implies y implies z, this y implies x implies z. This is what we want.

Now, look at this, p star q implies r this p implies q implies r. Now, if this star is also commutative in the case of t-norms, it is commutative, then what we get is x star y implies z is actually equal to x implies y implies z.

And now this by the curve commutativity of star you could write this as y star x implies z and here we get its y implies x implies z. So, these two are equal, these two are also equal. So, in the residuated lattice setting exchange principle comes to you for free. So, every residuated implication, that means, an R-implication obtained from left continuous t-norm does indeed satisfy the exchange principle.

It does not stop there. Just one more result. Let us start with the residuated lattice, L is the residuated lattice. If p joint p implies 0 is 1, this is the joint of the bounded lattice on L. If this happens then in fact, L becomes a Boolean algebra, not just a residuated lattice it becomes a Boolean algebra. But what is more interesting is in that case the t-norm of star that we have here it reduces to the meet of the corresponding boundary lattice.

So, in that sense, we do not obtain any new structure. So, residuated lattice in which this property is valid immediately becomes a Boolean algebra, and the star becomes the meet of the lattice.

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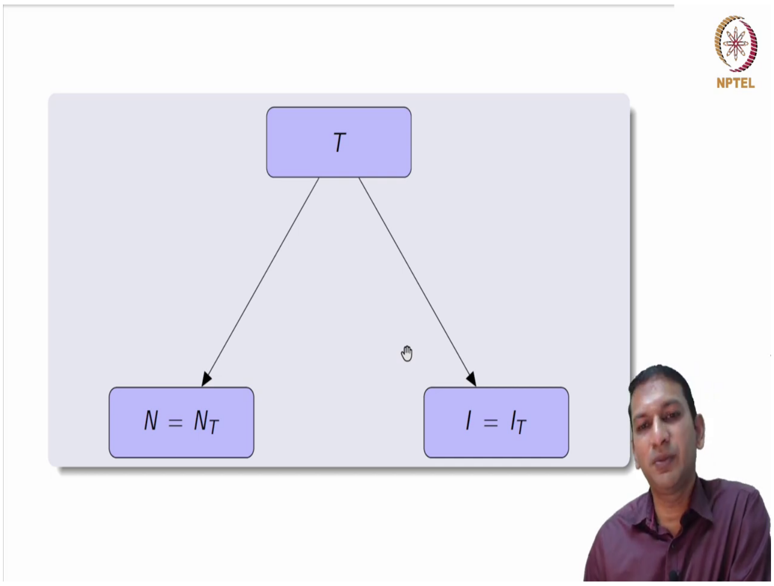

So, what have we seen so far? We have seen that from T, we can obtain the natural negation of T, N T and we also can obtain the R-implication using T which is the I_T. So, these are T is related or we can obtain N and I from T.

Interestingly, we can also obtain a T from an I.

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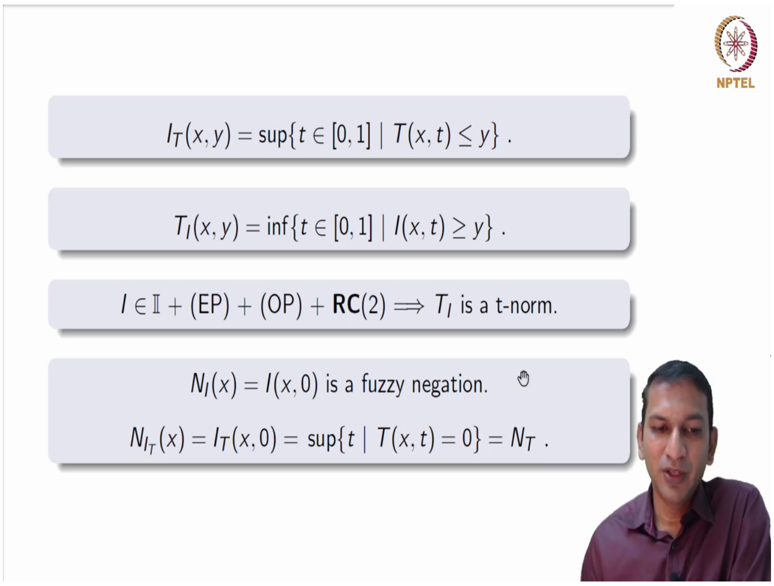

So, look at this is how we obtain an implication from a t-norm. If you rewrite the formula in this form given an implication line to (Refer Time: 31:58), consider this function T_I which is nothing, but in some sense the dual of it infimum instead of supremum and greater than or equal to instead of less than or equal to. That means, look at all the elements T such that I of x t is greater than or equal to y and pick the infimum. Now, this is a binary function of course.

It can be shown that if I is an implication that satisfies exchange principle, the ordering property and is right continuous in the second variable, then in fact, T_I is a t-norm. It is clear that for supremum to become maximum we need left continuity, and for infimum to become minimum we need right continuity. And that is how this right continuity comes into picture here.

So, any implication that satisfies exchange principle, ordering property, and desired continuous in the second variable, does indeed give you a t-norm using this formula. We already know that given an implication we can obtain a negation from it, a natural negation, and in the case of an R-implication, this natural negation is in fact, the natural negation that you obtain from the corresponding t-norms.

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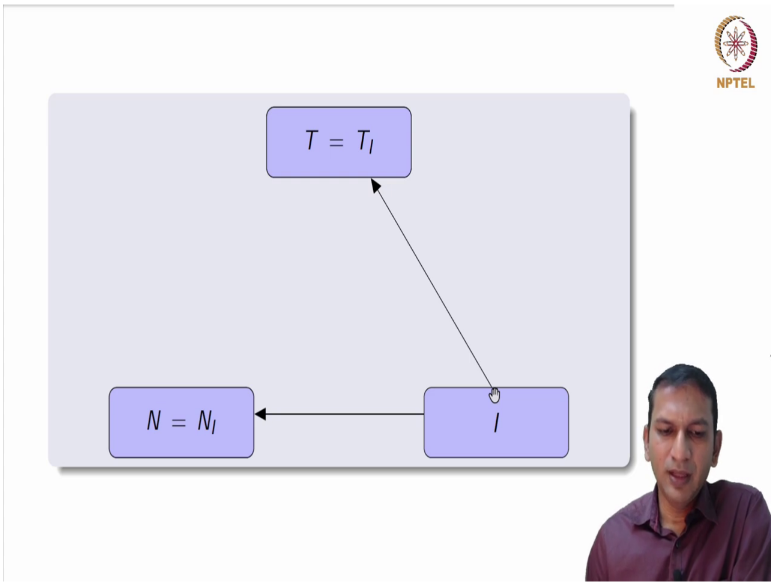

So, interestingly if you start with an I, you can get a N, you can also get a t-norm of course, with some conditions on I, not always. Or of course, you will always get some kind of a conjunction, but for it to be a t-norm I needs to satisfy EP, OP and also the right continuity in the secondary.

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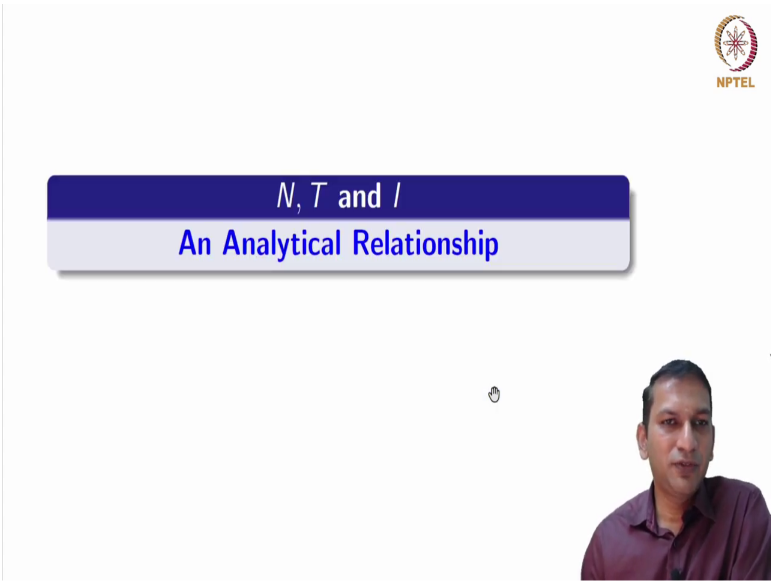

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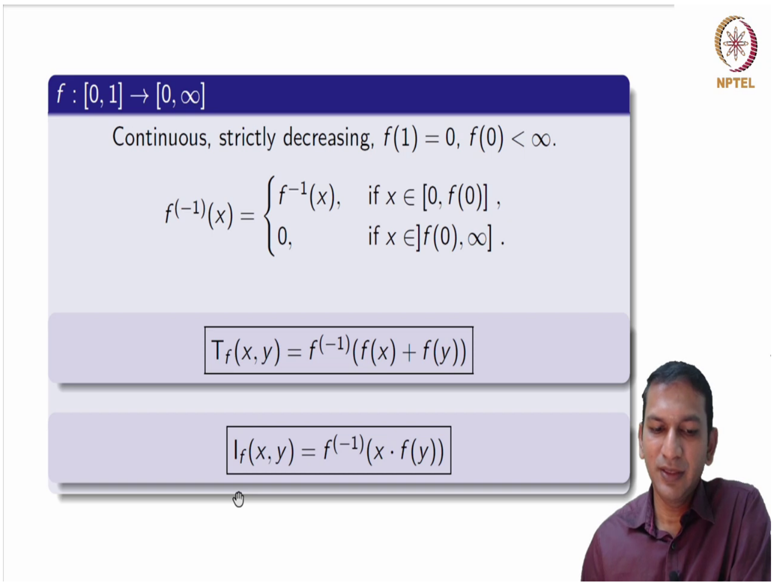

So far we have discussed in the setting of algebra. Now, let us look at some analytic relationships.

Please recall that if you have a function f from 0, 1 to 0 infinity which is continuous strictly decreasing, such that f of 1 is 0 and f of 0 is less than infinity, we could define the corresponding pseudo inverse like this. And we were able to construct a t-norm like this, T f as f pseudo inverse of f x plus f y.

And we could also construct from the same function f an f-implication, in that sense we call it the f generated implication or f-implication. So, it is from the same f, we are able to obtain both the t-norm and an implication. In this case, it is an f-implication.

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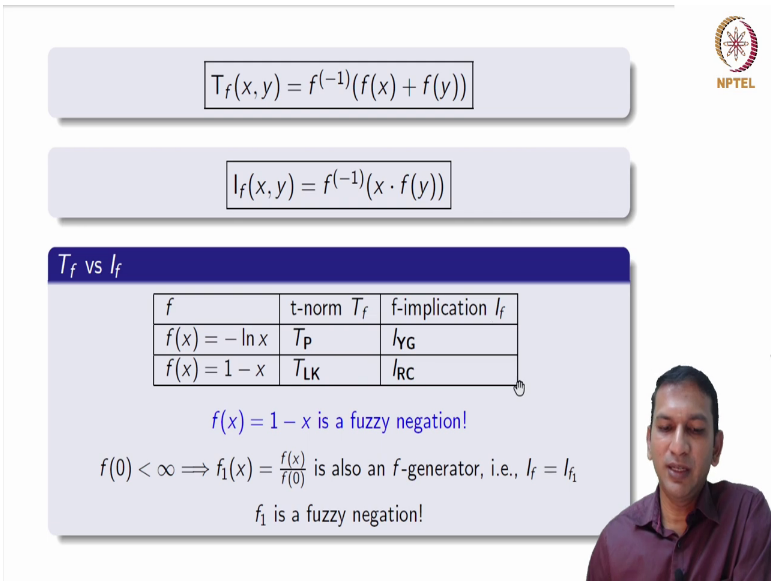

Now, how do these two match with each other? That means, if you take an f generator and look at the t-norm and obtain from it and also the f-implication obtained from it, how do they look like.

So, for the f generator minus lon x, where f of 0 is infinity, we obtain the product t-norm and in the case of f-implication we obtain the Yager's implication. If you consider 1 minus x, where f of 1, f of 0 is finite, we obtain the Lukasiewicz t-norm as the generator t-norm and we obtain the Reichenbach implication as the f-implication.

Now, what is interesting is note that f of x is in fact, when f of x is 1 minus x, it is in fact a negation. So, it appears that we are obtaining a t-norm and an implication from a negation. Now, recall that for these generators f, f of 0 can either be finite or infinite. In the case it is finite, we could also define such a function f 1 which is f of f x by f of 0 which makes it that f of f 1 of 0 is actually 1. It is also called the norm f-generator. This is also an f-generator.

What do you mean by saying it is also an f-generator? The f-implication obtained from f and I, f 1 they are essentially identical. They are actually equal. So, that means, if you take an f-generator, so that f of 0 is finite, then what you always get from that you could always get f 1 which is a fuzzy negation. That means, whenever f of 0 is finite it is a negation and from that negation you can always obtain a t-norm and an implication.

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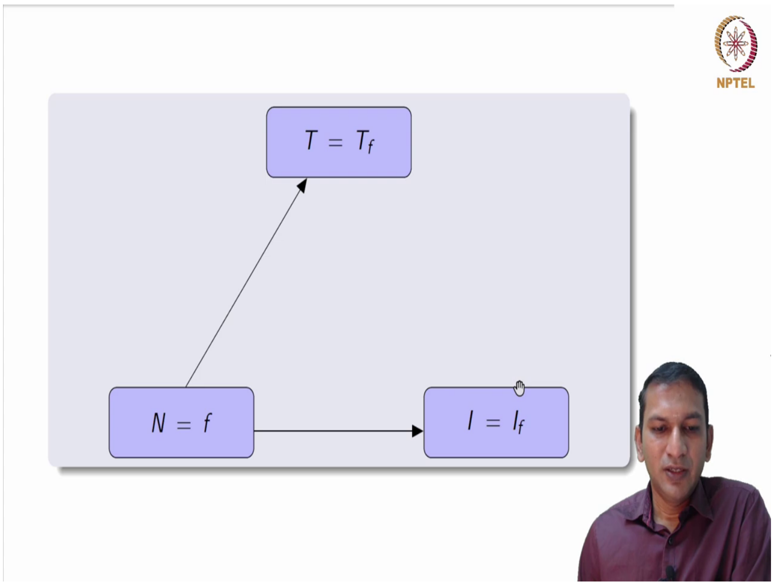

So, consider this negation. One way we have seen a way to obtain a t-norm and also an implication.

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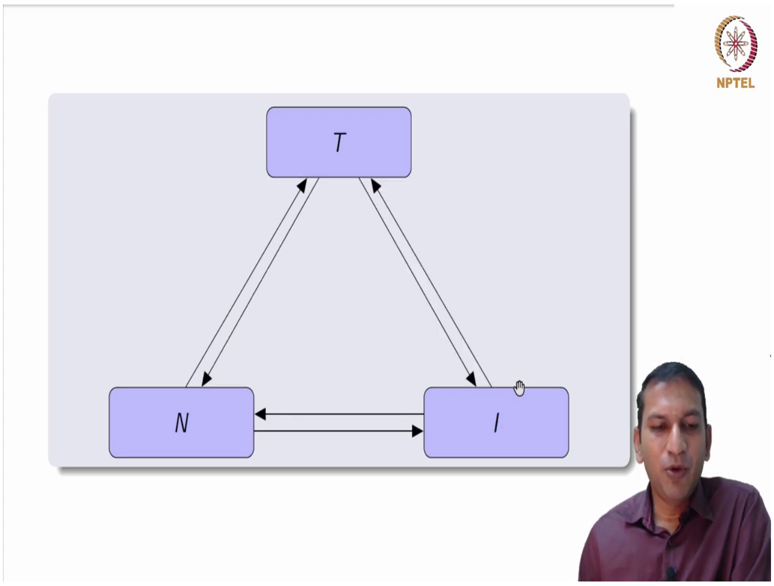

So, finally, what we have seen is among these basic fuzzy logic connectives, there seems to be some interesting relationships. You can move from one to the other, generate or construct the other two from one of them. Just like how we did in the case of classical logic (Refer Time: 36:29). Of course, there are many more conditions on these operations that has to be borne in mind.

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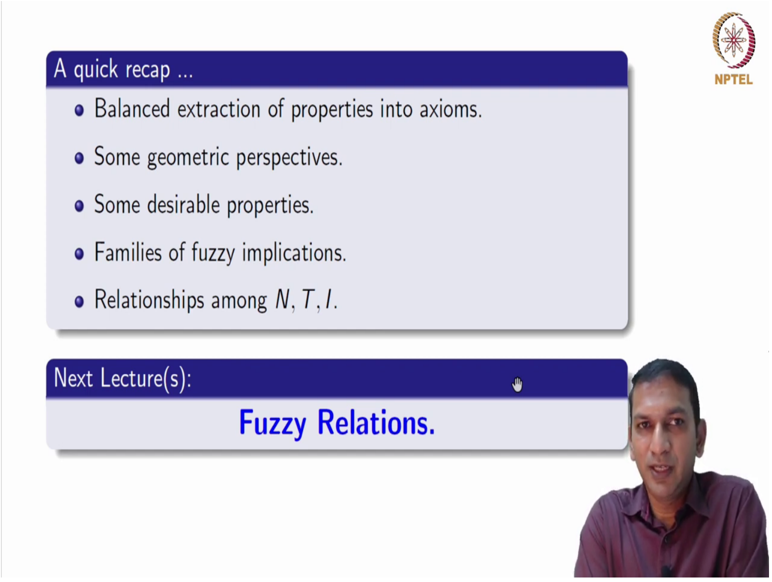

A quick recap of what we have done in this lecture or over the week itself. We have once again been very careful in picking the properties that we would want in the definition of fuzzy implication. As was mentioned in the very first lecture, initial days, neutrality property and exchange principle were also asked of a fuzzy implication. But now, we have just come with a bare bone structure, a bare bone definition of what a fuzzy implication should possess in terms of the axiomatic definition.

Looked at it geometrically, understood it from its 3D plot, listed out some desirable properties, looked at many families of fuzzy implications, constructed them. And in this lecture, we have seen some interesting relationships between these 3 basic fuzzy logic connectives of negation, triangular norms, and fuzzy implication.

What next? In next week, we will discuss at length Fuzzy Relations, the different types, special types of fuzzy relations, and how they are useful, compositions, and perhaps a peek into fuzzy relation of the equations also. Just a peek, but we will do fuzzy relation of equations at length in depth later on when we discuss interpolativity of a fuzzy relation effects.

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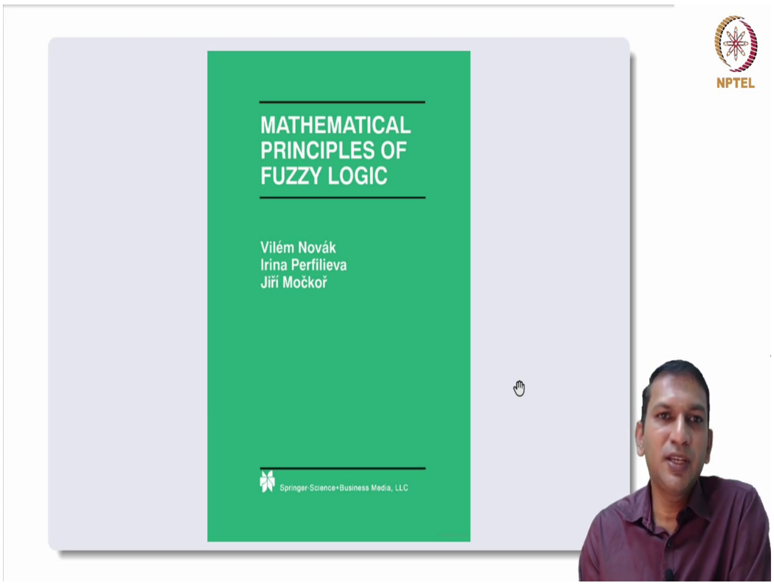

A good source for the topics that we have covered in this lecture especially about residuated lattices and the different properties that they possess, is this book by Professor Novak, Perfilieva and Mockor.

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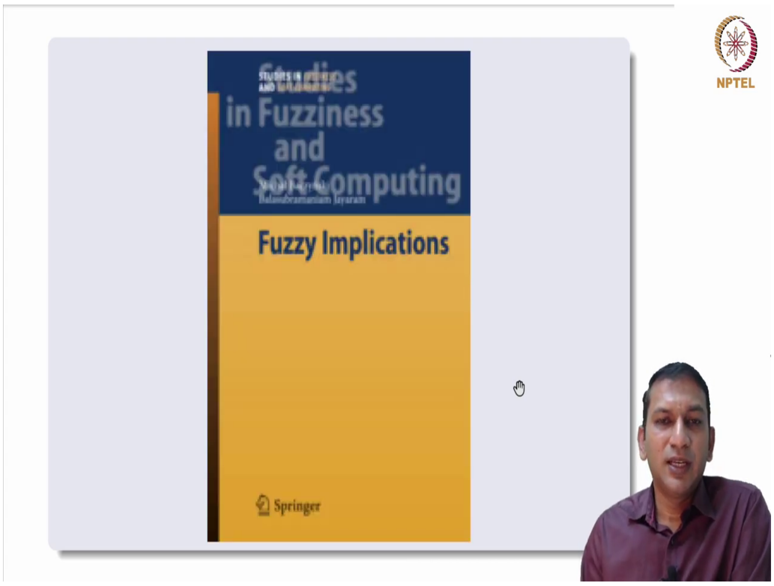

Of course, some related results on f-implication and T that can also be obtained from the book of Fuzzy Implications.

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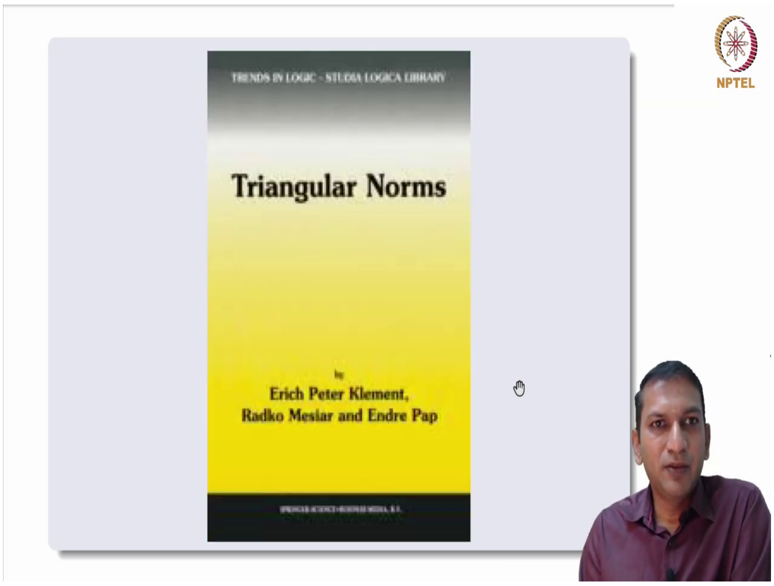

And some interesting ways of a representing R-implications from the corresponding generators if the original T is a generator t-norm, those formula can be found in the book of Klement, Mesiar, Pap titled Triangular Norms.

Glad that you could join us in this lecture. I am looking forward to seeing you again in the next lecture which is week 5, discussing Fuzzy Relations in Depth.

Thank you once again.