Our Mathematical Senses

The Geometry Vision

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Lecture - 07

Video 1F: the horizon line theorem

So, answering that question is going to lead us to another theorem. But before we get into that, let's try and answer the question. So the question we asked is, what is the image of these three sets of railway tracks going to look like? And hopefully, maybe, you guessed that since these tracks are not parallel to our original track, they're not going to share that same vanishing point. Instead, they're each going to have their own vanishing point. These tracks are going to have a vanishing point here. These tracks are going to have a vanishing point here.

And in this way, we're going to end up with three distinct vanishing points. But although they converge to separate vanishing points, you might notice that these vanishing points do share something. They have something in common. And in particular, they're collinear.

They all lie on the same line. And that brings us to the next theorem, which is the horizon line theorem, which states that all vanishing points of all lines parallel to the floor plane are all going to be collinear. That's the content of the horizon line theorem. Now, the question is, why is it true? So why are all of these vanishing points collinear? Why are all the possible vanishing points you'll get from lines that are parallel to the floor going to all lie on this horizon line? Well, it's useful to once again go back to how our vanishing points are constructed. And it's by taking sight lines from our eye and following the line in space further and further and further along until you get to a limiting sight line, which is parallel to that line in space.

So this limiting sight line is parallel to these railway tracks. This limiting sight line is parallel to these railway tracks. This limiting sight line is parallel to these railway tracks. And all of these railway tracks are parallel to the ground plane. So basically what I'm

trying to get at is a small question.

What is the collection of all lines from your eye which are parallel to the ground plane? What is that collection going to look like? Well, the collection of sight lines which are parallel to the floor plane, that'll form a plane. And so we're really looking at a whole plane coming out of your eye, a plane's worth of lines. That plane is parallel to the ground. So that plane is going to intersect the picture plane in a single line. And that line is known as the horizon line.