

Our Mathematical Senses

The Geometry Vision

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Lecture - 14

Video 3C: what does a perspective drawing reveal about the viewer?

There's one question we still haven't considered, which is, what does a perspective drawing reveal about the implied observer, the viewer? So a perspective drawing simulates viewing a three-dimensional scene through a window with one eye open in a fixed location. But does a perspective drawing tell us anything about the location of that fixed viewpoint? For example, can we determine anything about the implied location of the viewer in this painting? Is the viewer standing or are they sitting down or maybe are they climbing a ladder? What about this painting? Where is the viewer's eye level? This implied observer, are they sitting on a bale of hay on the ground? Are they standing? Are they viewing the scene from a window? What's the eye level? And this painting here is another good example. Again, where is the eye level? And also, what's the lateral position of the viewer's eye? In other words, is the viewer standing somewhere around here or maybe around here or maybe all the way over here? So I want to make a small definition that will help us discuss this. The viewpoint on the picture plane is the point on the picture plane closest to the viewer's eye. In this case, it will be this point here. The viewer's eye is closest to the picture plane here and this line is going to be perpendicular to the picture plane.

Notice that the vertical and horizontal components of the viewpoint are kind of significant. The vertical component is the eye level of the viewer and the horizontal component is the lateral position, the lateral viewpoint of the viewer. So it might be helpful to recall the horizon line theorem which stated that all vanishing points of all lines parallel to the floor plane are collinear and the line they lie on on the picture plane is called the horizon line. And the eye level on the picture plane, well, that is just the horizon line.

The horizon line gives us exactly that eye level of the viewer provided that the picture plane is perpendicular to the ground. If it's angled back or angled down, then things get a little more complicated. Technically the eye level of the picture plane is still the horizon line but if your picture plane is angled in a funny way, you'll end up with a mathematical horizon line which is different from your physical horizon line where you see the sun setting. So that's something to think about a little. That's a side problem to think about what happens if you angle your picture plane up or down but provided the picture plane is perpendicular to the ground, there's no confusion and the eye level of the picture plane is just the horizon line.

So here's a question. If a person's height is equal to the viewer's eye level, how would that person appear in the picture plane to the viewer? So in this image here, what can we say about the heights of the people in this picture? Well, here's the horizon line. All of them are, their heads go all the way up, exactly up to the horizon line. So their heights are all equal to the viewer's eye level. If a six foot tall person is viewing this and seeing the scene and everybody's head is just hitting the horizon, then all those people are a little less than six feet because the six foot tall person would have an eye level that's maybe five foot nine inches or something.

So yeah, but basically you can tell the heights of the people. Similarly, you can see whether people are taller or shorter than the viewer. All these people have the same height as one another and the viewer's eye level is at their knee level. All of these people have the same height as one another but the viewer's eye level here is at their chest level. And finally, here's the image we just saw where the viewer's eye level is at the exact top of their heads.

And artists play with lower and higher eye levels all the time to achieve various emotional or dramatic effects. So here's just a couple examples of a low eye level, a medium eye level, a high eye level. What about the lateral viewpoint? Where would it be in this painting, for example? So this painting we've already seen, the lateral viewpoint, or rather the vanishing point is somewhere here. Many objects are oriented in one point perspective in this image. And as a result, we have this nice clear vanishing point and the eye level will be the horizon line.

And the lateral viewpoint, well it definitely feels like the viewer is not over here. And since the vanishing point is here, it sort of feels like the viewer is standing here. And if we imagine viewing the image from this location, that somehow feels more right. And indeed if we imagine, in fact it's not hard to show that in a one point perspective painting, the viewpoint is exactly that one vanishing point. It's a little easier to see if we

imagine a person viewing the picture plane and viewing the scene through it.

This person here, just as their eye level was the horizon line, their lateral viewpoint will be here. And their viewpoint itself will be that one point. So the question becomes a little more complicated if there are no objects oriented in one point perspective. In that case, can we still determine the viewpoint? Is there any way to determine the viewpoint? And in certain circumstances, it's quite difficult, but in others it may not be as difficult. It requires us to know a little bit about the painting.

So here's one example of a two point perspective painting. And I'm actually not sure the easiest way to determine the viewpoint. If there was any object here that was oriented in one point perspective, that would give us a clear way. We just have to find that one vanishing point. But I don't see any objects that are obviously oriented in one point perspective.

On the other hand, I claim that if we know some object in the painting depicts an actual square, like maybe one of these cobblestones, we know in real life it's a square, we can use that information to help us find the viewpoint. So as a challenge, assuming the above picture plane depicts a square in two point perspective, can we determine the viewpoint on the picture plane? So assuming this is an honest square in real life, then in this perspective view, we have four vanishing points associated to it. These two edges go to the point D, these two edges go to the point B, this diagonal goes to the point C, and this diagonal goes to the point A. So using all this information can be determined the viewpoint on the picture plane. It's going to have to lie somewhere along the horizon line, because that's the eye level.

But as a hint, it's not any of the above vanishing points. It's not A or B or C or D. But you're going to have to use all of them to find it. So as another hint, it helps to consider a top-down view of the square and the viewer. So this is kind of what's going on as the person views the square in the picture plane.

The vanishing points B and D are here and here. Remember, B is the vanishing point corresponding to these edges, D is the vanishing point corresponding to these edges. And why are we getting the vanishing points in those particular places? Well, you can imagine looking further and further and further and further down along this line. And as you do that, you'll eventually get to a limiting sight line, which is parallel to this line and which will intersect the picture plane at B. Similarly, you can look further and further and further, further and further along this line and eventually get to a limiting sight line, which is parallel to this line in space.

And that'll hit the picture plane at D. So you get your two vanishing points that way. And as a hint, a first thing to consider, what is the angle θ made by the viewer's eye with the vanishing points B and D? What angle θ do you make? And once you've done that.

.. Okay, I'll give a little bit more of a hint because this is a tricky problem. That angle θ , well, we've seen that this line is parallel to this line, this sight line is parallel to this sight line. Well, these two lines, they form what angle here? This is a square, so this is a right angle. So θ is also going to be a right angle. So this can help us in the sense that we don't know where in space the eye of the observer is.

It could be here, but maybe it's here. Or maybe it's here. If we can find its location in space, or maybe it's here. We don't know where it is from this top-down view. But if we can find that, then we can certainly use that to find the lateral position of the viewpoint.

Wherever it is, maybe it's here. We just draw a line going straight up and see where our lateral position is on the picture plane. So we have to see where we are. Now we know that its angle with B and D is going to be a right angle.

So that's another hint. Maybe if you haven't seen this before, so we're looking for a point that forms a right angle with B and D. Well, the collection of all points that do that has a very special shape. And if you don't know what that shape is, then you can look up Thales' theorem. It's a very ancient theorem, which will give you another hint. But using that and using the fact that this is a square, there's still some more information you can get from the vanishing points A and C.

So I'll leave it there so that you have something to think about. But hopefully you can solve this problem. It is a bit of a challenge, though. So good luck. And as a recap, we've seen in this first chapter that families of vanishing parallel lines appear to converge when viewed in perspective.

And we've understood why that's the case and how we can use it. But it's still hard to understand what's changing and what's staying the same when we shift from one perspective to another. In the intro video, we saw that area, length, and angle keep changing under perspective shifts. And we also asked the question, is there any numerical quantity that stays the same? But more generally, in order to really understand what all is changing and what all is staying the same, these are the main questions we want to answer in this course. And in order to understand those, we need to define perspective shifts more precisely. So that's our next task.