Our Mathematical Senses

The Geometry Vision

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Lecture - 02

Why Do The Images of Parallel Lines Converge?

Video 1A: drawing parallel lines: a thought experiment

We've seen how when you view a scene around you, you're creating a two-dimensional representation of a three-dimensional space. That two-dimensional representation, it could be a photograph, it could be a perspective drawing, it could also just be the image in your head. Whatever the case may be, that two-dimensional representation has some very interesting features, like the fact that two parallel lines in space will appear to converge in that two-dimensional image. The first order of business is to understand why that's the case. Why do images of parallel lines appear to converge? In order to do that, I want to start with an even more basic question, namely, what are you doing when you see? When you see a three-dimensional scene, how do you create this two-dimensional image? What are you actually doing? When you witness or when you're taking in a three-dimensional scene, many, many light rays are coming to your eye from all of the different points in space. Somehow all of that information, all of the sensation of all of those light rays is somehow forming this two-dimensional image.

Where does this two-dimensional image live? Is it just in your head? And it is, it's being formed in your head, but in that case, how can we actually work with it if it's just in your imagination? Does it exist in any stronger sense? Can we somehow realize this two-dimensional image more concretely? And in fact, we can using the good old picture plane. So let's imagine a pane of glass sitting or standing in front of you. And this pane of glass, this picture plane, allows you to realize the image in your head physically as an image living in space. And the way it does that is that all of these different light rays are bouncing off all of these points in space to your eye, but they're all going through the picture plane.

And somehow the points that they go through the picture plane are quite significant. They form an image very similar to the image in your head. So in particular, if you were to draw on the glass exactly what you see, where you see it, then the image on the picture plane will be exactly the image in your head. Provided you're looking through one eye with the other eye fixed and that the, sorry, provided that you're looking through one eye whose location is fixed, the other eye will be closed. So just to put it another way, when you see you're letting your many, many sight lines paint an image on the picture plane, your many, many sight lines are all intercepting the picture plane different points and creating these image points of various points in space.

Now this actually gives us a setup that we can work with in order to carefully examine the geometry of vision and answer some of the burning questions that we've been asking. But one thing to notice is that the setup actually does feel kind of mind boggling when you think about it. There's so many light rays beyond, you know, not just the 10 that I've drawn here, light rays from every single point in space bouncing off into your eye at every moment, at every instant. So it's a completely mind boggling, unimaginable amount of data that you're taking in and processing all the time. And maybe that's one reason why it took so long to develop this model of vision.

And it's an interesting side note that the mathematician and philosopher Al-Haytham was perhaps the first to propose that vision is composed of infinitely many light rays projected from all the different points in space to your eye. And this idea was a crucial part of his theory of optics that he developed in the Book of Seeing, which he published a little over a thousand years ago in 1021 CE. So it was actually a surprisingly counterintuitive idea at the time. Almost every culture on Earth had a belief that something is emitted from your eye in the process of forming an image, not that many, many, many light rays are coming to your eye from every point in space. And that might be one reason why most cultures have a concept of an evil eye, something that you're sending out when you're looking, when you're viewing something.

But to come back to our problem, so sight lines create an image on the picture plane. So this model, we can use it to address our question, why do parallel lines appear to converge? And we're going to do this via a thought experiment. So imagine that you're observing a set of infinitely long, perfectly straight railway tracks. So you're standing on railway tracks like you would in real life, except in this thought experiment, the side rails are infinitely long, going on and on forever. But you have perfect vision.

So you can view as far down the railway tracks as you want. You can see a point that's a billion miles away, a trillion miles away, as far out as you want. And the question is, oh yeah, and of course, you're seeing all this with one eye in a fixed location, the other eyes closed. So what will the image look like on the picture plane? The real life version is something we've all seen before. These are not infinitely long tracks.

They're finitely long, but they're very long. And given our imperfect vision, we can't really tell the difference. They'll go out and out and out and appear to converge somewhere. But would the railway look any different if the tracks were infinitely long? So in this idealized version, I actually claimed that we'd get something very similar. Despite the infinitely long tracks and the perfect vision, we'd pretty much see the same thing.

And I'll try to convince you of this. After all, the fact that the side rails are now infinitely long wouldn't make them suddenly appear to cross each other. Already they seem to converge at this point. If we made them longer in real life, they're still parallel in real life. They won't suddenly appear to cross each other in the picture plane.

So the images of infinitely long lines must terminate. So that's a bit of a, the reason that I wanted to consider infinitely long lines is to get to this conclusion, which is a bit strange. And since space that are infinitely long will still have images in the picture plane which are finite, which end. So that's just one of several strange features of this very simple line drawing we get through our, when we observe railway tracks. So the first question that I want to look into a little more deeply is why do the images of infinite lines terminate? But beyond that, we also see that these two infinitely long lines, not only do they both terminate, but they both terminate at the same point.

They share a vanishing point. So why is that the case? And there's a third feature, which is also very interesting. This other family of parallel lines, which consists of these railway ties running in between the two side rails. They remain parallel in the picture plane. They're parallel in real life, but their images in the picture plane remain parallel.

So why is that the case? Why do some lines remain parallel?