

Our Mathematical Senses

The Geometry Vision

Prof. Vijay Ravikumar

Department of Mathematics

Indian Institute of Technology- Madras

Lecture - 03

why do the images of lines terminate?

Why do the images of lines terminate? So let's answer the question, why do the images of lines terminate? And to do that, let's go back to our setup, our framework with the picture plane and our sight lines, our infinitely long railway track and the image of that railway track on the picture plane. So let's kind of observe what happens to your sight lines as you look further and further down along the railway track. So to begin with, you're looking at this point here and you're creating an image point here on the picture plane. But let's see what happens if we look further down along the tracks. Well, okay, we're looking further down now. We're now looking at this point here and we're creating an image point here.

So we went from this to this. And if we look further down again to the next railway tie, we'll construct another image on our picture plane. And if we look further down again, we'll construct yet another image, which is a little bit higher. Our sight line, as we're looking further and further down, you can see that it's rising up.

And so again, I'll just play it in sequence. So we're looking further and further and further down. Let's look a little further still. Well, okay, I looked a lot further. But what's happening here? As we look further and further and further along, we will approach a limit.

In the infinite limit, your sight line will be parallel to the side rail of the railway tracks. It might be a little easier to understand from a different viewpoint. So here's a top-down view of the sight lines as you look further and further and further down the railway tracks. So in this image, you can see that here you're looking at this railway tie and

creating this image point. If you look at the next railway tie, you'll create this image point.

Look at the next railway tie, you'll create this image point. And from the top-down view, we see something else. These image points are coming closer and closer to this kind of center point here. We can't see if they're going up or down because we're at a top-down view. So we're not seeing them rising up in space, but we are seeing that they're coming closer and closer to the center.

And they are approaching a limit once again. And once again, the limiting sight line that we approach will be parallel to that side rail that we were following along. Once again, we'll look at it yet another time but from a different viewpoint. So this is a side view of the railway tracks. And that image of that first railway tie will give us an image point here.

The second railway tie will give us an image point here. The third will give us an image point here. And I skipped the fourth. The fifth one will give us an image point here. And here you can see very clearly that these image points of these points along the side rail are rising up as we look further and further down.

And our limiting sight line will be this horizontal one again. And as we've seen before, this sight line is parallel to the side rail which we are observing. So this suggests that we create a new piece of terminology, which is a vanishing point. So given a line L in space and a picture plane, we define the vanishing point of L to be the point on the picture plane which is pierced by the sight line parallel to L . So here in the picture, I have my sight line.

I have my side rail of the railway tracks here, which I'm calling L . And as I follow it along with my sight line, I get to a limiting sight line which is parallel to L . So this is my sight line parallel to L . And the vanishing point of L is this point here, which is pierced by my sight line parallel to L . Notice that the image of L in the picture plane is right here.

What I'm drawing here in red is the image of L in the picture plane. And that's going to terminate at this vanishing point of L . Although L itself is infinite, its image is finite and terminates at L . So hopefully this notion of a vanishing point makes sense. Now as a quick exercise, I'll leave this for you to do.

You can assume the railway ties are evenly spaced in real life. Just say that there's one unit between them. And assume that our observation point is also one unit from the

picture plane and one unit up from the ground. So in this scenario, can you work out the heights of the images of the railway ties in the picture plane? So these are the railway ties in the picture plane. Can you work out the heights of those images using this schematic diagram? So I'll leave that for you to work on.

And hopefully this answers the question of why infinitely long lines appear to terminate.