Our Mathematical Senses

The Geometry Vision

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Lecture - 01

Why Do The Images of Parallel Lines Converge?

Chapter One: The Power of Vanishing Points

In any work of two-dimensional art, a surface becomes a window to another world. And for as long as visual art has existed, artists have experimented with different ways of simulating depth in order to pull viewers through that window. Even the 30,000-year-old charcoal drawings of the Chauvet caves use intricate shading and layering to suggest three-dimensional depth. The window to another world could also be a Buddhist fresco on a wall of the Ajanta caves painted 1,500 years ago. Although two-dimensional, the image experiments with spatial depth through the play of light and the varied angles at which characters' faces appear. The window could even be an intricate portrait painted on a tiny 8-inch piece of paper in the court of Shah Jahan, just 400 years back.

Like many Mughal miniatures, it creates a sense of depth by portraying the more distant elements as smaller, softer, and blurrier. Artists never stop experimenting, and over the years they've discovered countless ways of representing depth. But one family of experiments is especially important for us as we explore the geometry of vision. In the 15th century, the urban European world contained more straight lines, more square tilings, and more windows.

And certain artists began treating paintings like literal windows to other worlds. In other words, they started painting lines, squares, and all other shapes as they would actually appear in your vision if you were observing the scene through a window from a fixed viewpoint. The technique they used for creating such paintings is now known as linear perspective. For comparison, here's a painting that does not feature linear perspective. An unknown English artist painted it in the late 15th century before the knowledge of linear perspective had reached England.

And here's a painting by Johannes Vermeer, created in Holland in 1665, well after the knowledge of linear perspective had arrived and matured. Notice any differences

between the two. Both paintings depict square-tiled floors, but which tiles appear more square? In the anonymous English painting, the square tiles all have right angles, which makes the floor look somewhat vertical. In contrast, Vermeer's tiles are all different shapes and sizes, and yet somehow they look like perfect squares, and our brains register the image as a depiction of a square-tiled floor. Why is this the case? Extending the edges of the tiles reveals the secret of their construction.

Each set of parallel lines converges to its own distinct vanishing point on the horizon line. We can even draw a third family of parallel lines, the diagonal lines of the tiles, and these will converge to a third vanishing point, also on the horizon line. In fact, any other line that is parallel to these diagonal lines in space is guaranteed to converge to their vanishing point. These basic features of linear perspective were discovered by artists in Italy in the early 15th century. This painting, from 1425, is thought to be one of the first to consistently use a fixed vanishing point for a full family of parallel lines.

As the knowledge of perspective spread, more and more paintings appeared that featured it. It didn't take long for artists to master this new tool, and yet, over the next few centuries, artists found that vanishing points and perspective kept offering fresh possibilities. Vincent van Gogh developed a totally new expressive style, abandoning realism, yet embracing perspective. The colorful, vibrant world of the painting seems built from a different material than our own, and yet, the clear, one-point perspective pulls us into that world. We can almost feel the brushstrokes running along the perspective lines towards the vanishing point at the door.

The surrealist painter Remedios Varro depicted haunting, dreamlike rooms. At the same time, she followed the rules of perspective quite precisely. Creating the impression of spaces that might actually exist somewhere, in spite of their magical subject matter. Around the same time, the famous mathematical artist M.

C. Escher created paradoxical scenes that strictly obey the rules of perspective, and yet defy our spatial expectations in fascinating ways. Escher also developed variations of linear perspective, such as curvilinear perspective. Notice how some of the vertical lines are depicted as curves in this image. And linear perspective is at the heart of optical illusions, like this street art drawn directly on the road in Kabul, Afghanistan. Viewed from the correct perspective, this anamorphic art gives the impression of a hole in the ground from which new towers are rising up.

Of course, not all artists use perspective in their art, and many enjoy consciously breaking the rules of perspective. For example, this collage painting by the Ghanaian-American artist Derek Forjor gives an accurate perspective view of a locker

room, but intentionally depicts the square tiles on the floor as actual squares. This creates a strange tension with the athletes sitting on the floor, while also coming full circle to that anonymous English painting we saw earlier from the 15th century. Over time, as artists experiment with perspective, they open more and more windows to more and more worlds. And yet it all started with a simple discovery that parallel lines converge.

Welcome to week one of the Geometry of Vision, the discovery of vanishing points revolutionized art by giving artists a range of powerful new possibilities to play with. But why do vanishing points work? And how exactly do we use them? This week, we'll focus on the answers to these questions. In particular, we'll try to precisely understand why families of parallel lines appear to converge to vanishing points, and how we can use vanishing points to draw in perspective. Now it's natural to wonder, how did 15th century artists discover the rules of perspective in the first place? Well, if your goal is to treat a painting like a literal window, what better way than to treat a window like a painting? For this, we'll take a cue from Albert Dürer, a German artist who clarified the rules of perspective for himself through careful experiments with a physical picture plane. Dürer's picture plane was really nothing more than a glass window, which he painted on directly while keeping one eye closed and the other eye in a fixed location.

As a first experiment in perspective drawing, let's use this physical picture plane to draw these newly installed railway tracks. Notice, the side rails of the tracks are infinitely long. They go on and on forever. How did I get this particular image? As my sight line traveled along the railway tracks, I kept marking the point where my sight line intersected the glass window. The image of the sight line was a little bit different.

It was a little bit more than a glass window, but it was a little bit more than a glass window. Where my sight line intersected the glass picture plane. Maybe this process gives you a clue as to why the parallel side rails converge in my image. Can you try giving an explanation? But be a bit careful. The horizontal railway ties that run in between the side rails, those remain parallel in my image.

So your explanation will have to account for that fact as well. After proving why the images of parallel lines converge to vanishing points, we'll look at how to use vanishing points to draw in perspective by going through a tutorial of one-point perspective and two-point perspective. But if you're feeling eager to try some drawing already, here's a drawing challenge. Can you draw a set of railway tracks with evenly spaced railway ties without taking any measurements? Let's be a little more precise. Start with a drawing of a single trapezoidal tile.

Now, using only a pencil and straightedge, can you extend your drawing to a perspective view of a full set of railway tracks with evenly spaced railway ties? But you'll have to be a bit careful. There's only one correct place to put the next railway tie. And this time, you don't have a compass to help you. Once you've completed the challenge, feel free to decorate the landscape around your tracks. Vanishing points will come in handy there too.

If you're finding the challenge difficult, don't worry. We'll see how it's done this week, after our perspective drawing tutorial, along with the previous drawing challenge. So without further ado, let's get started.