Real Analysis - I
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Lecture - 3.3
What are the Rationals

In the last module we discussed equivalence classes. Using equivalence classes and partitions we can study what the rational numbers are more precisely. So, this particular module will be intentionally a bit sketchy, but the details are not too hard.

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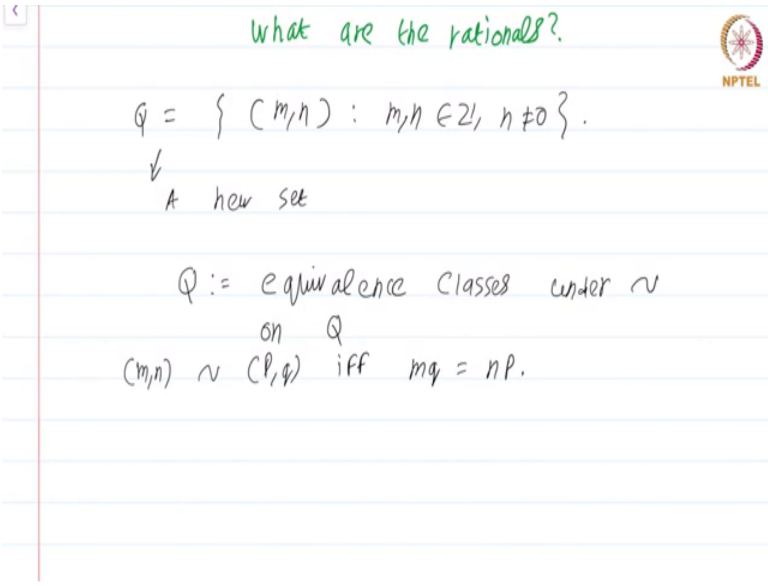

So, we have the set $\mathbb{Q} := \{(m,n) : m,n \in \mathbb{Z},\ n \neq 0\}. Aswesawinthelastmodulethe$ is a difficulty: several different elements in this set are actually identical.

So, what we do is instead we consider a new set and the new set is again by abuse of notation I just denote by $\mathbb{Q}$ again; I just define it to be equivalence classes under $\sim$ on $\mathbb{Q}$.

And what is $\sim$? $(m,n) \sim (p,q)$ iff $mq = np$. So, I have put a equivalence relation on $\mathbb{Q}$ from $\sim$ we saw this equivalence relation in the last module.

Now, I consider the collection of all equivalence classes and I declare a rational number to be an equivalence class. So, what needs to be done now is to equip this collection of equivalence classes with an addition and a multiplication; and how you do it is with what is known as a representative.

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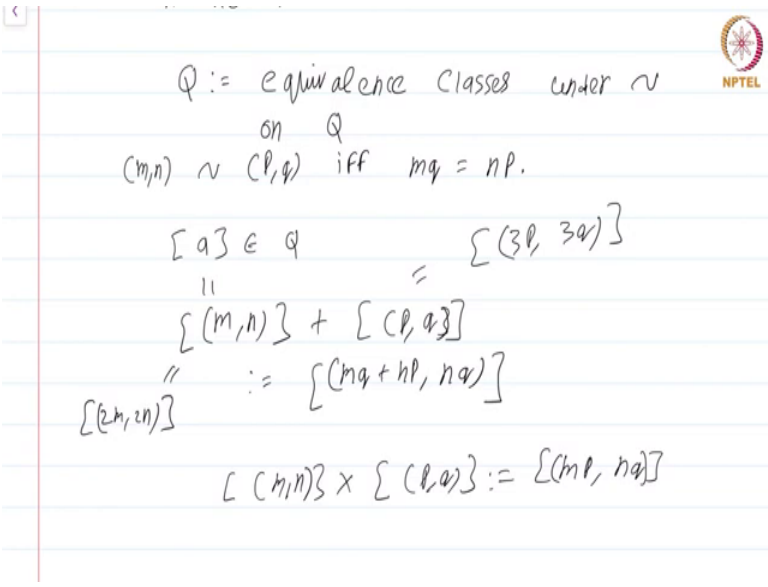

So, given any equivalence class, let us say $[a] \in \mathbb{Q}$ it can be represented as [m,n], where (m,n) is some ordered pair of integers with $n \neq 0$. What I do is if I take two such equivalence classes, I am going to define what addition is; the addition is just defined to be equal to $[(m,n)] + [(p,q)] = [(mq+np, nq)]$.

Now if you are observant, all I have done is taken LCM and done the addition in the rational numbers as we have learnt in middle school. The only issue that arises is that, this equivalence class can be represented by any particular member of that equivalence class; that means, I could have represented this [m,n] as [2m,2n], nothing is stopping me from doing that right.

And I could have represented this [p,q] as [3p,3q] nothing is stopping me from that right. Both are exactly the same equivalence classes. This means that the right hand side that I have written, I have written a particular equivalence class that should not change, if I change the particular choice of representatives for the equivalence class [m,n] and the equivalence class [p,q]if I had chosen a different representative, I should still get the same answer.

Now, that is not really difficult. You all know that it does not really matter which choice how I represent the rational number you get the same answer, but that needs a check, but it is just basic competition. Similarly, I can define $[(m, n)] \times [(p, q)] = [(mp, nq)]$.

This is just the standard multiplication formula in the rationals, again there is a check to be done irrespective of the choice of representatives of both equivalence classes, you must always get the same answer that needs to be checked. And that is an easy check that can be done simply through basic arithmetic.

So, once these checks are done, you can check that all the familiar properties that rationals numbers are supposed to have are indeed processed by our collection of equivalence classes; you have to check things like associativity of addition, commutativity of addition, associativity and commutativity of multiplication so on and so forth.

What you precisely need to check is that, the set $\mathbb{Q}$ as a structure known as a structure of a field, you will see more about fields in next week's modules. So, this is a representation of $\mathbb{Q}$ as a collection of equivalence classes; there are some checks that you need to do they are not really hard, I urge you to do them. This is a course on real analysis and you have just watched the module entitled what are the rational numbers.

Thank you.