Our Mathematical Senses

The Geometry Vision

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Lecture-57

Video 11B: homogeneous coordinates

So now we're ready to see how we can put coordinates on the real projective plane. So we need an easy way to refer to specific lines through the origin. Here's a line L. How do we quickly refer to it? Well, note that for any vector v in R3 minus the origin and any non-zero scalar lambda, the vectors v and lambda of v will lie on the same line through the origin. Here's a vector v lying on this line L. It's equal to v1, v2, v3.

If we multiply it by the number 3, the scalar 3, we get another vector, 3v, but it still lies on L. We can multiply it by negative 1 and get negative v. That still lies on L. So all three of these scalar multiples of v will all lie on the same line L through the origin.

In fact, as lambda ranges over the non-zero real numbers, we actually get every single non-zero point on the line L as different scalar multiples of v. And that's not too hard to convince yourself of. So given two vectors, v, which is equal to v1, v2, v3, and w, which is w1, w2, w3, let's write v tilde w if there's a non-zero scalar lambda such that v is equal to lambda times w. So if they're related through scalar multiplication, we write v tilde w. So this actually gives an equivalence relation on the elements of R3 minus the origin.

So we'll denote the equivalence class of v1, v2, v3 of that vector by bracket square bracket v1 colon v2 colon v3. So this is just a notation, but this equivalence class, v1, v2, v3, corresponds to the line L. Now maybe I'll go back a second and just clear in case you haven't, you're not very familiar with equivalence classes. Another way to, all this is saying is that this notation here, v1, v2, v3, this is really just the set of all scalar multiples of the vector v1, v2, v3. So it's the set of all lambda v where lambda is equal to lambda v.

So, that's another way of writing this equivalence class. It's just the set of all scalar multiples of v non-zero. So each equivalence class, it really corresponds to a line L because that set of all scalar multiples is giving us all of L except of course for the origin, but that's okay. So this corresponds to our line L and we'll kind of equate them. We'll think of L as really being this homogeneous coordinate here, v1, v2, v3.

And there's two ways, there's many different ways you can refer to this line L via homogeneous coordinates. You could write v1 colon v2 colon v3 or you could write w1 colon w2 colon w3 where w is this scalar multiple here of v. And you could take any other scalar multiple of v and use its coordinates written in this homogeneous form to refer to that line L. So there isn't a unique way of writing L in homogeneous coordinates. There's infinitely many ways.

You can take any scalar multiple. And that might seem like some weird drawback, but it's actually a big advantage and we're going to see why. So first here's a quick example. So if v is, I just want to give a concrete example. If v is the vector negative 1, 1, 2 and w is the vector 2, negative 2, 4, both lying on this line L, then it's easy to see that w is actually equal to negative 2 times v.

It's a scalar multiple of v. And w and v represent different points in homogeneous coordinates in R3. This is one point in R3. This is another point in R3. But the homogeneous coordinates bracket, minus 1, 1, 2 and bracket 2, minus 2, 4 represent the same point in RP2.

They both represent the line through the origin L. So L can be written as either one of these in homogeneous coordinates or you can take any other scalar multiple and you could just as well write L using that.