Our Mathematical Senses

The Geometry Vision

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Lecture - 06

Video 1E: the vanishing point theorem

Okay, so let's put all these observations together and state what's known as the vanishing point theorem. And the vanishing point theorem simply states that any family of parallel lines in space is going to converge to a shared vanishing point in the picture plane, unless that family is parallel to the picture plane, which is what we saw in the previous video. So in particular, if we look at these side rails of the railway tracks, any other set of parallel lines, like say another set of railway tracks that's parallel to our original one, well, what's the image of that going to look like? Well, it's in the same family as our original railway tracks. They're all parallel. All these side rails are parallel to one another. So as we look further and further along it, along one of them, we're going to reach a limiting sight line, which pierces the picture plane at that same vanishing point.

This full family of parallel lines will all share this vanishing point on the picture plane. That's what our theorem is saying. And the only exception is when that family is parallel to the picture plane, which is what we're seeing with these horizontal railway ties here. They're all parallel to the picture plane.

So as a result, they continue to appear parallel in the picture plane. But here's a natural question. What about other families? So other lines that are in this family will all share this vanishing point. But what about a different family of parallel lines in space, which is not parallel to the side rails of these railway tracks? For instance, let's say that we have two other sets of railway tracks, which are not parallel to our original railway tracks. What is the image of these three sets of railway tracks going to look like in the picture plane?