Differential Equations for Engineers

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Lecture 46

Uniqueness of solutions for wave equation

In the last video we have seen how to solve wave equation, wave equation in the full domain minus infinity to infinity and X is in minus infinity to infinity that is D'Alembert solution and when the wave equation is given in the semi-infinite domain that is zero to infinity and you have to provide a boundary condition at zero so that you can give in many ways either or you can fix that string so it is a it's a model for string when is when you give initial initially it is a displacement and its slope and you have to provide the boundary at that at one end of the string other end is (infini) infinite string so you don't have to give any condition but at one end either you can fix it or you can allow it to be free.

Free means its slope is zero, that is U X is 0 if it is fixed means displacement is zero so that is U is 0, so U equal to 0 is also we have we solved the equation we solve the boundary value problem initial boundary value problem for the wave equation in a semi-infinite domain zero to infinity when the boundary condition is U at U 0 T equal to 0 that means you fix the one end of the string so the displacement is zero.

So how did we do that? so we have say we have actually extended all the functions involved as odd functions to minus infinity-infinity and then we made use of D'Alembert solution finally so that we can automatically so the advantage of this is that boundary condition is automatically satisfy and then we found exactly what is solution ok.

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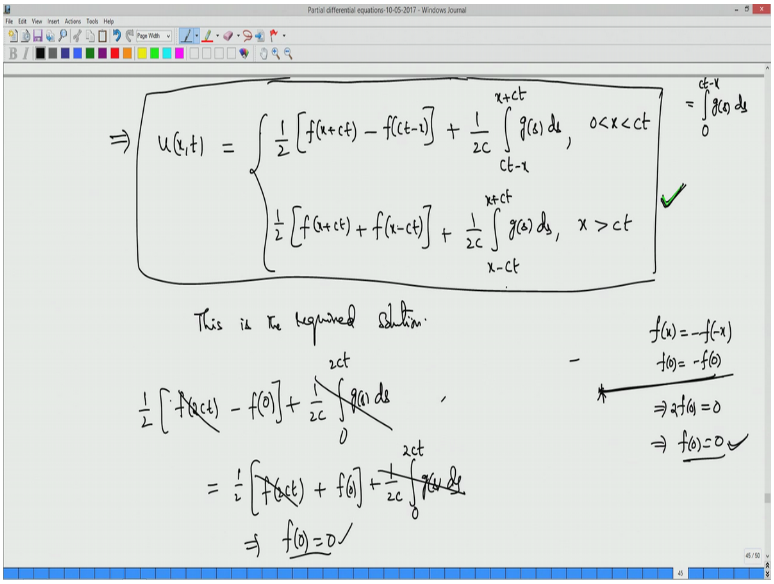

So we have seen that this is the solution U of X T is this so U of X T is this so this is your solution X is between 0 to C T and you have this solution and when X is greater than C T this is your solution and see you extended the functions FX and GX as odd functions that means F of minus X minus F of minus X and you put X equal to zero, so F of zero is minus F of minus 0 is 0.

So implies this F of 0 has to be 0 this is two into F of 0 is 0 that implies F of 0 has to be 0. So this has to be (cont) satisfy this is (also) this you can see if you want this to be a solution at X equal to C T the solution has to be continuous and it is also differentiable ok because it satisfy wave equation which is a second order in both the variables X and T it is twice differentiable function and F has to be twice differentiable X has to be twice differentiable function G has to be continuously differentiable one time differentiability is enough because U double dash is actually you will get G dash.

So this is only G needs to be only one time differentiable and F has to be twice differentiable function. So when you equate U is continuity U at X equal to C T here from the left side and U at X equal to C T from the right side when you equate you will see that F of 0 equal to 0 ok, so you can easily see that the half when you put F equal to X equal to C T F of 2 C T minus F of X equal to C T that is 0 plus 1 by 2 C zero to 2 C T G of S DS. This has to be same right, this has to be equal to that part when X equal to (when you) when you put X equal to C T from the right hand side so if you do that half times F of 2 C T now this is plus F of 0 plus this is same 1 by C T to 0 to 2 C T G of S DS.

So this gets cancel and this also gets cancel so what you are left with is F of 0 is 0. Similarly you can show that G of 0 is also 0 ok, so how do you do that? G of 0 means you have to differentiate not only that.

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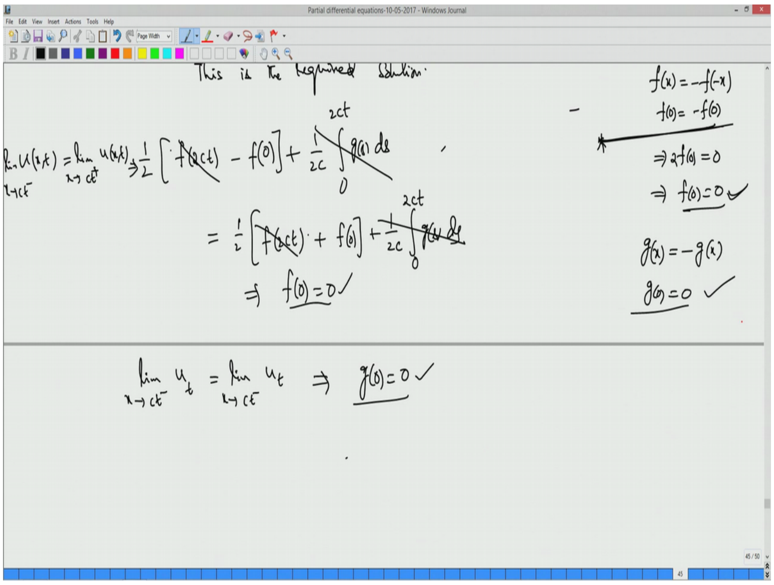

What you did is U at X T rather you should say U at the limits, limit X goes to C T from the left hand side minus U X T, this is equal to limit U X T as X goes to C T from the positive side, so this will give me this this gives me F of 0 is 0.

Now if you equate the limit U dash that is you can take U X or U T anything ok anything will work U T or U X ok U T as X goes to C T minus this is equal to limit U T this has to be continuous C T minus because it is twice differentiable if you want to say that U is the solution it has to be continuous and its derivatives have to be continuous, so this will give me actually you will see you can if you do the calculations you will see that G of 0 is also 0, because G is also you extend it as odd function when you put X equal to 0 you see that G of 0 is also 0 ok.

So that is how you can easy this are automatically satisfied if you want this to be solution this is the solution because U extended, U extended as an odd function that implies automatically G of 0, F of 0 at 0 that is clearly your satisfied because U is the solution that is how you found the solution ok, the same way now if your solution is, so you have seen from zero to infinity.

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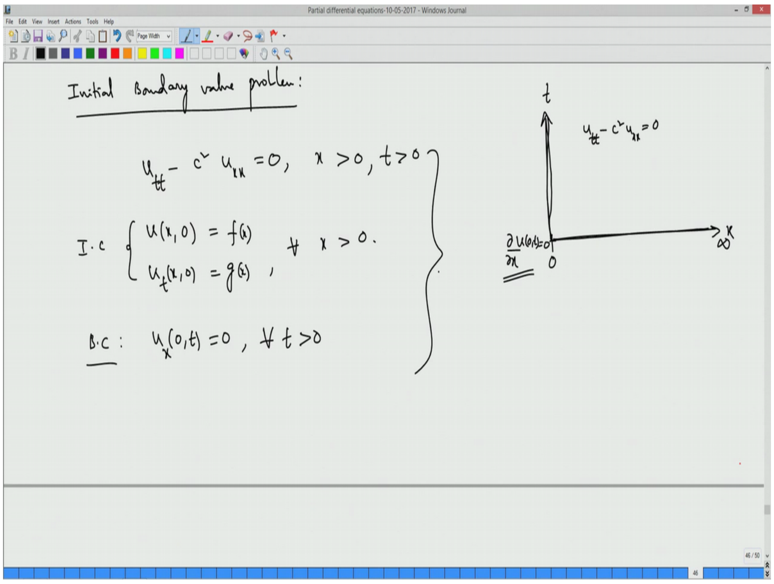

When this is when the string is given you attach now the boundary you allow it to be free that means U X at 0 T is equal to 0 that means dou U by dou X at 0 T is 0.

So if you give this condition this is your T this is your X so you have here U T T minus C square U X X is 0 inside this domain and you have two boundary two initial conditions here at T equal to 0 and here you provide this boundary condition this one. So that if you write so the boundary value problem initial boundary value problem. Now like U T T minus C square U X X equal to 0 X is only positive semi-infinite string T is positive and you have U at X 0 initial condition if F X, U T at X 0 that is G X.

These are your initial conditions and boundary data is U X at 0 T equal to 0 for every T and here for every X positive that is what is given. So such a boundary value problem such a problem we can solve gain by making use of the (())(08:24) by making use of the D'Alembert solution. If you want to use D'Alembert solution you have to extend this to the full real line.

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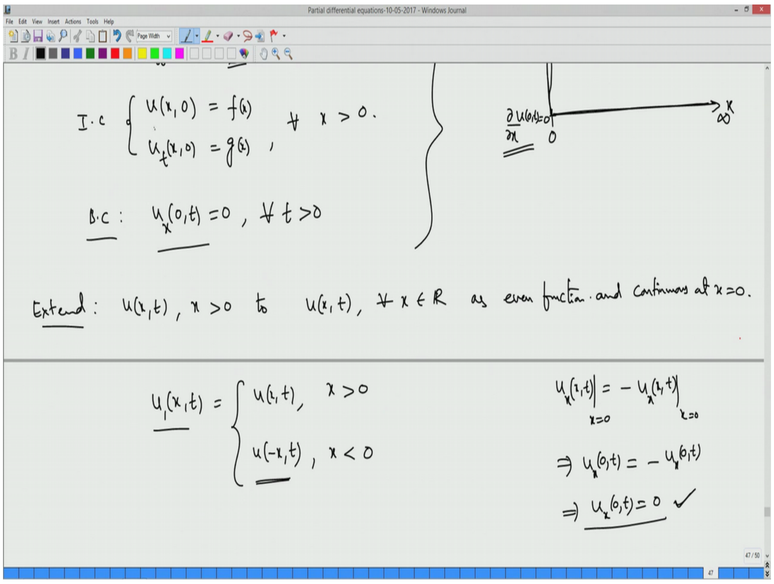

So because of this condition if I extend U of X T extend U of X T for X is positive extend this to U of X T for every X belongs to R in this fashion.

So extend it to, so how do I do that? so you call this extended function as U1 of X T that is U of X T if it is positive. I extend it extended to as even function. If you do as even function so U of minus X T, I know it is minus U for the positive side that is how it is define, X positive I know so if you write U of minus X T, X should be negative so now this is how I extend it knowing U in the positive side I define U1 new function that is for every X. If you do this you clearly say if you want this to be continuous function ok. So you want this to be a solution so if this is the solution of the equation it has to be twice differentiable and is continuous.

If you use continuity you can say that you differentiate U X of X T equal to minus U X of X T is what is the differentiability at X equal to 0 it has to be continuous that means U X at 0 T should be same as minus U X at 0 T that means U X at 0 T has to be 0. So if you extend as an even function and continuous at and continuous at X equal to 0 as this way in this way if you define you see that the boundary condition is automatically satisfied ok and clearly U of X T is satisfying the wave equation U of minus X T is also satisfy because you are twice differentiating minus two times minus sign will be plus so it doesn't matter so will this will also satisfy the wave equation and also initial condition.

It will satisfy same initial conditions ok at X equal to 0, at T equal to 0 it satisfy the same initial conditions.

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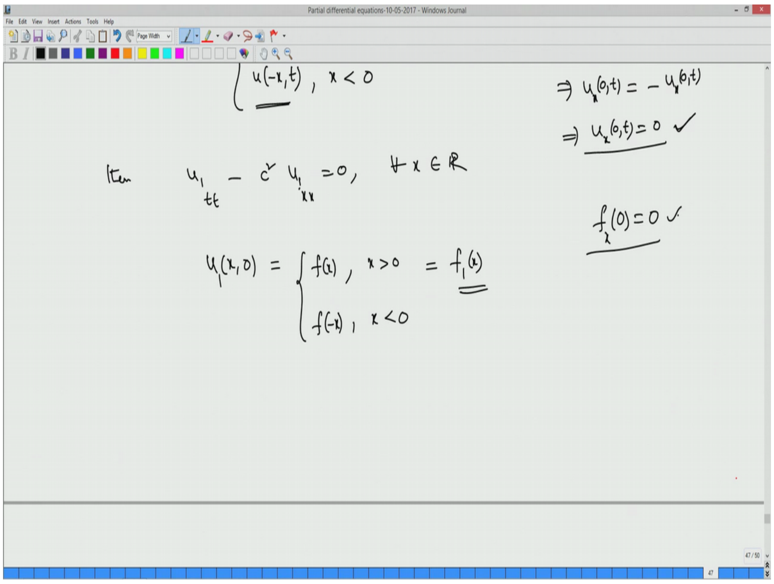

And then U satisfies U1 T T minus C square U1 X X equal to 0 for every (())(11:31) ok. Next this is satisfies the wave equation then initial conditions are also satisfied, so you can see that U1 at X0 is same as, what happens to U1 at X0? So you can easily see that this is U at X0 that is F X ok here also F of minus X ok.

When X is positive, X positive this, X negative is this so tis you call it F1 of X, so F1 is the function that is extending F X onto a negative side that is an even function ok so that you can see that F X if you extend in as an even function F X at 0 is actually 0 automatically ok (dou F) DF by DX at 0 is 0, so F1 is a function that is extended function of F X onto the full real line.

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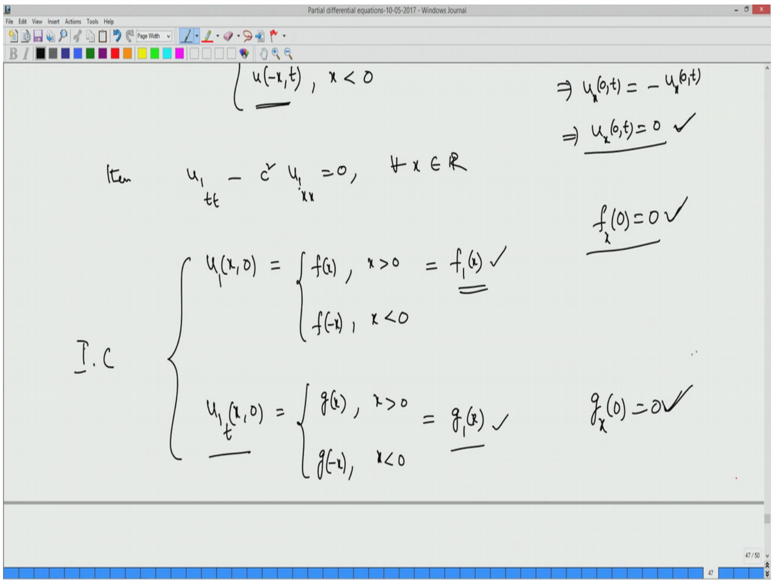

Similarly you have other initial condition U1 T at X0 this will be you can see this one so if you differentiate this with respect to T is here U T at X 0 that is G X again that is when X is positive and here also for negative also this side and x is negative.

Here also you can differentiate with respect to T that is U T minus X 0, U T minus X 0 is U T minus X so U T X 0 is G X, U T minus X 0 should be G of minus X so you have G of minus X here when X is negative so this is your G1 of X, so U T, U 1 T dou U1 by dou T at T equal to 0 is actually G1 function that is an extended function of G onto the full real line.

So you have for U1 or full real line it satisfies in the special domain it satisfies the wave equation and you have the initial data, these are your initial data and the boundary condition is now automatically satisfy ok and also along with that because they are extended as an even functions F at G these are automatically satisfied G X at 0 is also satisfy.

So (you) when you are given a problem for in the semi-infinite domain these F at G initial data should satisfy necessarily these conditions then only in that under those conditions you are actually giving the solution. So now it is direct evolution so this is now you have over full real line you have initial data.

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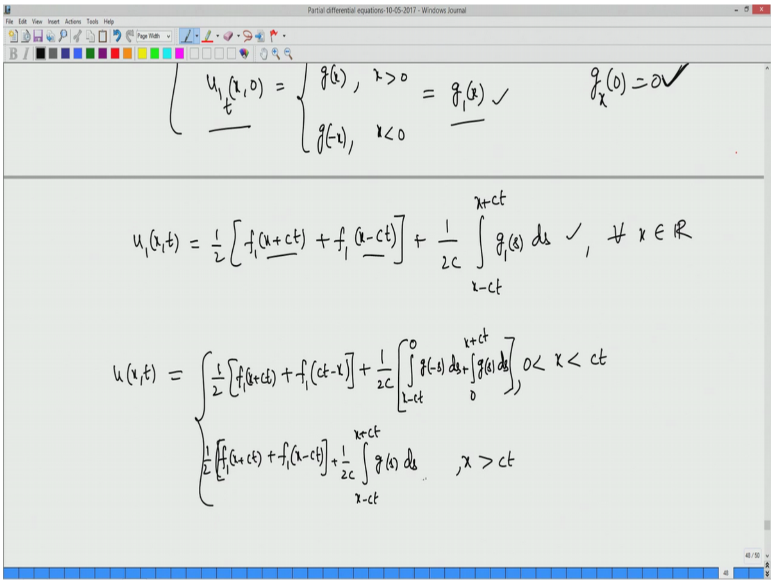

So U1 of X T is simply D'Alembert solution that is F1 of X plus C T plus G1 of X minus C T sorry F1 of X minus C T plus 1 by is 2C integral X knot to X should be X minus C T, X minus C T to X plus C T G1 of S DS, so this is D'Alembert solution.

So what is my U of X T? This is define for every X positive, for every X in real line because it is U1. Now you want only for X positive that is U, U is now you have to see when X is again like earlier when X is between 0 to CT this is always positive F1, here X minus C T is negative. So if it is negative F1 in the negative side that is you have to take F of minus of X. So in this case you have to write this as F1 of X plus C T plus F1 of C T minus X ok plus 1 by 2C now if you do this from 0 to X plus C T G1 is positive , so that is as it is.

Now when G1 is from X minus C T to 0, you split this integral to integral X minus C T to 0 and 0 to X plus C T, X minus C T to 0 G1 is negative so you have to write G1 of so you can write this, you can separate it split it and you write X minus C T to 0, G1 of S is negative, S is negative in this interval so it should be G of minus S DS plus 0 to X plus C T S is positive so it is same G of S DS, because G1 of S is G of S when S is positive.

So this is what you have for X is between 0 to C T, when X is greater than C T so what happens? In this case this is positive and this is also positive ok so F1 is positive when the ordinate is positive that means itself X1 is actually equal to F of itself so you can write directly F1 of X plus C T plus F1 of X minus C T ok plus 1 by 2C X minus C T to X plus C T G of S DS, because S is always positive between X minus C T to X minus C T is positive X plus C T is also positive so S is positive in this interval.

So G1 of S is G of S this is what you have.

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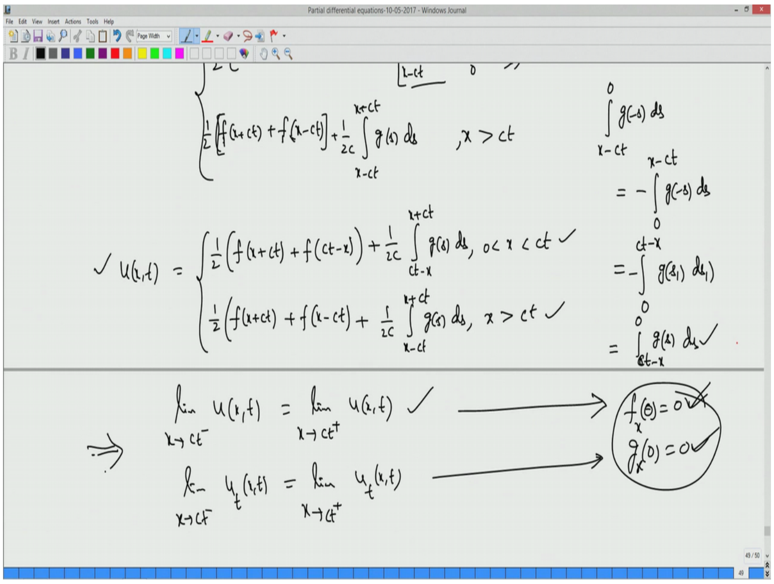

So if you actually rewrite U of X T is so this implies U of X be the solution final solution you can write it as half F X plus C T plus F of C T minus X and then this one plus 1 by 2C first integral this integral you can evaluate as X minus C T to G of minus S DS. Just do the change of our integration so you see that this is minus 0 to X minus C T G of minus S DS and now put G minus S as S as 1so minus DS will be DS1 and this is going to be 0 and minus X (minu) plus C T so that is C T minus X.

So this you have to replace for the first integral, 0 to C T minus X G of S DS, S1 is anyway dummy variable so we can replace with S plus 0 to X plus C T G of S DS this is for X between C T 0 on C T, X greater than C T is nothing to change so you have half F of X plus C T plus F of X minus C T plus 1 by 2C integral X minus C T to X plus C T G of S DS, this is for X greater than C T. Automatically FX at 0 is 0 satisfy GX is also satisfy these two conditions are satisfied if you just equate U both sides U a limit, limit equate this one like earlier U X T is you make this limit U X T now here X goes to C T plus here X goes to C T minus.

By this you will get this one first condition but if you do this in a limits U T of X T these limits derivatives of these are also continuity, continuity of its derivative that means C T plus as X goes to C T minus this will give me this condition so will see that automatically but we don’t have to do because that is how we have seen that ok. The way we have extended as a even function this are automatically satisfied that means you can easily now verify that U T is actually continuous ok. That means these are actually satisfied these is what that is how we extend it, these are all these FX and GX at 0 are 0.

These we already know, implies these you can see that these limits are continuous ok or in opposite way if you want this to be a solution this has to be satisfied that is already satisfied ok that is how we extended the (solu) the functions F function we extended as a even function. So if you are given a semi-infinite wave equation on a semi-infinite line you're initial data F and G should be satisfying these conditions necessary conditions that F derivative, (dou F) D F by D X at 0 and D G by D X at 0 has to be 0. So only for those such problems only for such initial data only we can give the solution like this ok.

This what we have seen so what we have (sown) shown so far is wave equation on a semi-infinite line on a semi-infinite line.

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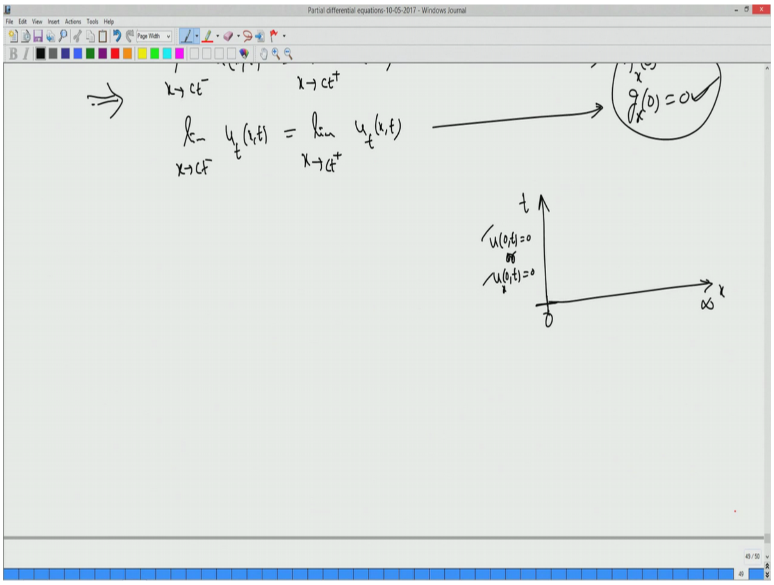

So this is your X and this is your T so zero to infinity is given ok. So on the boundary you provided either U at 0 T is 0 or U X at 0 T equal to 0. So this is what each of these cases we have seen how to find the solution from the D'Alembert solution we derive the solution ok.

Main problem is you know that when you are solving the wave equation you have the general solution that is C1 of X plus C T and C2 of X minus C T. X plus C T is always positive and X is positive and T is positive but X minus C T when X is positive as T goes to T becomes bigger and big X is fixed X minus C T can be negative. So you want to fix that C2, C2 function can be should be define on the negative axis so negative values you should know the you should be able to find that function even on the negative side.

So that is another approach actually we can do that so by directly applying the boundary condition ok by applying the boundary condition you can fix that C2 ok so that approach is actually more general as we see later on. So we have this boundary value problems for the wave equation either in the full domain or in the semi-infinite domain we have provided a solution ok, so we still don’t know whether this solution is unique or not.

So we can actually show in general there is energy argument approach (use) through this we can actually show that this initial boundary value problems solutions are unique that means you take two solutions to assume that you have two different solutions or possible then you take the difference and show that by this energy argument it has to be when you take the difference that is whatever that has to be zero.

That means the both the solutions have to be same so that means the solution is unique using that approach ok, so we will see that how we can show by energy argument how we show that the solution is unique ok for this initial boundary value problems.

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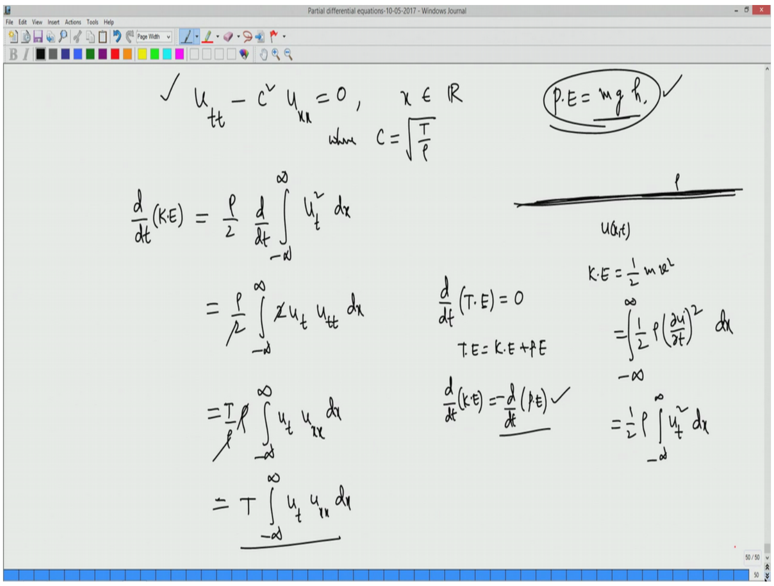

So to start with to do that we take this wave equation so U T T minus C square U X X is 0 ok so you can take first now uhh domain special domain as a full domain this also works for any domain ok.

So this is what you have the wave equation, so what is the kinetic energy? You know that it is it models it governs the governing equation wh`en the spring is attached for an infinite string you want to see the vibration of the spring, spring is governed by this equation so kinetic energy means U is the displacement U of X T is the displacement kinetic energy is half M V square right is what we know. So half assume that you have a string which is density is constant a rho so rho M is rho throughout and V is dou U by dou T this square.

This square and then you integrate this all along the special domain that is your kinetic energy throughout ok so you have a full minus infinity to infinity you have a string so you have a half M is there ok this rho, rho is rho into throughout this integration so you have a rho is constant comes out you have minus infinity to infinity U T T square sorry U T square, U T square is your kinetic condition sorry kinetic energy ok. So energy argument is you want to check whether this kinetic energy whether this conserves that means as a derivative of this whether it if it is zero or not.

You want to see whether this is the only energy ok, is the kinetic energy we know that there is a something called potential energy kinetic energy plus potential energy is constant right that is the total energy that should be conserved. So if you take this time derivative will see what happens. So because I (can) I don't know how to find this potential energy, potential energy what we know from the Newton's lies MGH so H I don't know how to find this exactly ok.

So will see will define this potential energy that is equivalent to this, so how do we do this? Find this derivative so I have 1 by 2 rho by 2 DD T of this minus infinity to infinity U T square DT no DX ok. So you take this derivative inside what you get is rho by 2 minus infinity-infinity 2 U T, U T T DX ok. So what is CC we already know that C is C is square root of T by rho we have seen this one T is the magnitude of the (surf) the tension of this string ok so where C is this.

So this is equal to now I know from the equation you can substitute for U T T so that is rho by 2 minus infinity to infinity so rho by 2 goes 2 2 goes so this 2 2 goes so this is simply rho U T into U X X is C square so rho C square C square is actually C square is T by rho that you can write T by rho into U X X DX, U T into U X X this is what you found this is equal to T times minus infinity-infinity U T into U X X DX. As I know that D D T of total energy is 0 because its conserved total energy is constant so when you take the derivative as for all times it should be same so their derivate should be zero.

So total energy is kinetic energy plus potential energy so this is zero you want so you already know that D D T of K E equal to something ok you got this is, this is what we have so this if we can write this, this as something like D D T minus D D T of PE if you can equate whatever you, so whatever the right hand side here should be minus D D T of PE like this in this fashion if you write so whatever is in this derivative inside ok that is the potential energy you can define which is equivalent to this MGH.

So will see how to do that, so this is they now you can use, you can use the integration by parts here.

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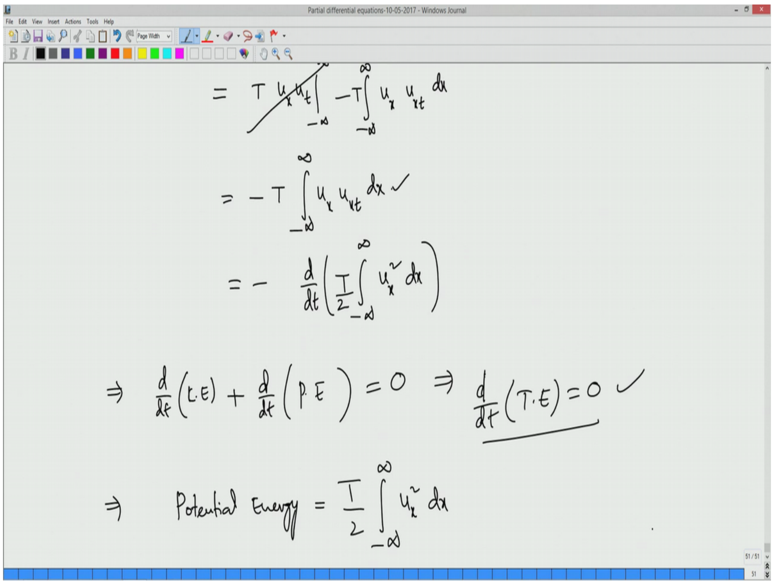

If you do that T U X U T minus infinity to infinity minus integral minus infinity to infinity T is there so T U X and you have to differentiate this U T with respect to X so you have U X T ok DX that is (())(31:13) so at infinity any you have a string you have a string of infinite distance so at infinity they have to there should be zero, the displacement has to be zero subsight right or the slope should be zero or velocity at infinity it extend has to be zero.

So that this will be zero so what you have is this minus T minus infinity-infinity this one how to write as I want to take the derivative out how do I write this derivative? You can simply say what you have is U X U X T DT so this I will write like this D DT of you can say D DT of T integral minus infinity to infinity U X square (D) this is D X by the way not D T this is D X ok, if you write like this is a half here so if you take this derivative inside what you get is 2 U X that 2 2 goes U X U X T DX this is what exactly you have ok.

So this is equivalent to this, now this implies D DT of KE plus now take this D DT of this something which I am defining as potential energy equal to zero. So this is D DT of total energy is zero, so implies I can define my potential energy so you can say potential energy , potential energy of the spring is basically T by 2 minus (inte) infinity to infinity U X square DX ok. So you can get this potential energy like this ok.

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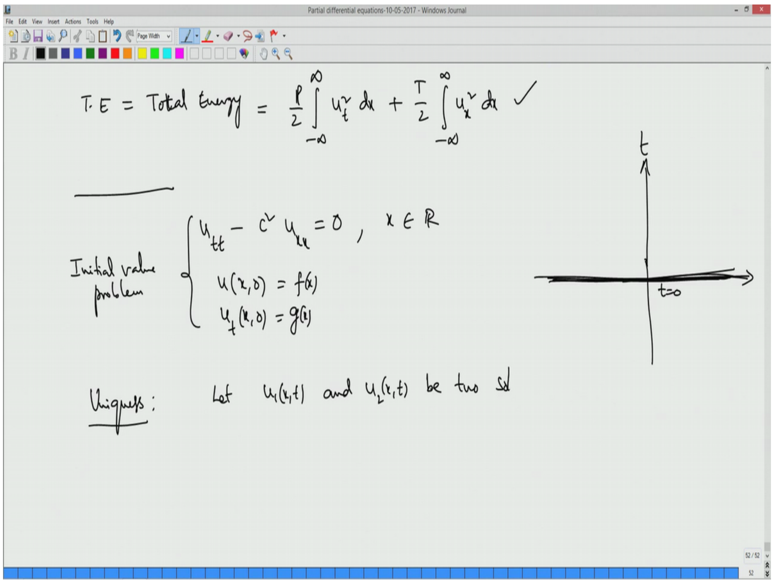

So once you say that then you have a total energy is simply half integral minus infinity-infinity rho is there rho by 2 U T square DX plus this potential energy is T by 2 minus infinity to infinity U X square DX ok. This is what is the total energy you can say total energy so you see that the total energy is zero so you that is how you find ok. If your string of infinite length minus infinity-infinity is satisfying the wave equation and then your total energy of that string is this one which is conserved, whose time derivative is zero ok, this is what we find.

Now what we do is let now you take this the problem so your problem is U T T minus C square U X X equal to zero X belongs to R and U at X0 is FX initial by data this is a you have two initial data that is FX GX this is the problem so you have a initial value problem for the full real line, so you have given this for this problem you have given D'Alembert solution, now you assume so you don't know whether this is that is the only solution ok.

So you have one solution now you have a solution that is D'Alembert solution you want to know whether this is the only solution you have for this problem ok. So initial value problem because there is no boundary here boundary is full this is T ok so T is this now X axis is and this is your T axis ok and this is the only boundary that is where you have initial value data T equal to zero you have provide, so you have for this you want to check whether uniqueness whether you have unique solution or not.

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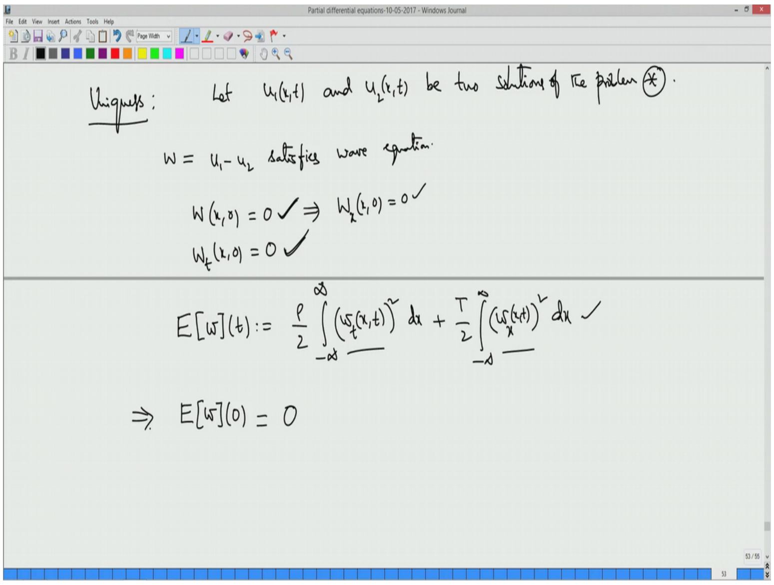

Let U1 of X T and U 2 of X T be two solutions so if you take them as two solutions then they satisfy these one ok so you can see that U1 T T so U1 U2 satisfy wave equation ok and what happens to the initial data U1, U1 satisfies that right, and W let us call this W that is U1 minus U2 also satisfies the wave equation ok. This satisfies wave equation.

The difference also satisfies wave equation clearly because it is a linear equation and because U1 and U2 satisfying this initial data W also satisfies so W at X0 equal to 0, W is U1 at X0 minus U2 at X0, U1 at X0 is FX, U2 at X0 is also FX because U1 U2 are two solutions of this initial value problem, the problem stop and W at X0 is 0 W T at X0 is also 0 because of this.

U1 T X0 is GX U2 T X0 is also GX so the difference ok W T is U and T minus U 2 T at X0 each is GX minus GX that is zero so this is what you (satis) W actually satisfying the wave equation with this zero initial data ok. Now I consider my energy function ok of lets define this E W of T I define for all times what is my energy so from this this my energy so instead of U you write this energy, energy is rho by 2, kinetic energy is minus infinity to infinity W T at X T square ok this square DX plus T by 2 minus infinity to infinity W X of X T whole square DX.

This what is the total energy, this is how I define actually we have right this is how we have seen. Now if you put W at X0 T equal to 0 E of W of 0 ok what if were T equal to zero W T at X0 is 0, 0 W T at X0 is also 0. So this implies W at X0 is 0 means W X at X0 is also 0 ok. So that means this is also zero this is zero this is from the second condition second initial data. So this is actually zero ok.

The way if you define this as (ener) this is your energy function initially the total energy is zero ok.

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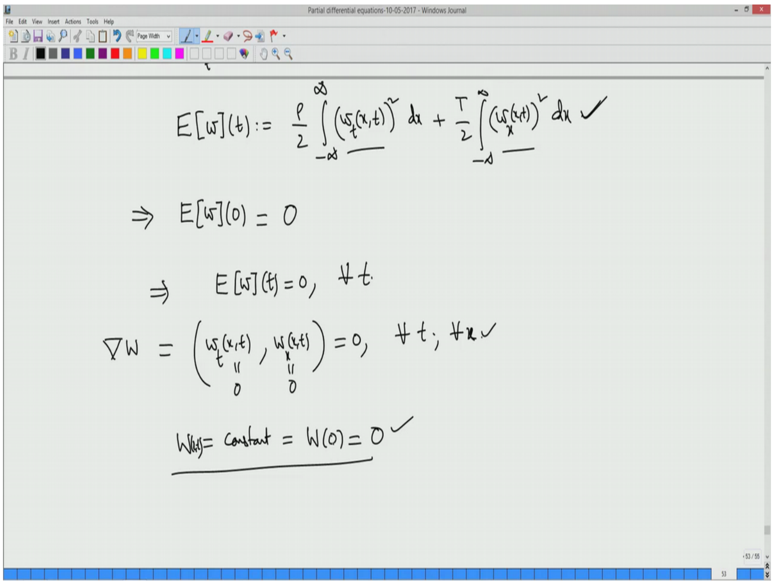

That implies so that implies E of because its energy is conserved E W of T has to be zero for every T that means and you can see what is that E W of T is zero means this is zero this whole thing is zero that means this is only positive integral, integral square function rho is positive T is positive so that means W T at X T and W X at X T so those partial derivatives have to be zero for every T ok, for every T and for every X has to be zero then only positive (something) see integrate is positive..

If this is integral with positive integrant is zero means integral integrant has to be zero ok so that is the idea so this is zero each point has to be zero so this is zero separately and this is zero separately which I am writing as this ok and this is actually gradient of W ok. If this is zero for every X and T, W has to be constant ok that is W at zero and which we already know that it is zero ok. W has to be constant, W, So W is basically W of T ok W of X and T right. so you can say simply W of X and T, X comma T has to be constant ok.

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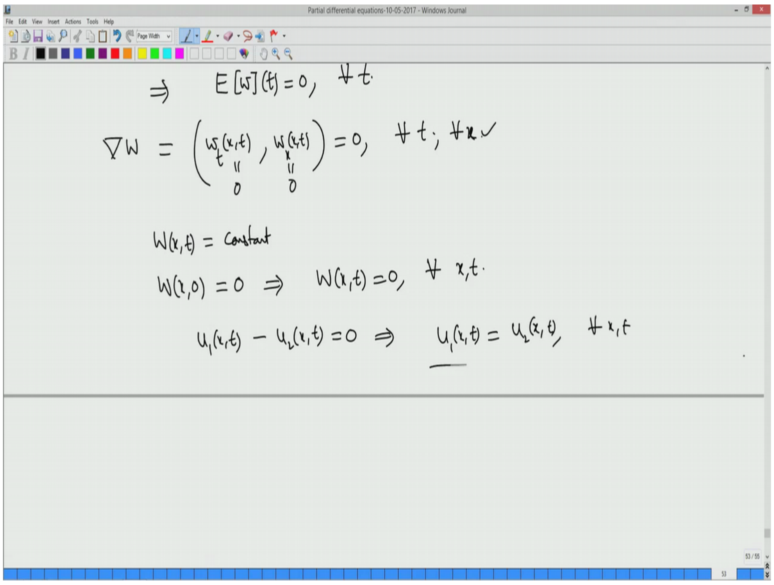

That means W at X0 right let me write this, is constant W at X T is constant but we know the W at X0 is 0, why? This is the initial condition W at X0 is 0 implies W at X T has to be zero foe every X and T. So what is W? U1 of X T minus U2 of X T is equal to 0 that implies U1 of X T should be same as U2 of X T for every X and T, so what does it mean? This is whatever if you assume these are if you assume two solutions of this initial value problem, they have to be same ok. This is how did I do that? this is through this energy argument.

So you know that potential energy is half M V square so that we have seen half M V square is I can easily write that is why I made it ok I could nicely write in terms of this displacement string displacement or string velocity ok string velocity you can write its kinetic energy I cannot make cannot write this potential energy so but instead I indirectly I could I know that the energy is conserved so I can if I can put it in this form I can I didn't wave what is why potential energy. So that is how I identified the my potential energy so implies total energy is here.

So through this total energy function if you assume two solutions two different solutions if you consider then they satisfy the this problem with this initial data ok with this initial data then you define the energy of this W function ok as this instead of U you write in terms of for this problem, for this problem what is the energy? Energy in terms of W T, W X square, so if you do this you know that because of the initial data the energy is zero at initially implies energy is conserved implies for all times say this is zero.

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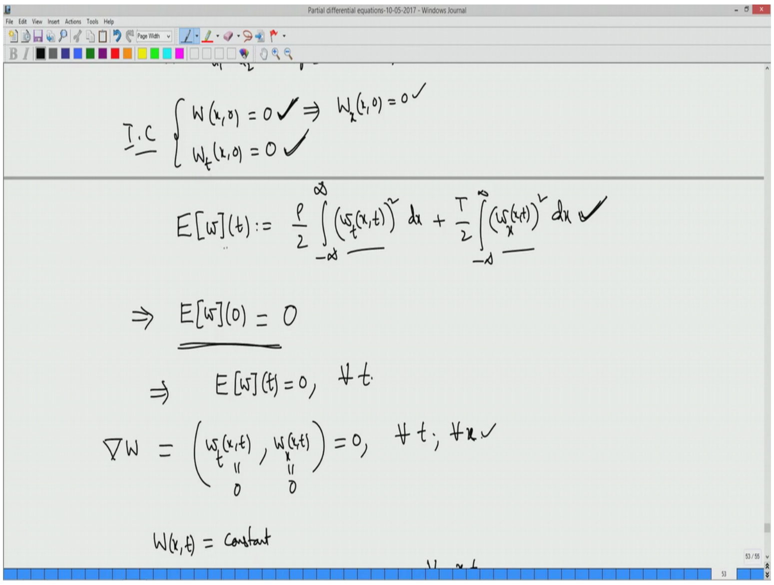

That implies if you write this E W of T is zero that means W T, W X has to be zero that is gradient of W is zero that means W is constant but from the initial data W at X0 is 0 that implies W of X T has to be zero for all times ok. So if this is constant that constant has to be zero so that is how I found this zero. So implies you have found that the solution is unique for the initial value problem.

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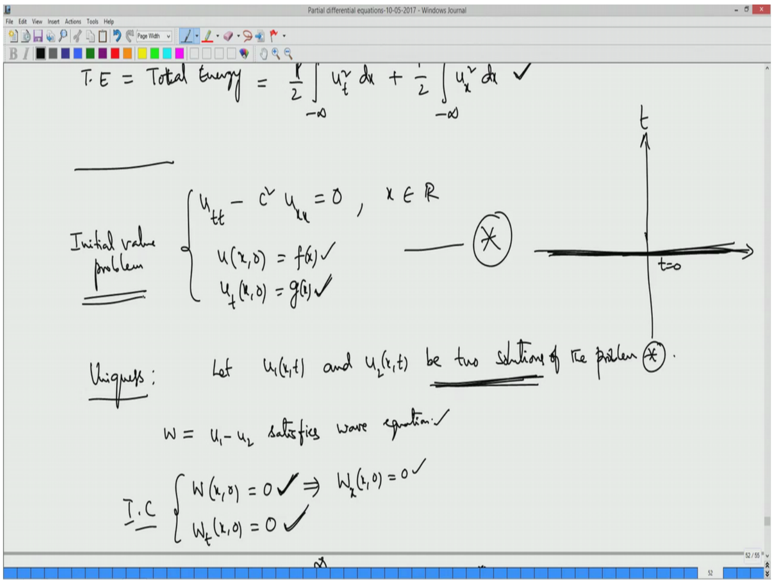

The same proof works even if you take the problem of if you replace this initial value problem with the problem on the semi-infinite domain with the boundary at X equal to zero ok.

Or any X domain with the initial data and the boundary data you can apply this energy argument and show that the solution is unique ok it is not dou that lets not repeat this but you can (repea) you can replace this initial value problem with any other problem.

So that means you have to take the wave equation you change this X domain, if you change this to zero to infinity you have to provide the boundary condition.

So that whole together you form the same argument you can show that (which) then the uniqueness (pro) say U1 and U2 satisfying that problem and you define these energy argument energy and you show that finally a W is actually zero identically for all X and T that shows that U1 and U2 you don't have two different solutions so that implies D'Alembert solution is the solution of the initial value problem of the wave equation over the full real line.

Similarly you can even for semi-infinite string equation that is wave equation on zero to infinity with boundary data whatever you have given so far two boundary data that is use is fixed string is fixed with the zero (initia) zero displacement at X equal to zero or its slope is zero that means it is free the edge is edge of the that means at zero you have tis string is free left free for which these two problems also solution is unique by the same argument you can show that solution is unique that implies you have given solution that should be the only the solution ok provided F at G satisfying the conditions ok.

If based on the boundary data if U X 0, U 0 T equal to 0, if that is your boundary data you extend it as an odd function so F at G has to satisfy F of 0 is 0 G of 0 equal to 0 so for those initial data you have the solution ok. Similarly for other boundary data that is U X at 0 T equal to 0 if you are given that as a boundary data for the semi-infinite string then for all F at G initial data with dou F by dou X at X equal to 0 and dou F, dou G by dou X at X equal to 0 has to be zero they are the necessary conditions for those F at G initial data you have the solution for the wave equation with initial data and the boundary data ok.

So for given only two types of boundary for the semi-infinite string ok.

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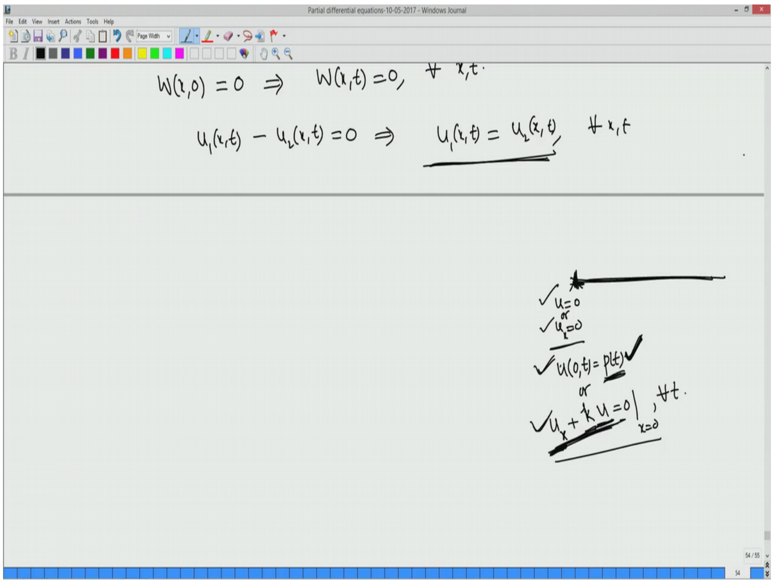

That is at X equal to 0 you have provided U is 0 and U X equal to 0 at this point you have initial data at T equal to 0 and at the boundary you have only here and these are the two either this or this is given and you can actually make a combination of this it is some constant equal to 0 ok. U X into some K or U or you can also give a different ways.

You can actually provide U at zero T is actually some it is non-zero condition you can give some function of T or U X plus K, K is some constant times U is also zero at X equal to zero for every T that is also you can give this is if you can attach this point at X equal to 0 with a spring of stiffness K you can attach so that this slope is actually proportional to the displacement with this constant ok that is what if you do with a spring of stiffness K if you attach, if you attach at X equal to zero this is the condition for the spring ok.

This one you can attach in such a way that this is U is displacement all the time changing ok, this maybe P T maybe all the time varying so it maybe some oscillatory function P of T ok. So we have seen if you are given these two boundary conditions you have seen how nicely you can extend it as a odd or even functions and get the solution from the D'Alembert solution. What happens if you do this way? If you are given this boundary data that may not work, so the whole general approach is just directly working from the general solution of the wave equation ok.

That we will see in the next video ok with some remarks so I will try to re-do this semi-infinite wave equation problems ok whatever we have done so far for these boundary value problems will do that approach and so that these other two boundary conditions general boundary conditions are similarly can be worked out ok. I will try to re-do whatever I have done for the semi-infinite boundary value problems ok initial boundary value problems for the wave equation on a semi-infinite line zero to infinity I will try to re-do in a more general approach ok, so that I will see in the next video and then we will move onto a finite string that is where we make use of our sturm liouville theory ok, so this are the things will see in the next few videos, thank you very much.