Differential Equations for Engineers
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Lecture No 27
Examples on Frobenius Method

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In this video, we will see, we will demonstrate with an example. We will demonstrate the case 3 when indicial roots are same, Ok, in that case how to find two linear independent solutions for an equation. Ok, so we will take an example here. So we consider, say let me write this as case 3, example for case 3.

So example is

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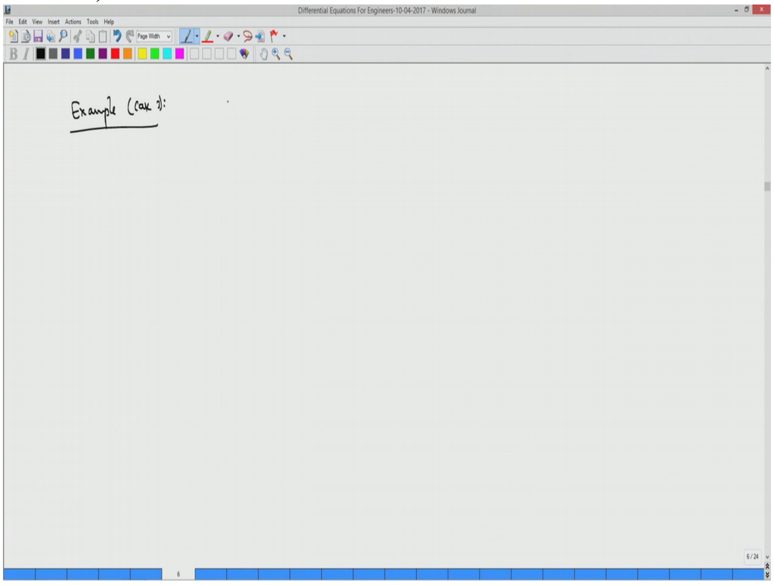

x square y double dash plus x square minus x y dash plus y equal to zero. So you see that x equal to zero,

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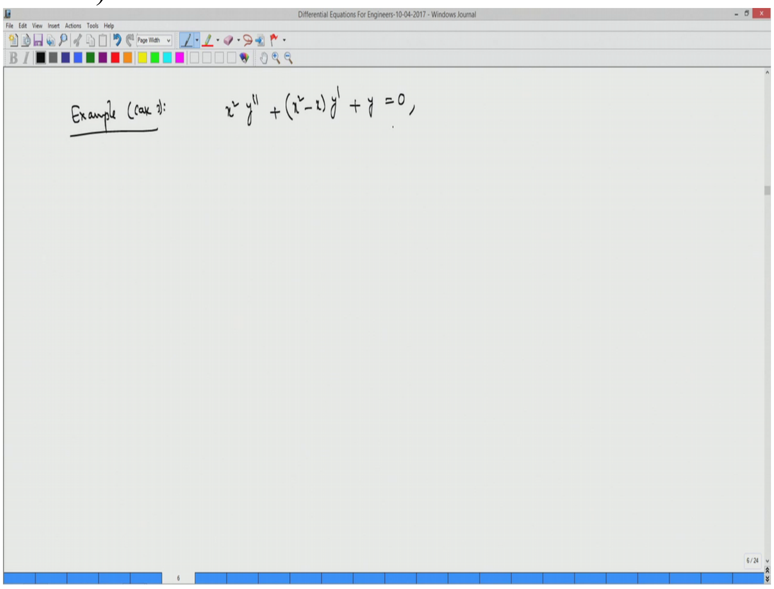

again x equal to zero is regular singular point because, since what is the reason, this is same, this divided by this, that will give me one minus one by x.

This limit x goes to zero, into x, actually minus 1 and which is finite, Ok. And this limit x goes to zero, this divided by, this coefficient of y that is one by x square into x square so that is 1, both are finite, Ok.

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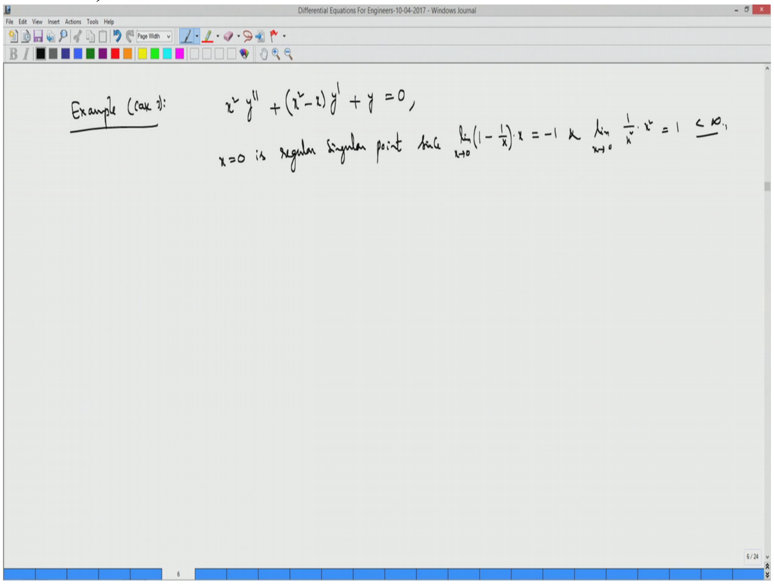

For that reason, this is a regular singular point.

So the domain of the differential equation is either positive or this is negative. So both the cases you can,

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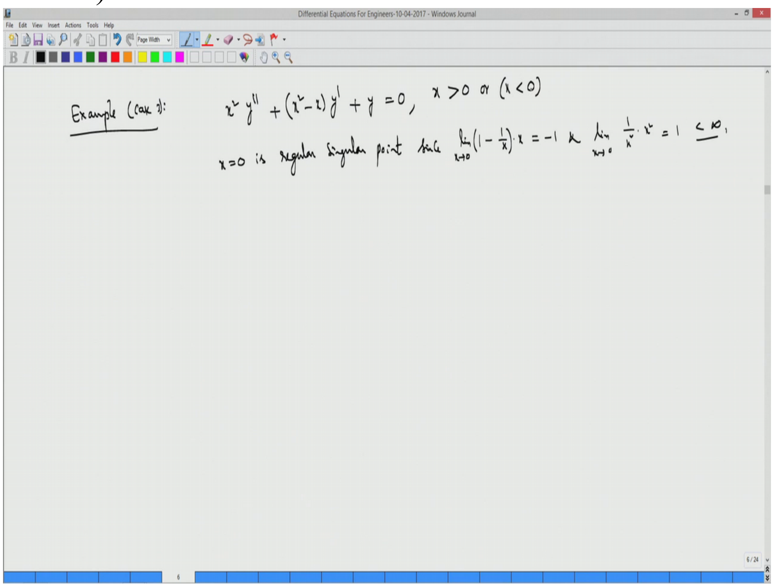

separately you can deal. So let me deal only for x positive. When x is less than zero, when you look for the special form of the solutions log x, you have to consider log is defined only for x, for x negative it has to be log mod x because

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log function is defined only for positive values.

So when x has negative values, you have to write a solution form as log mod x, Ok. So for calculation

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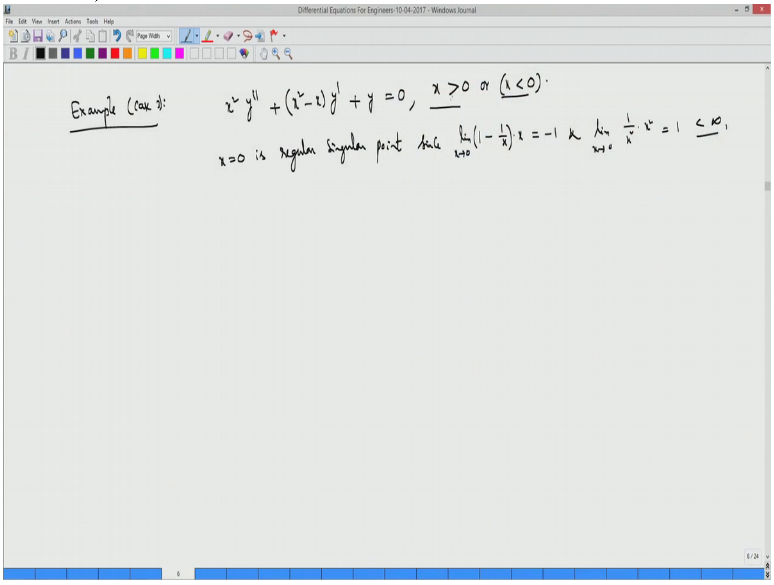

purpose it will be easy if you are in the domain x positive. So let me do this one. This case, so as usual, by Frobenius method because zero is a regular singular point, by Frobenius method, you look for solution in this form y one of x, Ok.

So look for, look for solutions of the form

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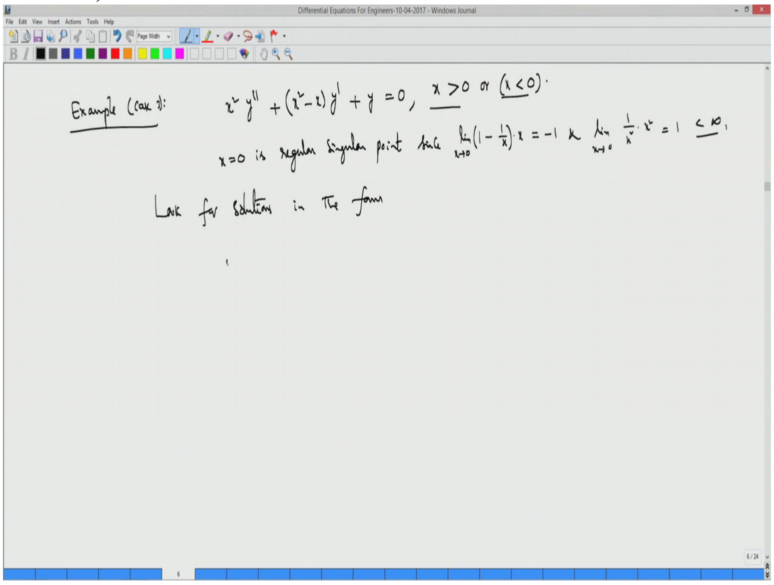

y of x as, by Frobenius method what we get is n is from one to infinity, c n x power n plus k. This is what we used earlier, right? So x power k into some series solution. So this is what we have written, Ok. For x positive and c zero is always we assume that is non-zero

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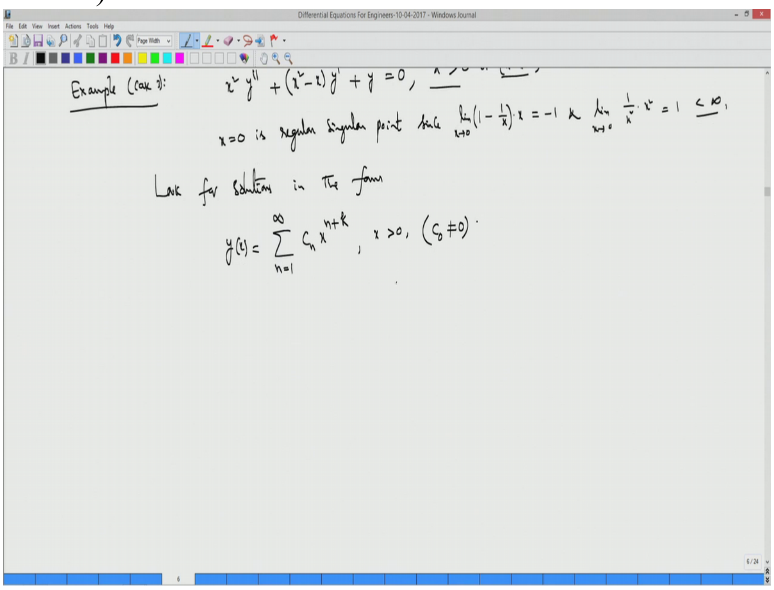

without loss of generality, Ok.

So if you do this, you substitute into the equations x square y double dash, it will give me, I have x square y double dash will be, n equal to, sorry this is from zero to infinity, so zero to infinity n plus k, n plus k minus 1 c n x power n plus k. Now you have minus 2 after differentiation y double dash and you are multiplying with this x square, Ok.

So this is actually your y double dash. And you are multiplying with x square. So x square if you multiply, the first term will be, so let me write this one here, so y double dash of x is this, Ok

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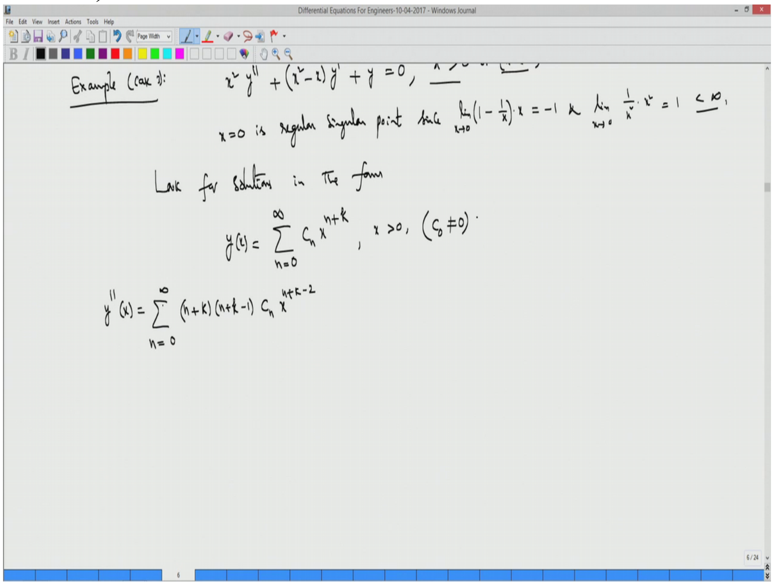

and we have y dash of x is, n is from zero to infinity, n plus k c n x power n plus k minus 1. This is what you have.

So now substitute all these

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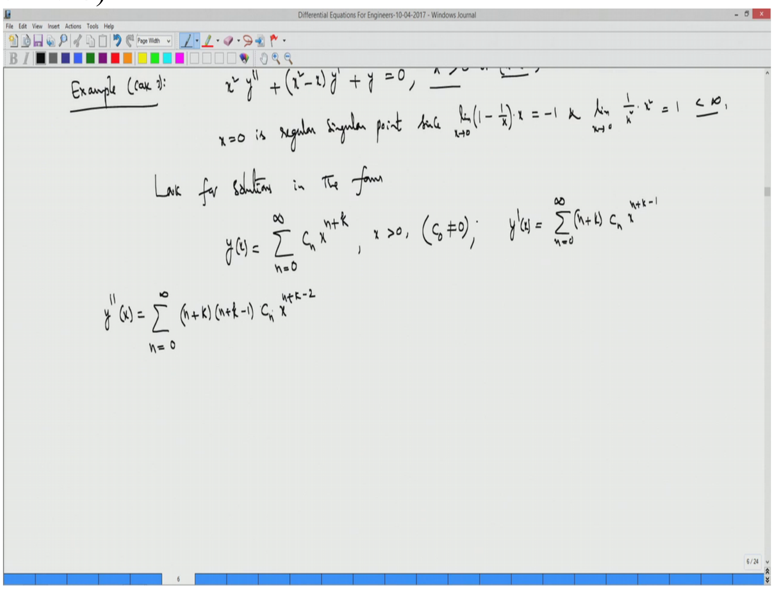

three into the equation, you get x square y double dash that is nothing but n is from zero to infinity n plus k n plus k minus 1 c n x power n plus k, Ok. Plus x square y dash, first x square y dash, that will give me n is from zero to infinity, n plus k c n x power n plus k plus 1. You are multiplying with x square, Ok. And then this n is from zero to infinity.

This is minus, minus x times y dash, that is n plus k into c n x power n plus k. Then plus y, y is n is from zero to infinity, c n x power n plus k is equal to zero.

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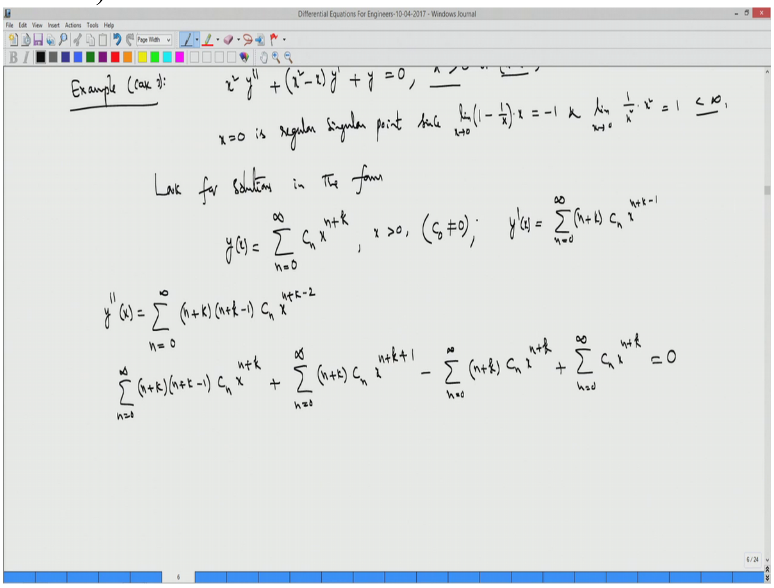

So this is what the equation becomes if you substitute these solutions, y, y dash and y double dash Ok.

By putting all these three into the equation this is what you get. So now you see what happens to solution. So this is a power of n plus k, n plus k, here n plus k plus 1. So you can make the syntax so that this also you can make n equal to n plus k. This is also n plus k, right. Powers are n plus k in these three series, only except this one.

This one you can make it by replacing n equal to n minus 1. If you make n equal to n minus 1, that minus 1 plus 1 goes, this is going to be x power n plus k. That is the idea. So you check this index n equal to n plus 1, n equal to n minus 1. So if I do, if I replace n equal to n minus 1 Ok, n minus 1, this is what it becomes.

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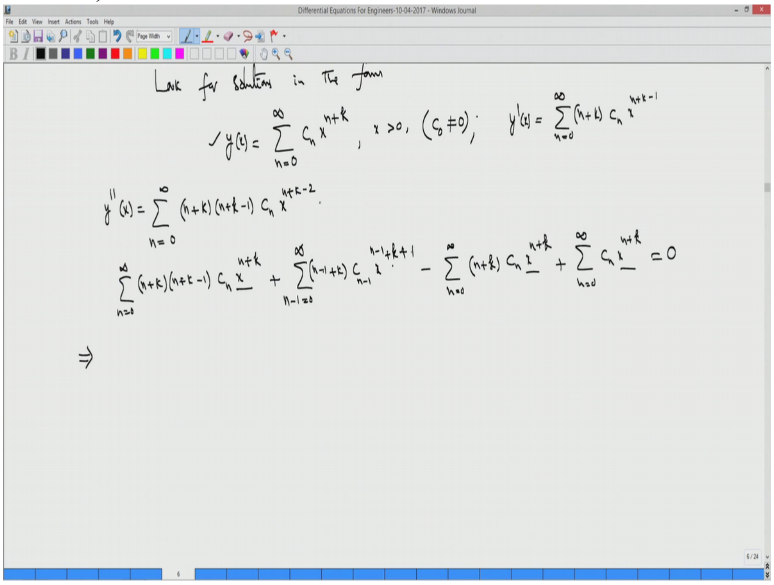

So that this goes together. So it is going to be n plus k.

So this is what, so

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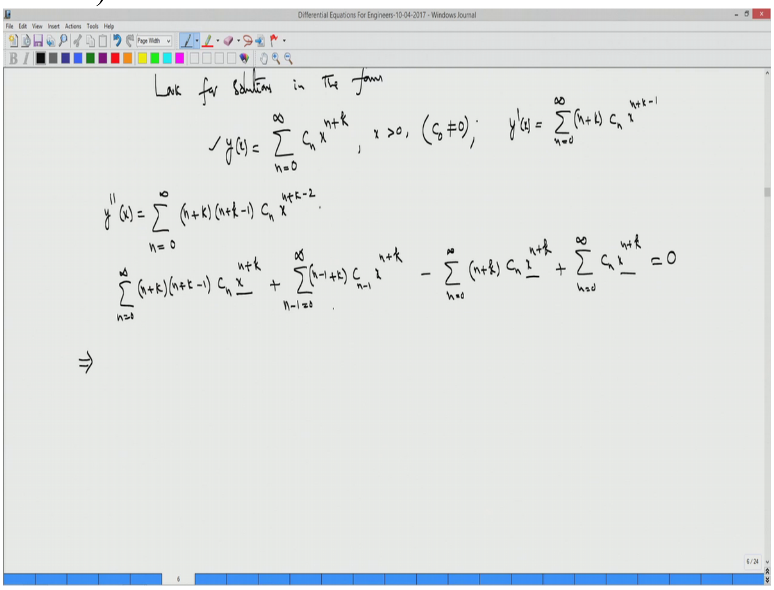

n equal to, so n minus 1 equal to zero means n equal to 1, n equal to 1

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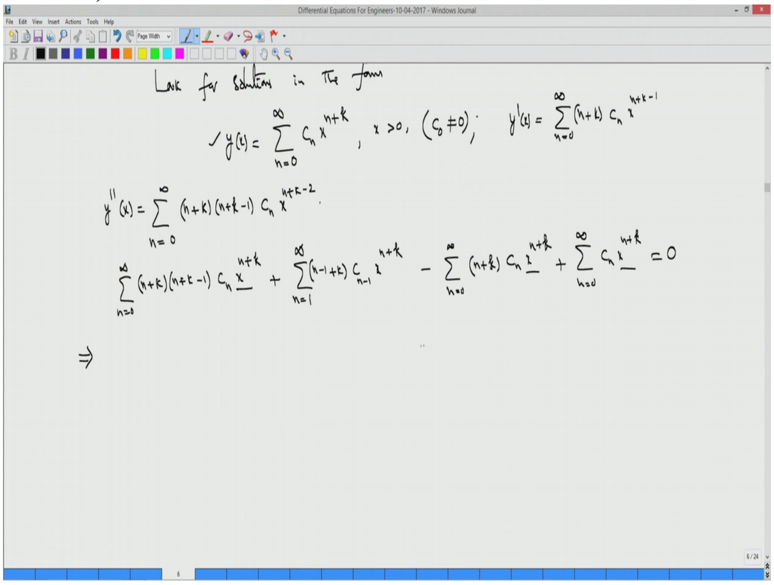

Ok. So this what, now the all the powers are same. So you have, except so what is the isolated term n equal to zero, common is n equal to 1 onwards everything is common. But n equal to zero, you have only here. So n equal to zero you write, if you write n equal to zero separately, you have k into k minus 1 c zero, c zero is non-zero and you have x power k plus this you write n is from 1 to infinity you have n plus k, where is equal roots, do we miss something?

Again I think I miss something. This is x square minus x, x square y double dash, x square y dash, x square y dash, this one, so, Ok, so what are your n equal to zero terms? So zero term is here. So here also is isolated. n equal to zero thing you write it separately. So you have minus, n equal to zero is k c zero and here x power k and here n equal to zero is plus c zero x power k. So this is your n equal to zero.

The rest you write from 1 to infinity, n plus k into n plus k minus 1, Ok plus, into c n, Ok, so c n plus, plus 1 into c n mi... so all the c ns so this is n plus, this term I wrote, from n is from 1 to infinity, 1 to infinity into c n, what else?

So you have c n terms of minus n plus k plus 1 c n plus n plus 1 n plus k minus 1 into c n minus 1. So this is what is my coefficient of x power n plus k, n is running from 1 to infinity which is equal to zero.

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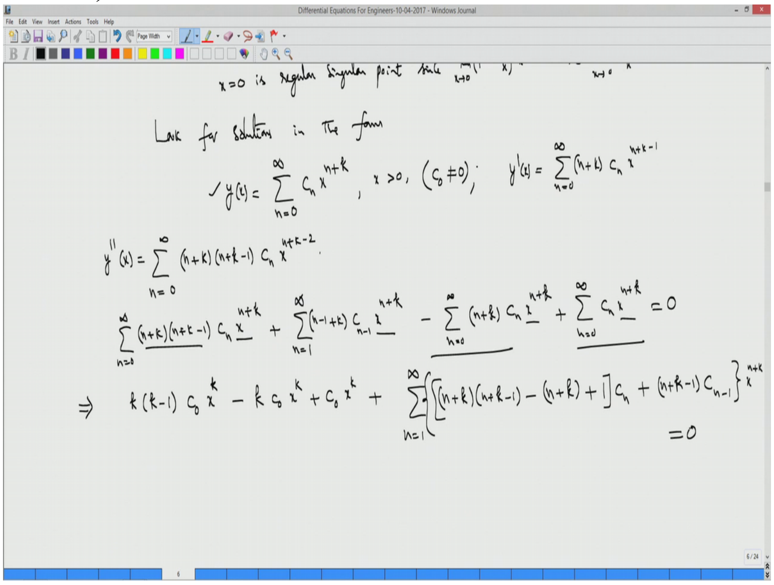

Ok. You can see that c is not equal to zero. Coefficient of x power k, we can make it zero, this will give me k into k minus 1 minus k plus 1 equal to zero.

So this will give me k square minus two k plus 1 equal to zero.

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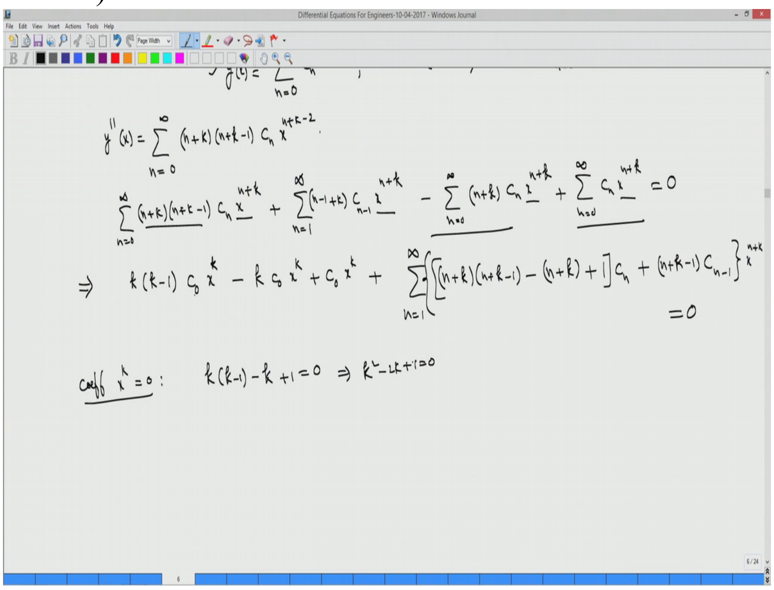

You will have two terms so k equal to 1 1. So these are

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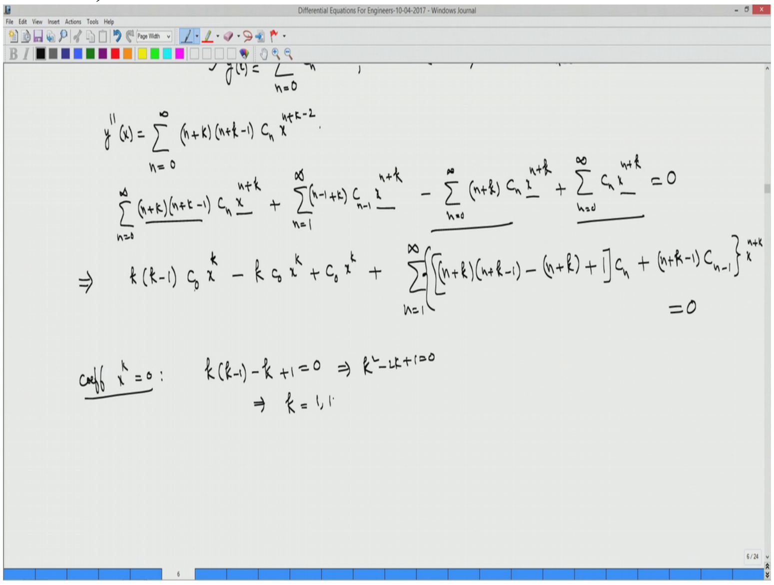

the roots. Ok so this is your indicial equation. So indicial equation has roots, roots k 1, k 2 both are same, Ok, so what happens to the remaining?

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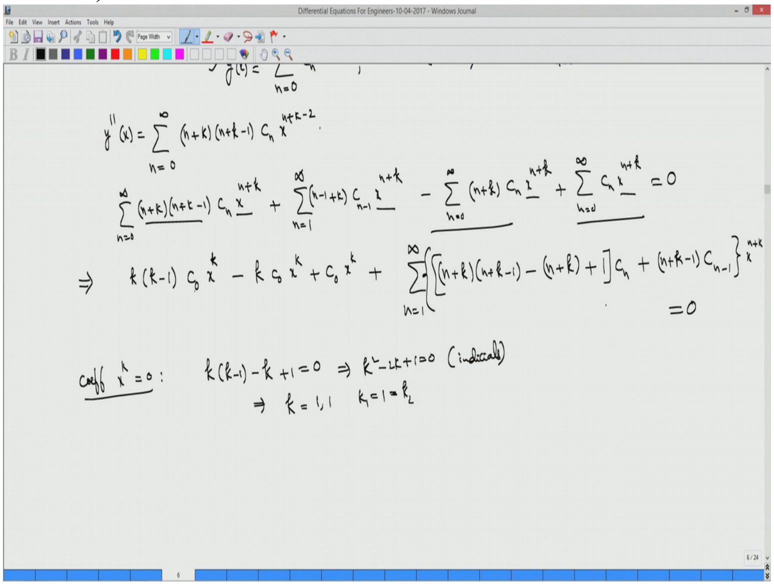

So remaining if you...so full left hand side is full series.

So if we equate the coefficient of x power k, we get indicial equation k equal to 1 to infinity,

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n equal to 1 to infinity. You will have a recurrence relation. So

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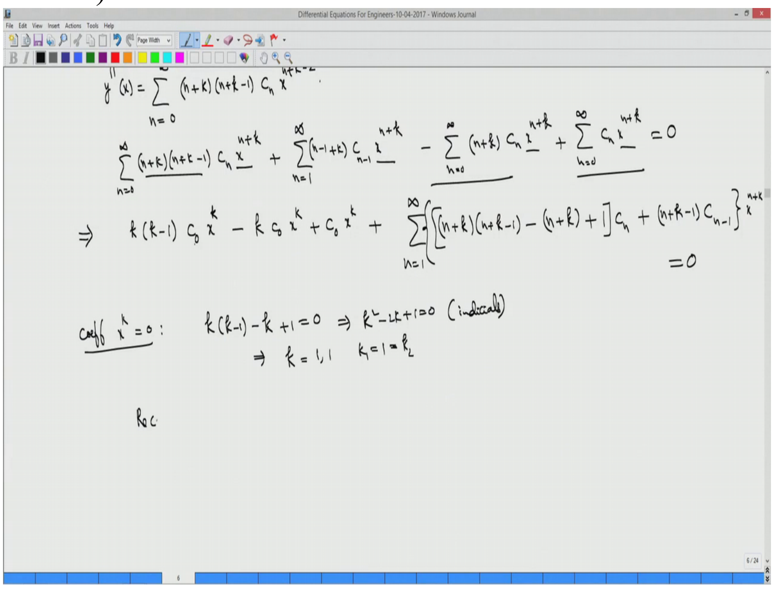

recurrence relation, recurrence relation will give me, so you have, so what is the coefficient of x power n plus k, for n is running from 1 to infinity? So you have n plus k which is common. So you have n plus k if you take it, n plus k minus 1, right?

So you have minus 1 minus 2 plus 1, Ok into c n plus n plus k minus 1 c n minus 1, which is equal to zero, this is running from 1, 2, 3 onwards.

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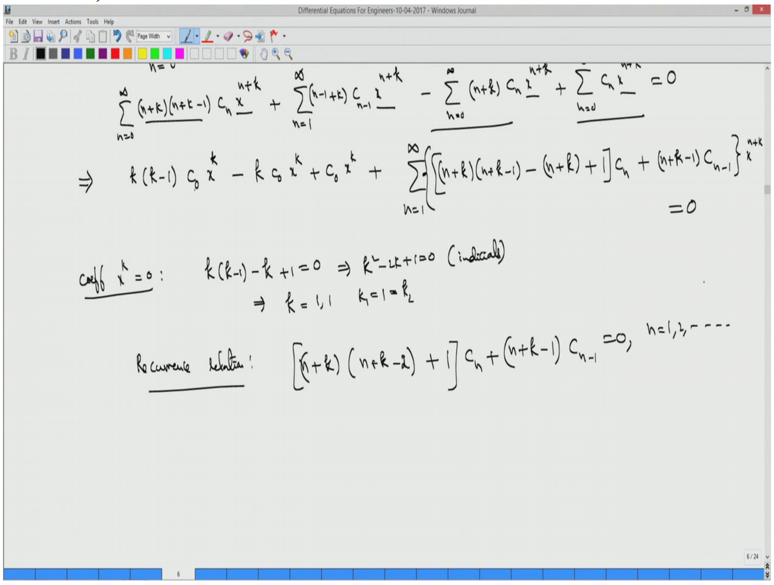

This is what you have. So recurrence relation now put k equal to 1. So k equal to 1, so you have seen in case 3, one root whichever is the bigger root, both are same, so doesn't matter so k equal to k 1. So k equal to 1 we put it so you will get n plus 1and if you put k equal to 1, n minus 1 plus 1, Ok? n square minus 1

So this is basically n plus 1 into n minus 1, is n square minus 1 plus 1, so together will give me n square. c n plus n c n minus 1, if I put k equal to 1, which is equal to zero, Ok? This is running from 1, 2, 3 onwards. But n cannot be zero. So

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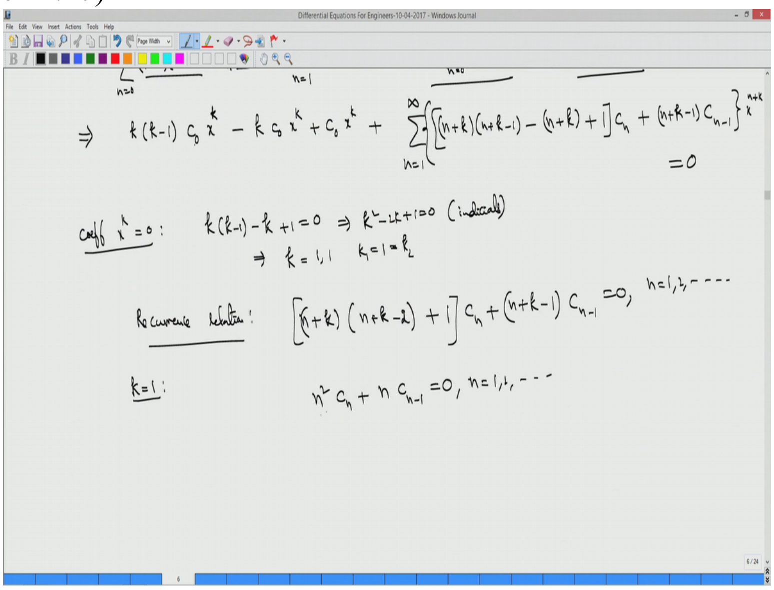

this implies n c n plus c n minus 1 is equal to zero running from 1,2 onwards.

So this will

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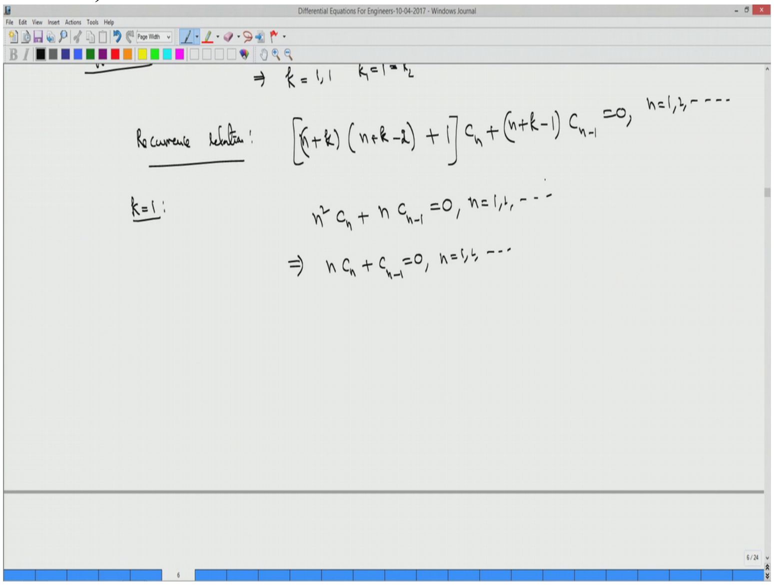

determine everything. So put n equal to 1 now. So n equal to 1 will give me c 1 is equal to minus c zero.

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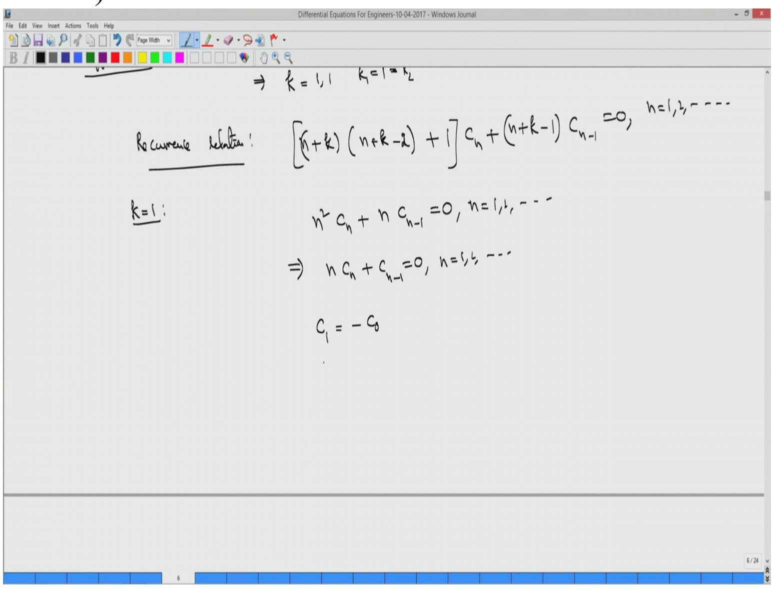

Put n equal to 2, so n equal to 2 if you put minus c 1 by 2, so it will give me minus, minus plus, c zero by 2.

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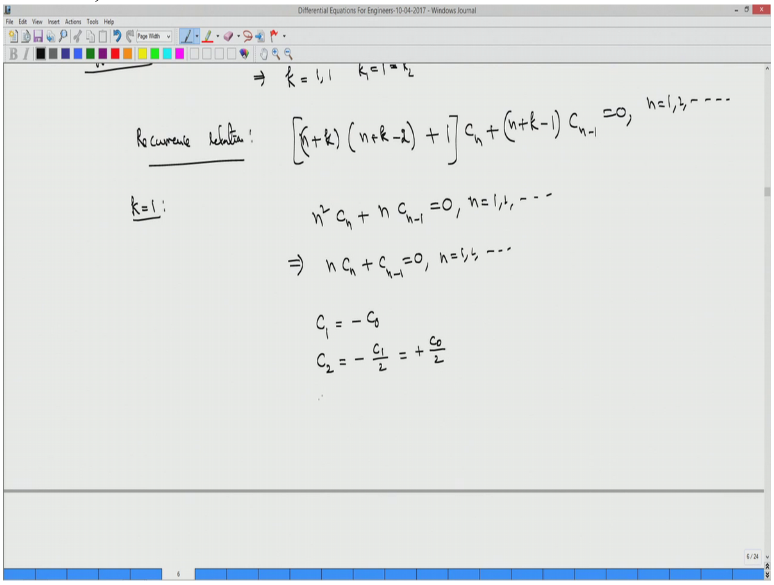

Again c 3 minus c 2 by 3, that will give me, c 2 is c naught, so c naught by 2 3 and so on, Ok. So this is basically, you have c zero by 3 factorial

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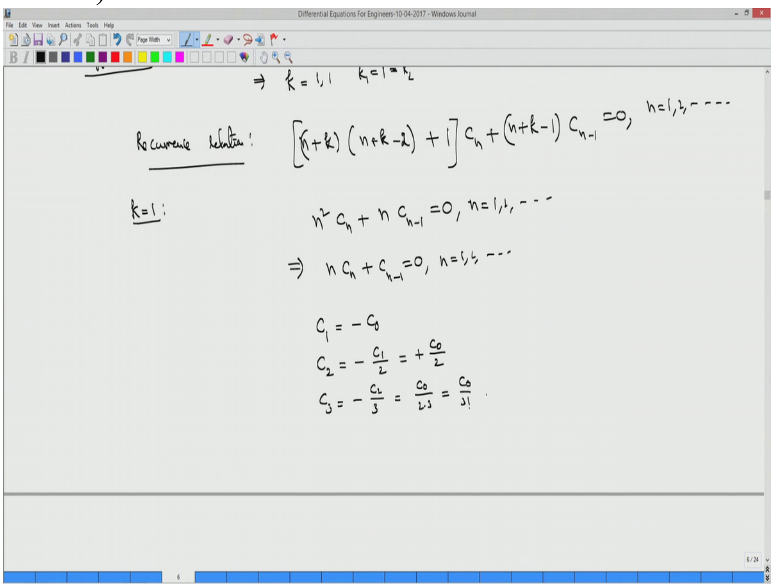

and so on. So you get c 4. c 4 is minus c 3 by 3, c 3 by 4, by 4 , it is going to be c zero by 3 factorial, 4 into 3 factorial So it is going to be c zero by 4 factorial and so on.

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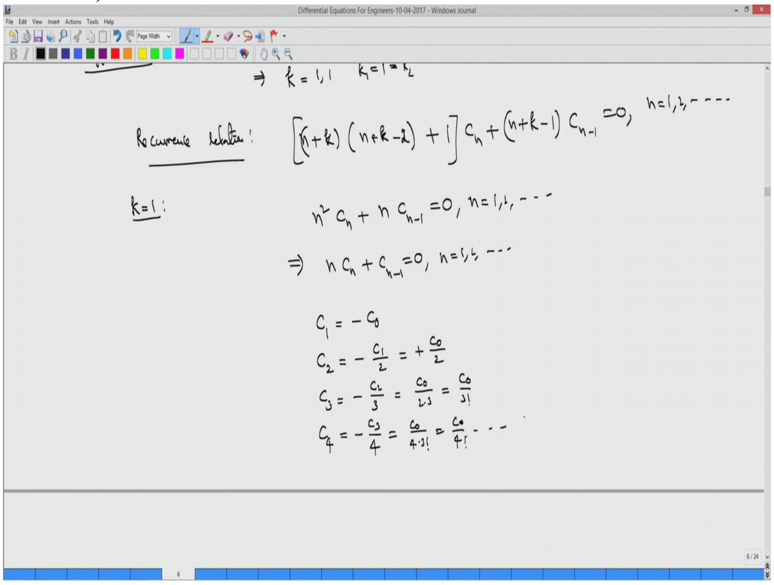

So finally what you are getting is...

so what is your first solution? So first solution is, for some k. k equal to bigger root which is 1 itself, which is equal root. So you have y x, put what is the solution? Solution is k equal to 1, you have already put. So x power sigma. So if we put x power k 1, so this is 1. Now sigma is running from c 0 plus c 1 x plus c 2 x square and so on.

So this

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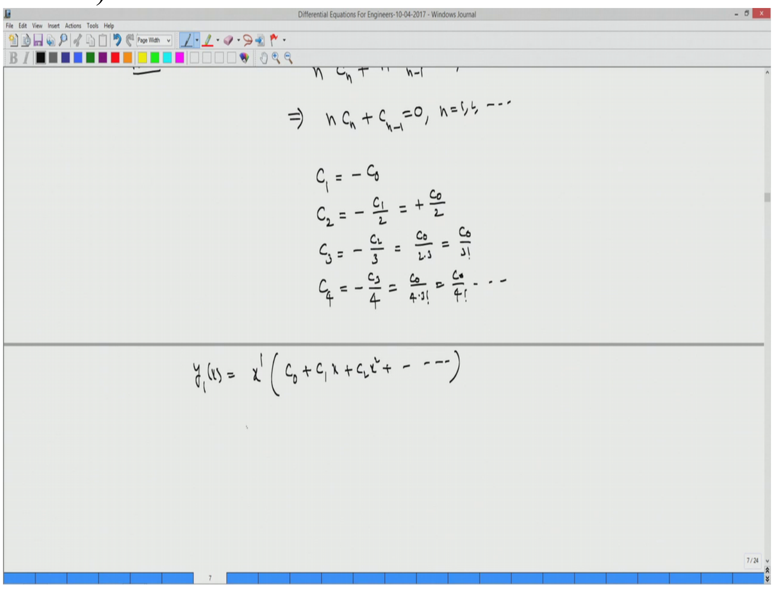

if you put it, x 1. c zero i fould not find anything, it is non-zero. c 1 is minus c zero x. c 2 is c zero by 2 factorial x square. c 3 is c zero by three factorial into x cube and so on,

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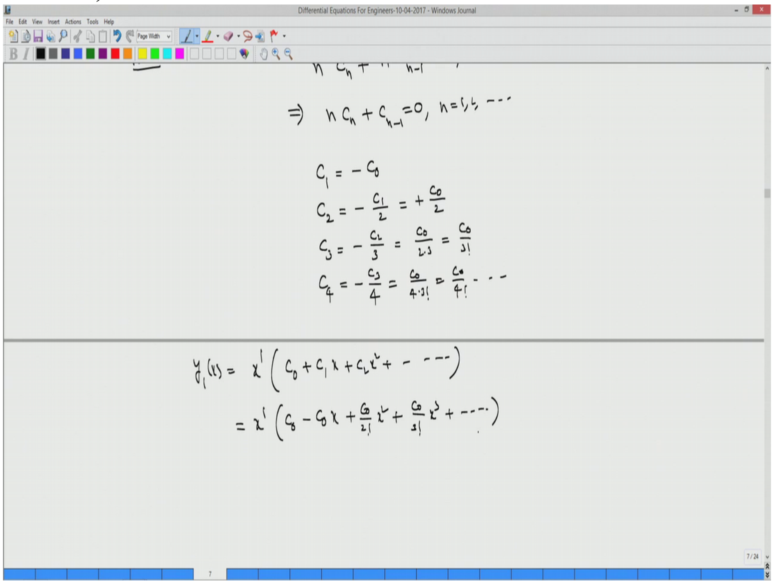

all are placed except this one. So this is equal to, what you have, you take c zero equal to 1, if you do that, you will get, you can write your y 1 x as x times, what is this point?

So this will give you, c 1 is this, c 2 is this c 4 is this, Ok. This is what you get. So what is this one? c 1 is, c 1 if we do that, x minus x square plus x cube by 2 factorial plus x power 4 by 3 factorial and so on.

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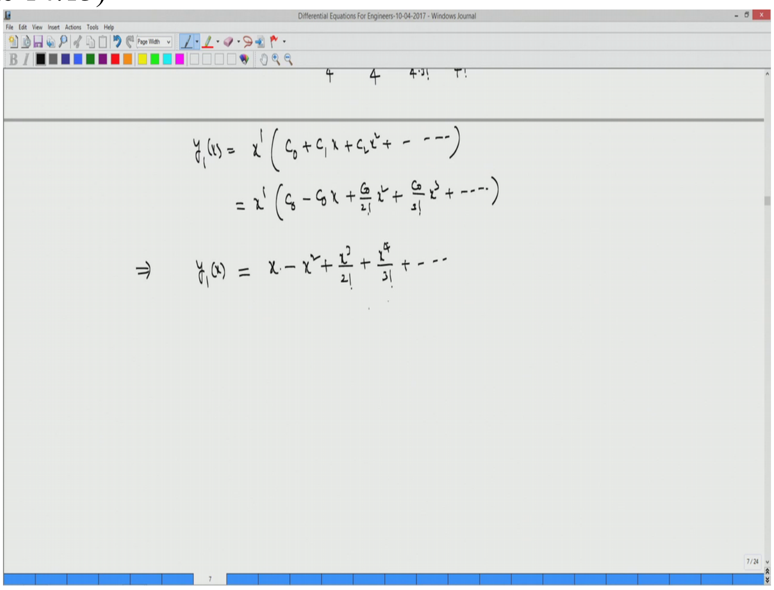

So this is actually equal to x, I take it out, 1 minus x, plus x square by two factorial, plus x cube by three factorial and so on, Ok.

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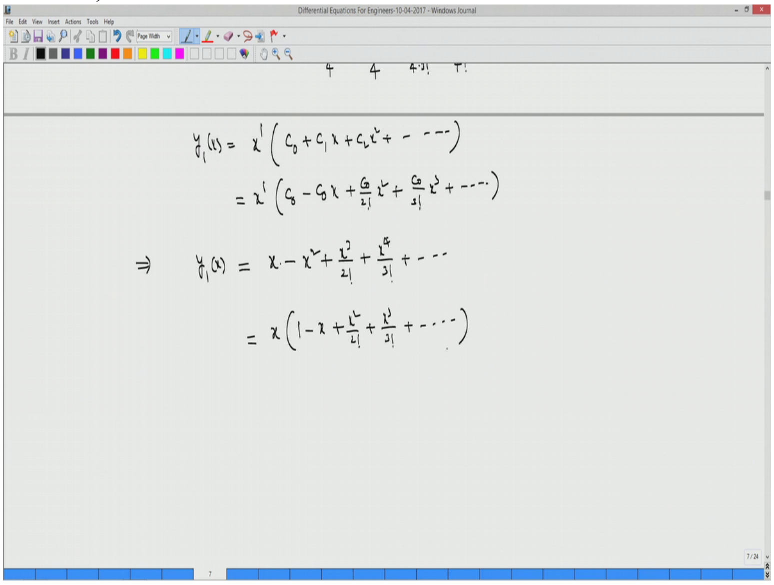

c 1 is this,

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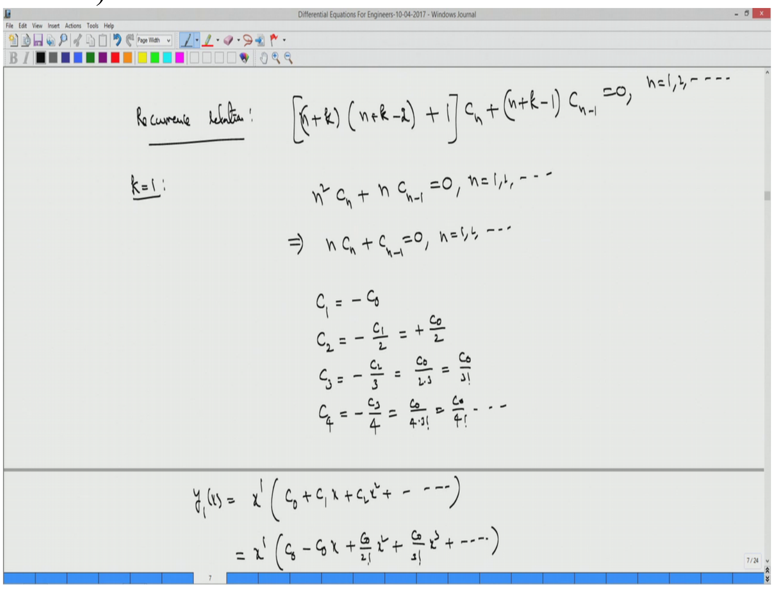

c 2 is minus c 1, so it is going to be plus. c 3 is minus c 2. But c 2 is that. c 2 is this. So you have, c 2 is this means we have minus right? So this is going to be minus. And c 4 is going to be plus, Ok. So you have c 3, you have issue here. c zero, c 1 x so c 1 is this; this is going to be minus c 3 and so on,

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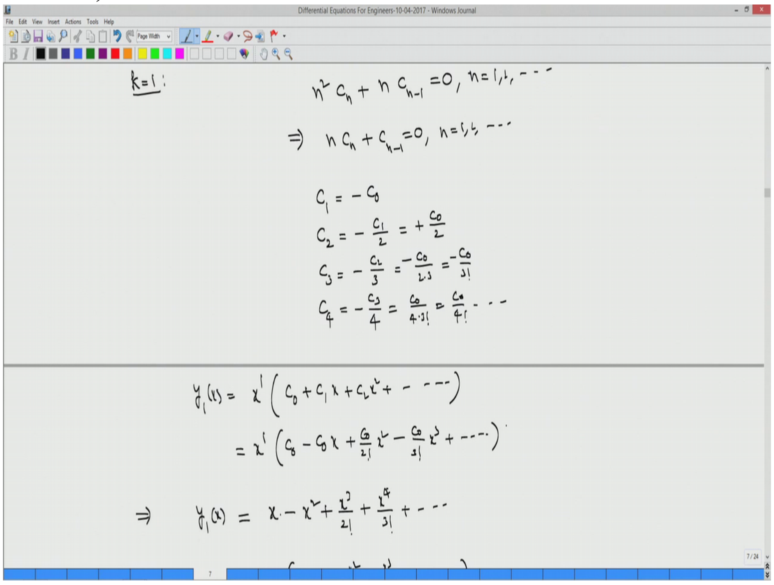

So you have minus here. This is going to be plus, this is going to be minus.

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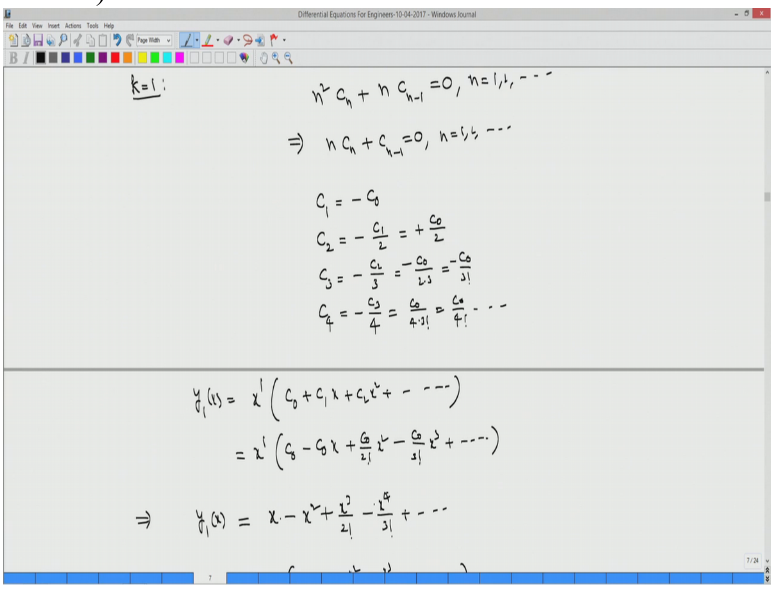

So what you get is, this is minus so this is nothing but x into e power minus x. So you have

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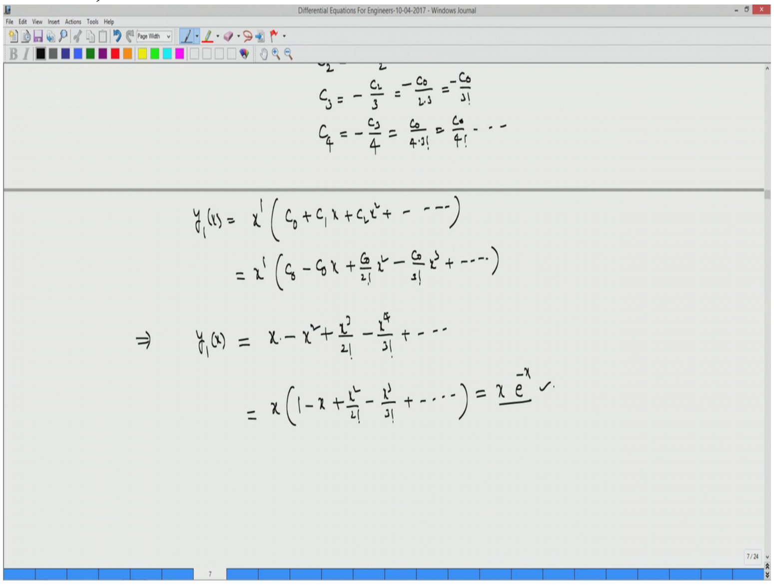

one solution y 1.

So you have got nicely, solution for y 1 is x into e power minus x. So first solution is when you put x equal to bigger root. In this case it so

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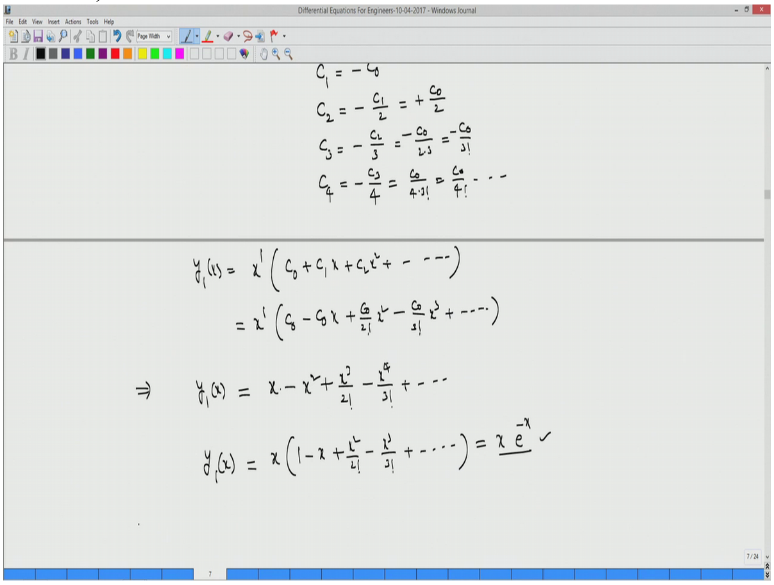

happened that both the roots are same, so you could put k equal to 1, you get one solution is this. So by Frobenius method if you get one solution in this explicit form x into e power x you should not go for the

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second form, the actual form Ok.

So immediately you have to go for,

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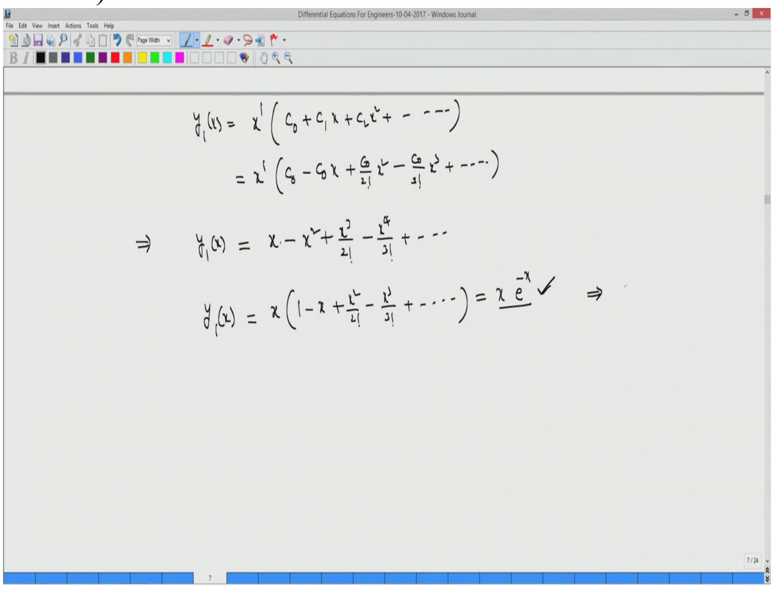

if you know one solution for any second order equation, homogenous equation, if you know y 1 as e power minus x,

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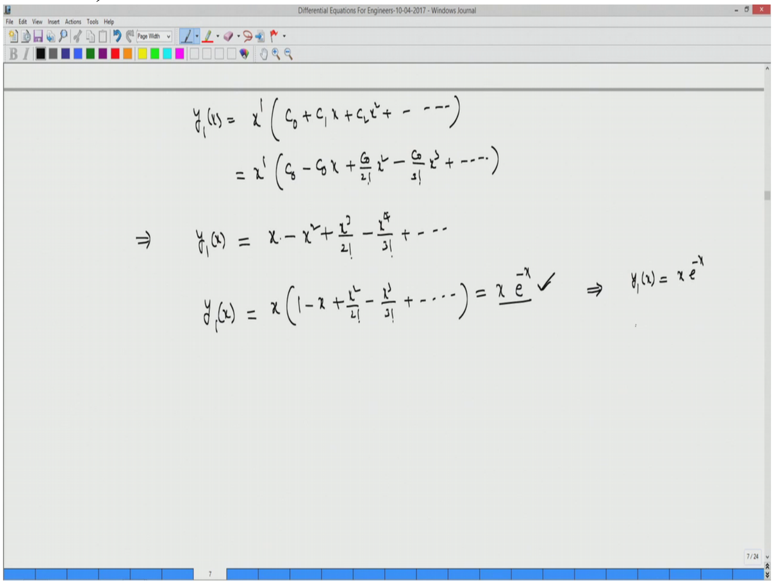

second solution is you know already, right? So you know how to find second solution if you are given one solution y 1, Ok?

So with that method you should find your y 2, Ok. So what is that? So that is y 1, that is x into e power minus x. y 2 is actually y 1 integral e power minus integral, what is your equation p, what is your p x after dividing a 2, a 1 by a naught, that will be your p. So in this case, the given equation is this. So this divided by this; that is 1 minus 1 by x is your p.

So you have integral is 1 minus 1 by x d x, this divided by y 1 square. So that is x square e power minus 2 x this d x. If you do

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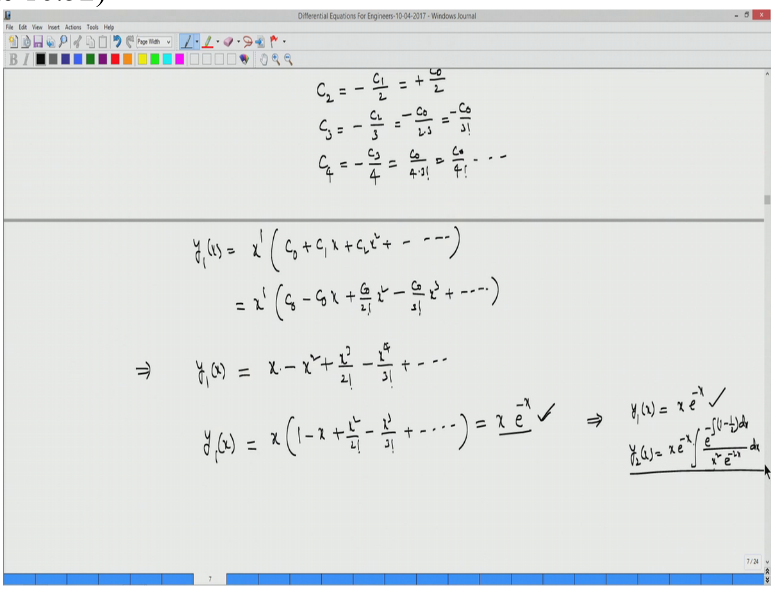

indefinite integration, you do like this; you get your second solution. This is how, you already learnt how to calculate, if you know one solution in an explicit form, you can find the other. Even if it is not explicit form, it is a series form, if you can calculate like this, if you can

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manipulate the series Ok in this formula you can actually get the second solution as another series, Ok? But mostly it is advisable, if you know first solution in an explicit form, then you go for this, from this formula you get the second linear indicial solution, Ok.

So just to explain you this method how to find the second solution,

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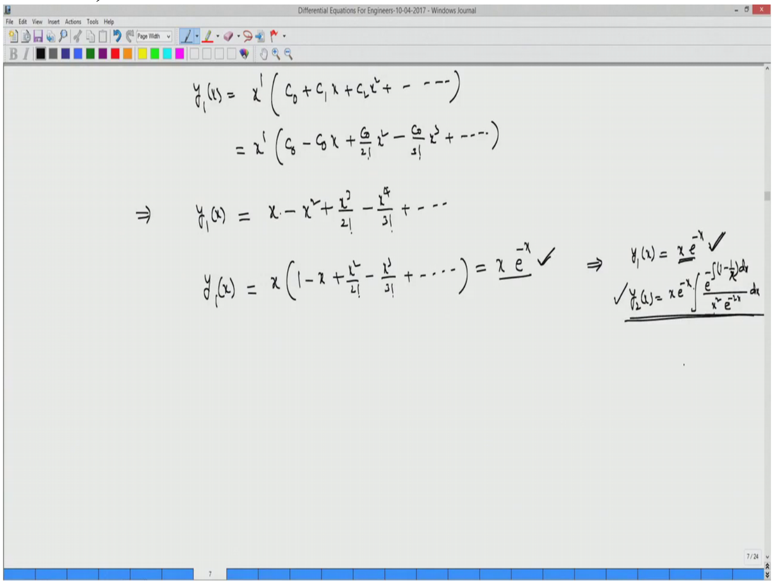

so if you actually do this by this method, once you know one solution if you find the other from this formula you can get your y 2 as from this formula Ok. But let me give you a second solution.

So because by, in the Frobenius method, if you know bigger root will give me one solution which happened to be explicit here, Ok. So second solution I can get it by this formula that is one method. Otherwise just to give you this Frobenius method Ok, this need not be the case always. In some other examples I may not get this explicit form. So I may get series solution as your y 1.

In that case how do you get

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your second solution? Again you can manipulate like this, horrible calculations you will have to go through. So to avoid that you will have to follow this case 3 Frobenius method. So

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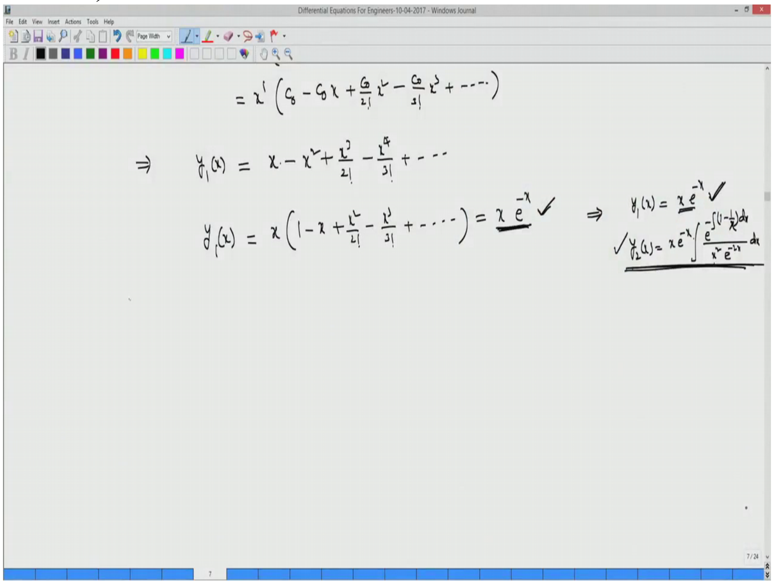

to get the second solution; to get the second solution, second linearly independent solution, so we

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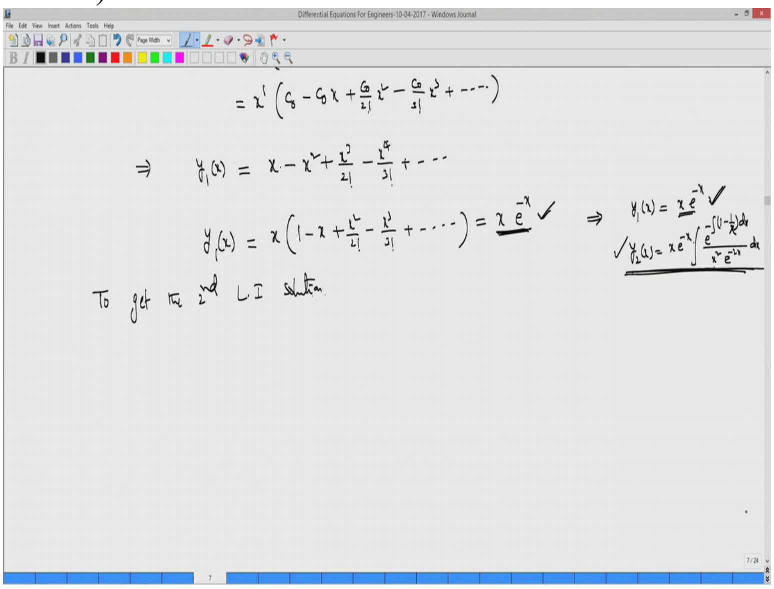

choose this form as y 2 of x as, even the roots are same. So what you have to do is it is in this form. So you, there is no arbitrary constant. So there is no a here.

So you have y 1 of x log x. You look for solution this, like this when the roots are same. Earlier when roots difference is non-zero integer I put one arbitrary constant a. Here I don't. I keep it as 1. Plus x power k 2, here it is 1 and then you go for n is from 1 to infinity, Ok. n is from 1 to infinity. This is what is the difference, some d n x power n.

So you have to look for solution in this form

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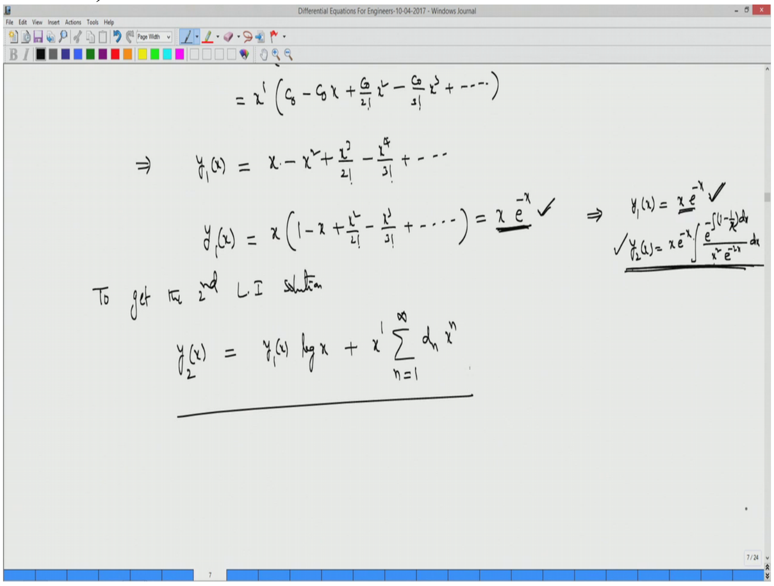

in the case when the roots are, the indicial difference, when the indicial equation roots are same, to get the second linear independent solution what you need is, this is your y 1, if it is given as a series. See you recognize this as a series in a nice form. If you don't know this, Ok, if you don't know this, ok then you can think of putting use this series here, y 1 log x so the case 2 you have a here and you have x power that smaller root, here small or big both are same so you have x power same, x power 1.

Earlier you were running for n is from zero to infinity. So here you have to make that change. n is only from 1 to infinity. That is the difference. Ok this is what you have to remember. So n is from 1 to infinity and there is no constant. When there is no constant, you should have the second series, series solution;

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the second solution will be starting from 1 to infinity, Ok.

So if I look for in this form,

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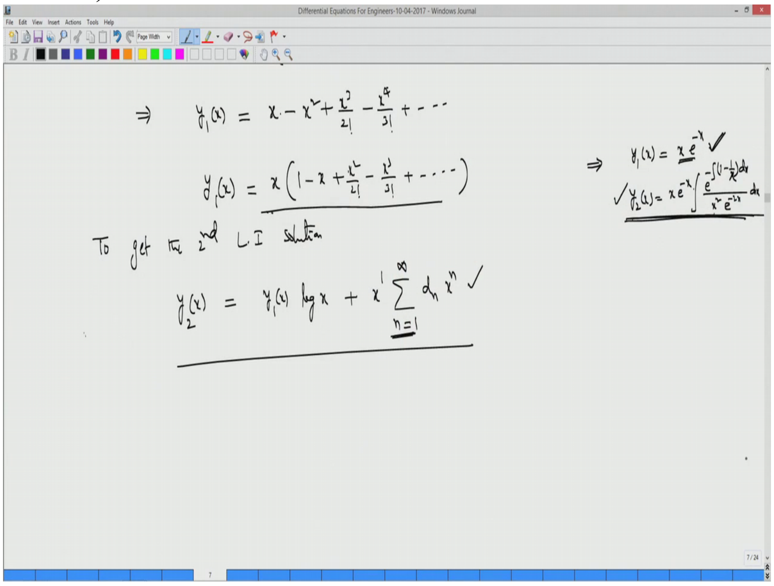

look for this form so you can get

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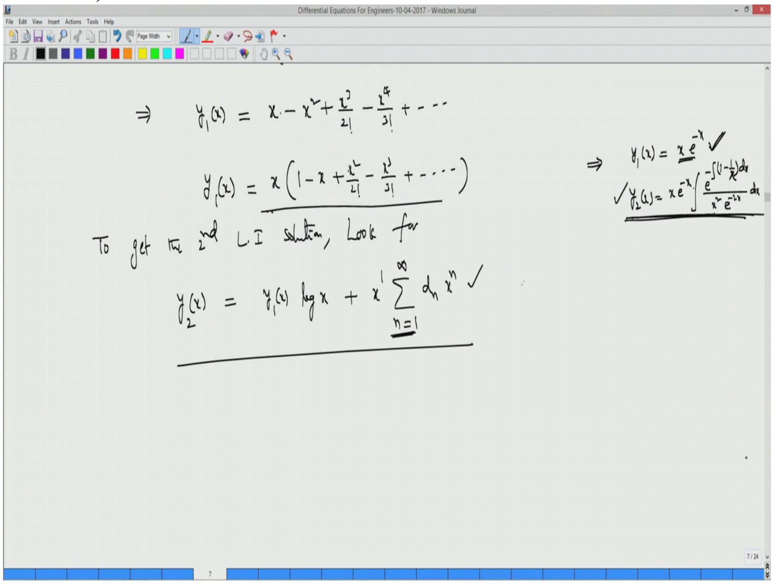

your, get the unknowns d n, get the unknown d ns, Ok?

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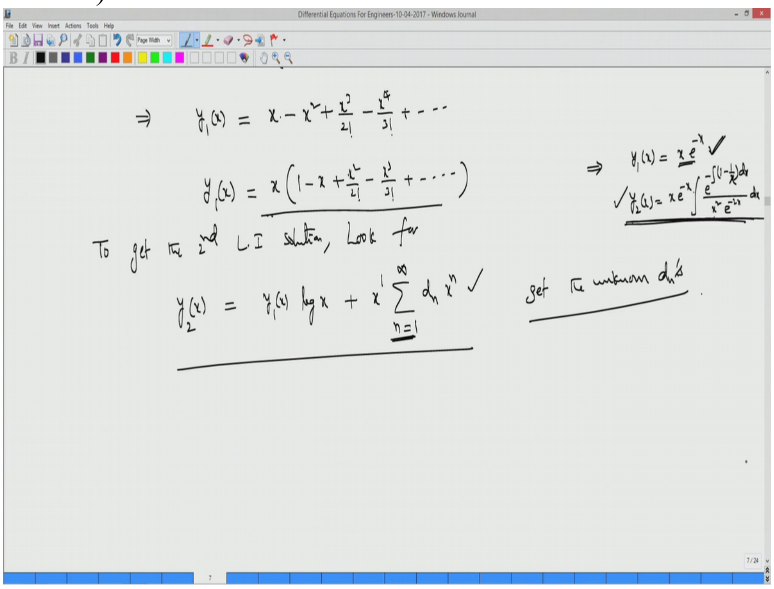

So that is the idea. So what you do? So again as usual, you substitute this equation into the given equation and get your, this is a routine procedure but let us do it.

So get your, so it is a second order equation, you need two derivatives. So you have y 1 x by x plus y 1 dash of x into log x plus if you differentiate this, this has one term and this whole series as another function. So if you can think of like that, you have n is from 1 to d n x power n. I differentiated only x Ok initially. Now I differentiate the series. n is from 1 to infinity so you have n d n x power n minus 1.

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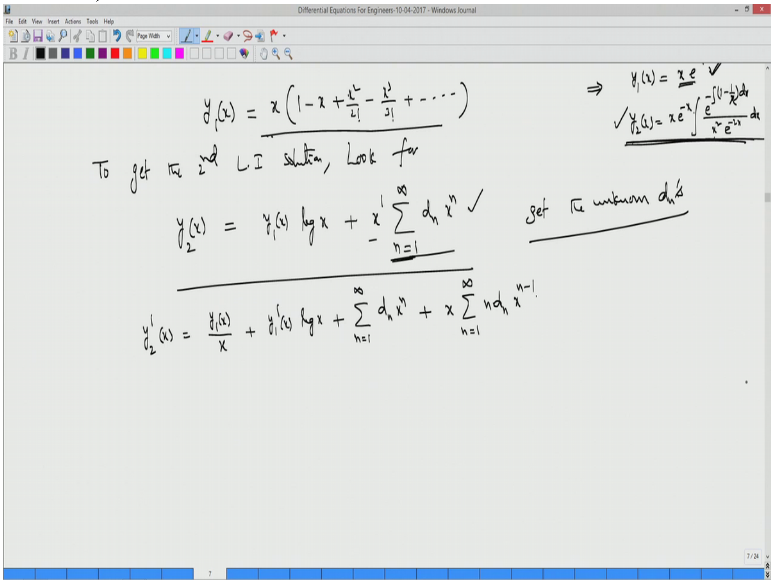

So what happens, it is starting from 1. So there is no change Ok, so index will not change. So n is running from 1, so you have d 1 x plus d 2 x square and so on. So

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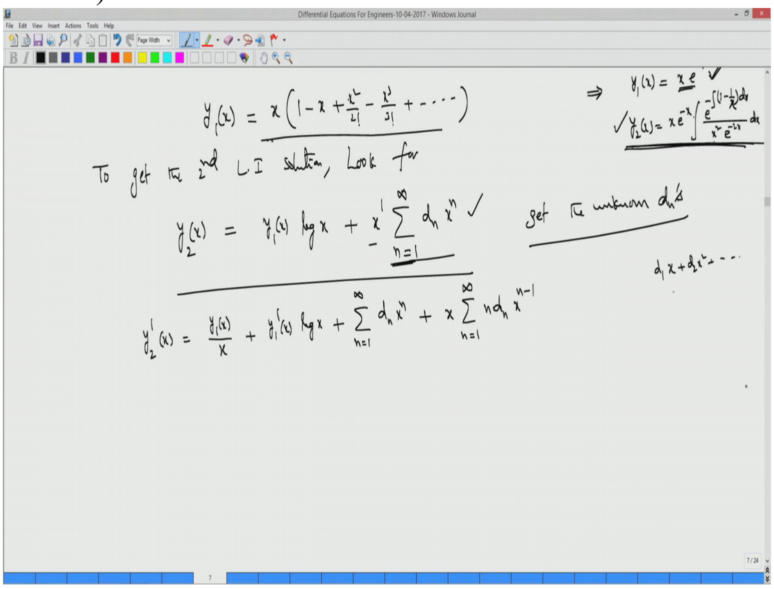

after differentiation we get d 1 plus d 2 x, 2 d 2 x and so on, 3 d 3 x square and so on. So this is nothing

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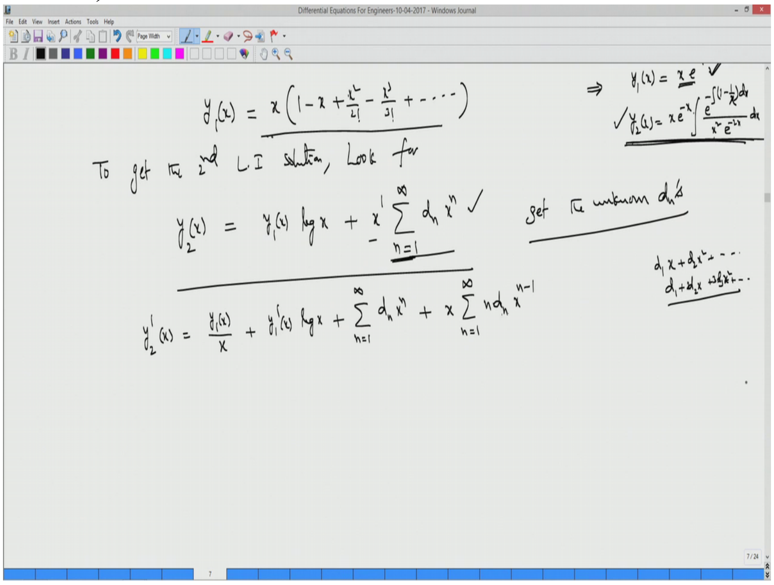

but this, Ok.

This is what your y 2 dash plus y, so you need y 2 double dash. So y 2 double dash is differentiate again. So you have minus 1 by x square y 1 x plus y 1 x, y 1 dash of x by x. Now you differentiate this once. You have y 1 dash of x by x plus log x y 1 double dash of x plus you differentiate this. So you get n is from 1 to infinity n d n x power n minus 1 plus n is from 1 to infinity n d n x power n minus 1. I differentiated this and keep as it is. That is what is this.

One more term is, you keep this and differentiate one more term. So now if you do this, n is from 2 to infinity, n into n minus 1 d n x power n minus 2. So there is, this is what, you differentiate

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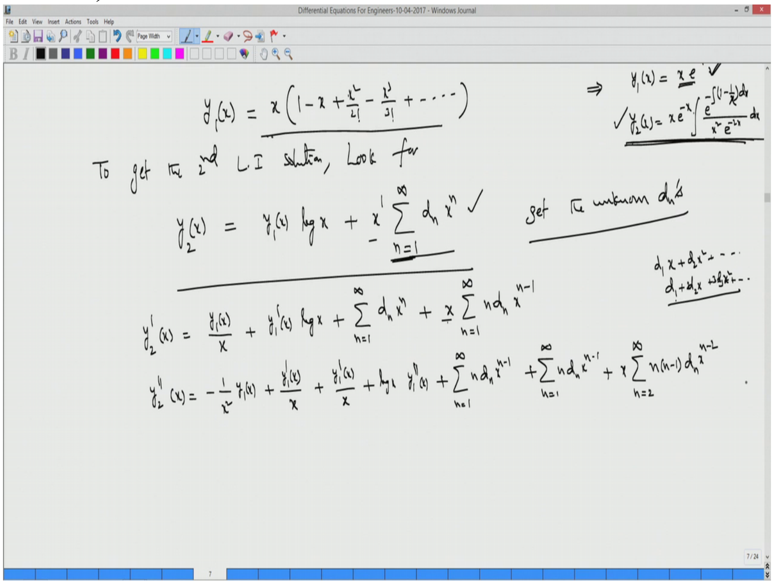

one more time. You get 2 d 2 plus 3 d 3 6 d 3, Ok, 3 2 d 3 x and so on. That is n,

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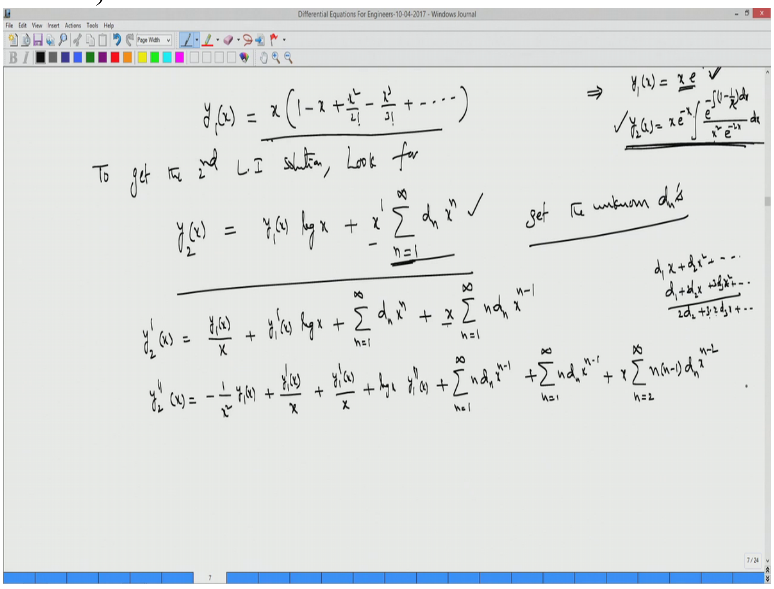

n minus 1 d n x power n minus 2. So is y, Ok. So this is y 2 double dash.

So substituting into the equation x square, x square what is your equation x square y 2 double dash plus x square minus x into y 2 dash of x plus y 2 of x equal to zero. So this is your given equation.

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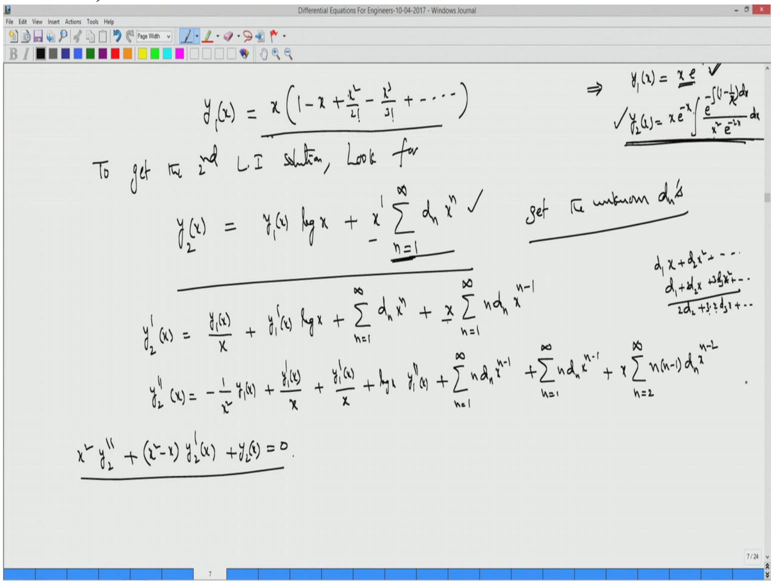

So substitute into this equation x square y 2 dash, if you substitute, so you have minus y 1 x plus x y 1 dash of x; see I can sum these together so you have two times, so I will put two x here, Ok. So 2 y 1 dash of x by x if you multiply so it is going to be x square this is what you get. Plus x square log x y 1 double dash of x plus now you have n is from 1 to infinity n d n x power n plus 1. If you multiply x power n minus 1 with x square you get this, Ok. So plus n is from 1 to infinity.

So you have, these are 2, so you have two times, 2 n, Ok so these are same so you can put it together. So you have this. Now here if you multiply x square with this, so you will get x cube, x cube if you add here, x power, x power n plus 1 so n is from 2 to infinity, n into n minus 1 d n x power n plus, n minus 2 plus 3 plus 1. So this is what you get, Ok.

This is what your first term here

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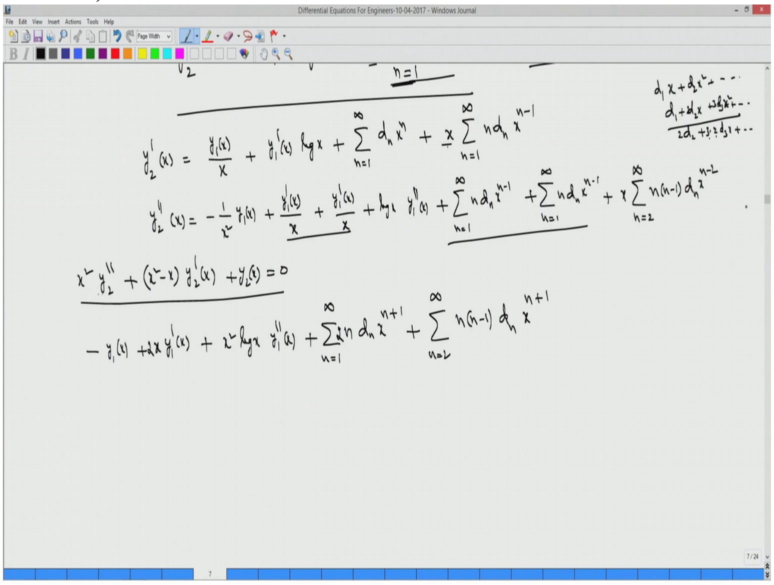

plus x square y 2 dash, x square y 2 dash is x y 1 x. x square y 1 dash of x log x plus x square y 2 dash, this one so x square if you do n is from 1 to infinity, d n x power n plus 2 plus if you do it here, so this will be, again you are multiplying with x cube, x square. So x square, x cube. So you have n is from 1 to infinity, n d n x power n minus 1 multiply x square and you have one more x here, so plus 3. So plus 3, it is going to be plus 2. That is what my x square y 2 dash.

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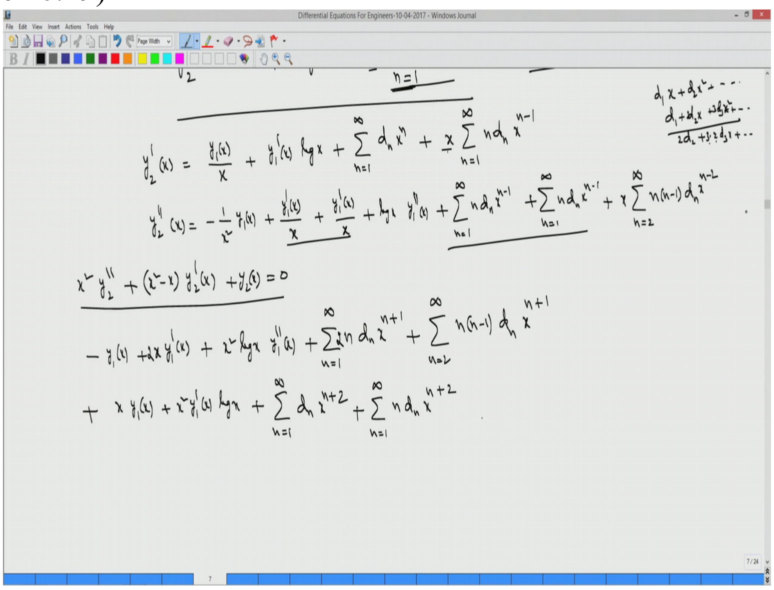

So I have minus, minus x y so minus y 1 of x multiplying x Ok minus x y 2 dash minus x y 1 dash log x minus you have x, you are only multiplying x, so you have n plus 1, n is from 1 to infinity. Similarly I have minus x into y 2 dash so that will give x square, so x square starting from 1 to infinity n d n x power n minus 1 plus 2 so it is going to be n plus 1. So this is my second, third term.

So that is minus x y (()); plus

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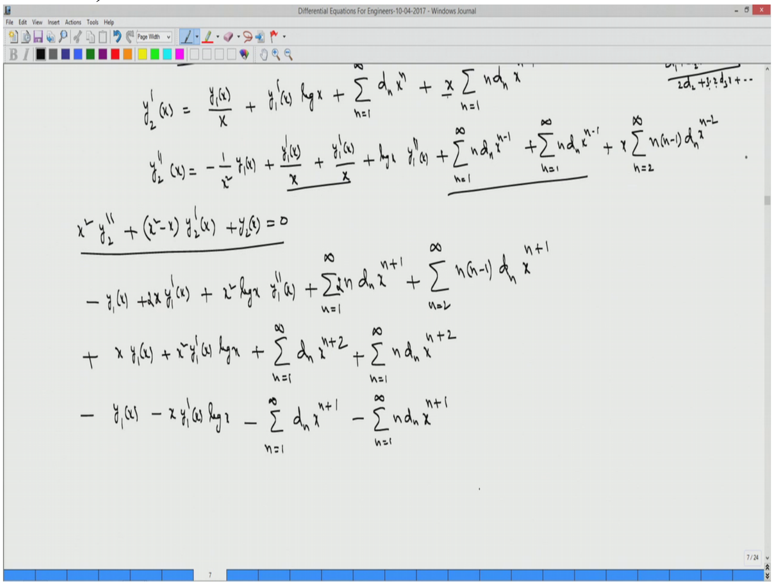

y 2. So what is your y 2, y 2 you already know, that is, y 1 x log x plus sigma n is from 1 to infinity d n x power n into power x so you have n plus 1. This is equal to zero.

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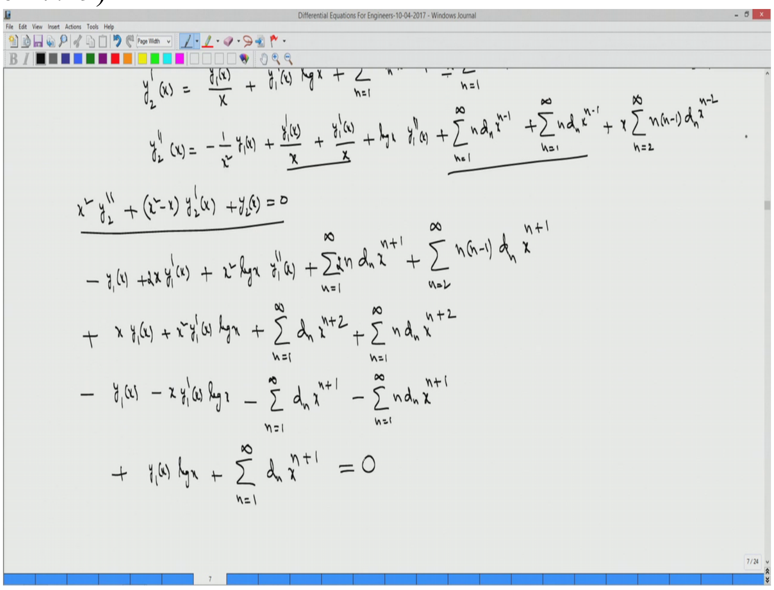

This is what if you substitute. So you see this, this term, this term, log x, the coefficient of log x, these 4 terms will be zero because y 1 is satisfying the equation.

So if you actually take log x out what you get is x square y 1 double dash of x plus x square y 1 dash of x minus x y 1 dash of x plus y 1 of x. This together x square minus x into y one dash of x. This is actually given equation, Ok. And this is zero.

So these four terms will become zero. So what you are left with is

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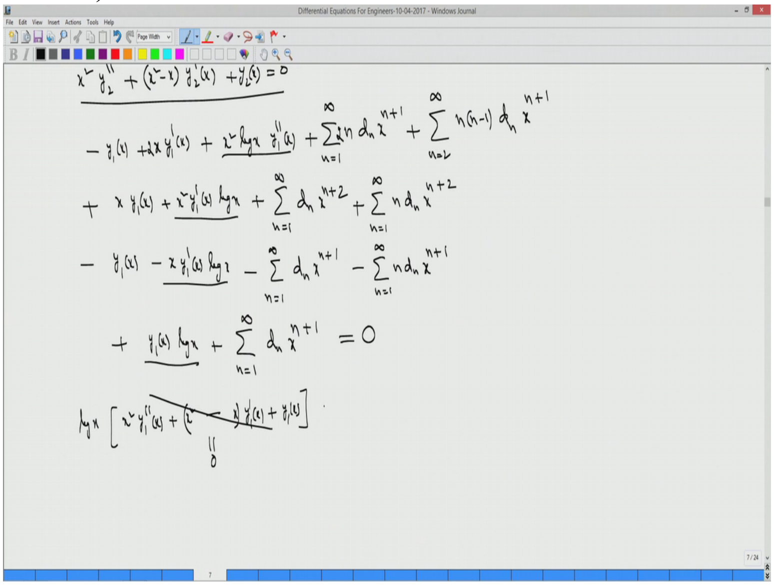

y 1 x, y 1 x I have minus y 1, minus y 1 so you have minus 2 y 1 x and then you have 2 x y 1 and of course I have plus x, plus x y 1 x and then these three terms I have written and look at the y 1 dash terms. So I have only 2 x y 1 dash of x. This is what you have and then write all other series terms,

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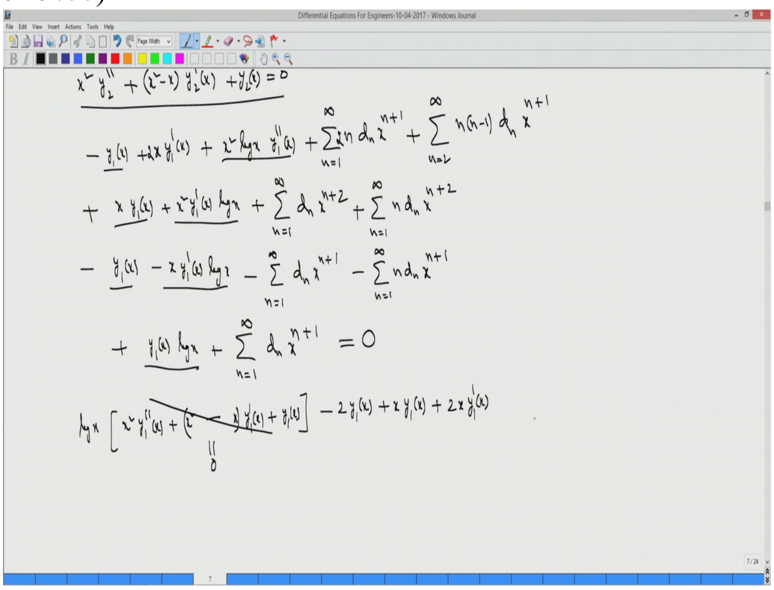

like 1, 2, 3, 4, 5, 6, 7. 7 series terms, some of them may go simply by, you can cancel them. For example n equal to 1 to d n x power n plus 1, this is minus and this is plus. This goes.

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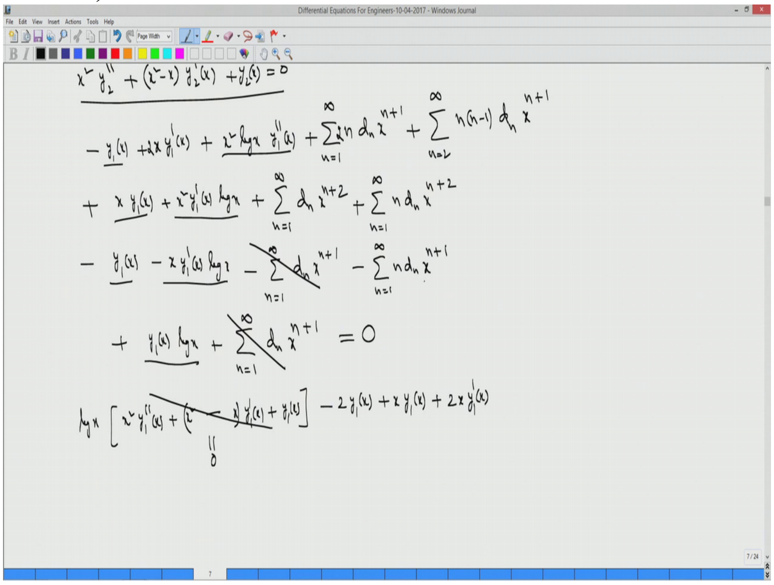

And here, n into n d n x power n plus 1, 1 to infinity and I have two times, so this is going to be this together, this and this I can write first as plus n d n x power n plus 1, n is from 1 to infinity. So these together will give me

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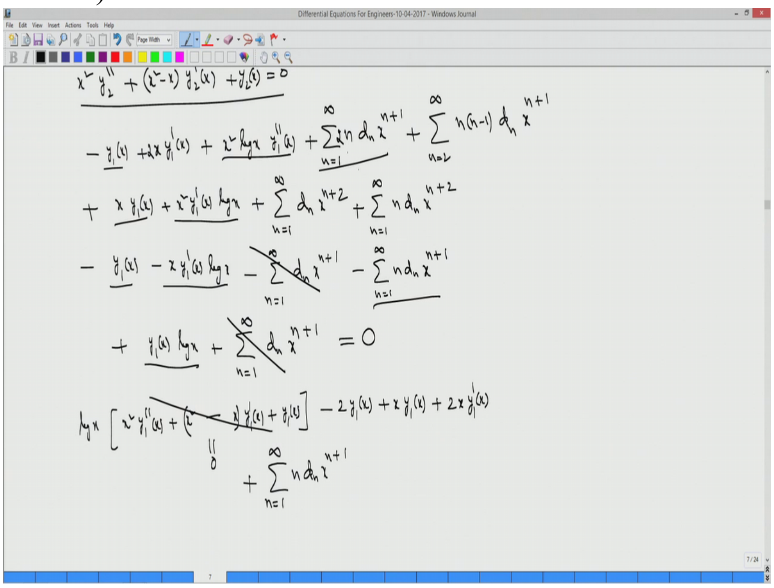

1, Ok. This I have written, now these two together. So what is left is only these three terms. This is minus n d n, that will not go. So let us write it as it is. So this is n is 2 to infinity, n into n minus 1 d n x power n plus 1 plus n is from 1 to infinity d n x power n plus 2 plus sigma n is from 1 to infinity n d n x power n plus 2. I have, so 2 I have written, 2 cancel so I have 4 so 3, 1, 2, 3, 4 so is equal to zero.

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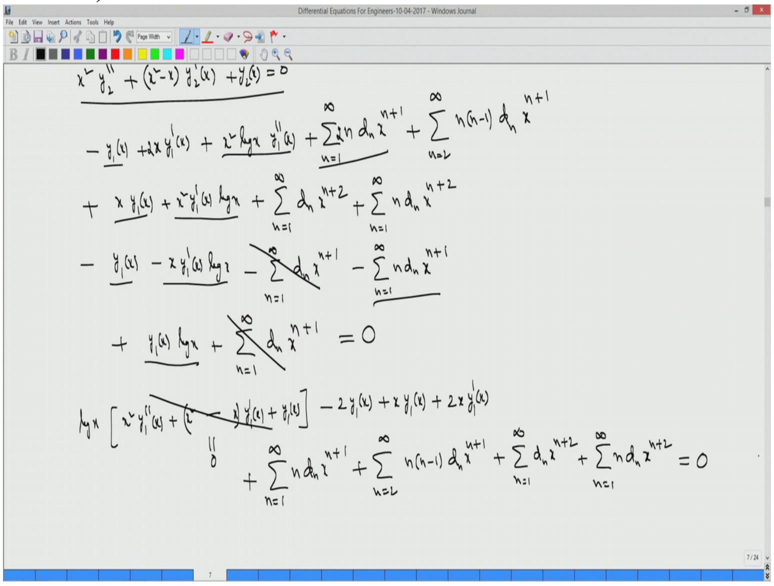

This is what you have. So the equation is this. This has gone now this is the equation.

So you know what is your y 1.y 1 is simply,

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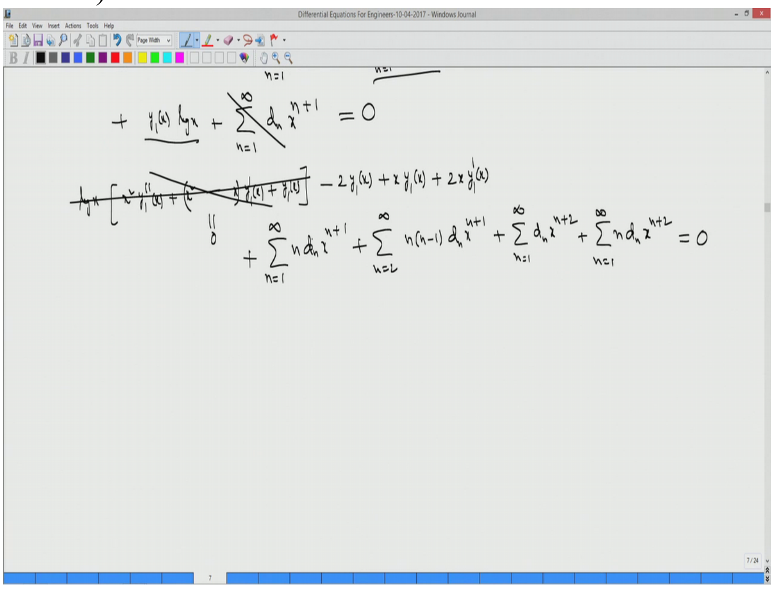

so if already know y 1, y 1 of x is x, x into e power minus x Ok, so you have x minus x square by 2 or x square by 1 factorial plus x cube by 3 2 factorial plus minus x power 4 by 3 factorial and so on, that is what you have, right.

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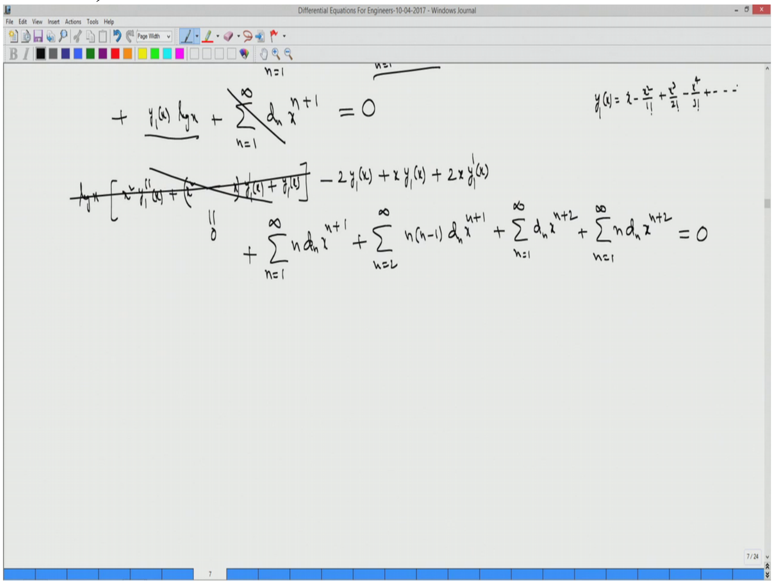

That is exactly your x into e power minus x. That is your this thing, Ok.

So if you differentiate y 1 dash of x, so assume that you don't know

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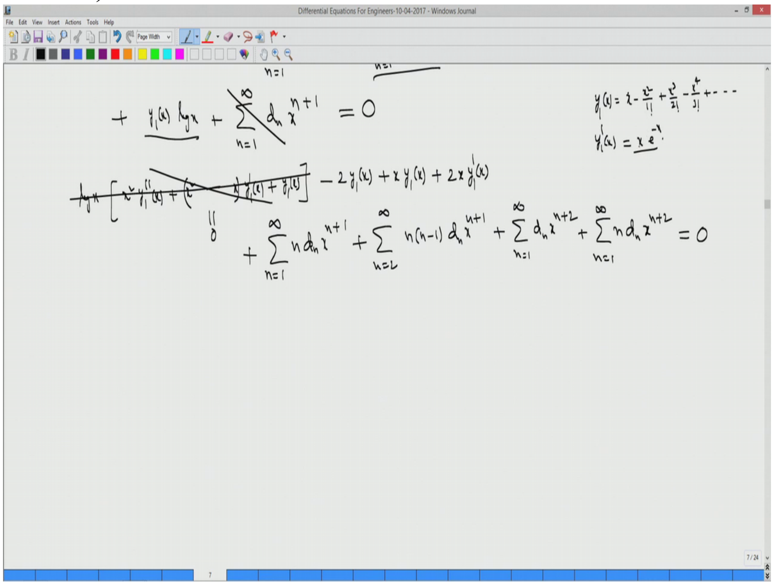

this form, Ok. So what you do, you differentiate, you need derivative of y 1. So that is going to be minus 1, 2 x plus 3 x square by 2 minus 4 x cube by 3 factorial and so on.

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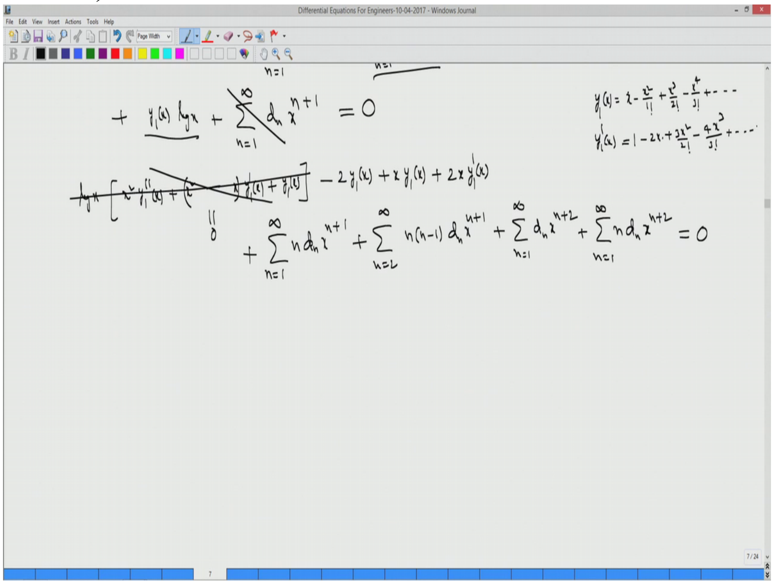

That's all, you substitute here now. If you substitute, so let me write this. So minus 2 times y 1 I write it as minus 2 x plus 2 x square minus 2 x cube by 3 factorial, 2 factorial, Ok minus 2 minus minus plus 2 x power 4 by 3 factorial and so on. That is your first series.

Now I write x into y 1. So instead of

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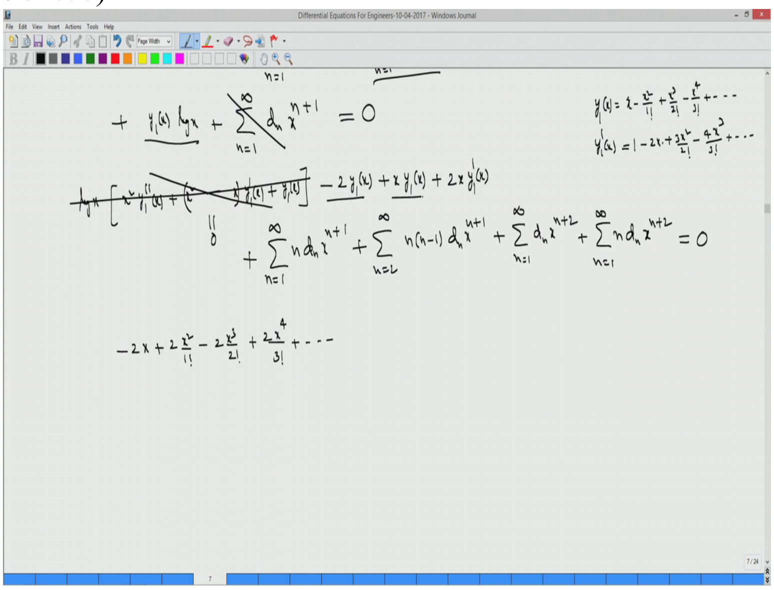

minus 2, I put x. So we have x square minus x cube by 1 factorial minus plus x cube by so x cube will be x power 4, we are multiplying with x, x by 2 factorial plus minus x power 4 by x power 5 by 3 factorial and so on. That is your second term.

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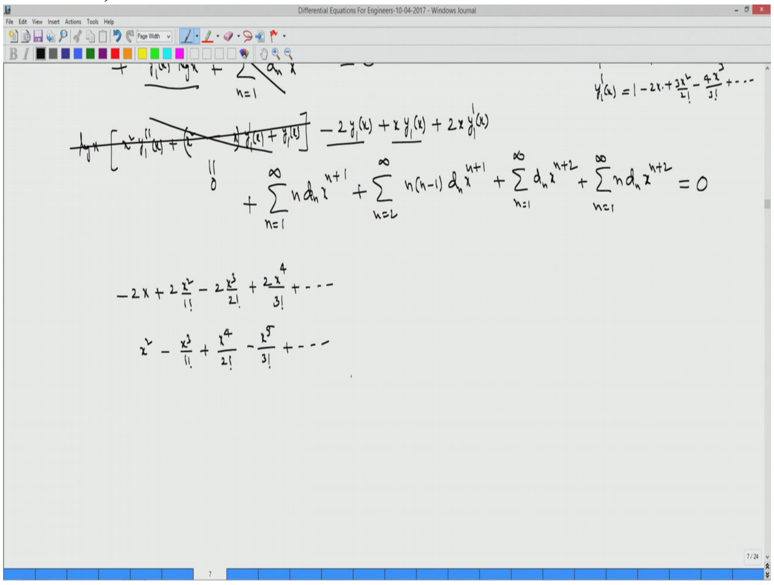

Now you multiply with 2 x with y 1 dash. So you have 2 x minus, simply multiply 2 x, you have 4 x square plus 6, 2 x right, you are multiplying with 2 x, 6 x cube by 2 factorial minus 8 x cube by 8 x power 4 by 3 factorial and so on.

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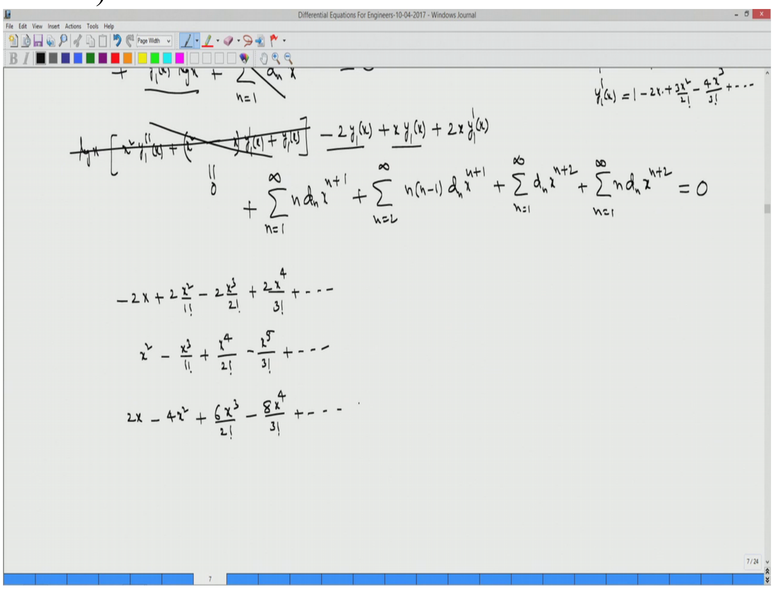

This is what you have, Ok. Multiply 2 x for this y 1 dash.

Rest you write as a series, 1, 2, 3, 4. So four more series you write, so n equal to 1,so you have d 1, starts with d 1 x square plus d 2, 2 d 2 x cube, 3 d 3 x 4, 4 d 4 x power 5 and so on.

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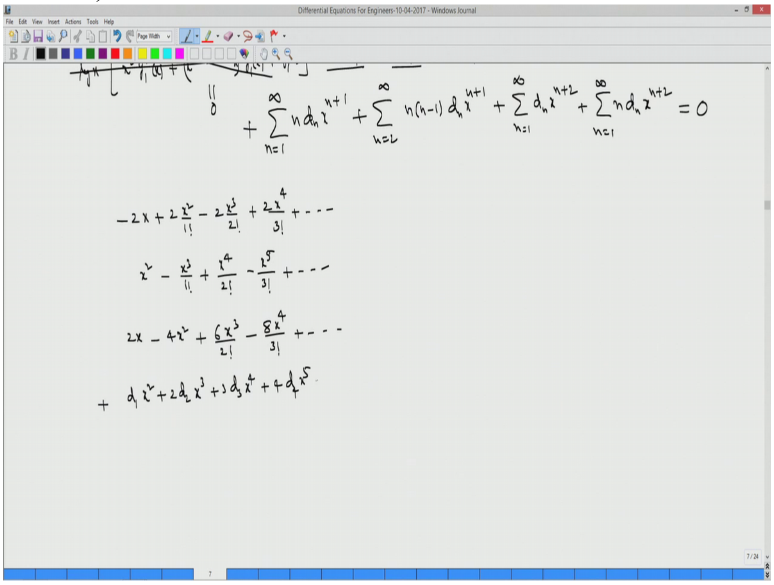

That is your this term. So what happens to this? n equal to 2 is going to be 2 d 2. x power x cube plus n equal to 3, 6 d 3 x power 4 plus n equal to 4, 12, 4 into 3, 12, 12 d 4 x power 5 Ok and so on. That is this term.

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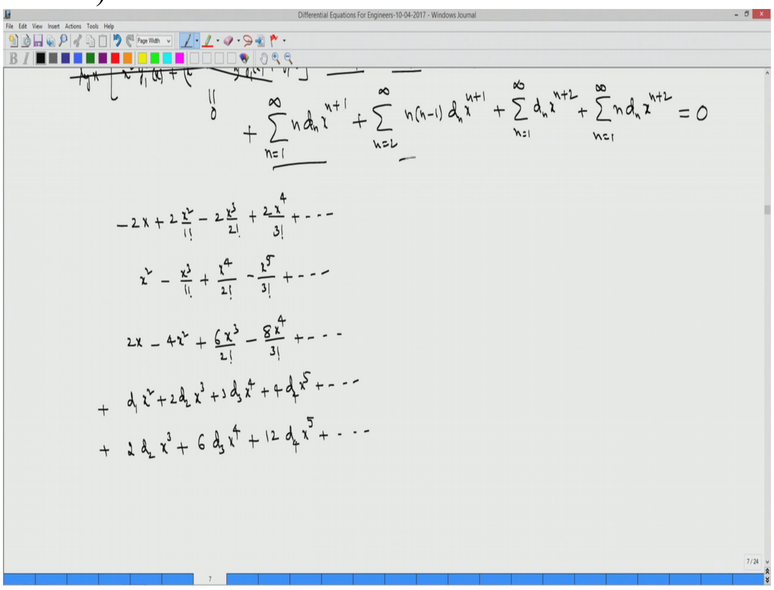

Now what about this, this will give me n equal to 1 d 1 x cube plus d 2 x 4 d 3 x 5 plus d 4 x 6 and so on. Where is my penultimate term? So the last term is series n equal to 1, so we have d 1 x power 3 plus 2 d 2 x power 4 plus 3 d 3 x power 5 4 d 4 x power 6 and so on. So now I have written everything, Ok, this whole thing equal to zero.

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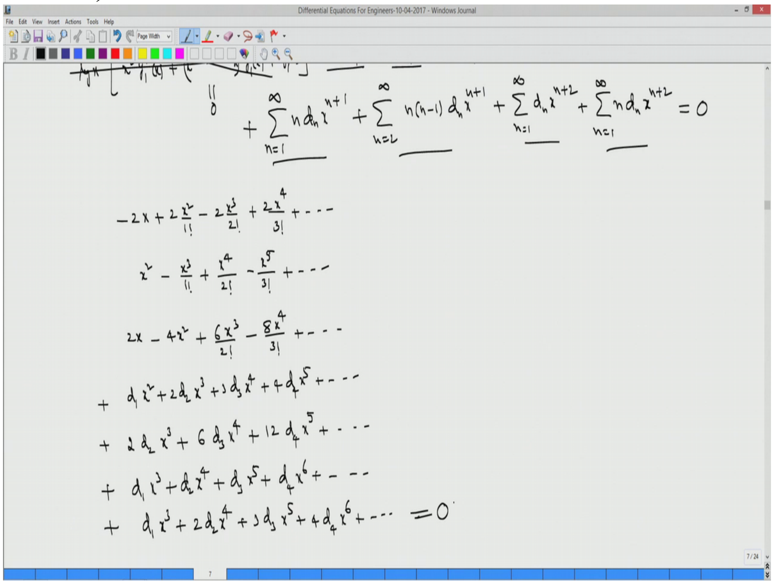

Now what is that? So you have something like, so you see that some series, you only have some series form, some series equal to zero, c n x power n equal to zero. Ok. Possibly some negative terms, so n is from may be some fixed number let us say, minus l to infinity.

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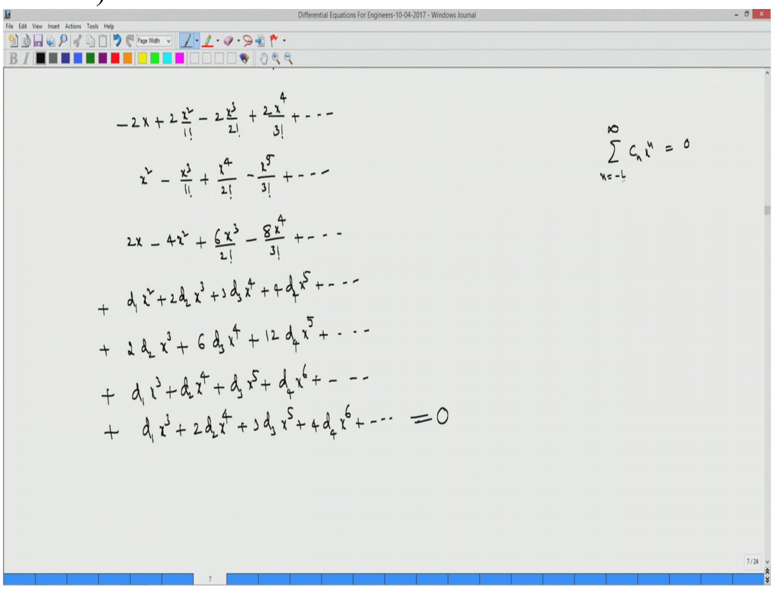

So all these c ns are zero from n is from minus l to infinity, positive Ok, up to zero, minus 1, zero, 1, 2, 3 are not, Ok like this.

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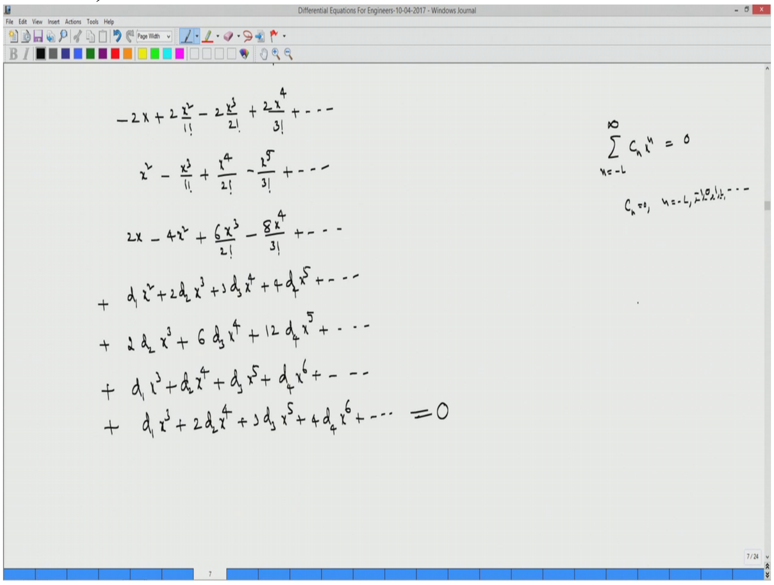

So that is what you have to do.

So the coefficient of, what is the lowest coefficient? It is the constant. Do you have the constant? No I don't have any constant here, right? Everything involves the power. So lowest power is x which is zero. Make it zero. So if you do that, minus 2

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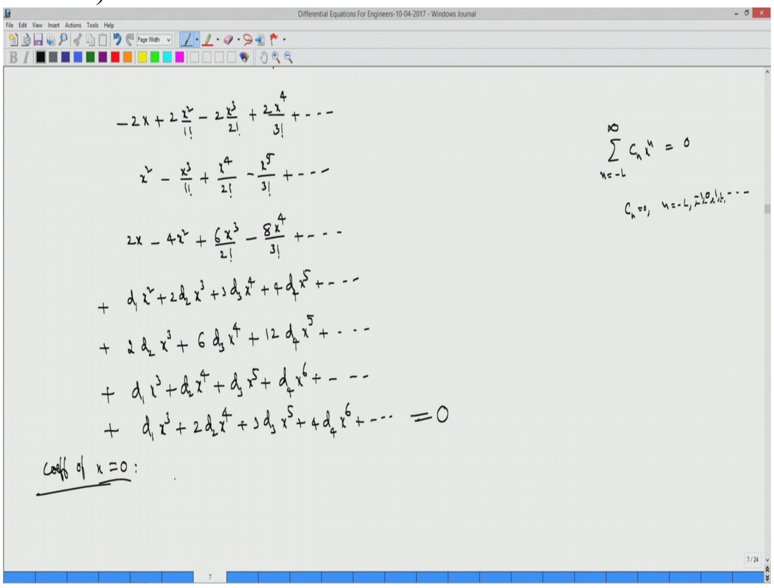

plus 2 x plus 2 equal to zero, why? So I don't get anything, nothing I get, Ok?

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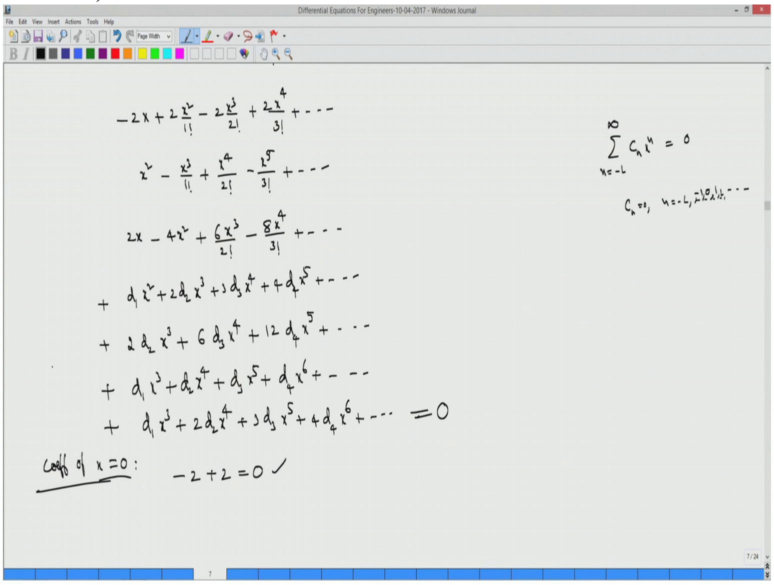

Right? So there is no x here.

Coefficient of x square equal to zero will give me, so you have 2 there

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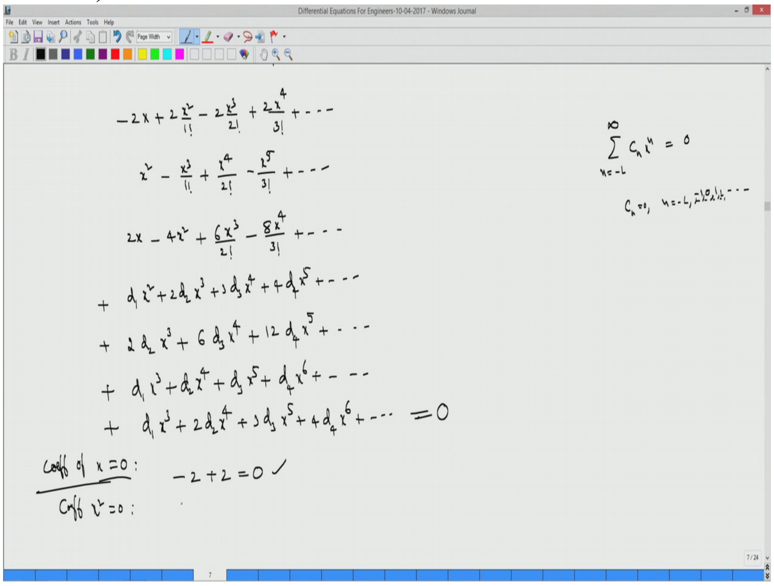

plus 1 minus 4 plus d 1 equal to zero, so that will give me 3 minus 1, so d 1 equal to 1,

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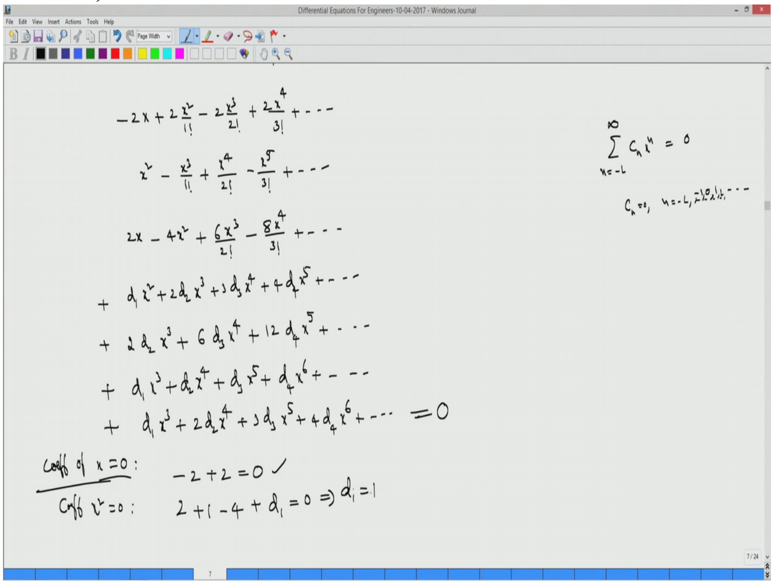

Ok. And then what happens to, what happens to coefficient of x cube? Coefficient of x 3 is zero. Now you make it zero,

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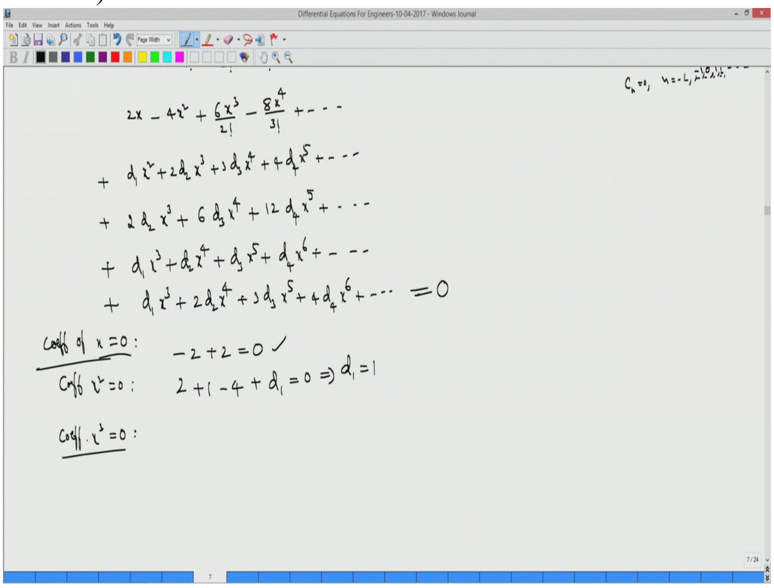

so you get 3 from here, 3 plus 2 d 2 plus 2 d 2 plus d 1 plus d 1 equal to zero. So I know what is my d 1. So you have 4 d 2, d 1 is 1, so you get 2 plus 3, so 5 minus 5, so you get d 2 as minus 5 by 4,

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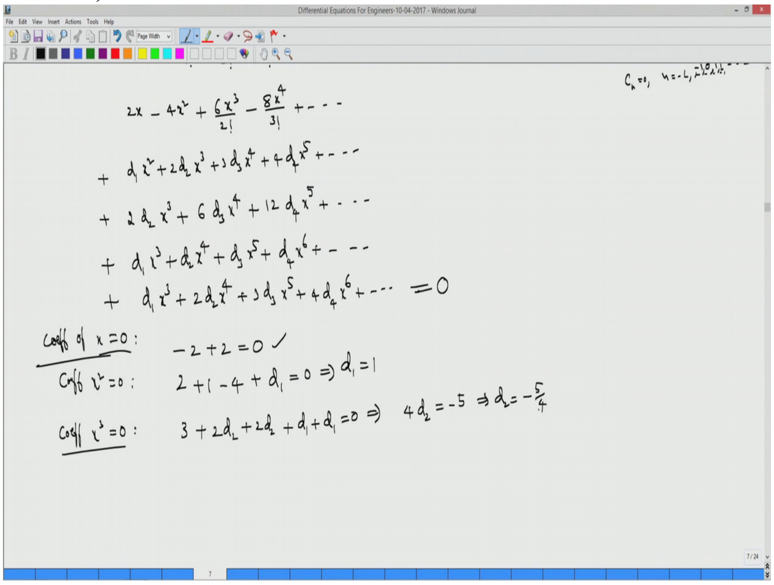

right? So 3 plus 2. Now you have coefficient of x power 4. Make it zero. This will give me

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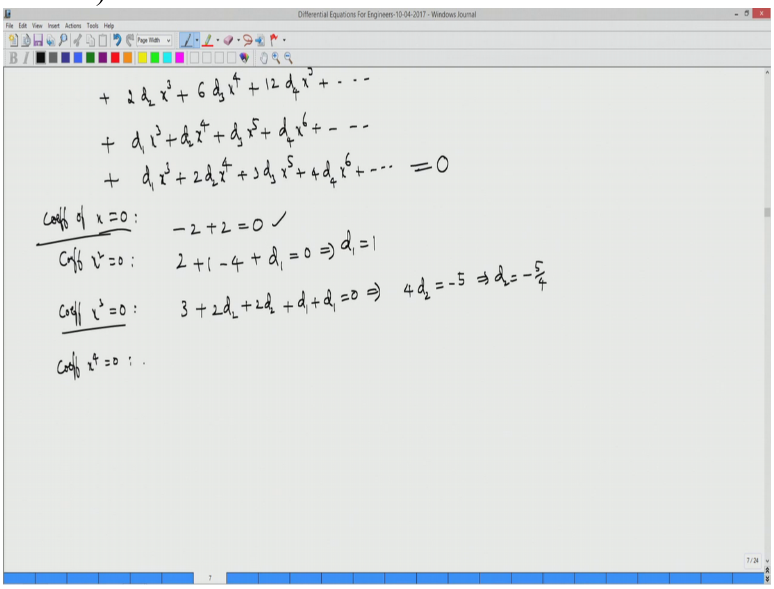

coefficient of 4.

So coefficient of 3 is gone right? Oh no, no wait. So I think I missed one term. Coefficient of x, this one zero, coefficient of x square I have 2 1, 3 minus 4 plus d, that is Ok. Coefficient of x cube minus 2, minus 1, minus 1, Ok, so you have minus 2, you have minus 2, minus 2 plus 3 plus 3 plus 2 d 2, this is what you get. So you have 2, 2 goes and you have 3. So minus 3, this actually minus 3 by 4,

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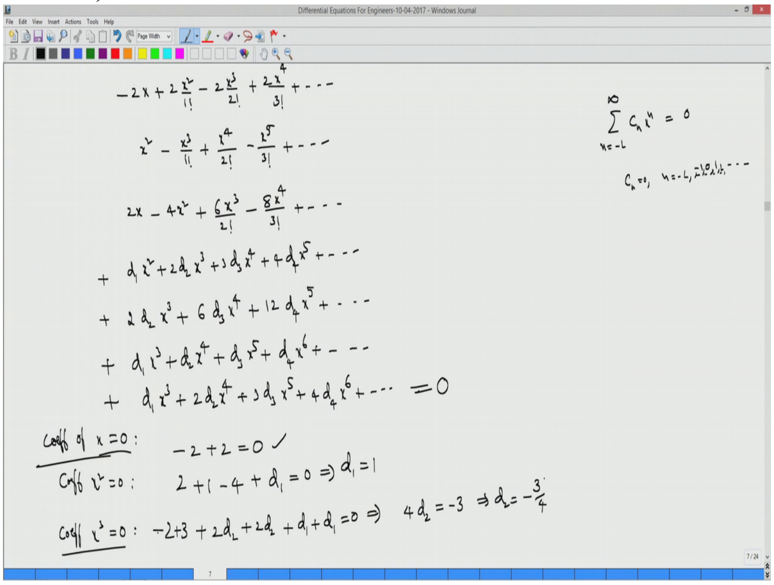

And now coefficient of x power 4 you get 2 by 6, first one is 2 by 6, 2 by 6 plus half, 2 by 6 plus half that is your second series. And then minus 8 by 3 factorial, so that is 4 by 3, minus 4 by 3, now you have coefficient of x cube is 3 d 3, 6 d 3 plus d 2 plus 2 d 2 which is equal to zero. So this exactly will give me, I know my d 2. d 2 is 9 d 3 plus 3 d 2. So 3 d 2 is, 3 d 2 so this will give me minus 9 by 4.Then this will give me 18, 6 plus 9, 24.Ok so what is the value? So here 15 minus, so 15 is 9, minus 9. So minus 9 plus 9, so it is going to be half, equal to zero.

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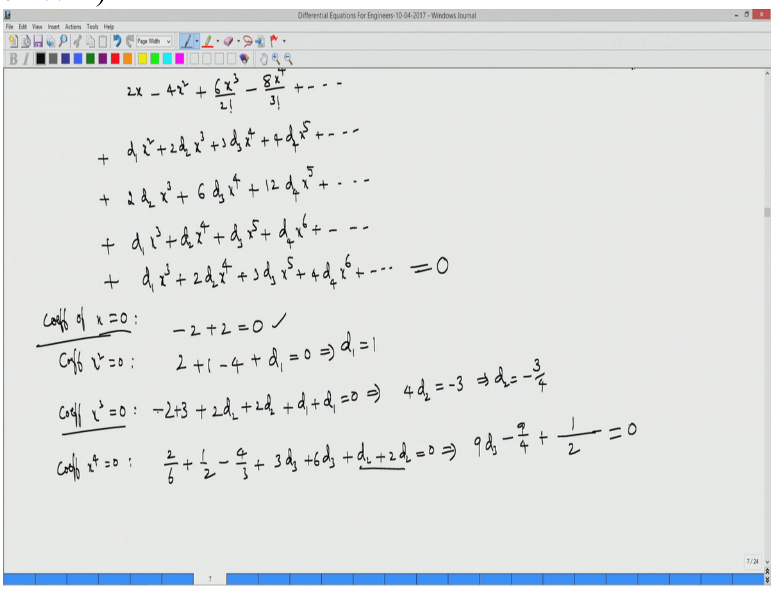

So this will give me my d 3 as, so you have 2 minus, so what you get? You get, so you have 4, you can make it 2 here. So it is going to be minus 8 by 4, so minus half, right, no minus 7. It is going to be minus 7 by 4, so 7 by 4 divide by 9, so 7 by 36,

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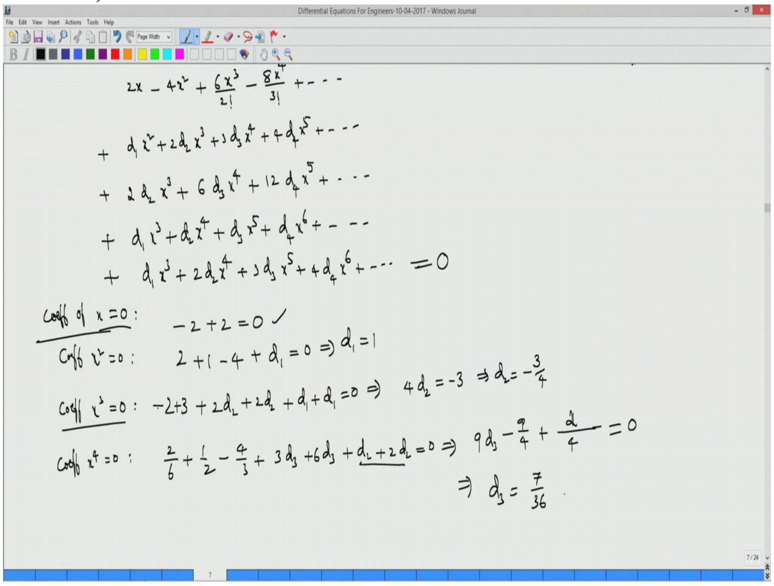

Ok so that is what you get. So d 3.

So like this you can go on and so on.

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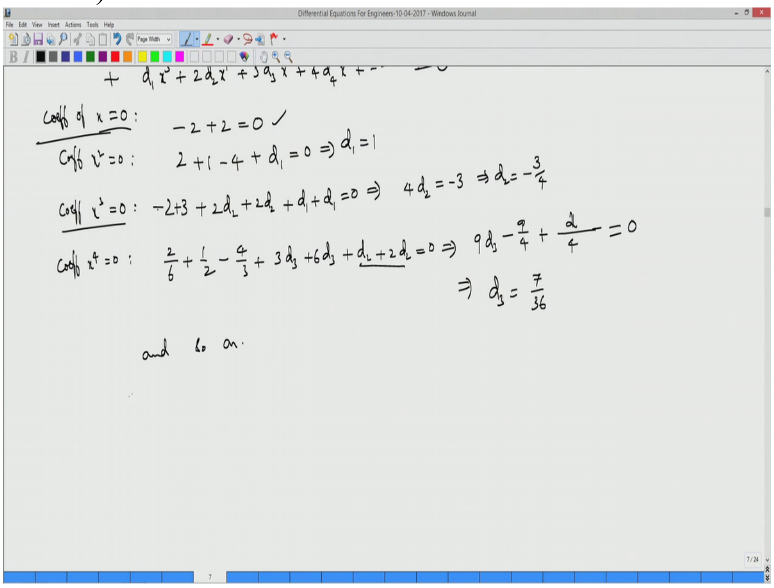

So what is your y 2 of x? y 2 of x is y 1 x into log x, there is nothing to find here, plus x times, So you have d 1, d 1 x right, so n is running from 1 to, 1 to infinity. So you have d 1 x plus d 2 x square plus d 3 x square x cube and so on.

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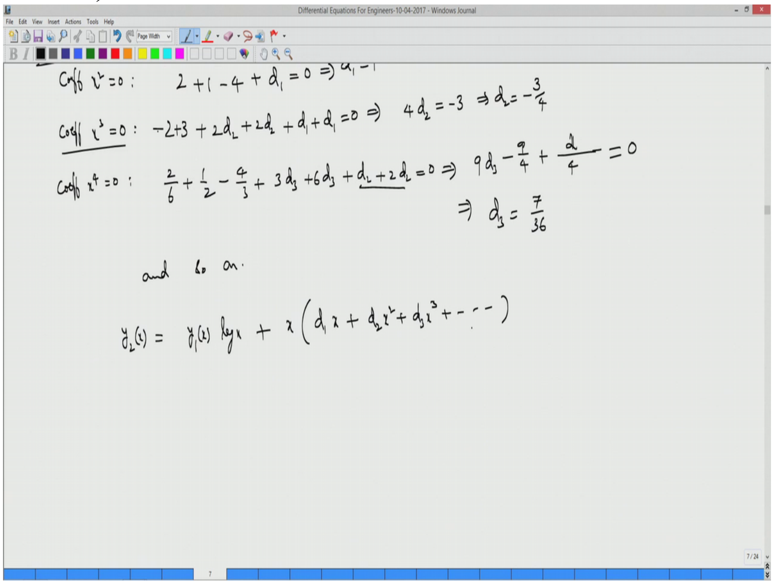

So what you get is, y I know is e power minus x log x. If you don't know this form, you can write it as a series, Ok. This series into log x plus d 1. So d 1 is 1. d 1 is 1, so you have x square plus d 2. d 2 is minus 3 by 4, minus 3 by 4 x cube plus 7 by 36 x power 4 d 3 and so on. So this is your solution.

This is your

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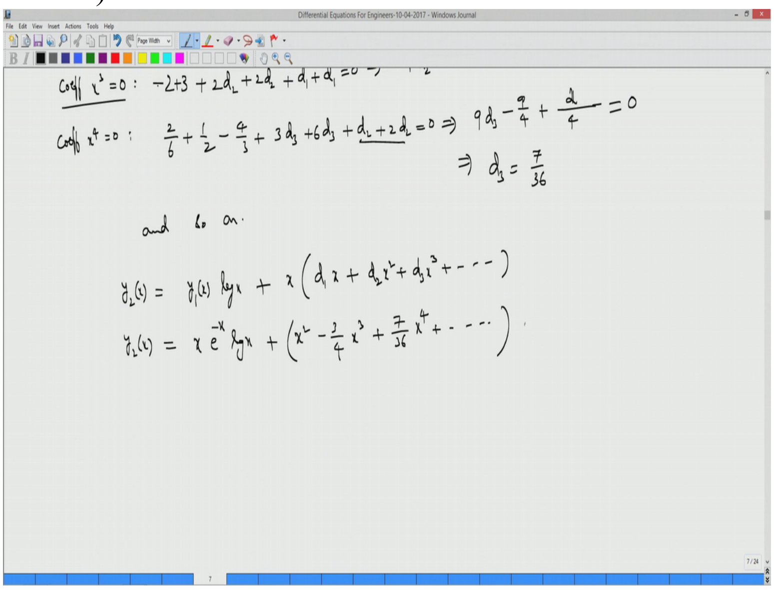

second linearly independent solution. So having known your y 1 as x into e power minus x, you got second linearly independent solution in this, for this given equation is this one. So, so this is how you can find, if your roots are same for the indicial equation, get your first solution as the series solution, usual series solution and the second solution, it will look for always in this form,

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y 1 x into log x plus to x power root into, now the series is, starting from 1 to infinity, Ok and like this you look for , you will be find all the constants involved, all the d ns you will be able to find, once you substitute this form into the equation, Ok?

So when the roots of the indicial equations are same, you can always follow this method. And in the process when you see that first equation, first solution is explicit form; you can go ahead and get the second solution by the linear method. If you know one solution, you can find the other solution by a formula which you know, by Abel's formula, Ok. So now we know any second order linear equation with variable coefficients. If you cannot find any indicial method, you can go ahead and find, when x equal to zero is regular singular point, you can find solutions either side, x positive or negative side, Ok. Two linear independent solutions always find and then finally you can write the general solution as linear combination of this y 1 and y 2, Ok?

So we will see, we will apply this Frobenius method, we can apply this Frobenius method to find the general solution of equation when zero is a regular singular point for certain good, some practically very important equations like Bessel equation. So we will apply this to the Bessel equation in the next video and see how the solutions of Bessel equations, they are known as Bessel functions, Ok, so we will see in the next video.