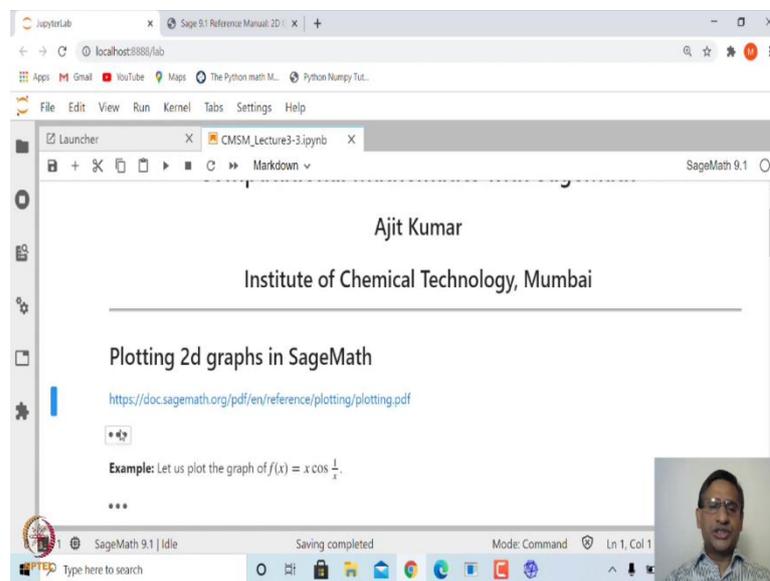


Computational Mathematics with SageMath
Prof. Ajit Kumar
Department of Mathematics
Institute of Chemical Technology, Mumbai

Lecture – 18
2d Plotting with SageMath

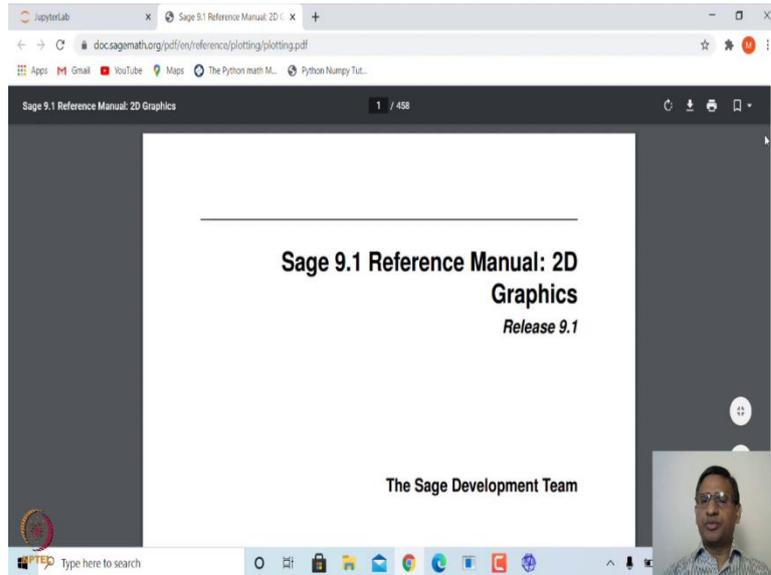
Welcome to 18th lecture on Computational Mathematics with SageMath. In this lecture we will look at how to plot graphs in 2d using SageMath.

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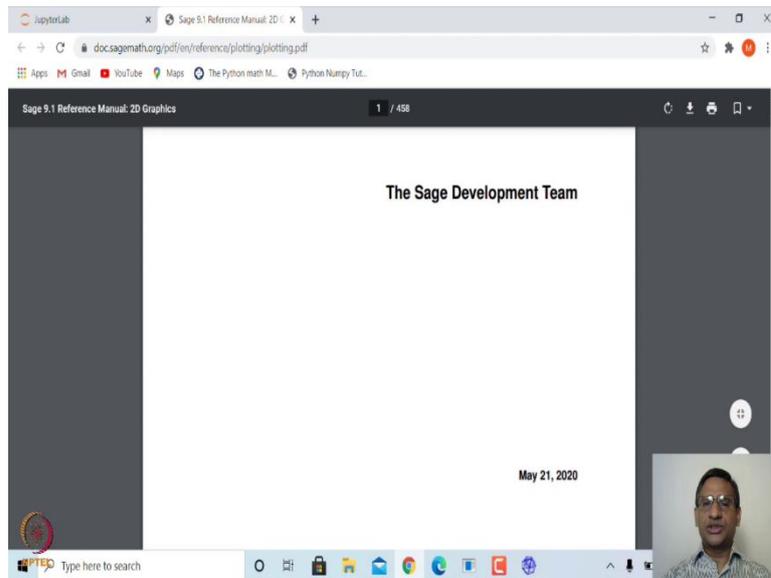
So, first of all you can look at this reference manual from sage website. So, for example, it is a reference manual on plotting.

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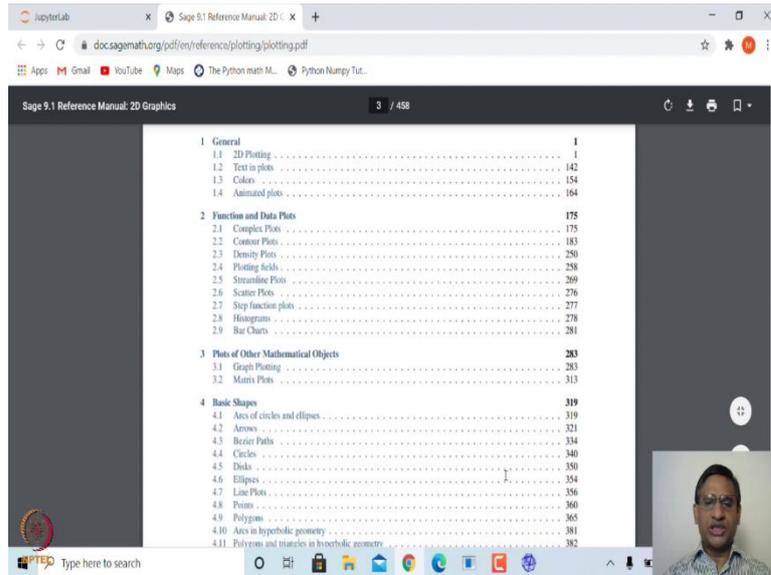
So, if I click on this, it will open a reference manual which is Sage 9.1 Reference Manual for 2D Graphics.

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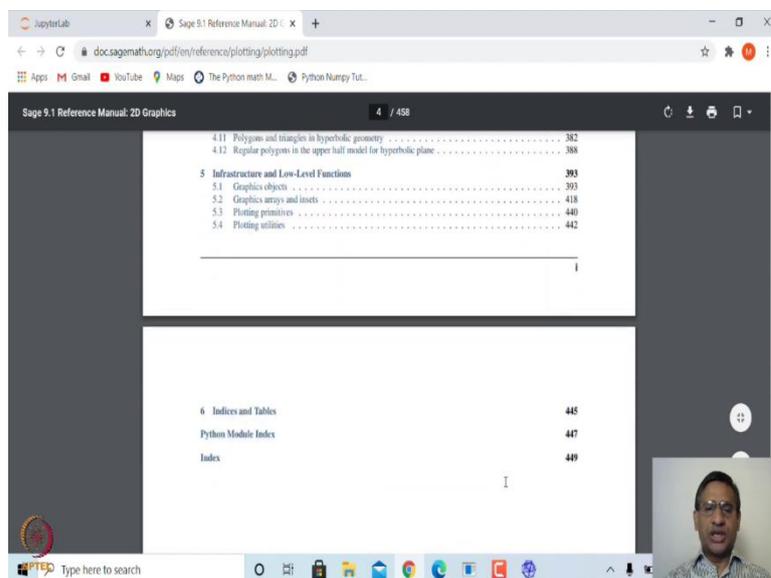
And if you go through this, this is 458 pages reference manual and this will give you all possible plotting in 2 dimensional space.

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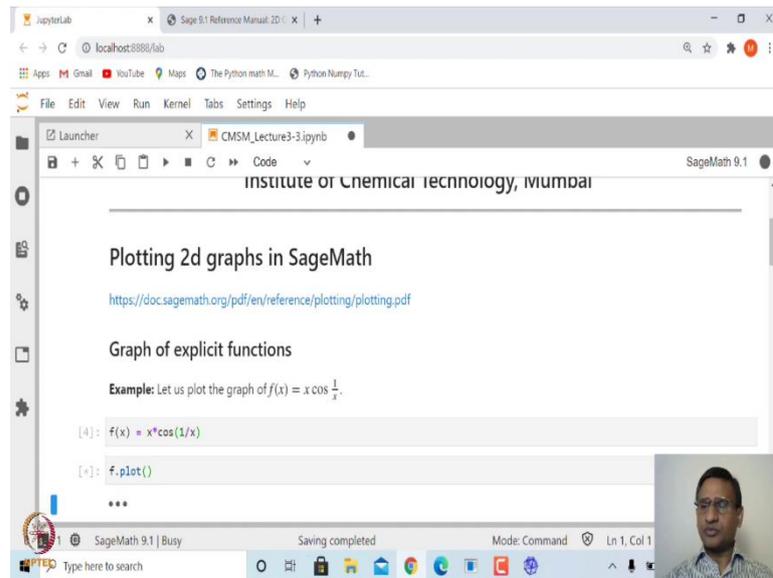
So, you have for example, general 2D plotting then functions and data plot of other mathematical objects basics shapes and then you also have some graphics, graphics array and things like that.

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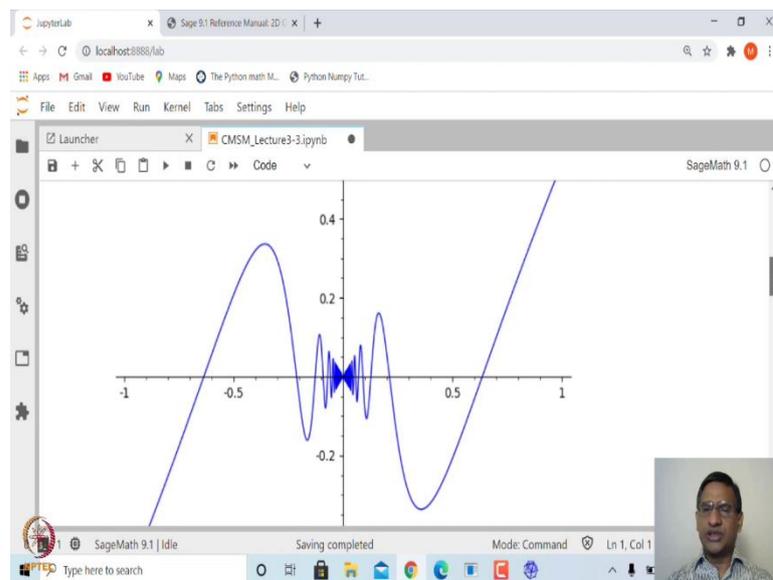


So, you can refer to this reference manual for more details. So, let us look at our worksheet.

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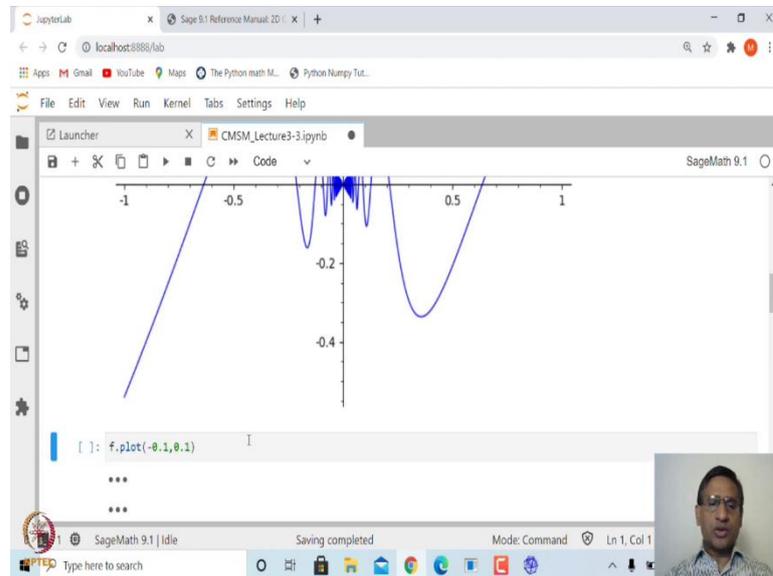


So, first thing is let us plot graph of a function which is explicitly defined that is $y = f(x)$ and $f(x)$ is explicit explicitly defined in terms of x . So, for example, let us look at a function $f(x) = x * \cos(1/x)$ and let us plot graph of this function. We have already seen plotting graphs using matplotlib so in fact, you can also plot graphs using matplotlib from sage worksheet itself. So, let us look at how do we plot this graph. So, first you need to define this function.

So, let us define $f(x) = x * \cos(1/x)$ and then in order to plot this graph again you can use `f dot tab` and you will see an option called `plot`. So, if I say `f dot plot` then it will plot graph of this function in the interval -1 to 1 . That is the default interval it will take since

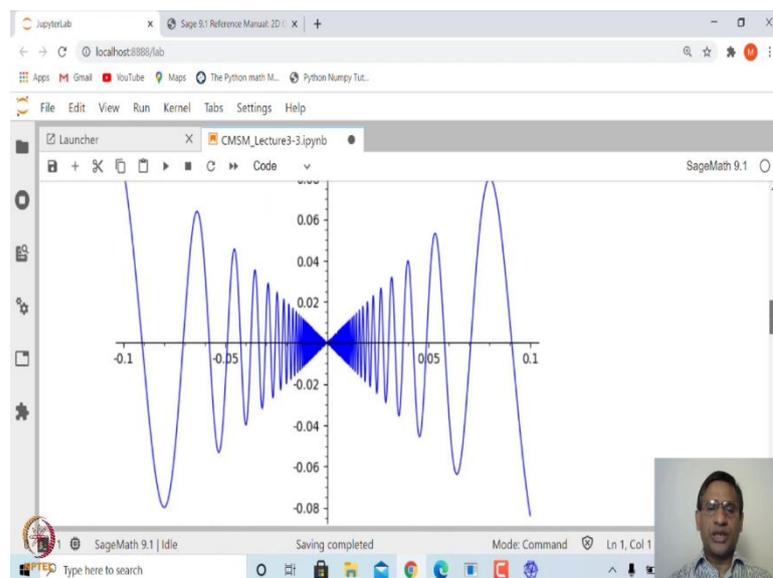
we have not specified any interval here, it will take -1 to 1 and it plots the graph in blue color and it has some default options.

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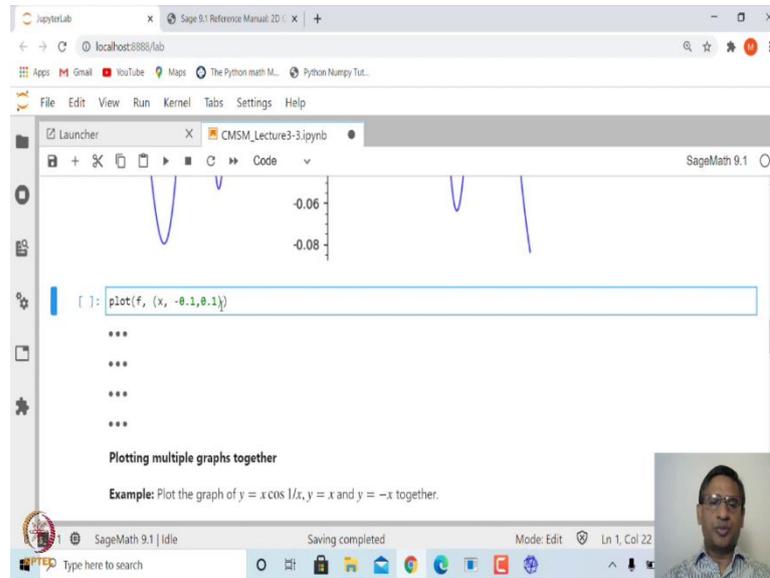
So, if you can also give the domain in which you want to plot. So, for example, let us say if I want to plot graph near 0, then we can mention the interval from -0.1 to 0.1 ('f.plot(-0.1,0.1)').

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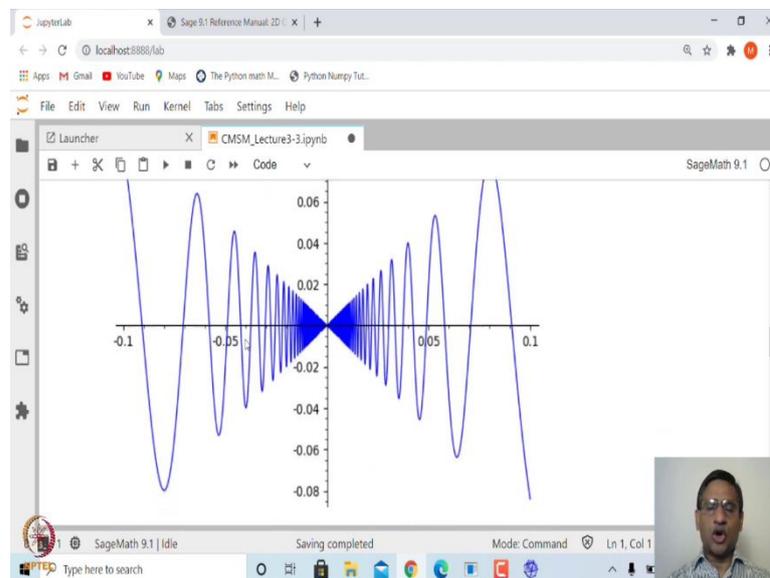
So, this is a graph of the same function in the domain -0.1 to 0.1 .

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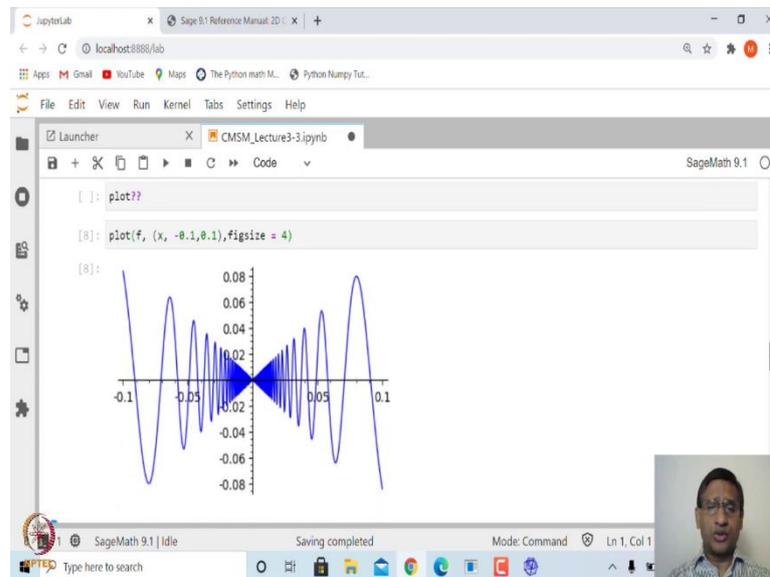
So, you can also plot graph of this the same function using plot function and inside that you give the name of the function and then give the domain inside round bracket. So, x varying from -0.1 to 0.1.

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So, that is another way of plotting graph. Now you can change the various options of this graph. For example, you can change the color, you can change the plot style, you can change the thickness, you can make gridlines all such options also exist.

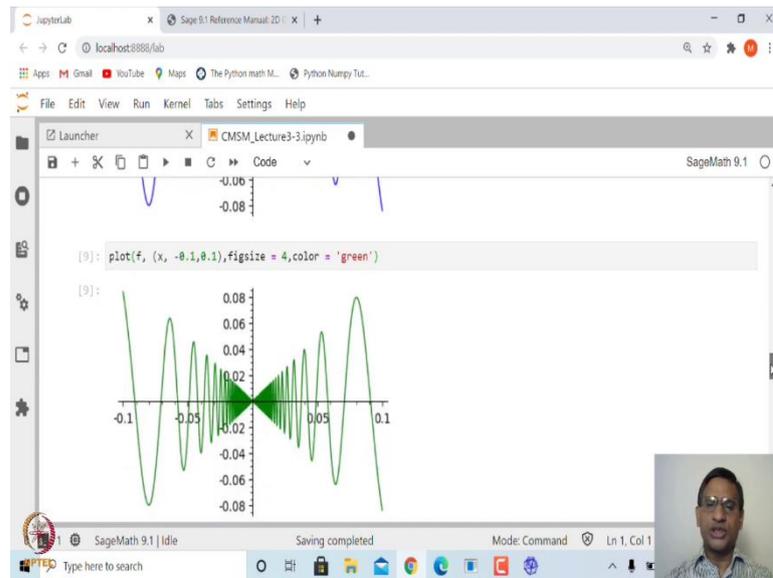
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So, you can for example, look at help on plot. So, if you say plot and then question mark then it will open an inbuilt help manual or you can say double question mark. So, you can go through the help document on plot which will also tell you what are the other options that you can use along with plot.

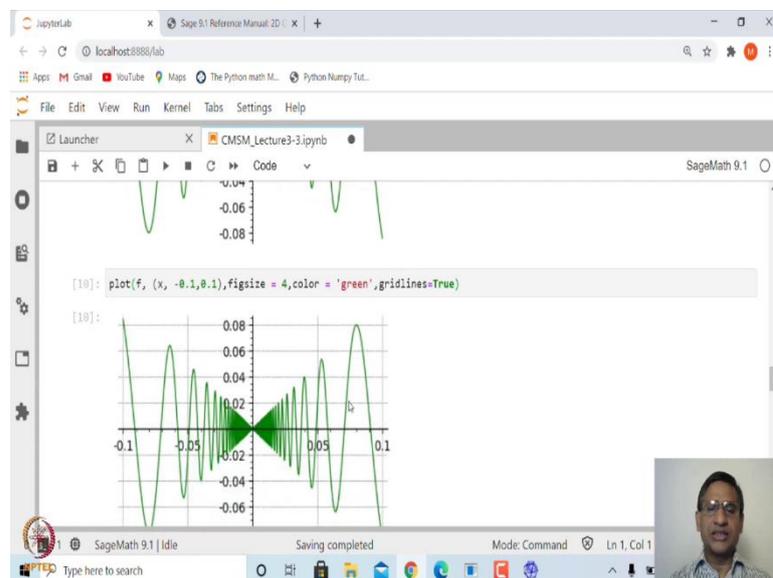
So, suppose for example, if I want to reduce the figure size by default it gives you figure let us say like this which looks quite big on the screen, suppose you want to reduce the figure size you can mention the width you can mention the height. Let me mention only one parameter for the time being I can say $figsize = 4$ and then it will reduce the figure size. So, depending upon what size do you are looking for you can change the size.

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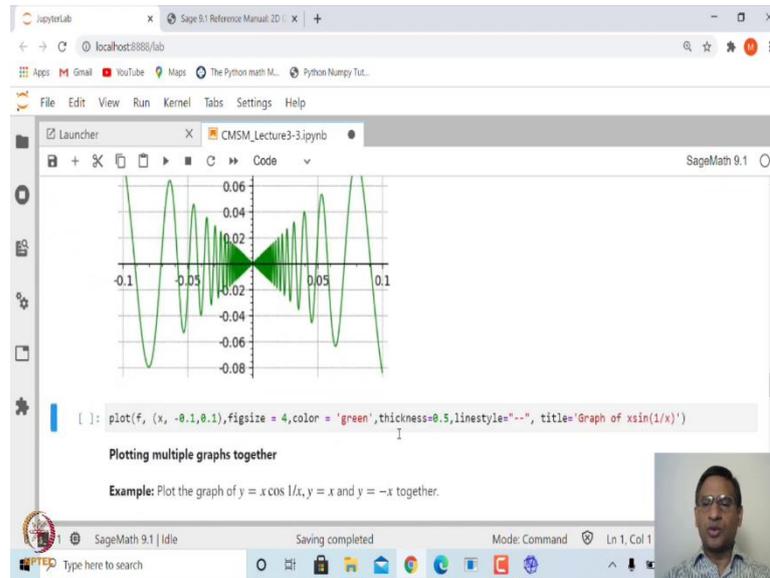
You can also change the color. So, suppose if I say color inside this plot, if I say color and inside single quote green or you can also give double quote if I say green then it will plot in green color ($color = 'green'$). So, this graph is in green color because we supplied this option green.

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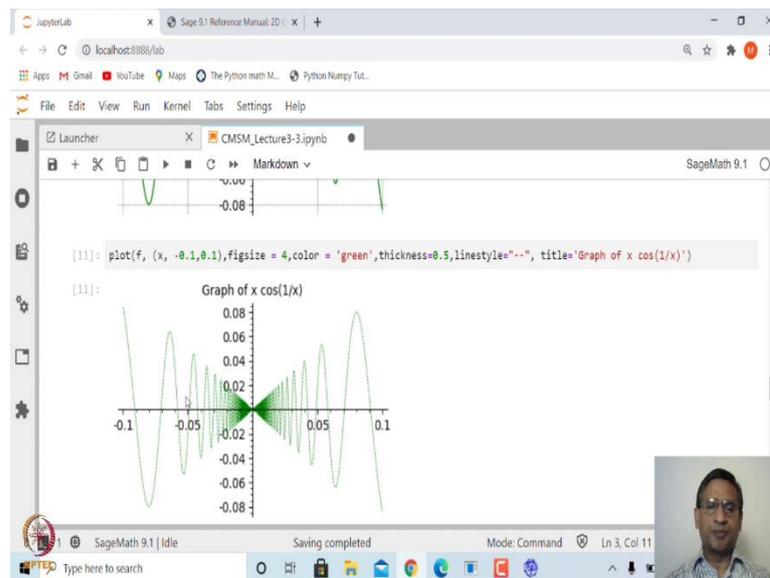
Now, suppose you want to also put gridlines. So, you can say $gridlines = True$, this will add gridlines. Of course, it will put gridlines default gridlines, but you do have an option to put grids at whatever interval do you want. So, you can look at plot options by getting help on plot and look at how you can change the gridlines.

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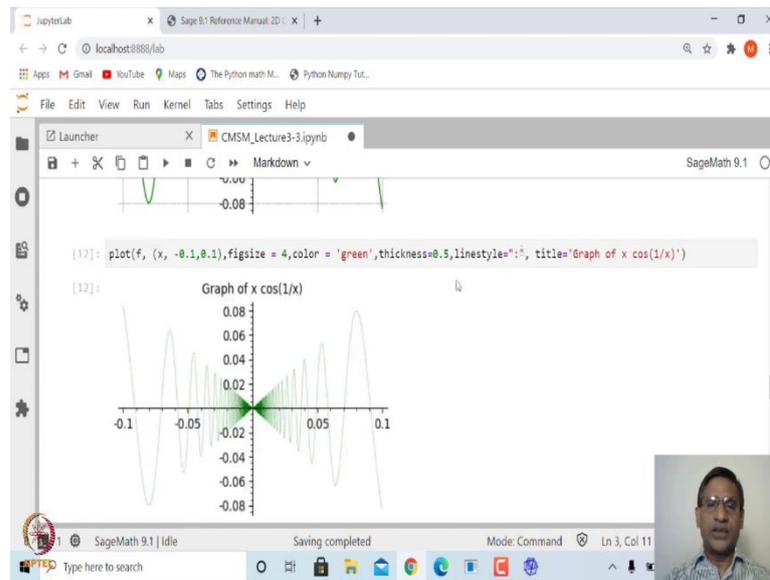
Similarly, you can more give more options for so, for example, if I want to reduce the thickness by default the thickness is 1, if I let us say *thickness* = 0.5 then it will reduce the thickness and let us say we want to plot linestyle like dash dash (*linestyle* = " - -"). So, it will plot dashed linestyle and you can also mention the title.

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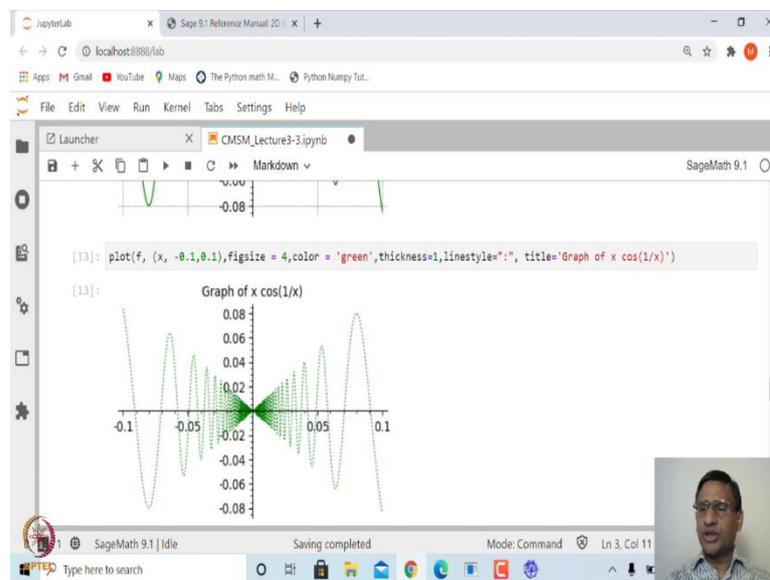
So, here the title is graph of $x \cdot \cos(1/x)$. So, that is the other options. So, it you can see here this thickness has reduced, it has also given the title of this graph and it is also plotting in dash linestyle.

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If you want dotted linestyle you can mention colon here inside this linestyle you can say colon and then it will give you dotted line.

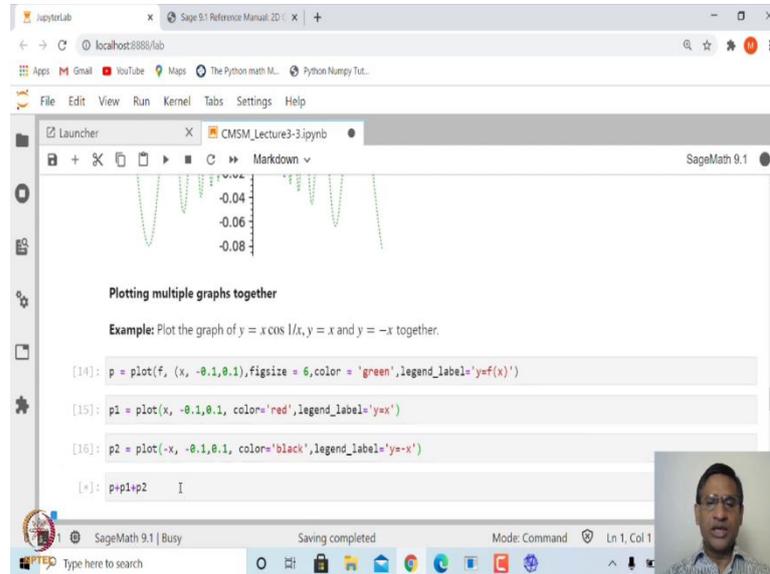
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So, let me just to visualize it properly, let me give the thickness default value which is 1 now you can see here this is the dotted. So, you can explore other options inside this plot and do all kinds of annotations. Now suppose you want to plot multiple graphs in the same window.

So, for example, we have plot a graph of $y = x * \cos(1 / x)$ and suppose inside this graph we want to add graph of $y = x$ and $y = -x$. How do we do that?

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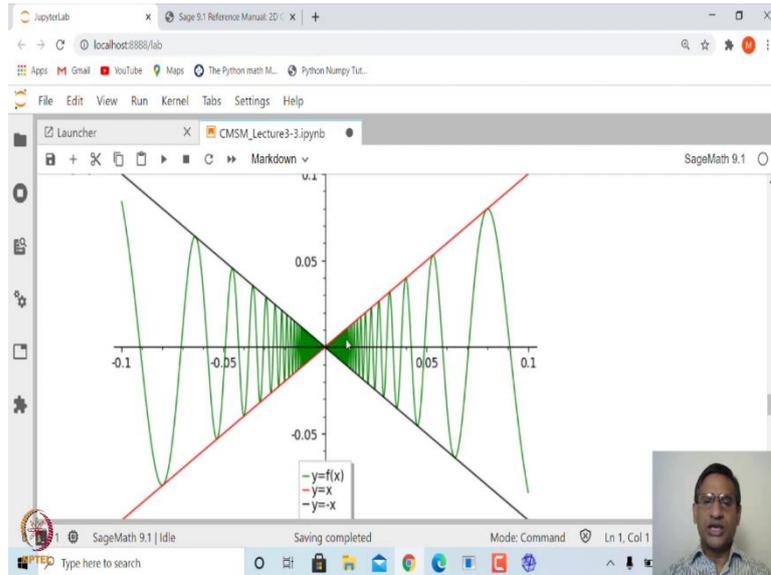


So, this is again very simple the one way to do is first plot this graph and store it in f. So, plot f from -0.1 to 0.1 figure size is 6, the color is green and we will also put legend so, that we can distinguish which curve represents which function.

So, that is the legend label. So, let us store the graph of the function $f(x)$ in p and then let us plot graph of x this function is x and this is in red color and level is $y = x$ that is in p1 and then in p2, let us plot graph of $y = -x$ and this in black color and the legend is $y = -x$.

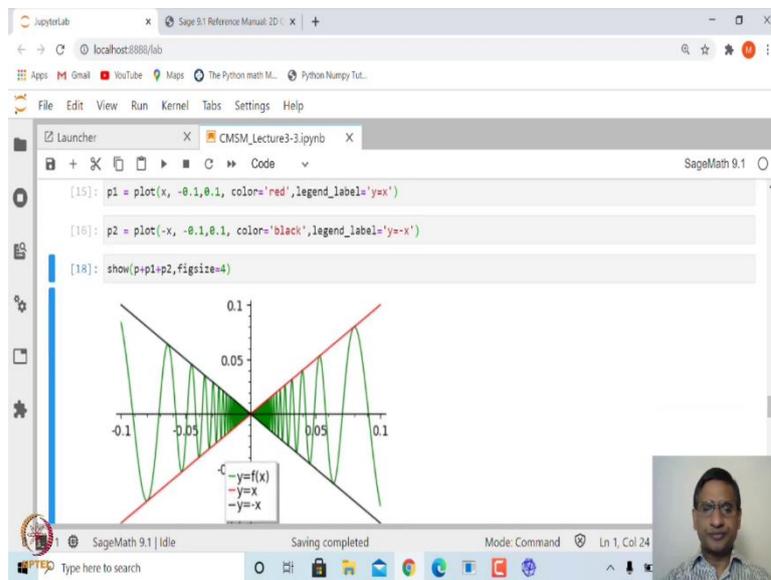
Now, what you need to do is just write $p + p1 + p2$. So, what it has done is it has plotted this graph and stored in a variable p, in object p and similarly p1 and p2 and when you add all these things together $p + p1 + p2$ it will give you the graph of all these functions together. So, that is how the graph looks like.

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So, this is red is $y = x$ and black one is $y = -x$ and this green one is $y = f(x)$ that is the legend. Again you can change the position of this legend depending upon the space.

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So, for example, if I want to reduce the figure size, I can also ask it to show and then in the bracket I can again I can say $figsize = 4$. So, then it will reduce the figure size. You have an option to give figure size inside plot or you can also give figure size inside show function. Let us look at the next example.

(Refer Slide Time: 09:29)

Piecewise functions

Example: Plot the graph of the function $f(x)$ which is $1 + x^3$ in $[0, 1]$ and $1 + x$ in $[-1, 0]$.

```
[19]: f = piecewise([(0,1), 1+x^3), ([-1,0], 1+x)]);
```

```
ValueError
Traceback (most recent call last)
<ipython-input-19-2d6206c391a1> in <module>()
----> 1 f = piecewise([(Integer(0),Integer(1)), Integer(1)+x**Integer(3), ([-Integer(1),Integer(0)], Integer(1)+x)]);
/opt/sagemath-9.1/local/lib/python3.7/site-packages/sage/misc/lazy_import.pyx in sage.misc.lazy_import.LazyImport.__call__
(build/cythonized/sage/misc/lazy_import.c:3686):()
 352     True
 353
 354     return self.get_object>(*args, **kws)
 355
 356     def _repr_(self):
```

Suppose, you have functions which are defined in pieces what we call as piecewise defined function. So, for example, let us say if I have a function $f(x)$ which is $1 + x^3$ in the interval 0 to 1 and $1 + x$ in the interval -1 to 1 and we want to plot graph of this function. So, it has an inbuilt function called piecewise you can look at help on piecewise, piecewise question mark and the way if one needs to define is you have to define all the pieces inside square bracket that is you have to make a list.

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Piecewise functions

Example: Plot the graph of the function $f(x)$ which is $1 + x^3$ in $[0, 1]$ and $1 + x$ in $[-1, 0]$.

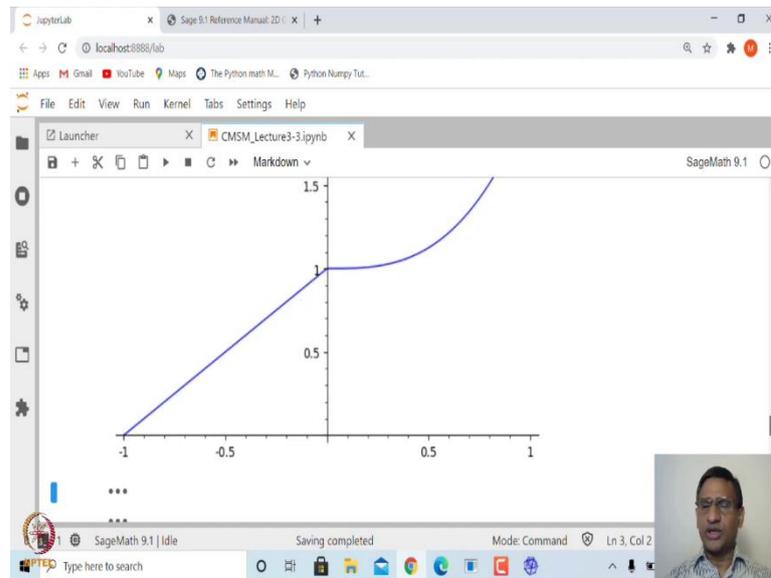
```
[20]: f = piecewise([(0,1), 1+x^3), ([-1,0], 1+x)]);
```

```
[*]: f.plot()
```

So, and then the first part is the first component which is between 0 to 1, it is defined as $1 + x^3$ and in the second piece it is defined from -1 to 1 it is defined as $1 + x$. Though here it is written as square bracket -1 to 1 and here it is 0 to 1.

So, this is open bracket alright. So, here also one should make these changes or let me write here 0 to 1. So, that is the function which is defined now. So, I think there is a problem because at 0 it is defined two different values that is why it is a problem. So, let me define this as 0 to 1 itself and then how do we plot this graph? So, you can simply say f dot plot.

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So, it will plot graph. So, you can see here from -1 to 0 this is this component and from 0 to 1 it is this component. So, that is the how you can define piecewise functions and you can plot this graph.

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```

Example: Plot the graph of the function  $f(x)$  which is  $1 + x^3$  in  $[0, 1]$  and  $1 + x$  in  $[-1, 0]$ .

[20]: f = piecewise([(0,1), 1+x^3], [(-1,0), 1+x]);

[21]: f.plot(-2,2)

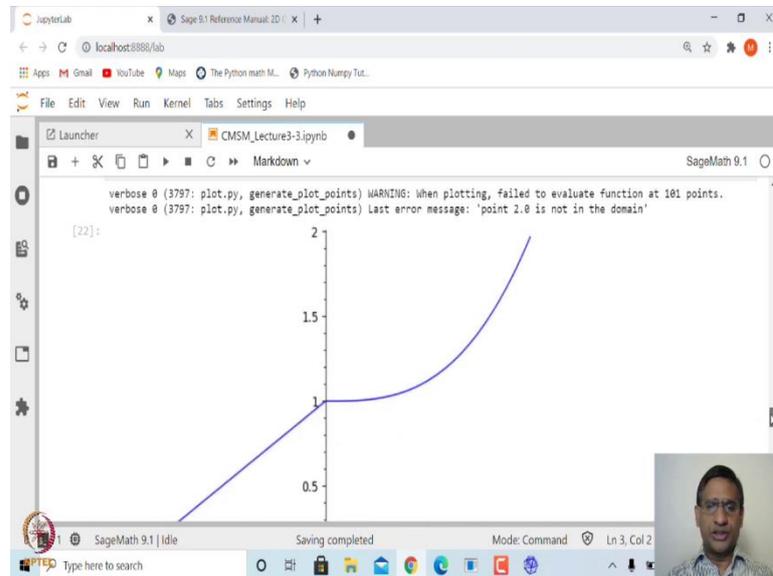
verbose 0 (3797: plot.py, generate_plot_points) WARNING: when plotting, failed to evaluate function at 161 points.
verbose 0 (3797: plot.py, generate_plot_points) Last error message: 'point 2.0 is not in the domain'

...
...
...
...
...

```

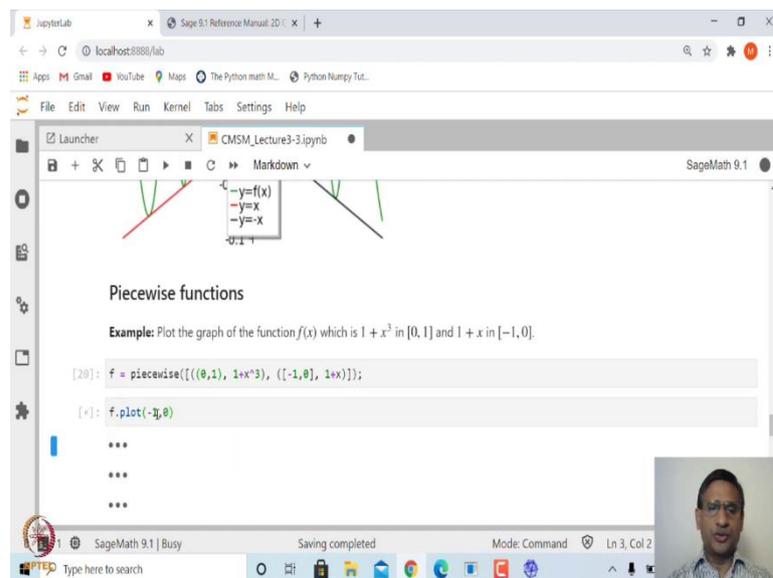
Of course, if you try to plot between let us say -2 and 2, then it will not work because this f is not defined outside -1 to 1.

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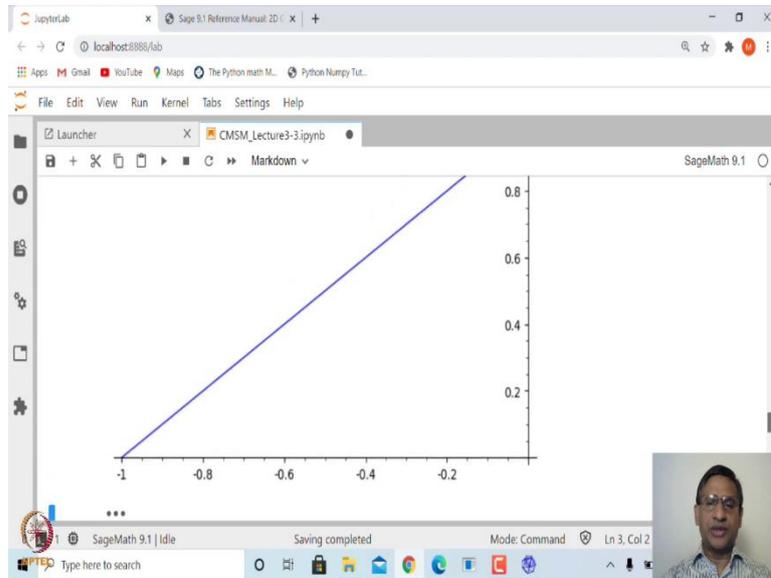


So, that will give you an error you can see it will give you it says that these points are not in the domain though it is plotting. So, let us make it from -1 to 0.

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So, similarly suppose you are -1 to 1 .

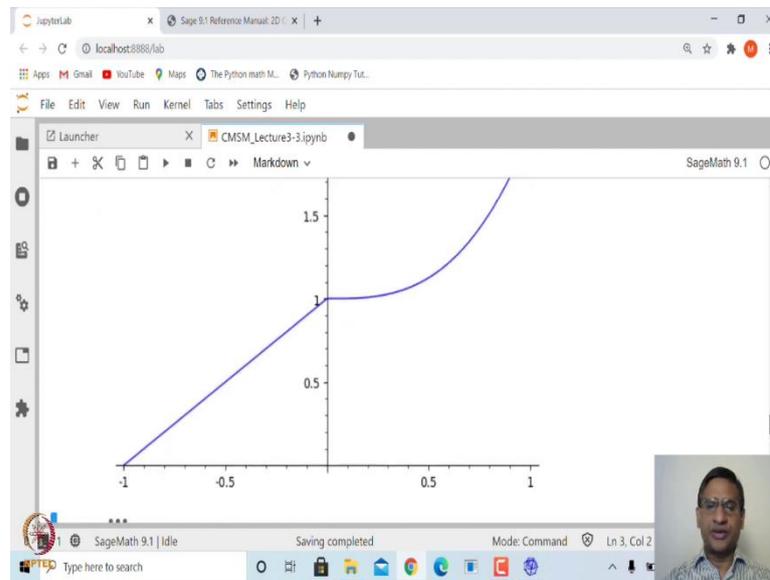
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The screenshot shows a JupyterLab window with a SageMath 9.1 kernel. The main area displays the text 'Piecewise functions' and an example: 'Example: Plot the graph of the function $f(x)$ which is $1 + x^3$ in $[0, 1]$ and $1 + x$ in $[-1, 0]$ '. Below this, the code is shown in a code cell:

```
[29]: f = piecewise([(0,1), 1+x^3], [-1,0], 1+x));  
[*]: f.plot(-1,1)
```

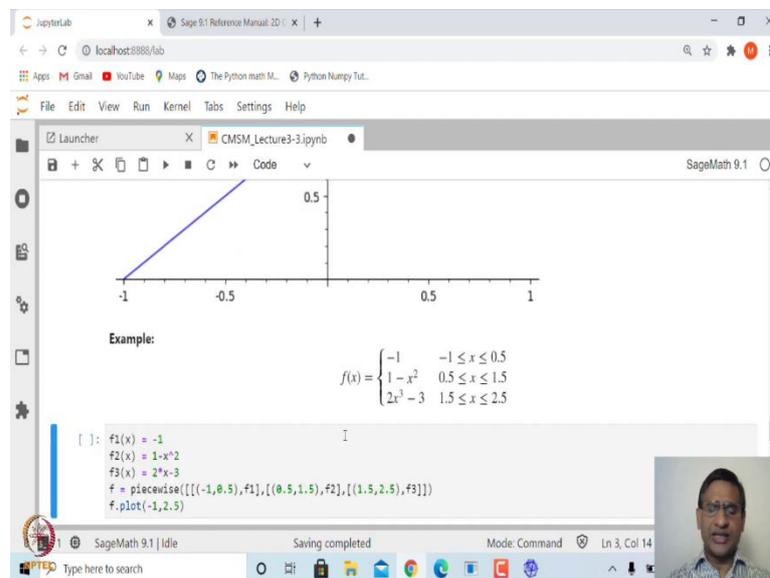
The code cell is followed by three dots, indicating the plot is not yet visible. The interface includes a file browser on the left, a top menu bar, and a status bar at the bottom indicating 'SageMath 9.1 | Busy' and 'Saving completed'.

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Suppose, you want to define a function which is defined in three pieces.

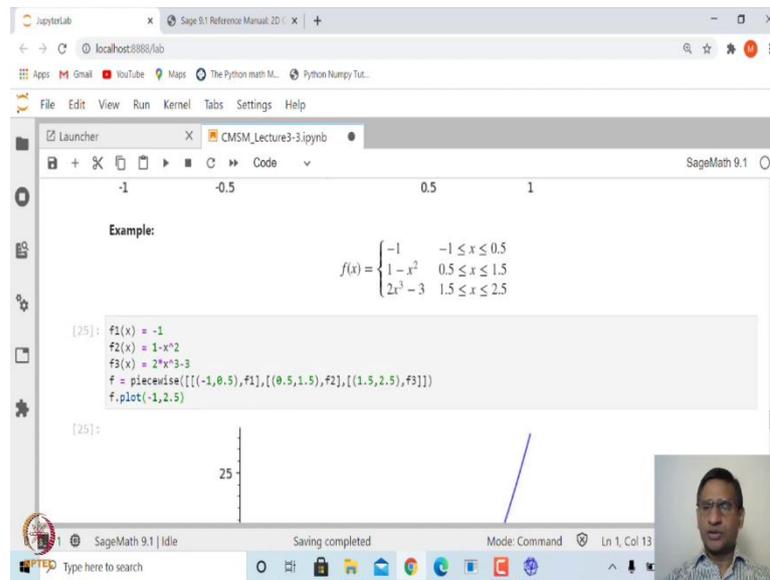
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So, let us say you have an example $f(x)$ which is equal to -1 between -1 and 0.5 equal to $1 - x^2$ between 0.5 and 1.5 and $2x^3 - 3$ between 1.5 and 2.5.

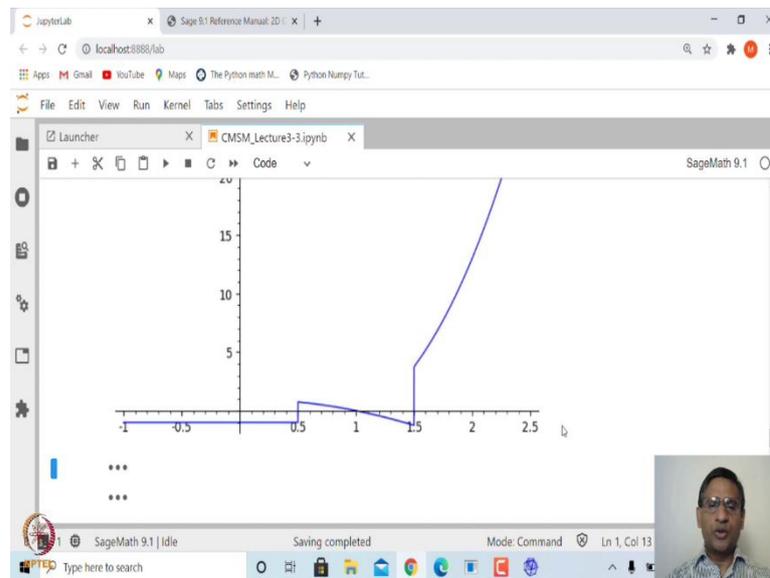
So, again you can define it in a similar way as we have done here you have to have one more component, but you can define them separately and then combine together that also you can do. So, I have defined $f_1(x) = -1$, $f_2(x) = 1 - x^2$, $f_3(x) = 2x^3 - 3$.

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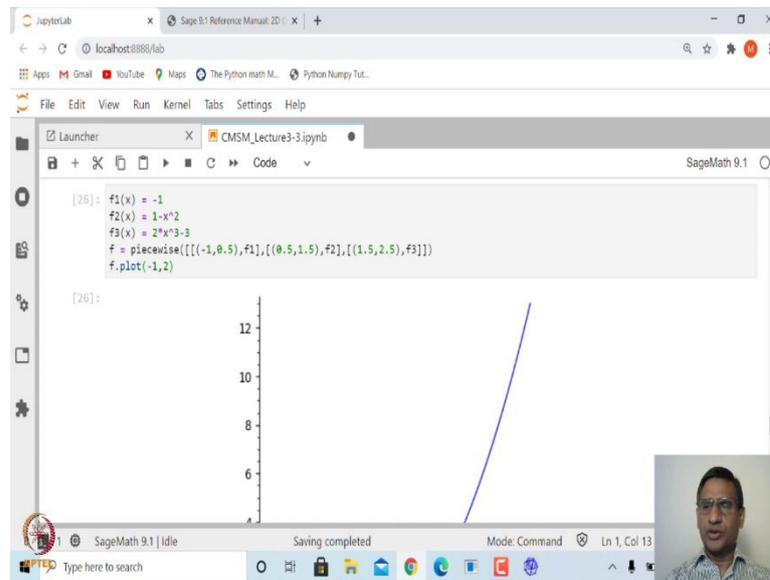
So, this is $2x^3 - 3$ and then you say f is equal to piecewise and in the first this is the first component the first you need to give the interval and then the f_1 , the next one is the second component and the third component and everything is defined inside a list. So, it is a list of each pieces and then we can simply say f dot plot and from $-1, 2.5$ ($f.plot(-1,2.5)$).

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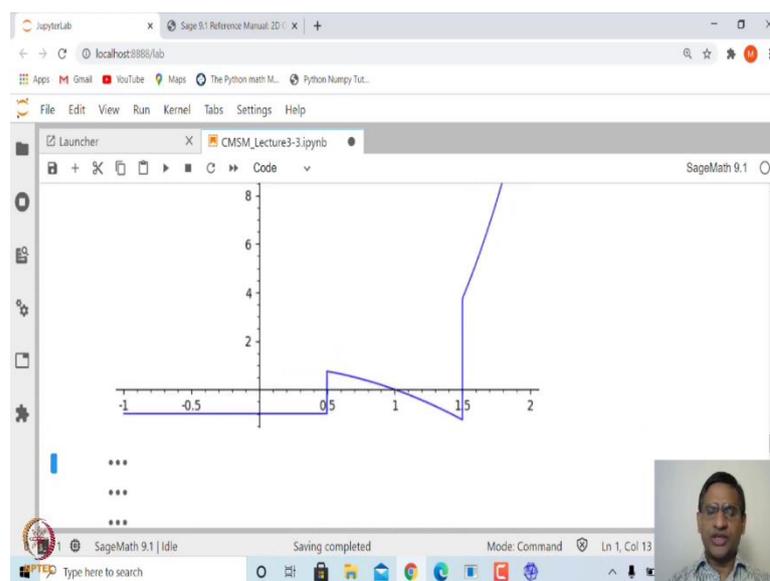


So, that is the graph of this function. Now you can see here this let me plot only up to 2.

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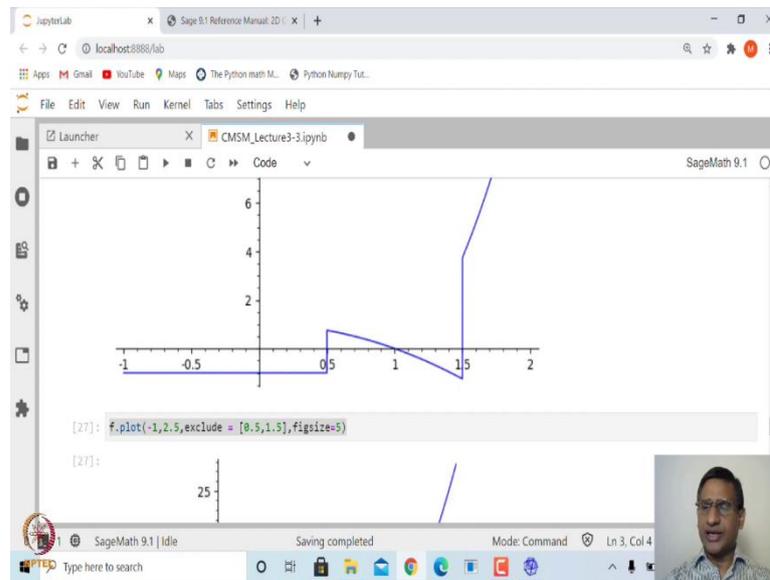
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So, that this will look slightly better this is slightly better, but you can see here from this piece to this piece there is a jump and that jump is indicated by this vertical line though vertical line is not part of this graph.

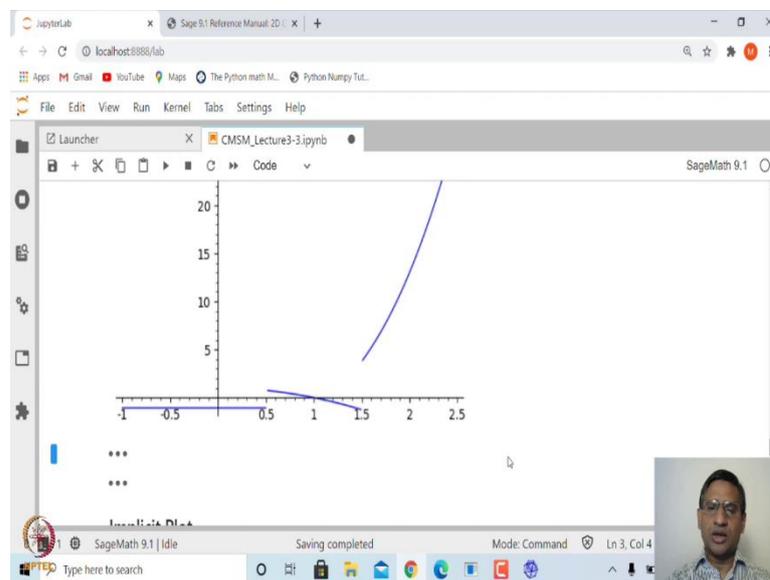
So, there is a way to tell sage that do not plot these vertical lines. Similarly, this vertical line and this vertical line you can you can suppress these vertical lines and the way to do is you can mention exclude.

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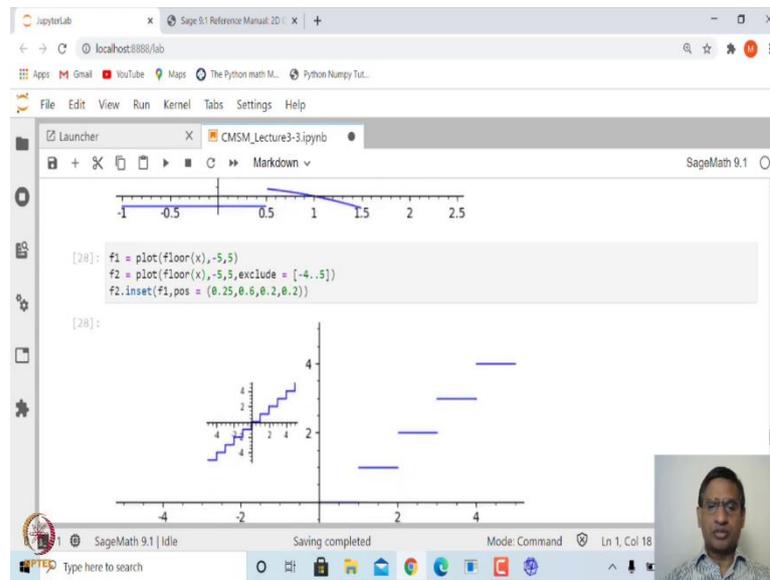
Exclude what should it exclude? It should exclude the points at 0.5 and at also at 1.5 and then if you plot this then you will not see these vertical lines, that is what you can see here.

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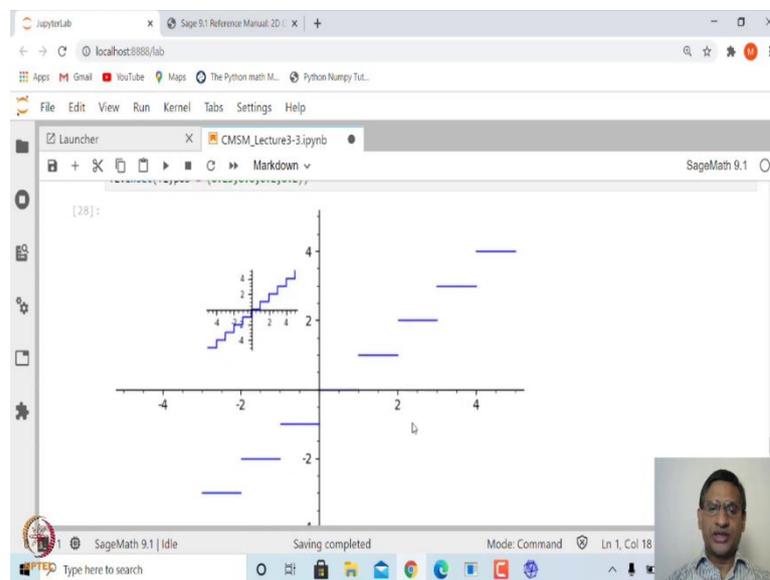
So, those vertical lines are gone only these pieces are shown.

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So, you can also for example, plot graph of floor function. So, this is greatest integer function.

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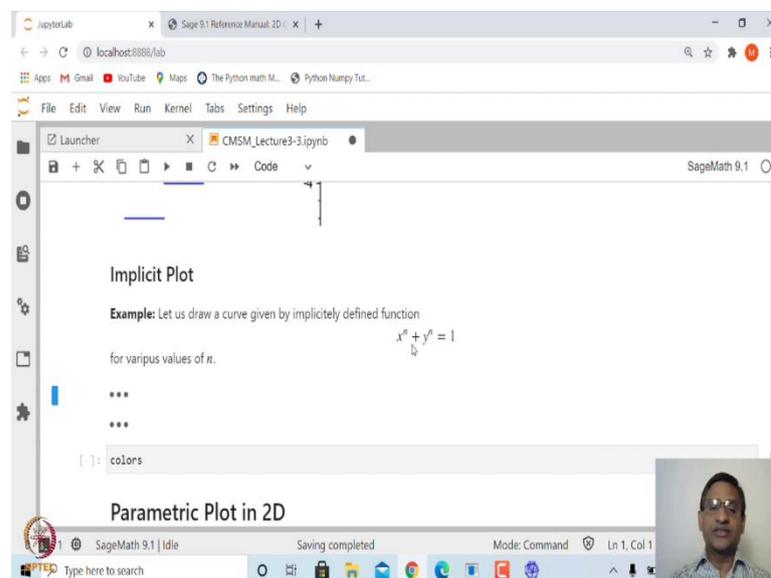
And so, this is the graph of greatest integer function they are just also known as step function and inside this we can plot graph of another function which can get inserted inside this.

So, what is that? I have this is the graph of this floor function floor x from -5 to 5 and f2 is a graph of floor function from -5 to 5 and with this condition f2 excludes this integer

points from -4 to 5; that means, this will exclude the integer points that is why you dont see these vertical lines.

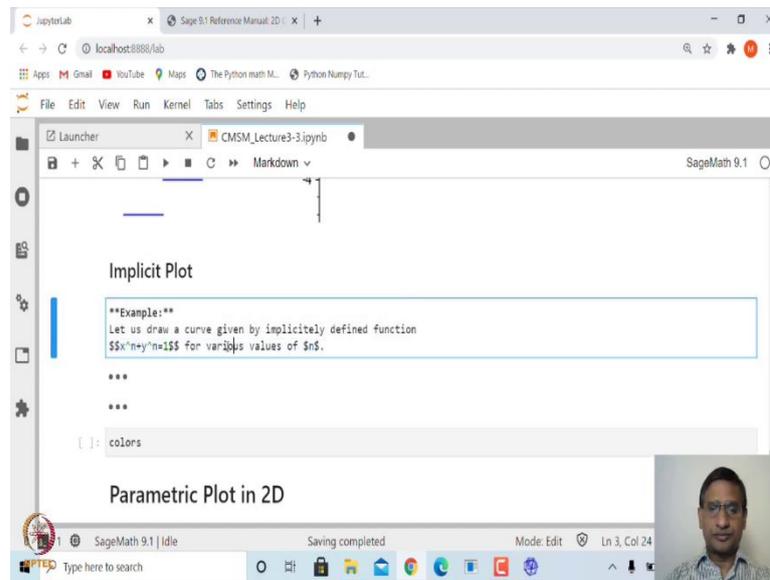
And then what you need to do? In f2 you are inserting f1 i.e. graph of f1 and you need to mention the position. This position will be in terms of a scaling factor in terms of x axis, y axis, height and width. So, if I plot this in this main plot that is this step function, it is inserting another step graph of step function. So, and you can insert maybe one more. So, maybe try to insert something here and as an exercise try to insert, let us say graph of ceiling function in this particular region.

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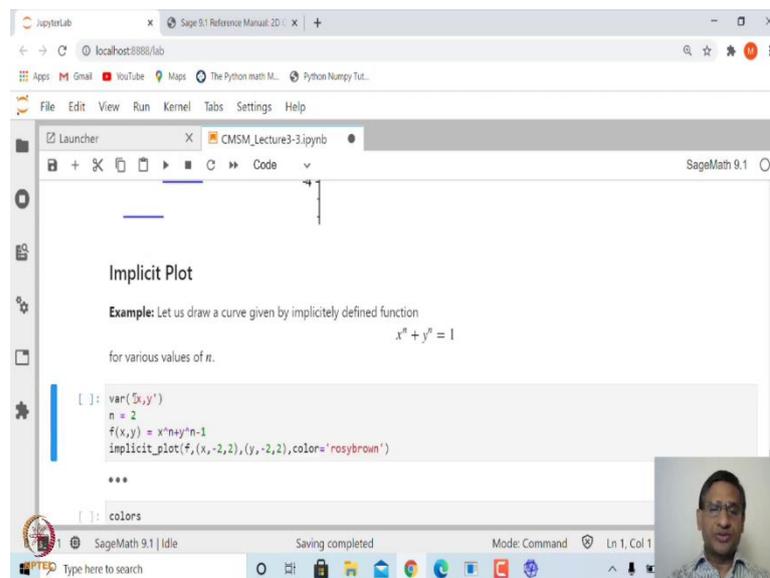


Now, suppose you want to define graph of function which is implicitly defined y and x are defined implicitly. So, for example, let us look at this example we want to define or plot graph of a function which is defined by $x^n + y^n = 1$ for various values of n.

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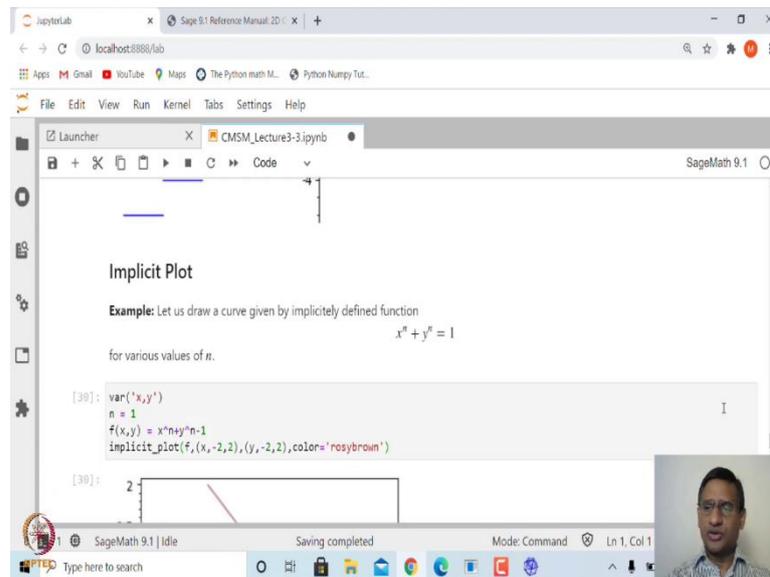
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For various values of n . So, for example, if $n = 1$, it will be $x + y = 1$ which is straight line if $x = 2$, $x^2 + y^2 = 1$ which is a circle and things like that.

So, let us look at how we are going to plot. So, first we will define this x and y as two variables. So, earlier we had y equal to $f(x)$ explicit function, now I have implicit function. So, y is also a variable. So, first we need to declare x and y variables. As I said in the last class x by default is thought of as a variable in sage. However, it is good idea to redefine that because in case you have stored x as some value then it may conflict.

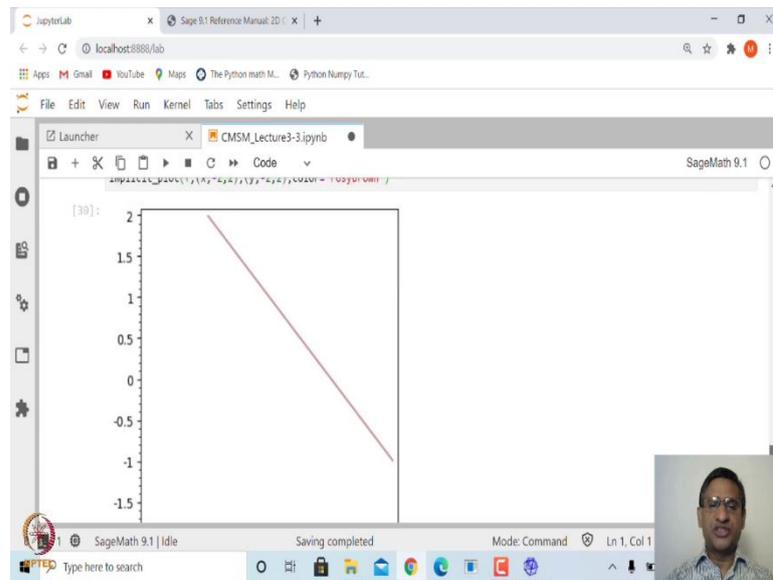
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So, now let us say we want to plot for $n = 1$ and $f(x) = x^n + y^n - 1$, this is -1 and then implicit underscore plot that is the name of the function which you can use. So, again you can see here implicit function this is combination of two words and in SageMath this is defined as it is combined as giving underscore in between these two keywords.

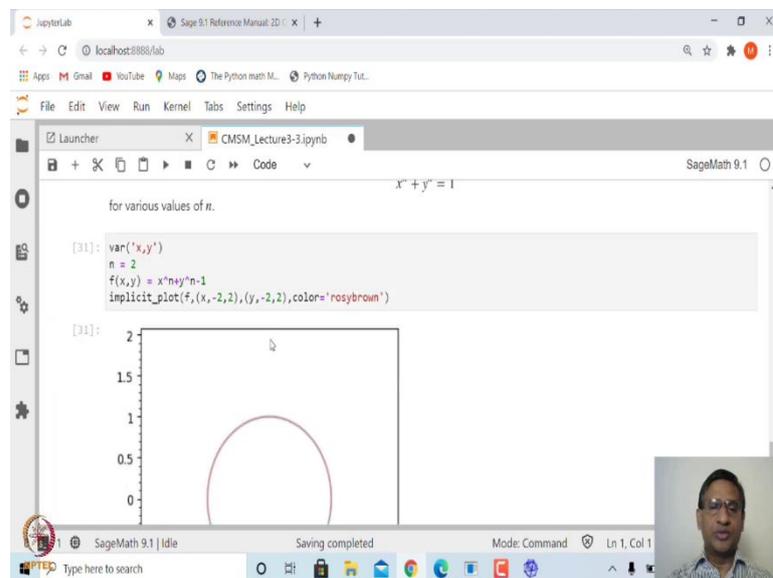
So, the what are the arguments? A plot implicit underscore plot $f(x)$ varying from -2 to 2 and y also varying from -2 to 2, color I am just taking another color called rosy brown. And there are several colors which are available I will show you in a minute and let us first plot the graph of this function.

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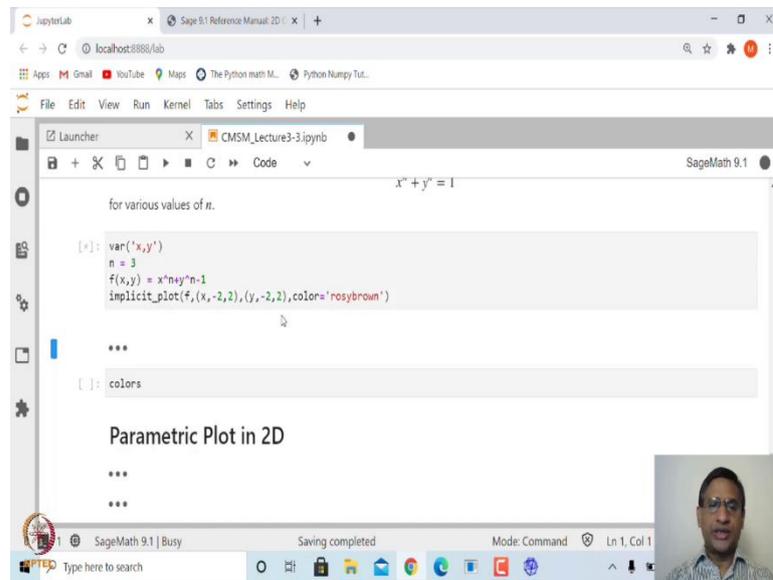
So, that is the graph of the straight line $y = 1 - x$.

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Now suppose, if I say instead of 1 if I take $n = 2$ it will be a circle.

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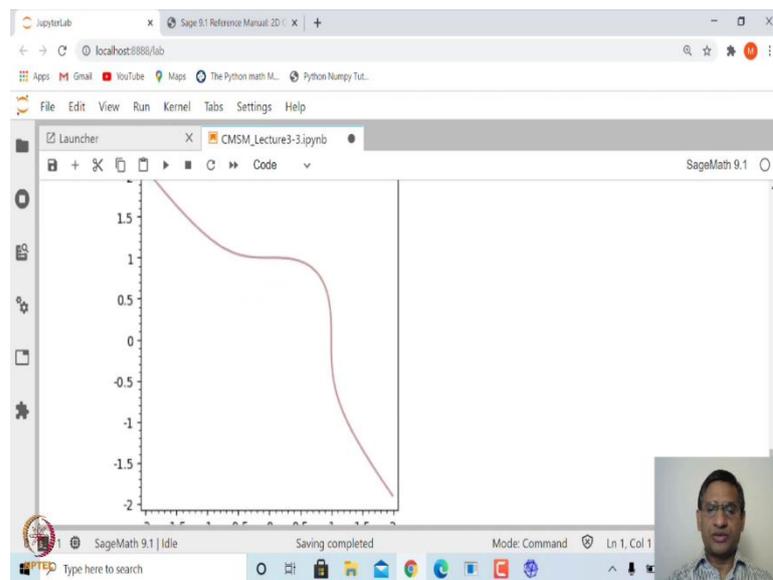


The screenshot shows a JupyterLab window with a SageMath 9.1 kernel. The code cell contains the following Python code:

```
for various values of n.  
[*]: var('x,y')  
n = 3  
f(x,y) = x^n*y^n-1  
implicit_plot(f,(x,-2,2),(y,-2,2),color='rosybrown')  
  
...  
[ ]: colors
```

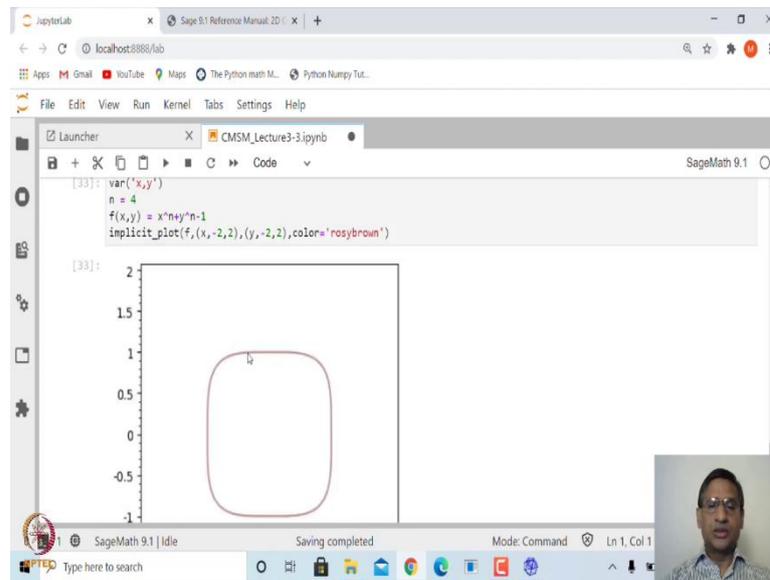
The output area shows the text "Parametric Plot in 2D" and a small video feed of the presenter in the bottom right corner.

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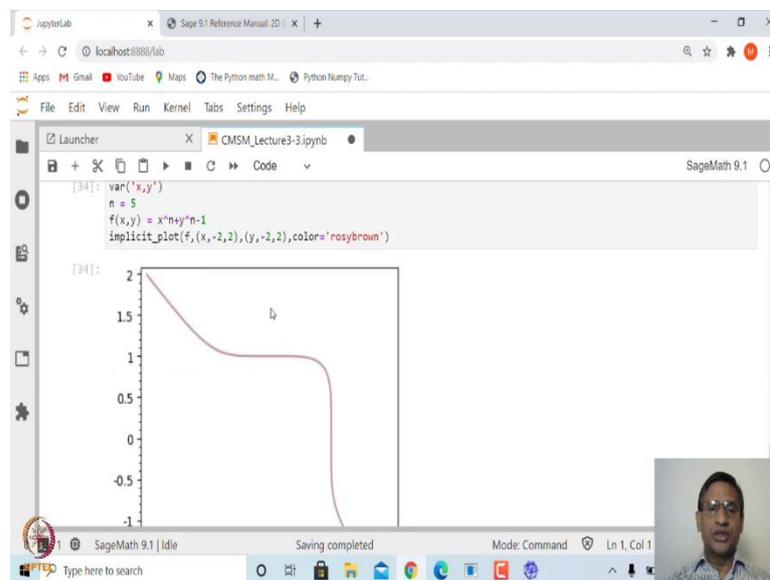
If I take $n = 3$, then let us see, what we are going to get? We will get this kind of curve.

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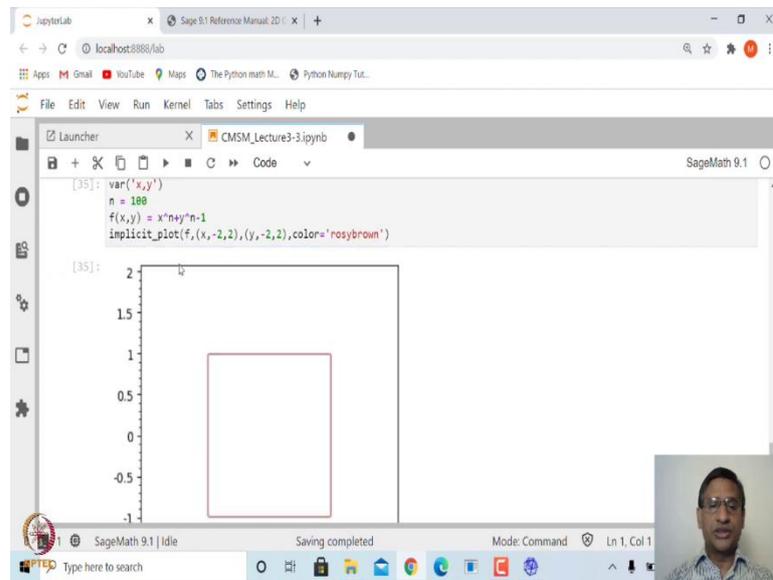
And if I take for example, $n = 4$, then this is not a circle, but it is a square with rounded corners and if I take $n = 5$.

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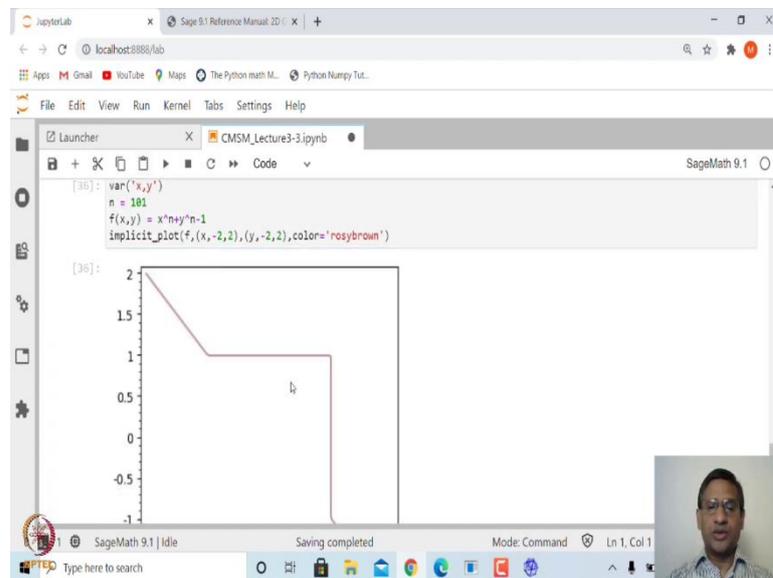
$n = 5$ then again you will see this curve. So, that is how you can plot this implicit curve for various values of n .

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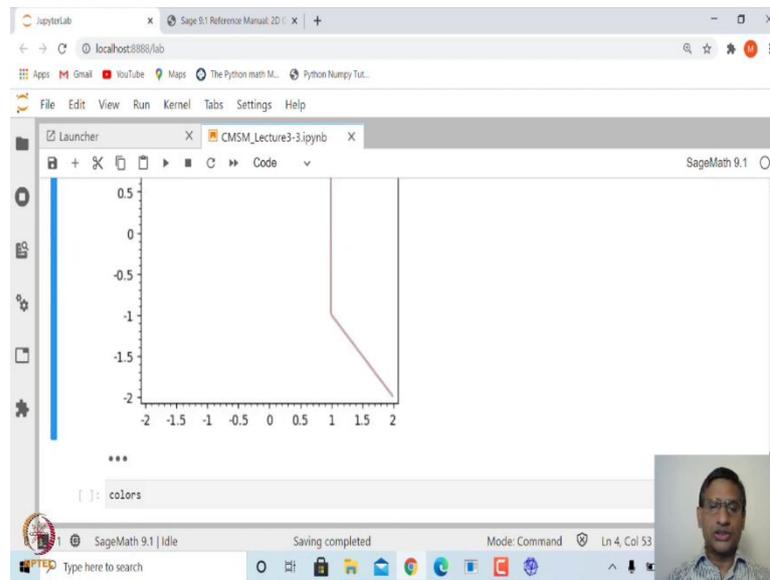


For example, if I take $n = 100$, now in this case it is almost like a square.

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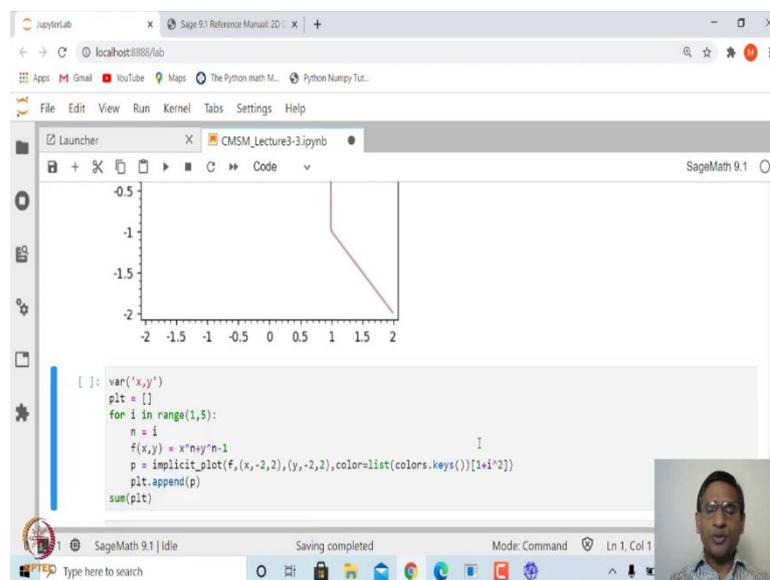


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If I take 101, then it will be again open curve. So, you can explore various values of this and of course, you should try to see why you are getting graph when n is even, you are getting closed curve whereas, n is when n is odd you are not getting a closed curve.

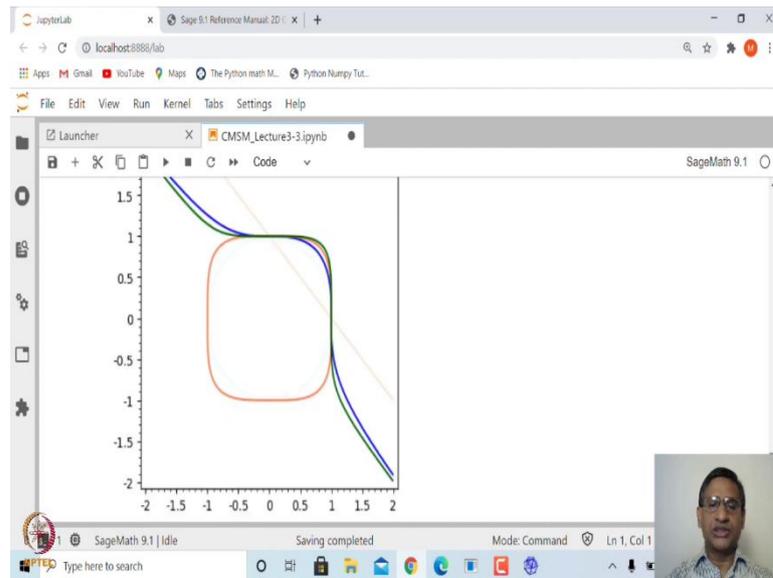
(Refer Slide Time: 19:24)



So, suppose we want to plot for example, graph of this for $n = 1$ to 5 all these together. So, we can use simple for loop. So, we can again define the variable x and y and let me create an empty list and inside this list we will keep on adding this plot for $n = 1, n = 2, n = 3$ up to $n = 4$ or $n = 5$ let us say.

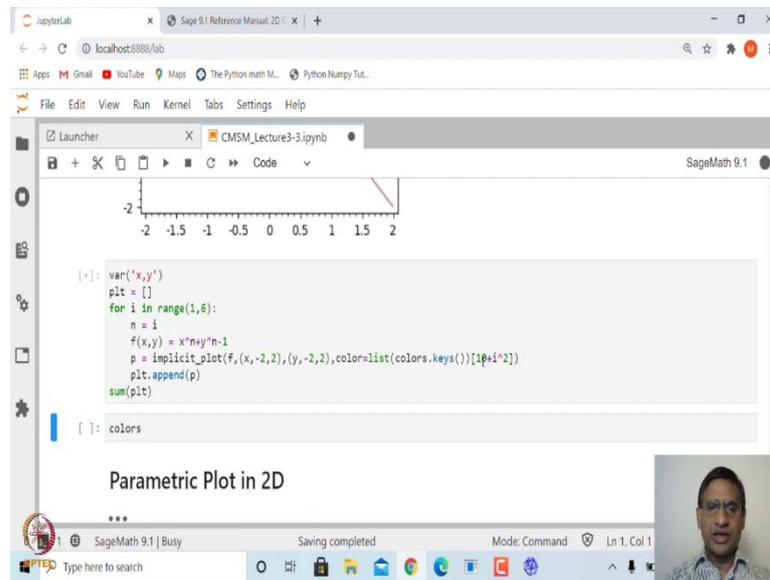
So, let us say we want it up to 5. So, for i in range 1 to 6 let us say $i = n$, $f(x)$ is defined like this and then the same thing which we used earlier inside this we are saying $p =$ implicit plot this. The color is taken from this color list, I will show in a minute what we I am doing and then you append this in the `plt` list and then at the end you sum all of these together then you will see this all these graphs together.

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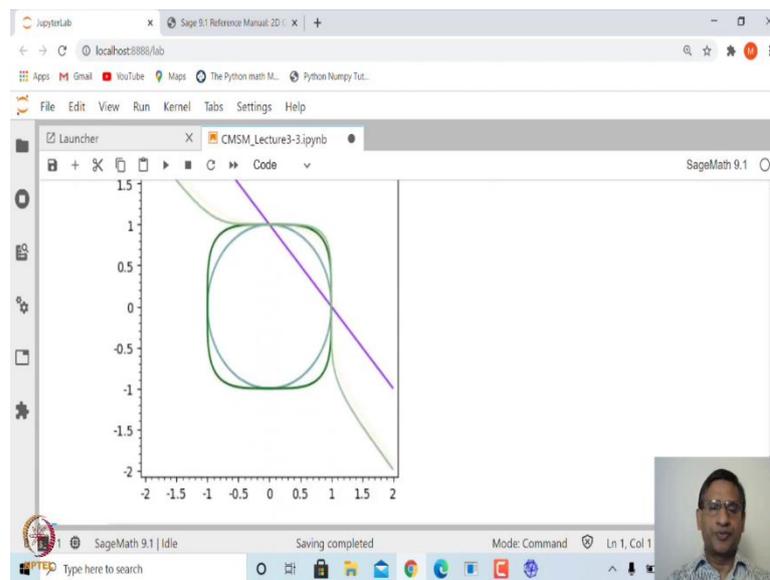


So, you have for $n = 1$ straight line, for $n = 2$ it will be a circle which is in somewhat in light blue color. So, it is not visible let me change this something here is instead of this, I will make it let us say 10 and then see whether it improves yes now you are able to see the circle also.

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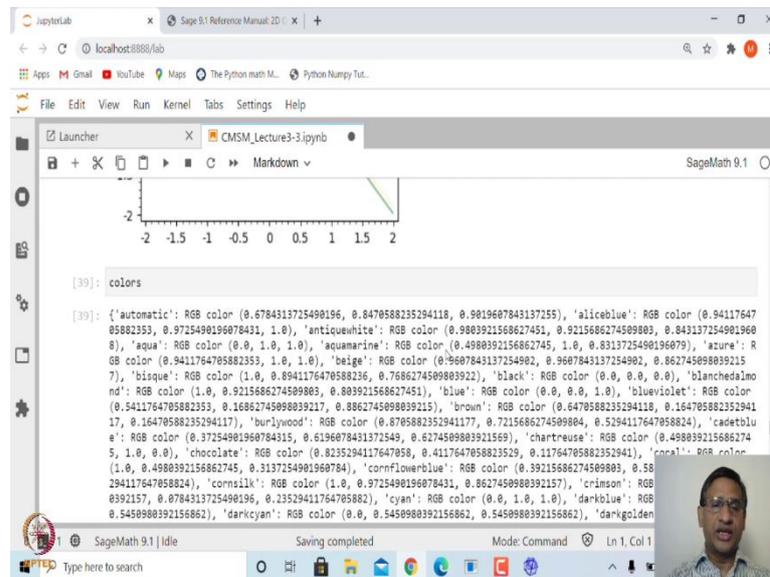


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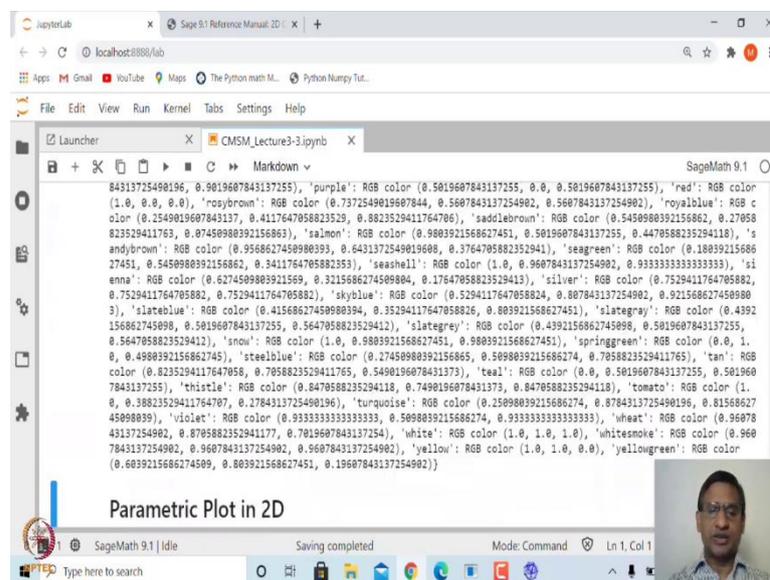
So, if you want to make use of different colors there is an you can look at colors and these colors are defined as a dictionary.

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So, you can see here by default this is the color and then you have various colors like antique white then you have there are so, many colors cornflower blue, dark blue so many of them. Now, there is, I think about hundred colors more than hundred colors are there.

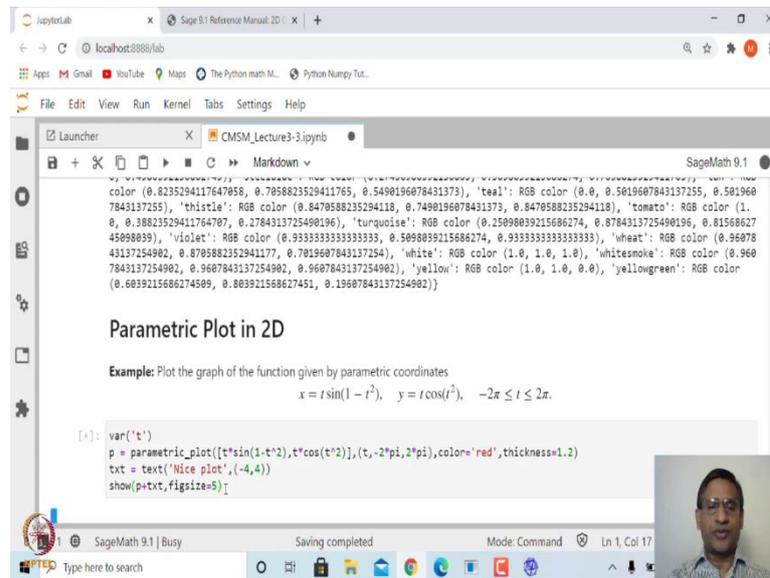
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So, you can go through this list and make use of whatever you want. So, that is what exactly I have done here. So, we took the keys of these dictionary colors and then converted into that keys as a list and then we are using $10 + i^2$. So, $10 + i^2$, for $i = 1$ it is $10 + 1 = 11$.

So, it is we are taking it as 20 first one in sorry 11th one in the list then for i equal to 2 it will be 4. So, 14th one and then next one will be 19th one and things like that. So, that is how you can make use of different colors.

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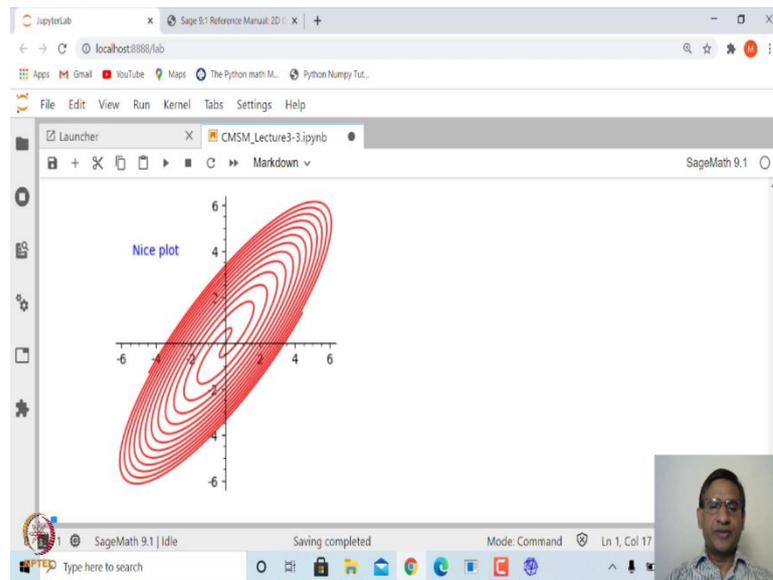


Now, suppose you want to define or you want to plot a function which is defined by parameter. Let us say for example, we want to plot graph of a function which is defined by parameter t and x coordinate is $t \cdot \sin(1 - t^2)$, y coordinate is $t \cdot \cos(t^2)$ and t varies between -2π to 2π . How do we do that?

So, there is an inbuilt function called parametric plot and again we will declare t as a variable. So, inside parametric plot now you have to give each coordinate namely x and y inside square brackets. So, we make a list of x coordinate and y coordinate and then mention what is the domain of t it varies from -2π to 2π color is red and thickness is let us say 1.2. And let me also add a text inside this plot and that I am calling.

So, there is a function called text with the name of the text is let us say 'Nice plot', that this is the 'Nice plot' will be added inside this plot and the coordinate at which this text will appear is $-4, 4$ and then you ask it to `show(p + txt, figsize = 5)`.

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So, let us see what the curve we get. Yeah, that is the very nice curve and this is nice plot is added here. So, that is the annotation you can add a row, you can add line, you can add points all these things can be done very easily and very efficiently.

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Similarly, suppose you want to have a plot graph of function which is defined by polar coordinates. So, here the function $r = 1 + \cos(s \cdot \theta)$ and s can take values let us say 1, it can take value 1.01, 1.02, 1.05, 1.5 etc.

So, for some of these values we will try to plot graph of this function and theta varies between 0 to 200π . So, let us see how we are going to plot? So, in order to plot polar graph there is a function called *polar_plot()*, polar underscore plot. So, here instead of theta I am defining this variable as t if you want you can define this as theta also. Let us say theta and so here wherever you have t, I have to say theta, theta here also theta.

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The screenshot shows a SageMath 9.1 JupyterLab window with the following content:

Polar Plot

Example: Plot the graph of the function given by polar curve
 $r = 1 + 2 \cos(s\theta), \quad 0 \leq \theta \leq 200\pi$
 for various values of $s = 1, 1.01, 1.02, 1.05, 1.5$ etc

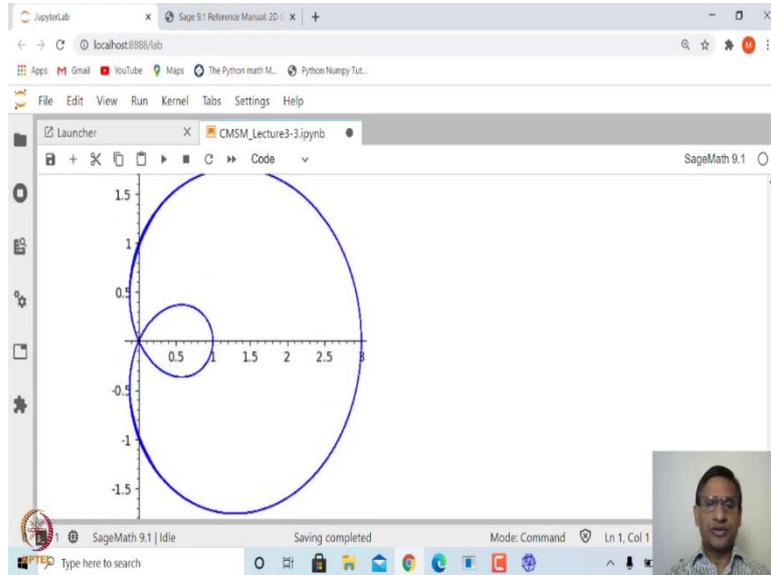
```
[*]: var('theta')
s = 1
polar_plot(1+2*cos(s*theta),(theta,0,200*pi),plot_points=5000,thickness=0.5)
...
...
```

Region Plot

And so, polar plot and then you need to give the right hand side of r which is $1 + \cos$ of 2 times cos yes I am taking as 0.5. So, let me make it s here and I will define s = point. So, to begin with s equal to 1 let us say $s = 1$ and then thickness is 0.2.

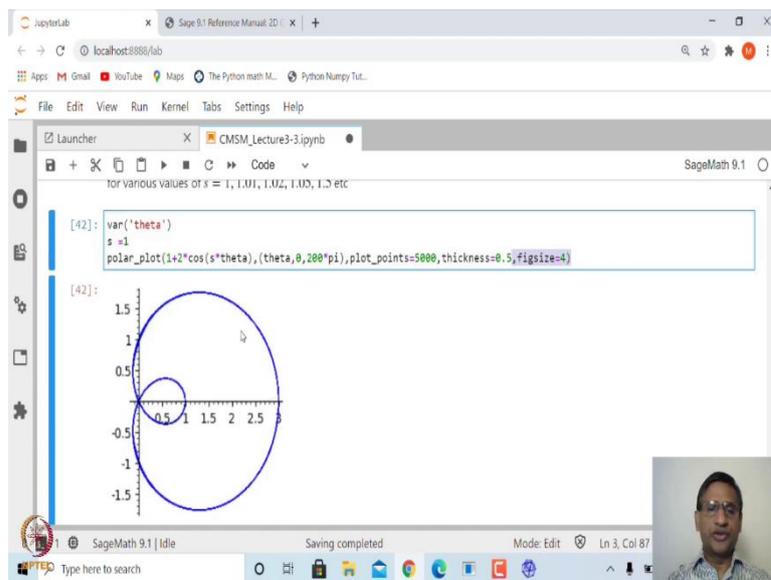
So, make it or let me make it thickness is 0.5 and when you actually when sage plots graph, it actually plots the number of points and then joints. So, here I am saying that in. So, in order to make this plot smooth we are taking about 5,000 points by default it may take about 200 points or 100 points one can find out. So, let us plot the graph.

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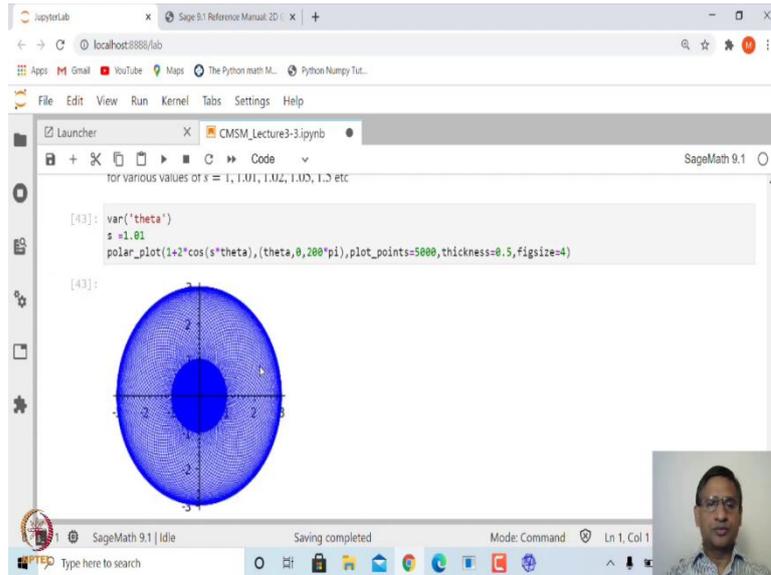
So, this is how the graph of this function looks like let me reduce the figure size.

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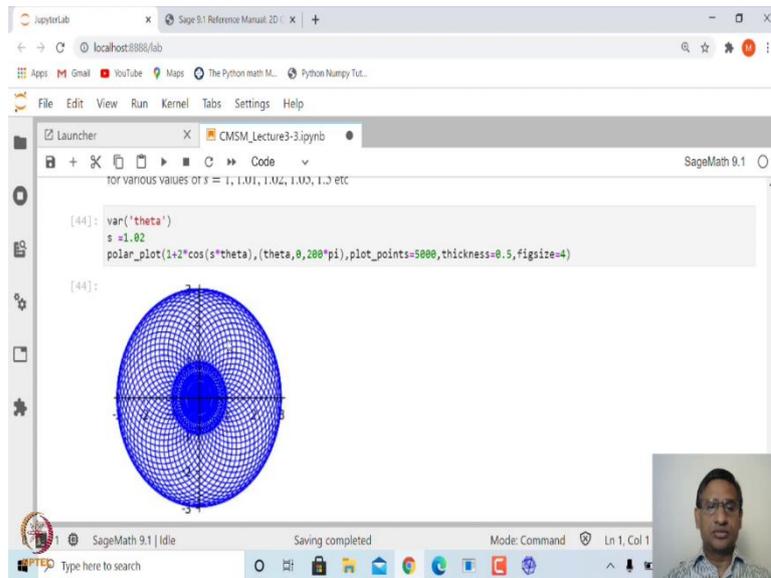
So, $figsize = 4$ now it is much better. So, this is graph of $1 + 2 * \cos(\theta)$ this is the, suppose now s instead of 1.

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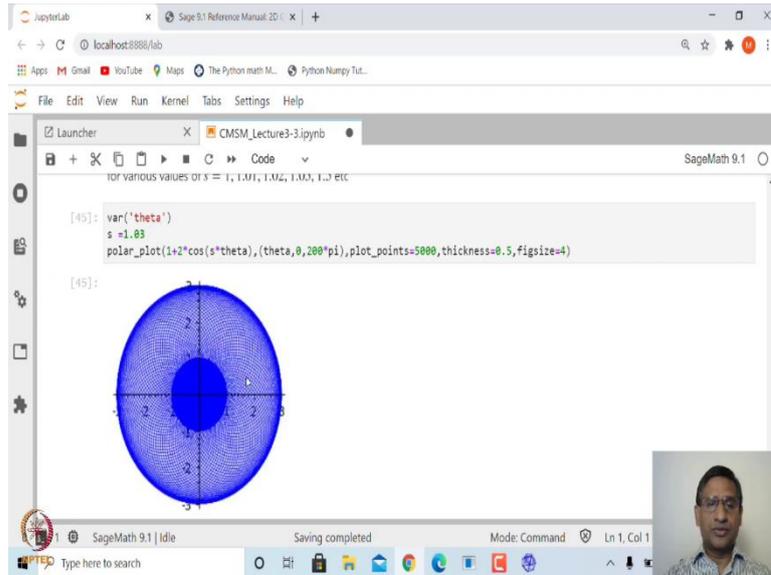
Let us make it 1.01 and then the graph is completely different you can see here what is the change it has done.

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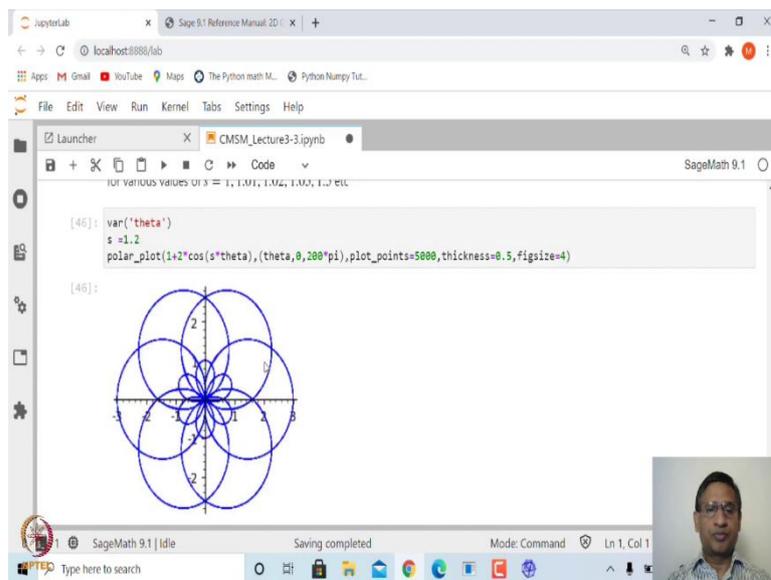
And if I say 1.2 this is how it looks like.

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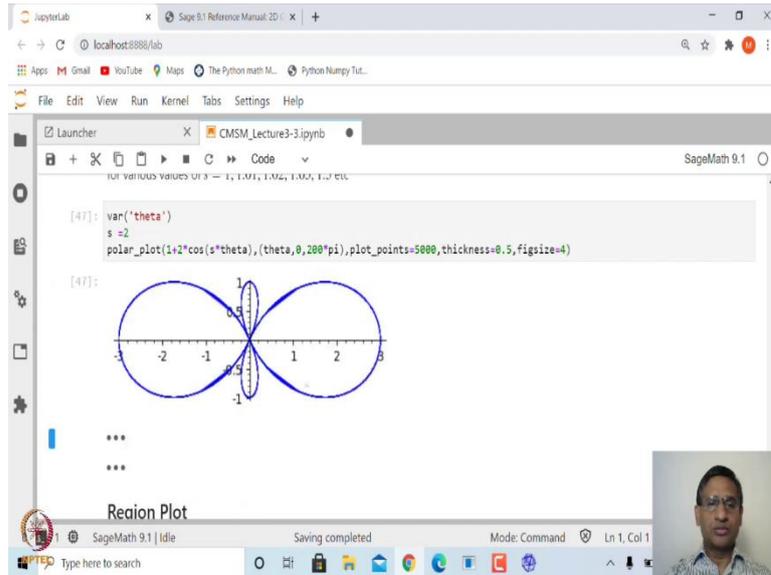
If I say 0.03, this is how it looks like.

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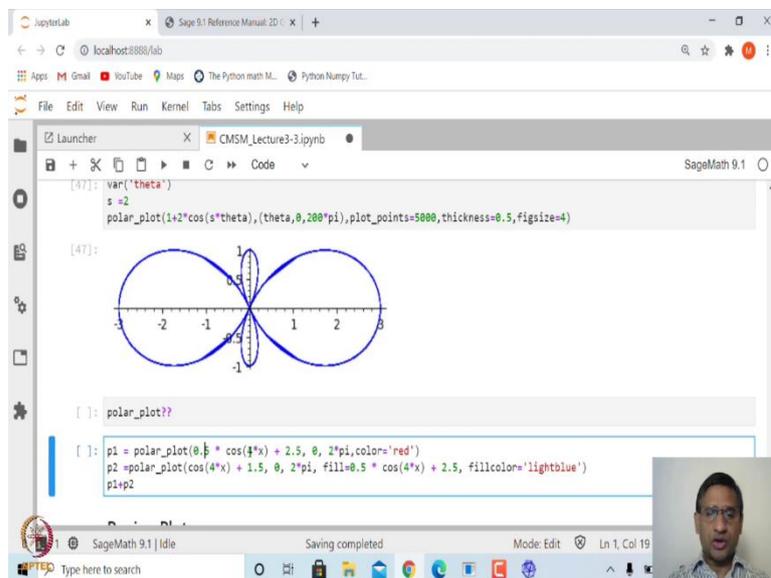
If I say let us say 1.2 that is again completely different.

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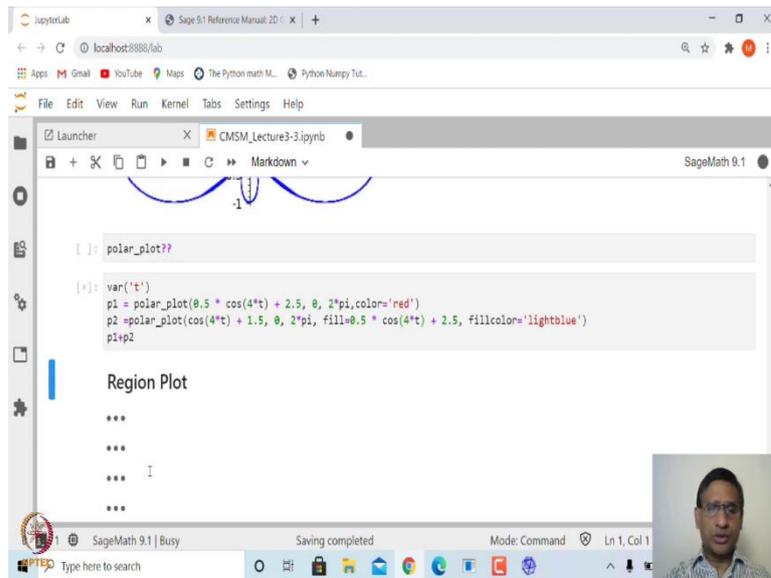
And if I say for example, 2 so that is the graph. So, you can see here for small change in s , the graph of this function looks completely different. So, this is advantage of using this kind of software in order to plot graph with some parameter, by hand you will not be able to kind of visualize so many things.

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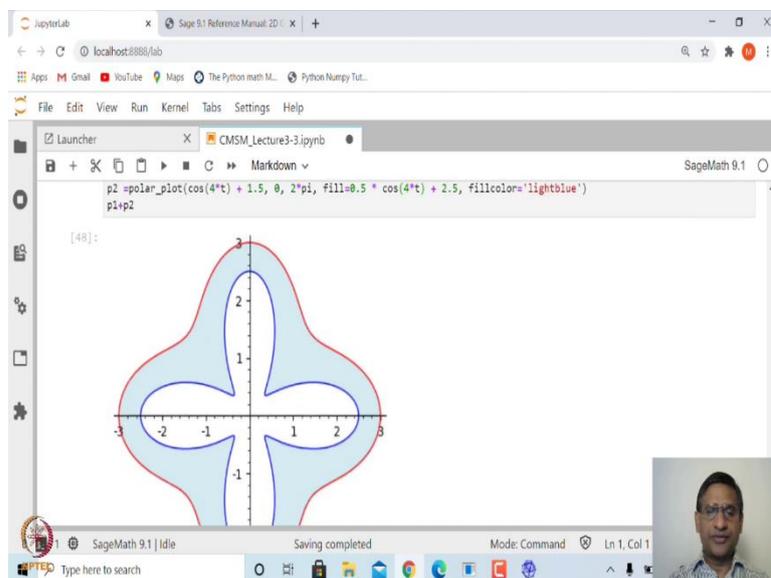
So, you can look at help on `polar_plot` and see what are the other options it supplies. Let us look at one more graph, this graph is graph of again polar function which is defined as $0.5*\cos(4*t)$.

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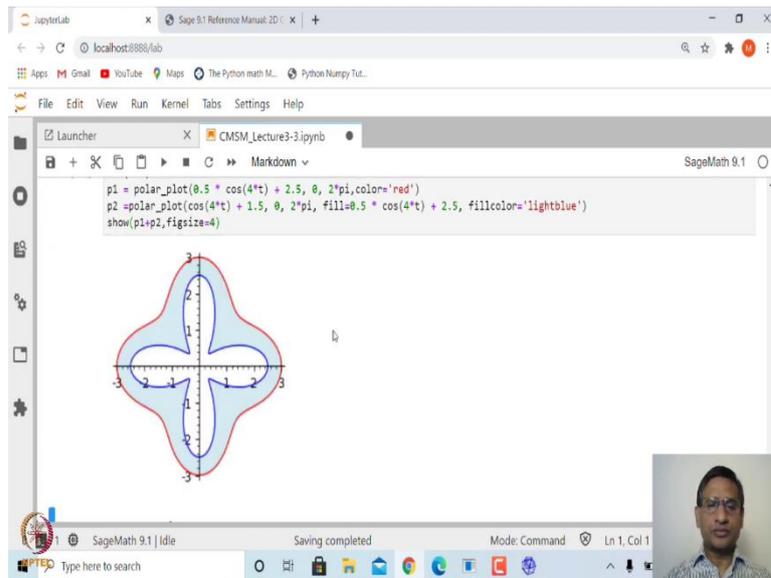
So, let me make it again in terms of variable t , it will be much better. So, it is not confusing here t and then and this also here t . So, this also here t . So, what is this graph? So, let us plot and then we will see what graph does it create.

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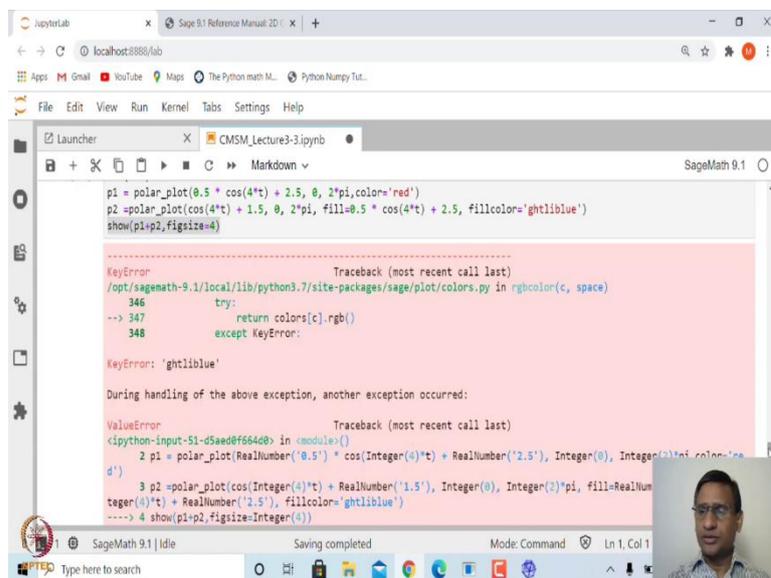
So, this is the graph. So, you have two graphs actually two curves, two polar curves.

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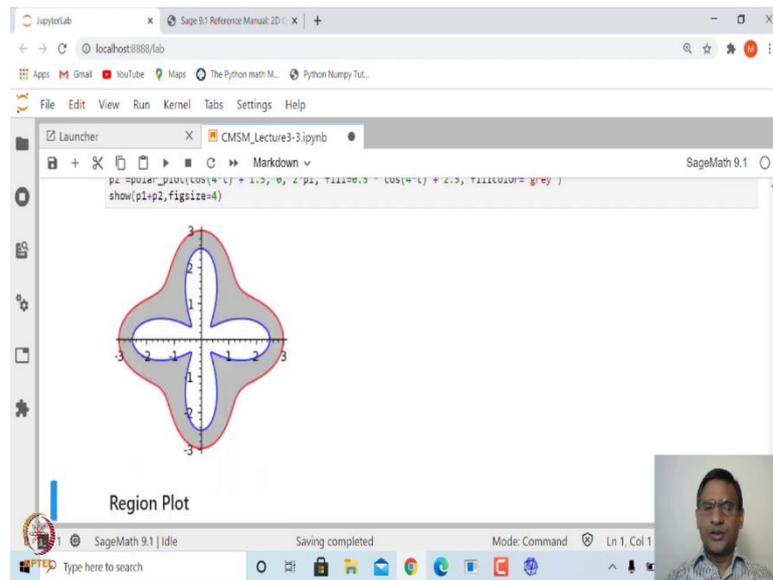
Let me again reduce the figure size. So, show this with $figsize = 4$. So, you have one blue curve and one is red curve and the red one is for example, graph of the polar function $r = 0.5 \cos(4t) + 2.5$ and the blue one which is inside that is actually $\cos(4t) + 1.5$. So, it is actually just shifted and then there is a filling. So, fill it is filled that the two curve in between and the filling color is light blue that is what it means.

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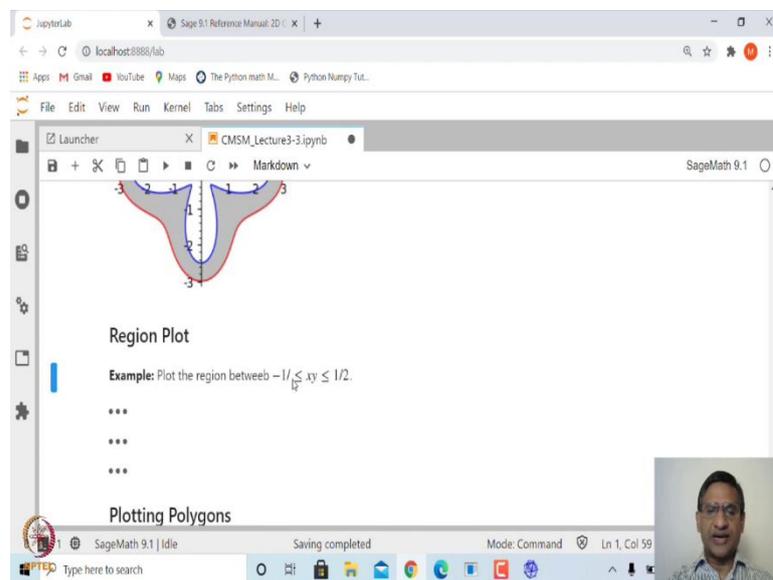
So, when you plot graph or function there is also an option to fill which will allow you to fill the area between two curves. So, when for example, when you want to explain or study let us say area under a curve or area between two curves, you can plot such graphs and using this fill option. So, that is it. Wait, something went wrong.

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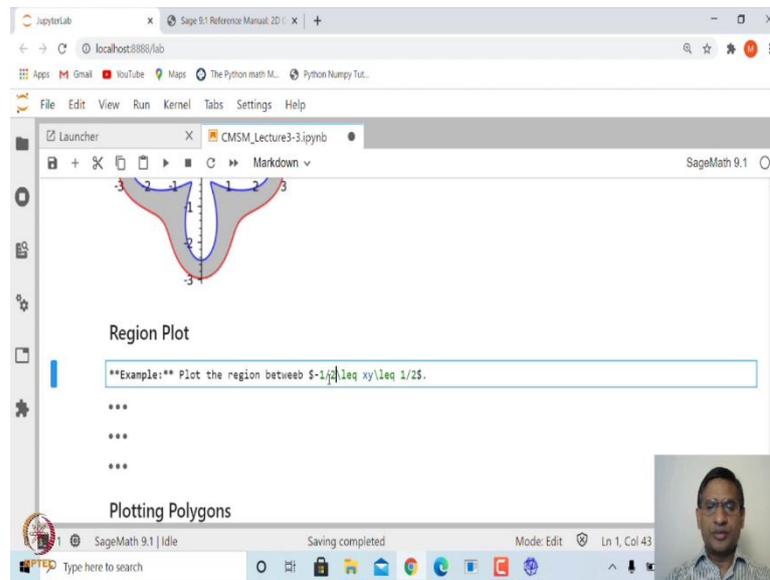
This is I think let me call this as for example, let us say grey. So, this is the next let us look at how we can plot region.

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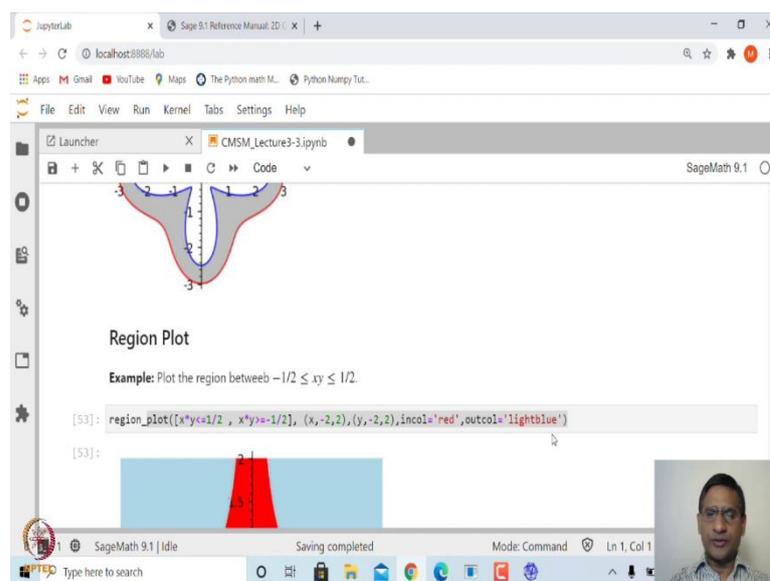


So, if you have to plot let us say region between $x*y$ lying between $-1/2$ and $1/2$.

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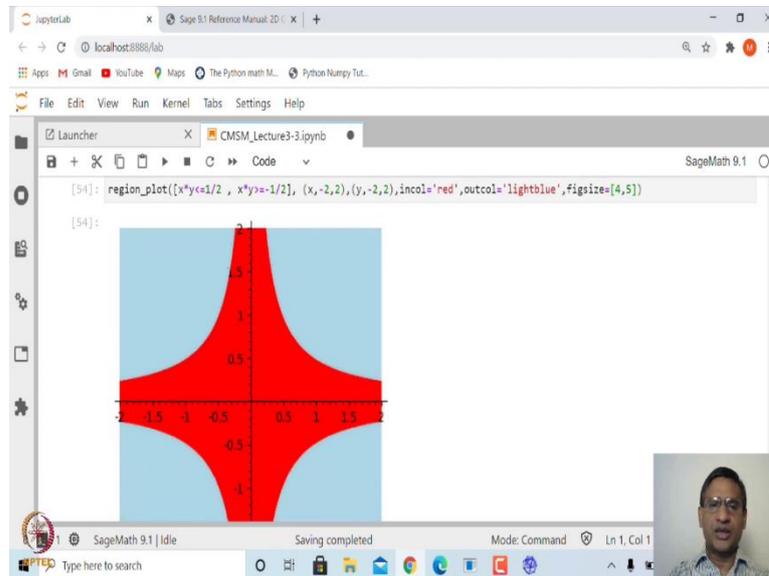


There is a small typo here $x*y$ between $-1/2$ and $+ 1/2$ and let us say take x value between -2 and 2 , y between -2 and 2 . So, how do we plot that? So, there is inbuilt function called `region_underscore plot` and inside this the first two curve, here first curve is $x*y$ is greater than equal to $-1/2$ and then second one is $x*y$ less than equal to $1/2$.

So, that is the first curve $x*y$ less than equal to $1/2$, and other one is $x*y$ greater than equal to $1/2$, x varying between -2 and 2 , y also varying between -2 and 2 and in color and out

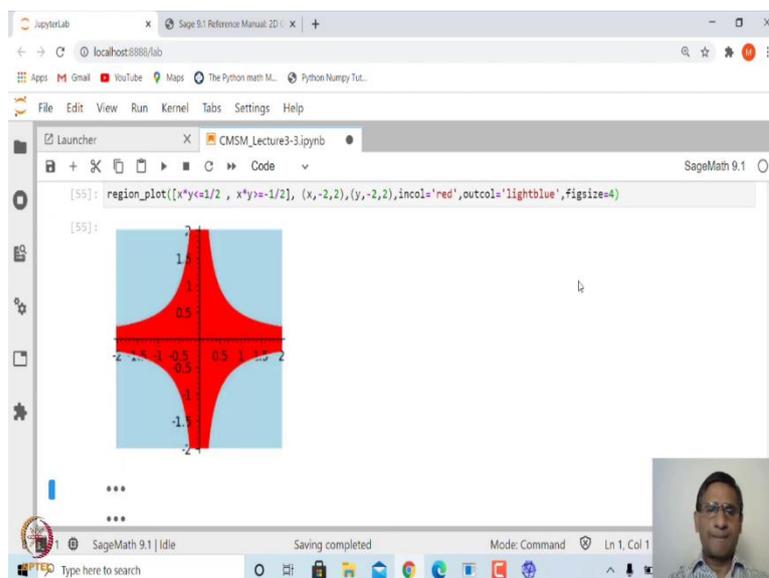
color there are two options here. So, let us plot and then we will see what is the meaning of this.

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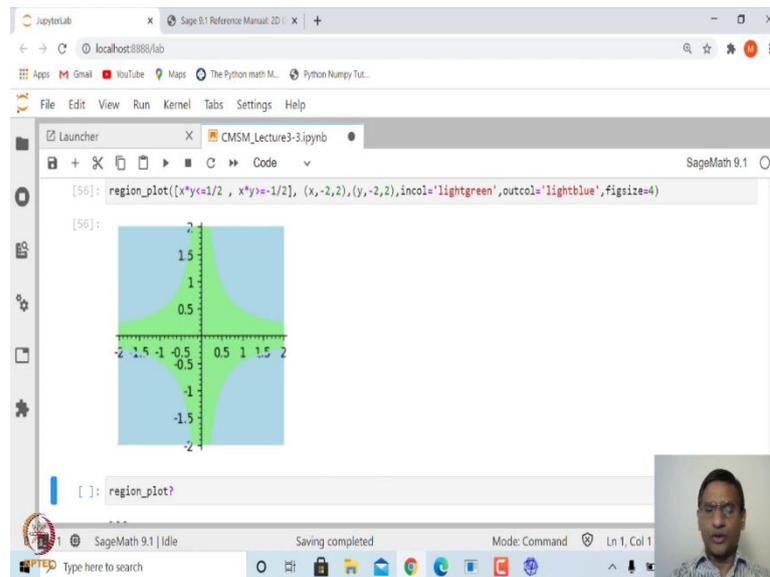
So, when we plot again let me reduce the figure size, $figsize = 4$ if we could mention for example, 4, 5. So, on x scale it will be 4, y scale it will be 5.

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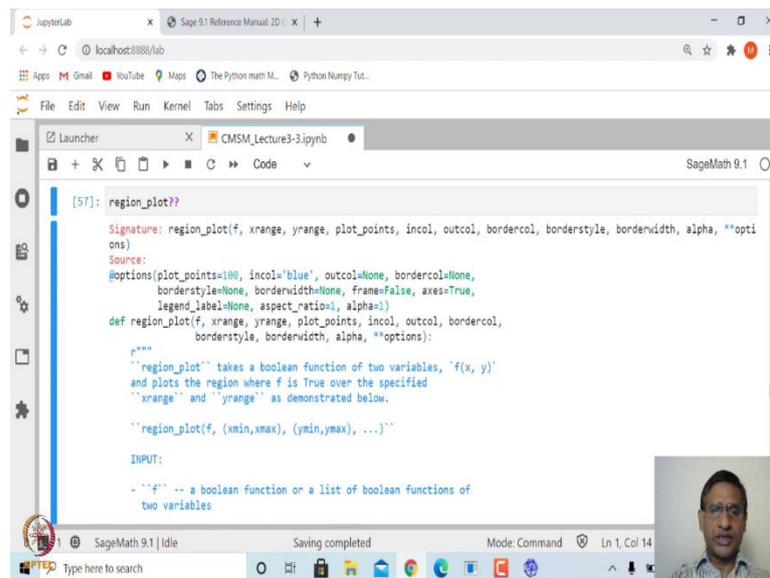
So, this is let me make it 4 itself so that yeah. So, what is it doing? It is inside this it is plotting in red color and outside this in light blue.

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So, I mean I can say light green and then inside this light green outside this light blue that is the meaning of course, you could add the curve also here. So, if you want you can create another one plot for $x*y = 1/2$ and other one for $x*y = -1/2$ and then put it in different color that also you can do.

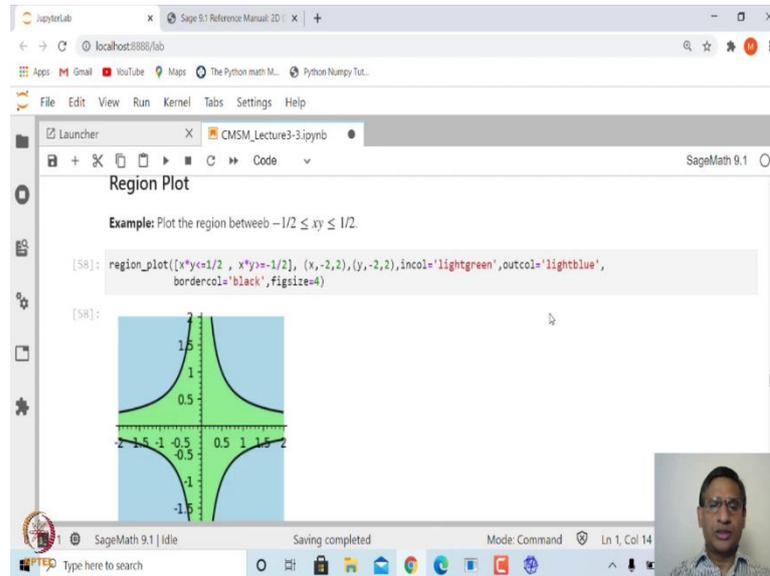
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And you can take help on region plot question mark or double question mark, double question mark will give you detailed help. So, if I say double question mark on region plot then this is what you get. So, you can see here this is the source, this is how the function

is defined and these are the options by default inside color is blue, out color is none and you border color also you can mention border color. So, for example, if I mention here, border color.

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So, if I say here let me put it here border color is equal to let us say black, then you can see the border is now black. And so, there are many other options inside this aspect ratio frame if you want a frame by default frame is false.

If you want to plot this inside a frame you can mention the region color and this is how the function is defined region plot. This is some documentations, this is what you get when you can give question mark.

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```

sage: region_plot([x == 0], (x,-1,1), (y,-1,1))
Graphics object consisting of 1 graphics primitive
sage: region_plot([x^2 + y^2 == 1, x < y], (x,-1,1), (y,-1,1))
Graphics object consisting of 1 graphics primitive
...
from sage.plot.all import Graphics
from sage.plot.misc import setup_for_eval_on_grid
from sage.symbolic.expression import is_Expression
from warnings import warn
import numpy

if not isinstance(f, (list, tuple)):
    f = [f]

feqs = [equify(g) for g in f
        if is_Expression(g) and g.operator() is operator.eq
        and not equify(g).is_zero()]
f = [equify(g) for g in f
     if not (is_Expression(g) and g.operator() is operator.eq)]
neqs = len(feqs)
if neqs > 1:
    warn("There are at least 2 equations;"
         "If the region is degenerated to points.")

```

And at the end it will also tell you how this function is defined. So, that is the, that is the body of this the function.

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```

g.add_primitive(C contourPlot(xy_data_array, xrange, yrange,
                             dict(linestyles=linestyles,
                                  linewidths=linewidths,
                                  contours=[0], cmap=[bordercol],
                                  fill=False, *options)))

return g
File: /opt/sagemath-9.1/local/lib/python3.7/site-packages/sage/misc/decorators.py
Type: function

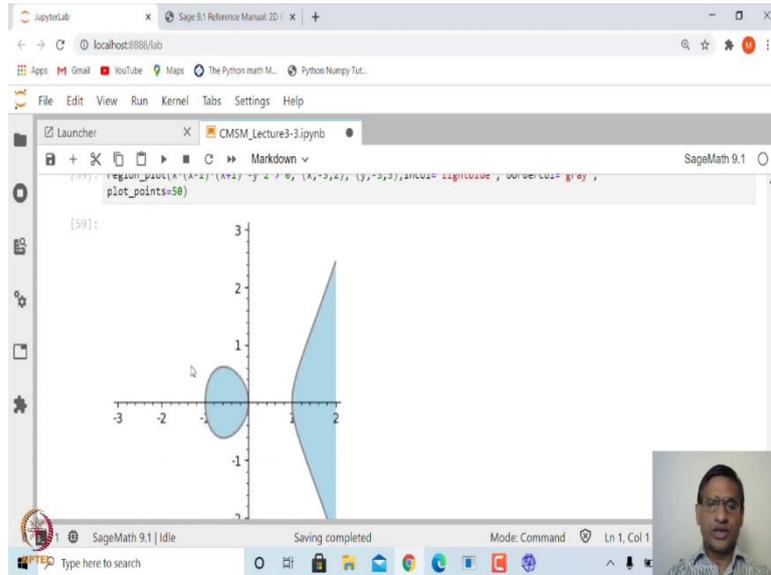
[59]: region_plot(x*(x-1)*(x+1) - y^2 >= 0, (x,-3,2), (y,-3,3), incol='lightblue', bordercol='gray',
plot_points=50)

[59]:

```

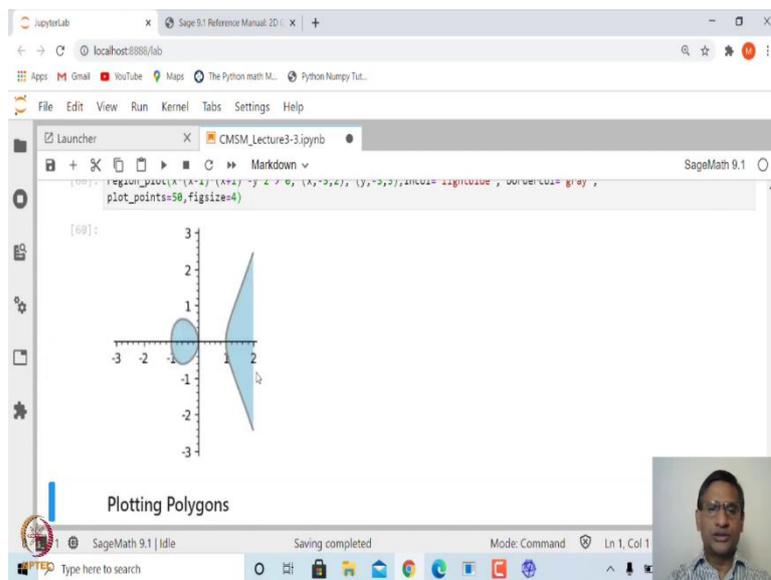
So, correct. Similarly, let me show you plot of another region which is $x * (x - 1) * (x + 1) - y^2$ greater than equal to 0 with x lying between -3 and 2 , y lies between -3 and 3 .

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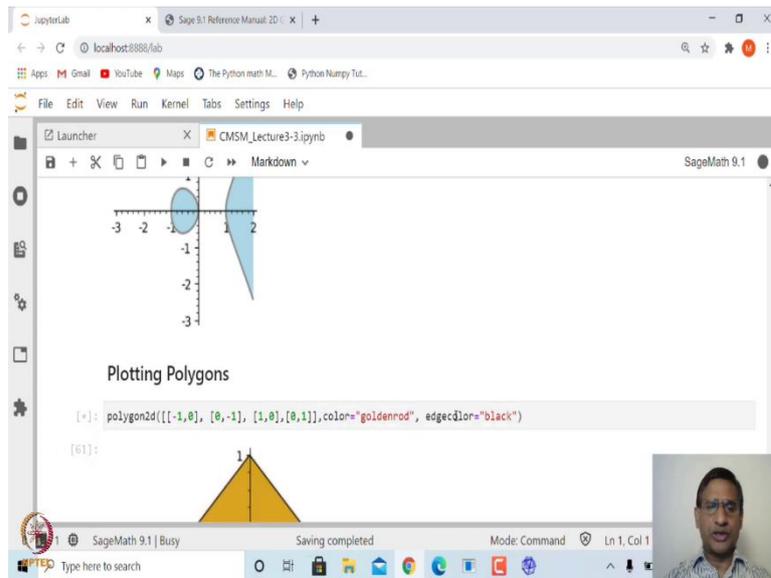
So, that is the region. So, you have here this closed region and then this is outside region.

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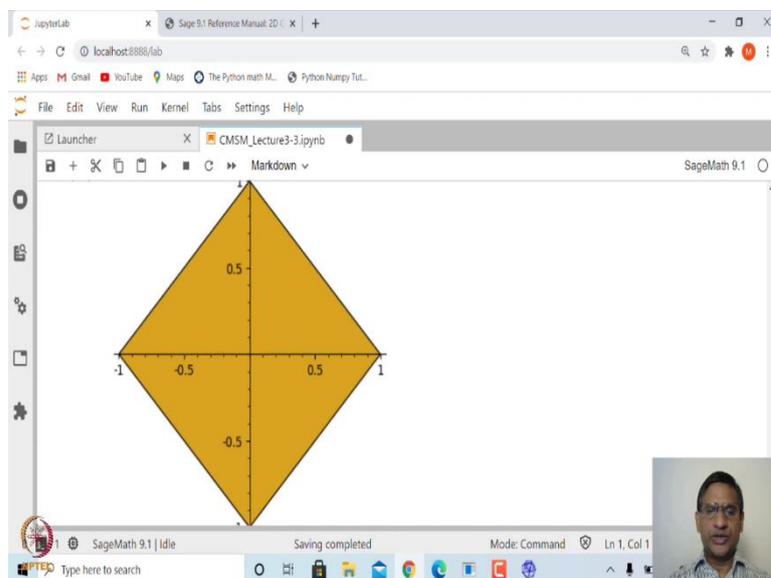
Let me again reduce the figure size, $figsize = 4$ you can see that is the region. Similarly you can also plot graph of a polygon.

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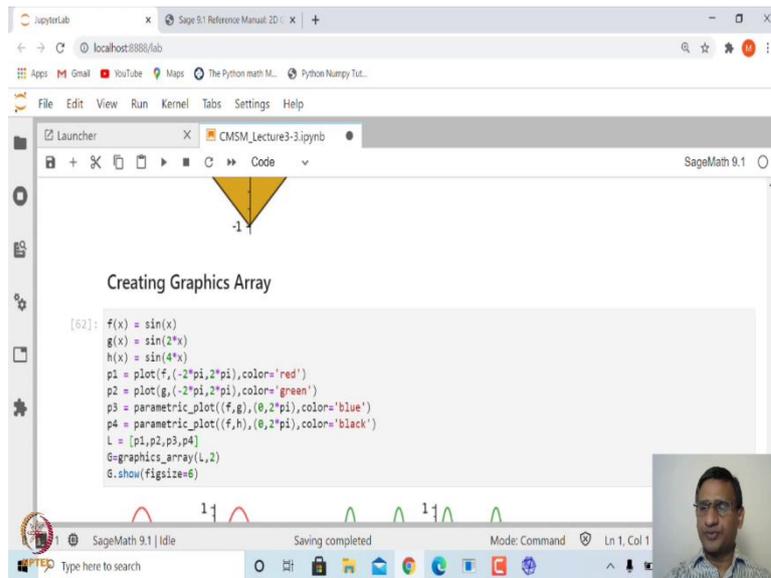
So, suppose I want to plot graph of a polygon as we have all you need to do is mention the corner points. So, here $[-1, 1], [1, -1], [1, 0], [0, 1]$ and the color is in golden red and edge color is black.

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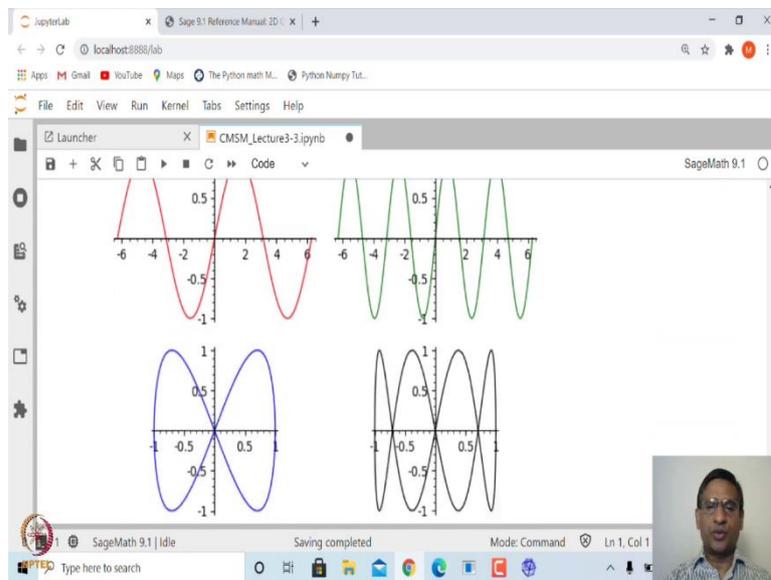
So, it will plot this, this is a trapezium. Similarly, you can plot let us say we saw in case of in case of matplotlib we could plot graph side by side in a matrix form.

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So, here also that option is there. So, suppose let me first run this and then I will explain how it is being done.

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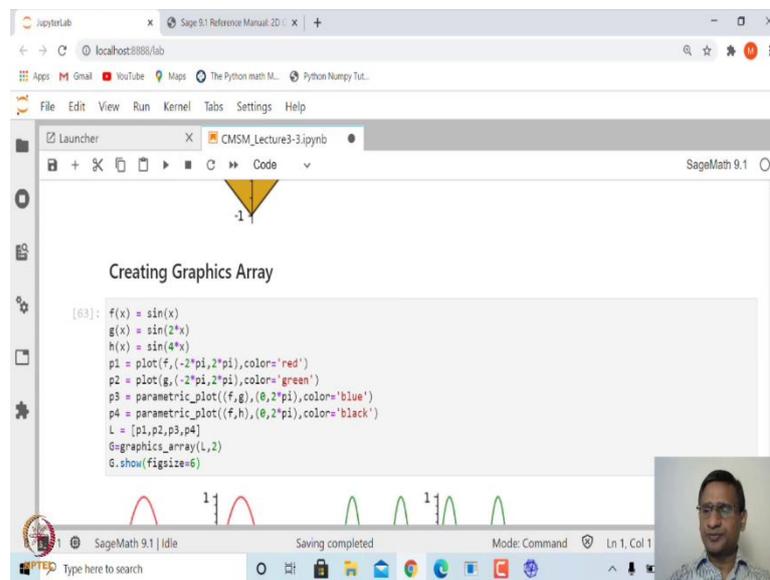
So, you can see here there are four graphs this is 1 graph, 2 graph, 3rd, 4th and how it is done? So, it is done using what is called graphics array what you need to do is you create an array of graphics.

So, here we are taking $f(x) = \sin(x)$, $g(x) = \sin(2x)$ and $h(x) = \sin(4x)$ and the first one p1 is graph of $\sin(x)$, the second one is graph of $\sin(2x)$ and the third one is actually a parametric plot whose first coordinate is $\sin(x)$, second coordinate is $\sin(2x)$ and

the fourth one is graph of again parametric plot whose first coordinate is $\sin(x)$ and second coordinate is $\sin(4x)$.

The color are taken as red, green, blue, black and then you create a list of all these graphics which you have created p1, p2, p3, p4 and then put it inside graphics array graphics array and you need to mention what should be the dimensions. So, here dimension is 2 which is 2 by 2 and then you simply say ask it to show.

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```
[63]: f(x) = sin(x)
g(x) = sin(2*x)
h(x) = sin(4*x)
p1 = plot(f, (-2*pi, 2*pi), color='red')
p2 = plot(g, (-2*pi, 2*pi), color='green')
p3 = parametric_plot((f,g), (0, 2*pi), color='blue')
p4 = parametric_plot((f,h), (0, 2*pi), color='black')
L = [p1, p2, p3, p4]
G = graphics_array(L, 2)
G.show(figsize=6)
```

This gives you the graph of these four functions together the four graphs together first two are explicit function, second two are parametric plot. So, and you can create as many as you want. So, these are some of the standard plots generally we come across in two dimensions, in the next lecture we will look at how to plot graph of a function in three dimensions some of the standard plots we will see. So, I will see you in the next lecture.

Thank you very much.