Our Mathematical Senses

The Geometry Vision

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Lecture-28

Video 6D: Solution to the Shadow drawing challenge

Okay, so let's see one solution to the shadow drawing challenge. Here's one way to draw the shadow. So the main question to start with is just where do the light rays hit the ground plane? We can't draw the shadow unless we know where the light rays are hitting the ground plane. For example, this top vertex of the triangle, is it going to land here or maybe up here or maybe down here? All of those seem vaguely plausible, but which is the correct place? Can we draw any part of the ground plane that's going to help us determine which of those was correct? Well, so far we have this point on the ground and this line on the ground. So we have a few points on the ground plane and we even have this line here. So we have some information about the ground plane.

So using that, can we get any more information about the ground plane? Are there any more points or lines that we can draw? Well, there's one obvious way to get some more. It's to connect the dots of what we already have. These two points are on the ground, so this line is on the ground. Similarly, this point is on the ground and this one is, so we get another line on the ground plane.

So we have two lines on the ground plane. Where do we go from there? Well, let's just forget about the triangle for one second because I think we're onto something here. Can we try and draw the shadow of the sign board first instead? Maybe that'll help us. So it has to lie in this kind of cone here. But where do the top corners project to? Well, if we go from the light source to this corner, it's going to be somewhere along this line, but where is it? Now, it's important to notice here that the street lamp post and the edge of the sign board, they're both vertical and they're therefore both coplanar.

So this line and this line are coplanar. So this line and this line are also coplanar. The

light ray L will lie on the same plane as the line M on the ground. Well, in that case, that means that these lines actually do intersect. This wasn't just an illusion.

These weren't just skew lines that we were viewing that seem in this two-dimensional representation to intersect. These actually intersect in space. So we get that L intersects M at a point P. Any two coplanar lines are going to intersect. That's one of the virtues and joys of working with extended planes and extended space.

So these lines intersect here, and that means that this point P is the projection of this point up here. And similarly, we can project this other corner of the sign board to this point here. So now we get two more points on the ground plane. What can we do? We can connect them and get a line on the ground plane, which is the image, the shadow of this top edge of the sign board. So not bad.

We've gotten the full shadow of the sign board. So I'll just erase all my projection lines and we can see it as a shadow. It's an actual shadow of the sign board. But I'll erase it now so that we have a little more space for what we're going to do next, which is return to the triangle. Now as it is, the triangle is kind of floating in the middle of the sign board.

It's not touching or connected to anything else. We don't have any incidence relations that we can observe easily since we're only seeing the segments of this triangle. So let's extend them a little bit so that we can see where we can get some intersections, which we can then work with. So let's start by extending this edge of the triangle to where it hits the top and bottom of the sign board. So that gives us two new points to play with.

Maybe we can project these two points. Well actually, this bottom point here where it intersects the ground plane, what's the image of that? What's the shadow of that going to look like? Well, it's already on the ground. So its shadow is just going to be itself. That point is going to project to itself. So we already know where that point goes to.

What about this point up here? When we project that to the ground plane, where does it go? Well, we already know that this line goes to this line. So this point of intersection of the two lines goes to this point of intersection here. So we get this line as well. And now we can just connect these together. And there we go.

We get the image of this edge. We're like one third of the way there. We've gotten the image of one of the triangle's edges, its shadow. Let's do another one. Let's extend and project another edge.

So I'm just going to extend this edge down to the ground plane, this edge that I'm coloring yellow. Well, again, the part on the ground plane is done. Its projection is there itself. It's already on the ground. So we just have to worry about this top point.

Where does the top point project to? Well, where does this ray of light hit the ground plane? We know that this yellow edge intersects this red edge. So its shadow intersects the red shadow. So its shadow is right there. So connecting those, we get our yellow shadow. Okay, now there's only one edge left.

This edge here. So as a first step, I would like to connect to extend it till it meets the ground plane. But we have a problem. This one isn't meeting the ground plane within the signboard. We're going to have to extend our signboard, but that's fine. Let's extend our sideboard, signboard, let's extend our line where the signboard meets the ground plane.

And where is it? So now where is this going to meet the ground plane? Well, again, there's another nice thing about the fact we're working with extended spaces and planes. It looks a little bit like maybe this edge is actually parallel to this edge and this edge. So it won't actually meet, maybe it won't ever meet the ground plane in real life. But it'll eventually meet the ground plane at a point at infinity, even in that case. So we don't have to worry about whether it's parallel or almost parallel or not parallel.

Eventually it's going to meet this ground plane edge somewhere. Either at a concrete point or at infinity. So it meets it somewhere. And the projection of that point to the ground plane is done, just as before. So we have to worry about the shadow of this point.

And what's the shadow of this point? Well, it intersects the red line, so its shadow intersects the red shadow, so it's right there. So we can connect the dots and get our blue shadow, our blue edge shadow. And there's a lot of lines here, but actually we've pretty much done it. Let's erase our sight lines.

We have our shadow triangle. We have all three edges in shadow form. So there it is. That's all there is to it. And let's erase some of our extraneous lines just to get a better view. You can nicely see there's the triangle, there's its shadow.

So of all the millions of possibilities, we figured out which one is the correct one. So I just want to bring back some of these lines and ask, is there anything in this image that relates to Desargues' Theorem? The two triangles, the shadow triangle and the actual triangle, are in perspective from a point. They're in perspective from this light source.

So by Desargues' Theorem, they must be in perspective from a line somewhere. So which line are the triangles in perspective from?