Our Mathematical Senses

The Geometry Vision

Prof. Vijay Ravikumar

Department of Mathematics

Indian Institute of Technology- Madras

Lecture-31

Video 6G: how to prove the converse

As a final challenge, I want to look at the converse to Desarque's theorem and give a hint about how to prove it. So Desarque's theorem's converse says that if two triangles in P3 are in perspective from a line, then they're in perspective from a point. As before, it's easier to prove this when the triangles are not in the same plane. And then, after that, we can deal with the coplanar case via a lifting. So I'm just going to give a starting hint and leave the rest as an exercise for those who would like to try it. So the statement we're going to prove is if two non-coplanar triangles in P3 are in perspective from a line, that's what's depicted here, then they must be in perspective from a point.

Namely, we need to show that the lines little a, capital A, little b, capital B, and little c, capital C are all concurrent at a single point. So as a hint of how to do that, I want to point out that the yellow lines span a plane. They're coplanar. We saw that already.

I want to give that a name. Let's call it the yellow plane. You can see it drawn here, depicted here. Similarly, the blue lines span a plane, which we'll call the blue plane. And I'm depicting it here like this, just shading it so you can visualize it a little bit.

Finally, the red lines span a plane, which we'll call the red plane, depicted like this. So let's try and visualize all three at once, for what it's worth. And can you show that these three planes intersect in a single point? And if they do, what is the significance of that point? Now, I recommend as a first step, you can consider the pairwise intersections of these three planes. What are the pairwise intersections of these three planes, and what's their significance? So think about these questions and see if you can use them to show that these triangles are in perspective from a point. So good luck with that as well.

Thank you.