Basic Algebraic Geometry
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Module 05
Lecture No 41
Any Variety is a Smooth Manifold with or without Non-smooth Boundary

Let me make a couple of remarks. See,

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what we did was we start with X, an affine variety. It is irreducible closed subset of affine space,

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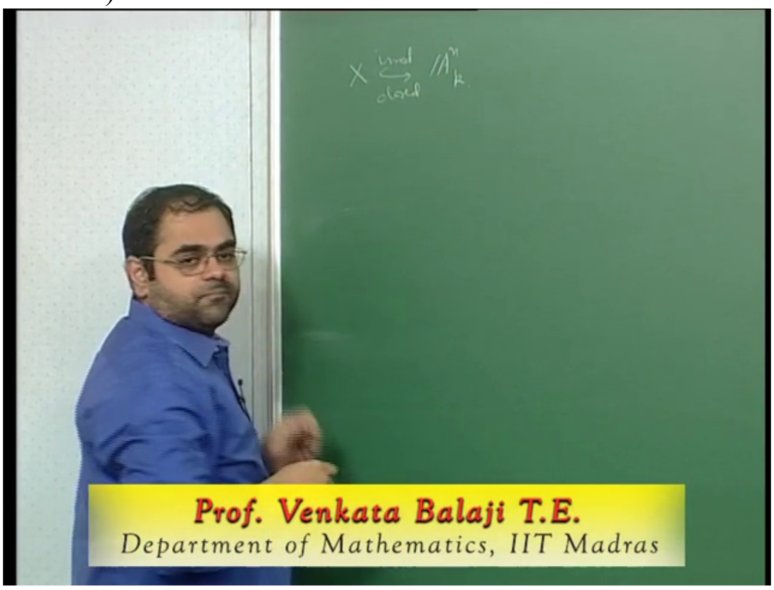

Ok and you, and we took a point P of X, the point given by coordinates lambda 1 to lambda n, alright.

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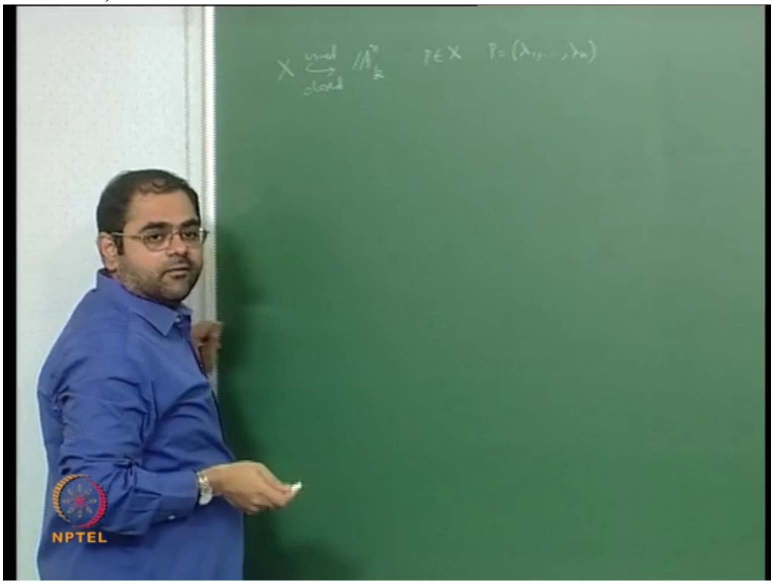

And we had of course, the affine coordinate ring of X is, just the affine coordinate ring of, affine space divided by the ideal of X,

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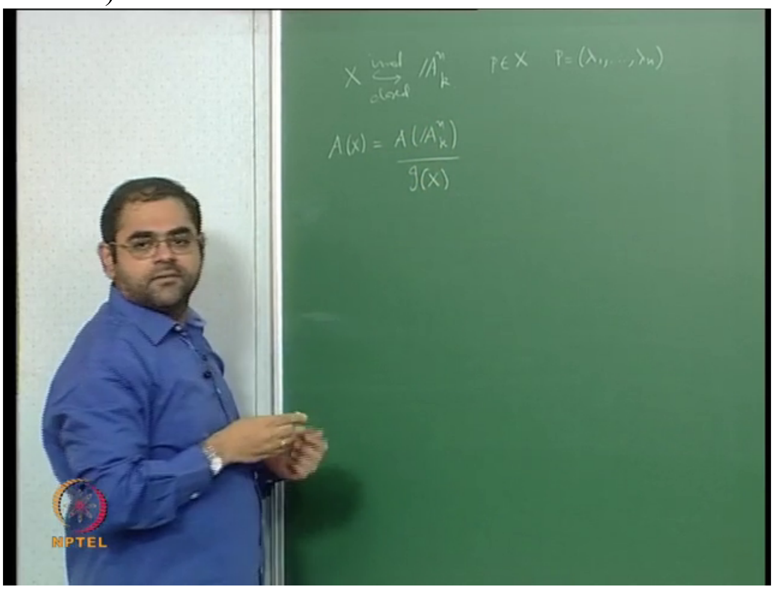

Ok. And that is just k X 1 et cetera X n divided by the ideal of X, Ok.

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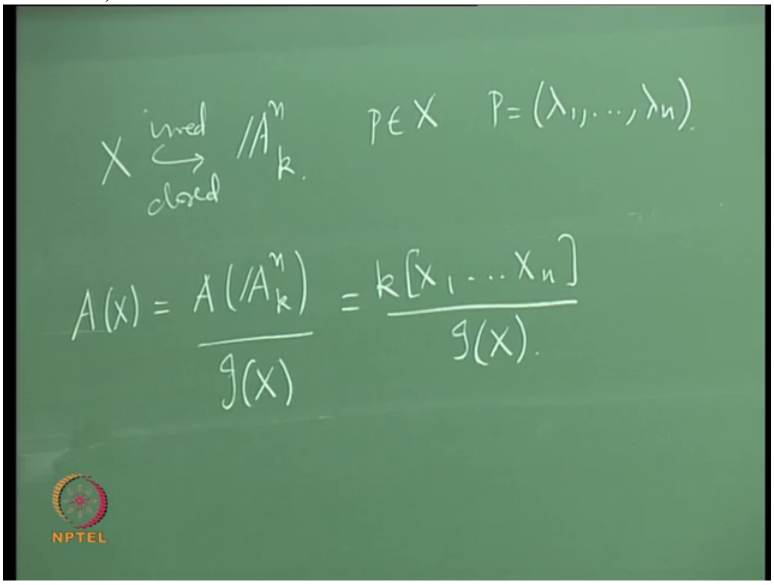

And of course if you calculate the, so you know the, there is the maximal ideal corresponding to the point P in the, this is the maximal ideal of P in the whole, the whole affine space, Ok, the affine coordinate ring of the whole affine space.

And then so we defined, we defined the essentially the gradient function, the function that associates to each

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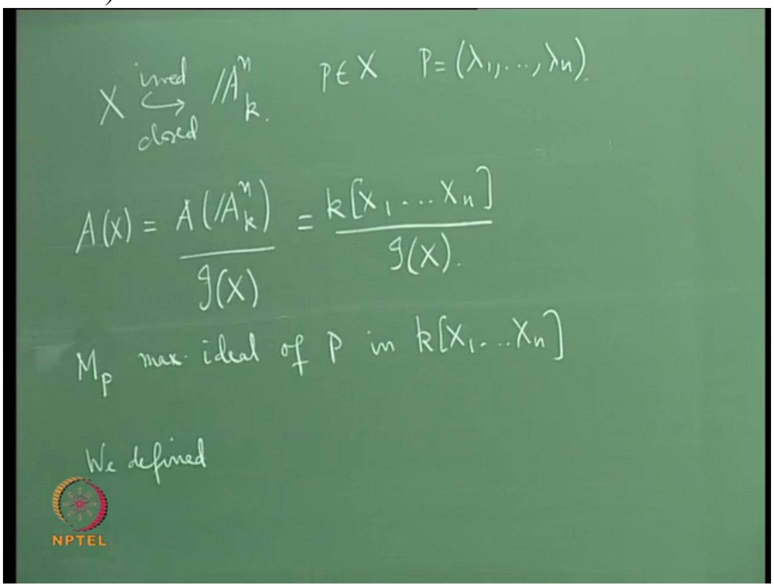

polynomial its gradient at the point P which is given by, you know k X 1 through X n to, I think I called the map psi, and that takes, so this is into k n which takes any g to, you simply take the gradient, namely you take the partial derivatives of g

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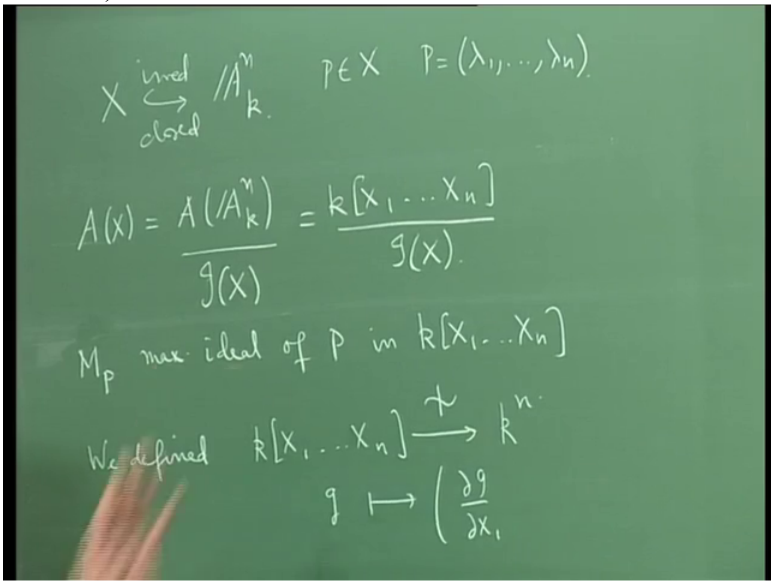

with respect to these n variables and you evaluate at the point P.

So this is dau g by dau X 1 at P, and so on, dau g by dau X n at P and what we noticed

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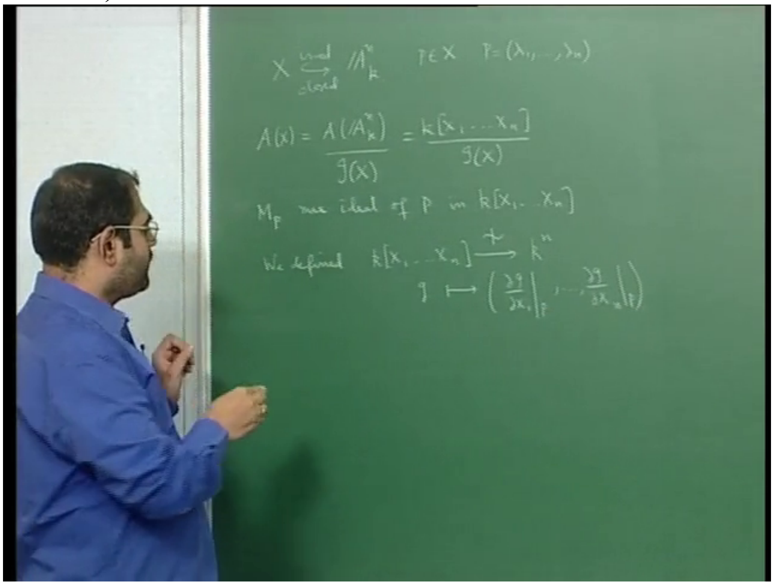

is that psi is k-linear, psi maps the generators x i minus lambda i of M P which is the subspace of the polynomial ring. Because everything that we are doing with, I mean, all rings we are dealing with, when we are worried about varieties, all the rings of regular functions

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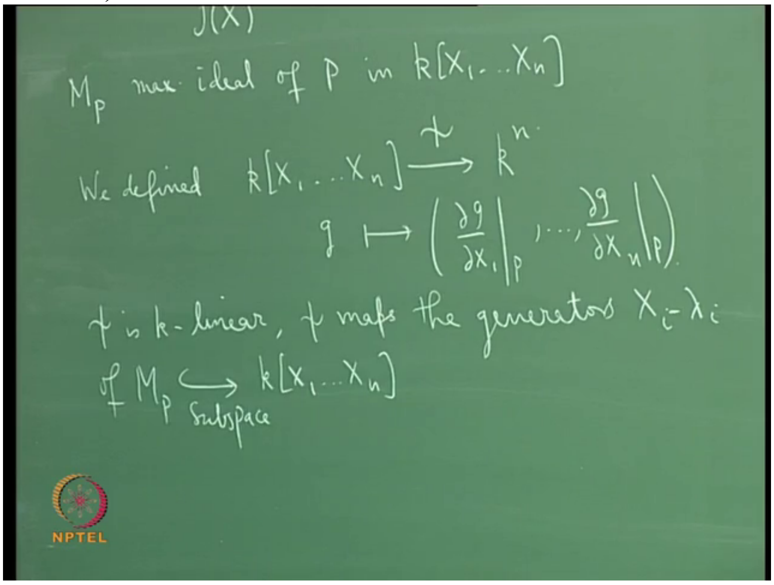

and affine coordinate rings, they are all k-algebras, Ok and therefore their ideals, ideals in such rings will automatically become k subspaces, Ok.

And this M P is generated by these,

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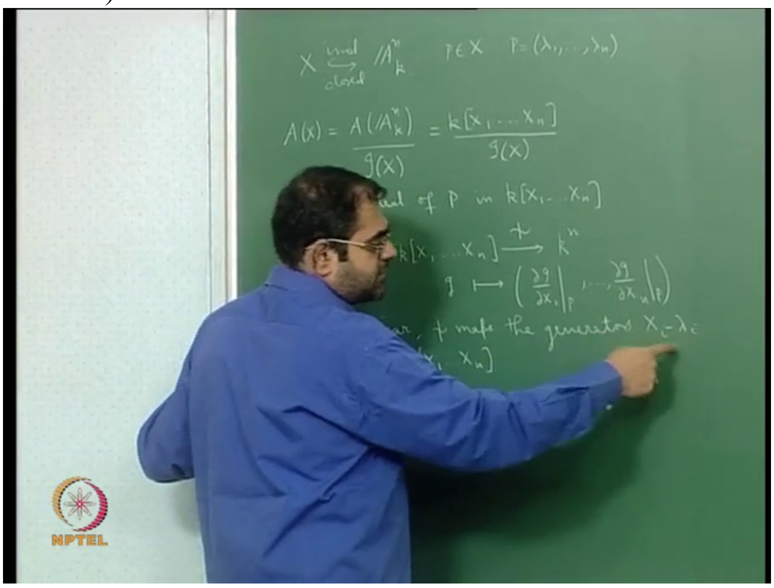

by these generators, and if you take the gradient of these at the point P, you will get the standard basis, Ok. So, so it means that the image, so the image of M P,

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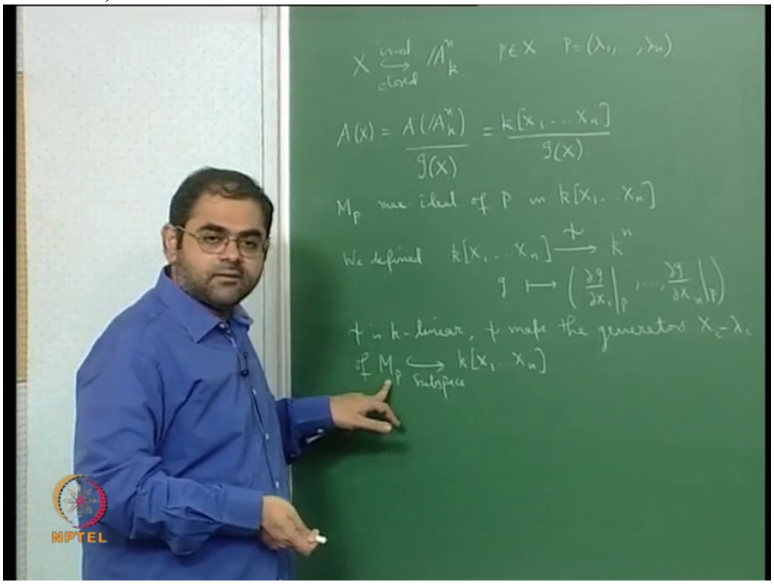

of the subspace M P contains

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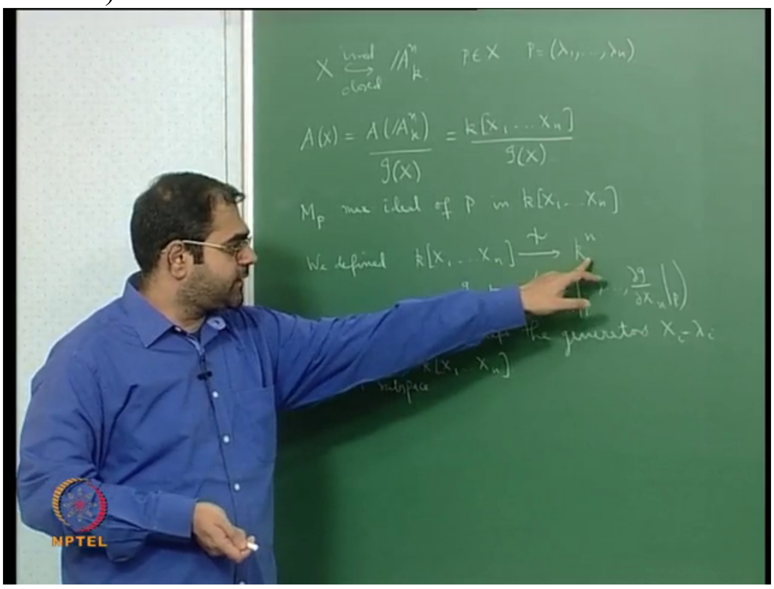

basis of k n, therefore it is surjective and what we proved is that the kernel is exactly M P squared, Ok.

psi restricted to M P, from M P to k n is surjective, that is onto, also kernel of psi restricted to M P is exactly M P squared, the square of this ideal which is, which is also a subspace, mind you, of M P and so what we got was an isomorphism, we got a k-linear isomorphism which I think I also called,

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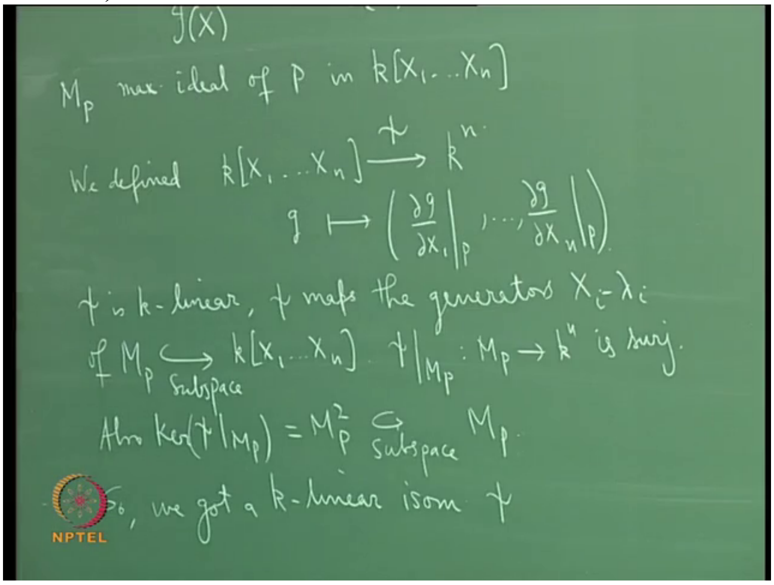

what did I call it? I also called that as psi probably, the k-linear isomorphism, from M P, mod M P squared to k n. psi dash. Ok.

So we got this psi dash which is from M P mod M P square to k n, alright

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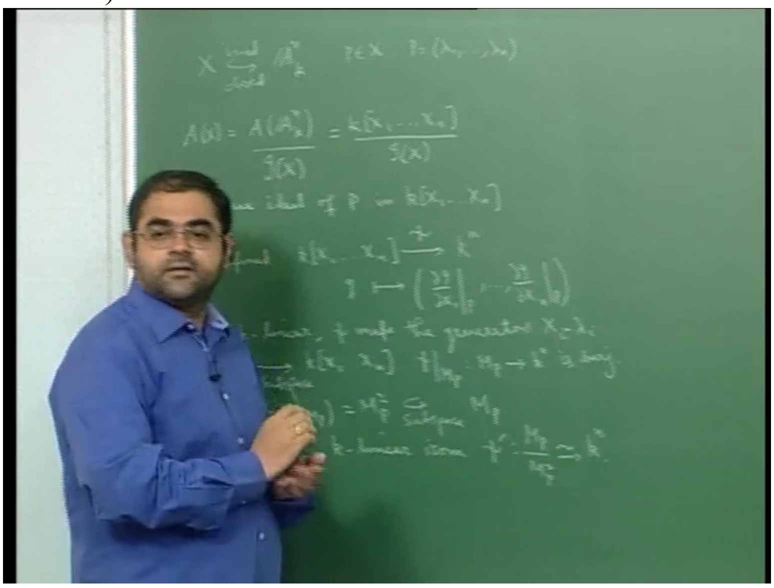

and now, now you see, now the, the fact was that the, so if you calculate, if you take I X, if f 1 et cetera up to f n are generic I X, you take a finite set of the generators for the I X, and that is true because I X is a, it is an ideal Noetherian ring, polynomial ring is Noetherian and therefore every ideal is finitely generated.

So I X is finitely generated and you pick a set of generators, then, then you know if you, if you calculate, then the rank of the Jacobian of this set of generators which is dau f i by dau x j at the point P, this

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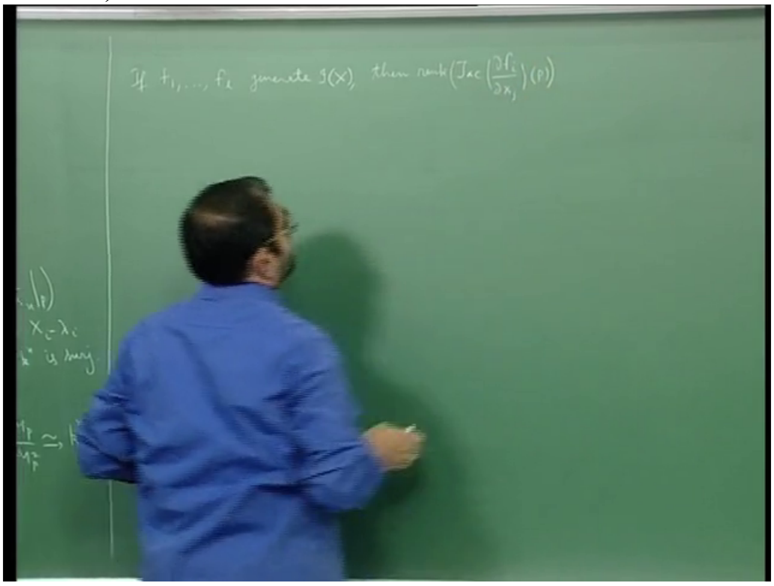

turns out to be exactly the,

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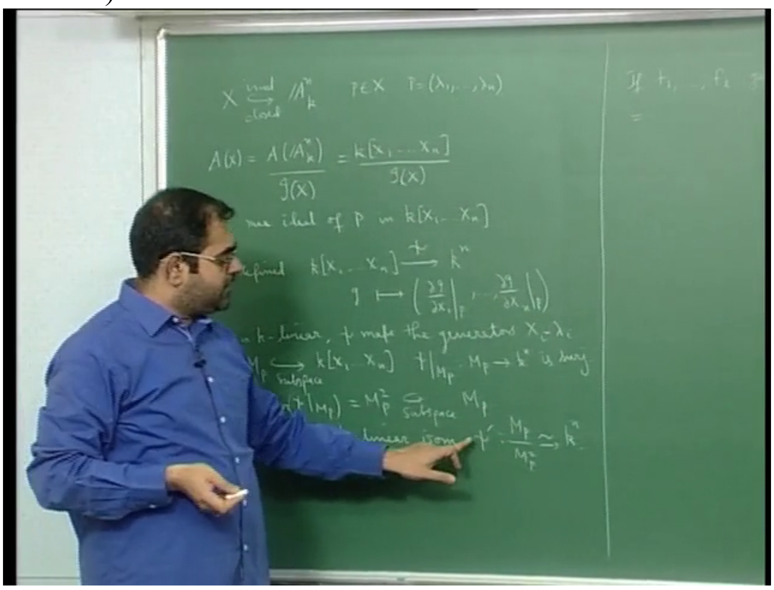

the image under psi prime of the subspace given by I X plus M P squared by M P squared. Ok, is equal to dimension, it is going to be dimension of the psi prime of I X plus M P squared over M P squared, which is subspace of M P over M P squared.

Mind you, I X, I of X is

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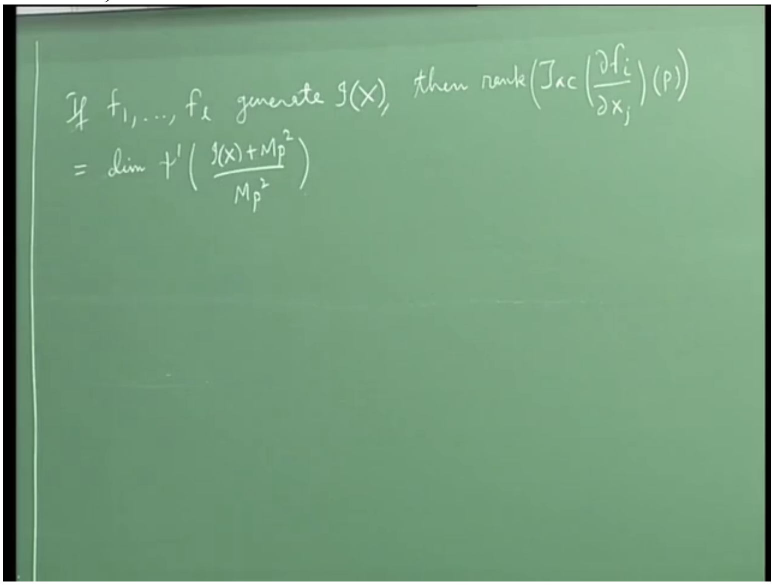

contained in M P because P is a point of X,

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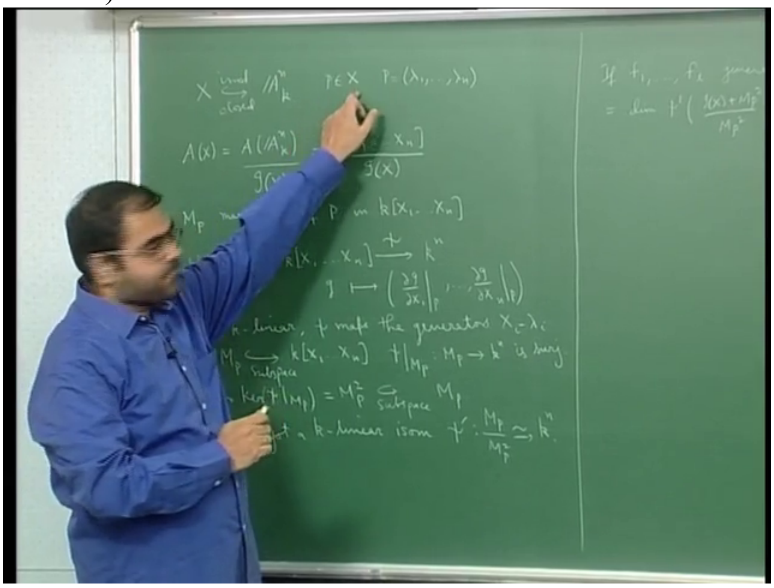

Ok. So this is a dimension of this. And this is well, this is one fact. Then the other fact was that, is that, if you take M P mod M P squared and you divide by the subspace which is I X plus M P squared mod M P squared.

The claim is that it is isomorphic, it is k-linear isomorphism of this vector space with M P mod M P squared, Ok where M P is the unique, is the unique maximal ideal, is the unique maximal ideal in O X P which is of course given, it is given by the localization of affine coordinate ring of X at the ideal, at the maximal ideal, that corresponds to P in X which is given by M P sub x, Ok.

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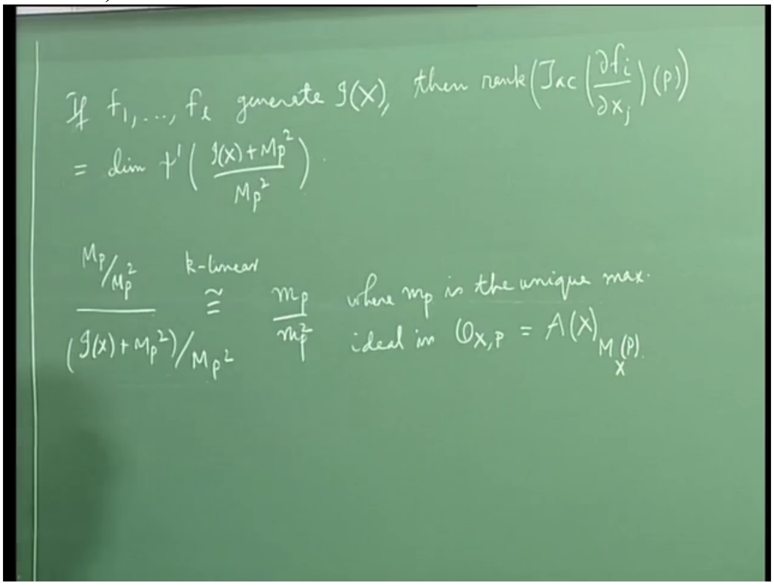

And where of course, M that is M P sub x is just, you know it is M P mod I X. It is the maximal

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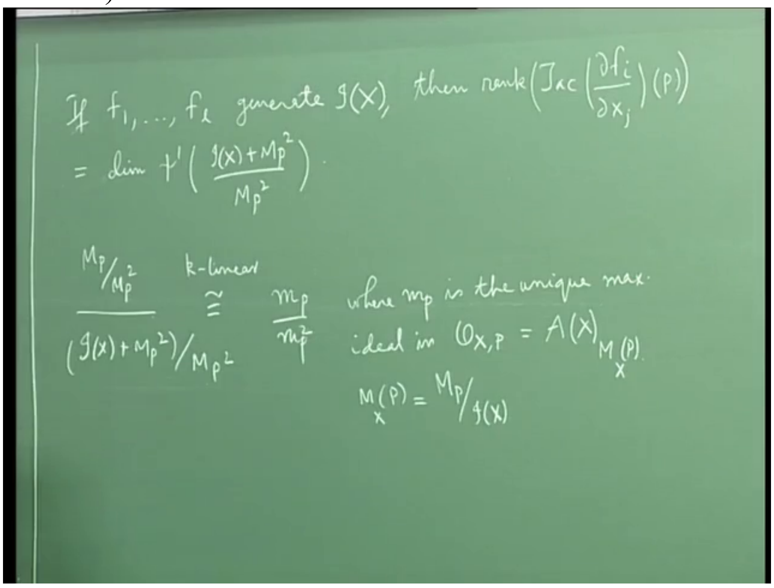

ideal in the affine coordinate ring of X corresponding to the point P, Ok. Just like points of,

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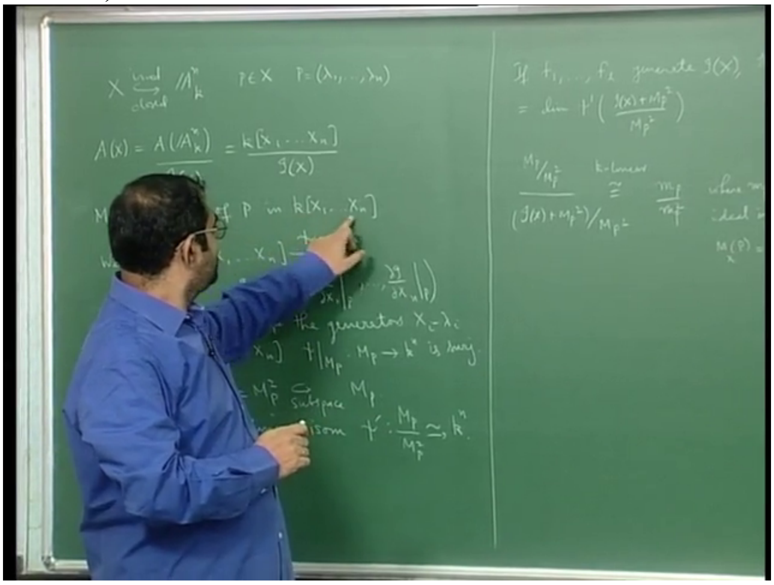

just like points of affine space correspond to maximal ideals. In this ring which is the affine coordinate ring of affine space, which is 0:10:56.4.

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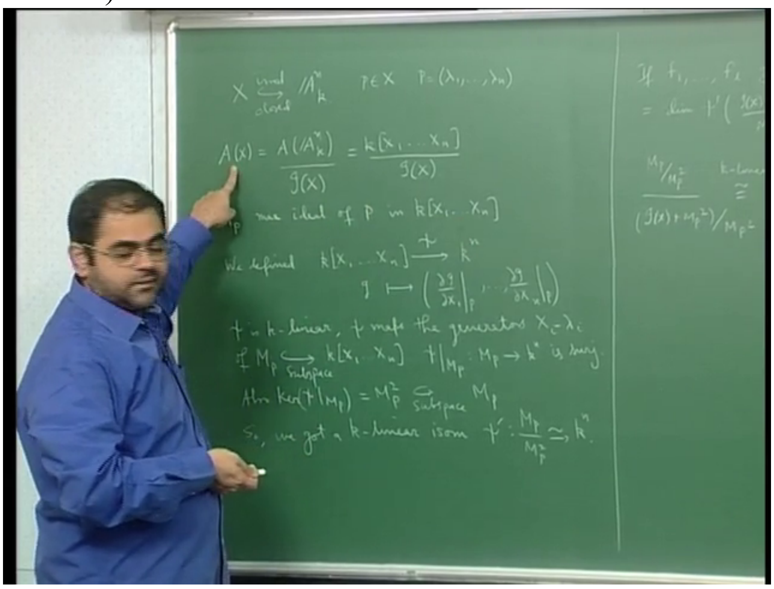

The same way the points of, the points of X are

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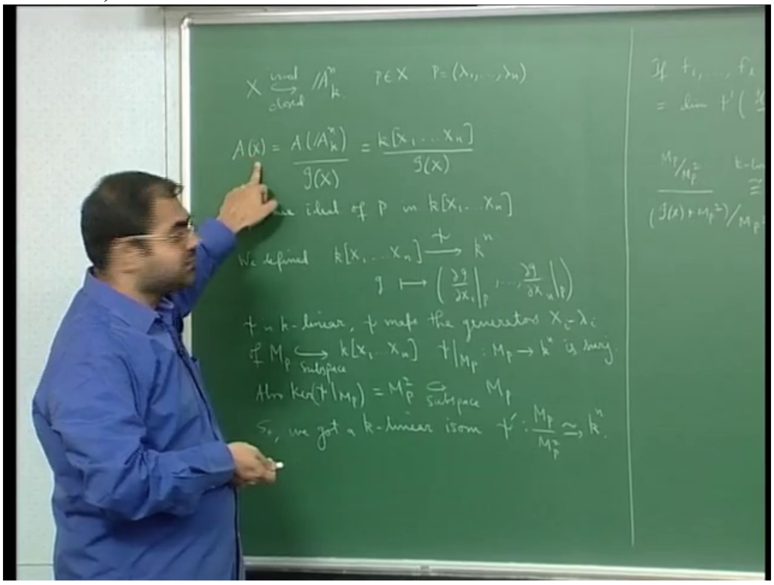

in bijective correspondence with the maximal ideals of A x, Ok and that, the corresponding maximal ideal of A x is governed by taking a maximal

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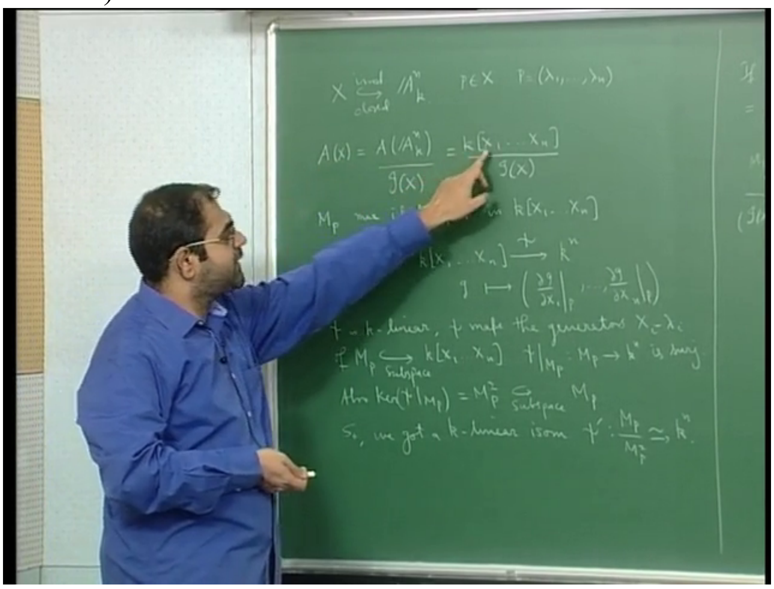

ideal of that point in this polynomial ring and going modulo I X. Ok, and that is why maximal ideal of the

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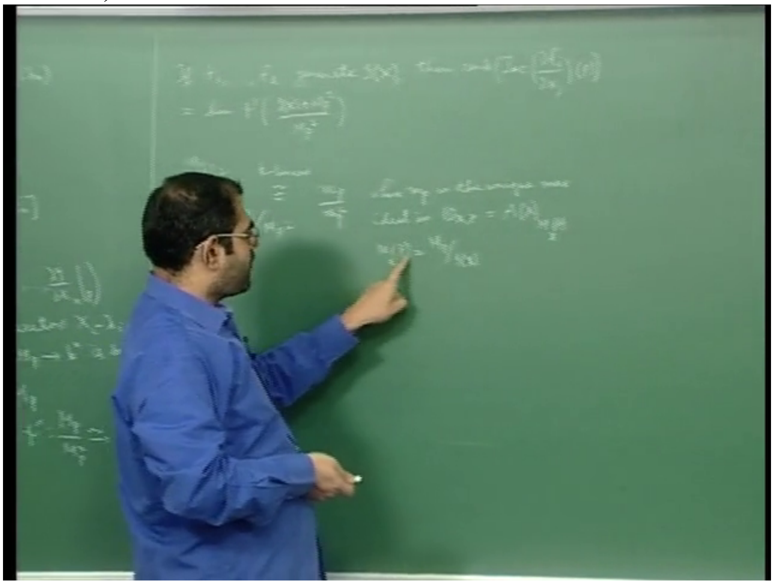

point P in x is just the maximal ideal of the point P in the, in affine space modulo the ideal of X, Ok.

And because of this k-linear isomorphism and because of this, what you get

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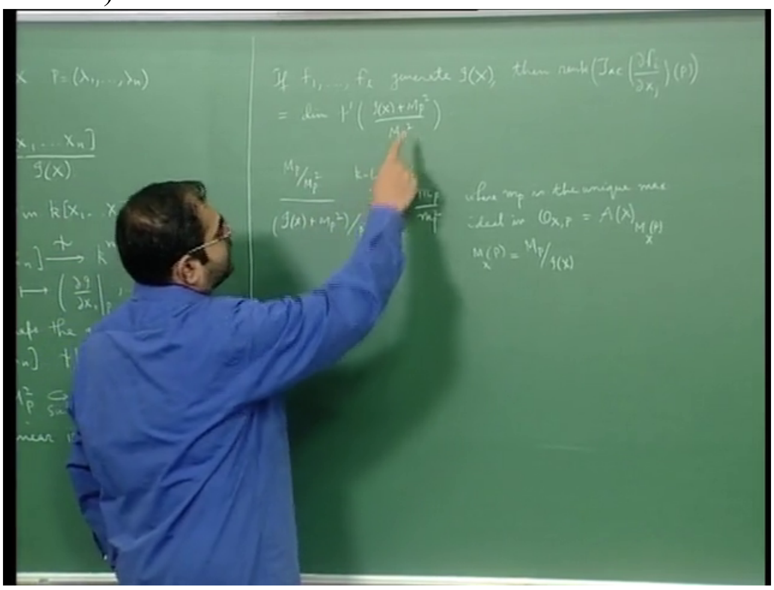

is that if you, so the dimension of this minus the dimension of this is equal to the dimension of this. The dimension of this is same as the dimension of its image under psi prime because psi prime is an isomorphism and restricted to its subspace it is injective, right

And therefore what you get is, so the offshoot of these two things is that the rank

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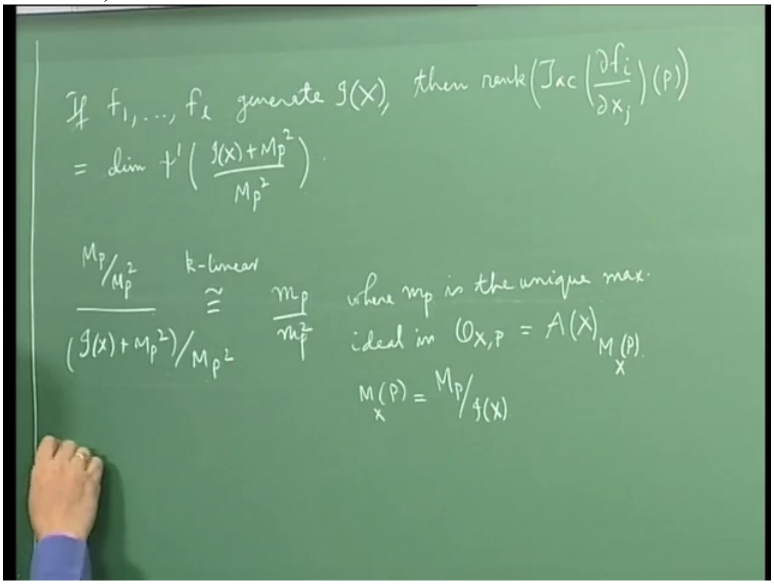

of Jacobian of dau f i by dau x j at the point P, this is equal to dimension of this which is equal to, well, basically dimension of this, dimension of this minus dimension of M P mod M P square, Ok.

So this would be equal to dimension of capital M P mod M P squared minus dimension of small m p mod m p square, right. Of course dimensions are all over as k vector spaces. Ok.

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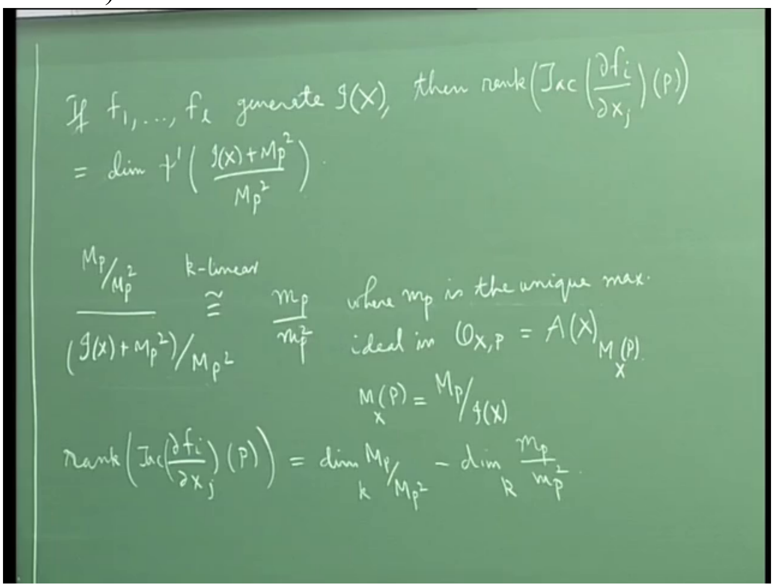

And, but of course M P mod, capital M P mod capital M P square is isomorphic as vector space to k n so its dimension is n.

So you end up with getting n here minus dimension of M P, mod M P squared and, and well

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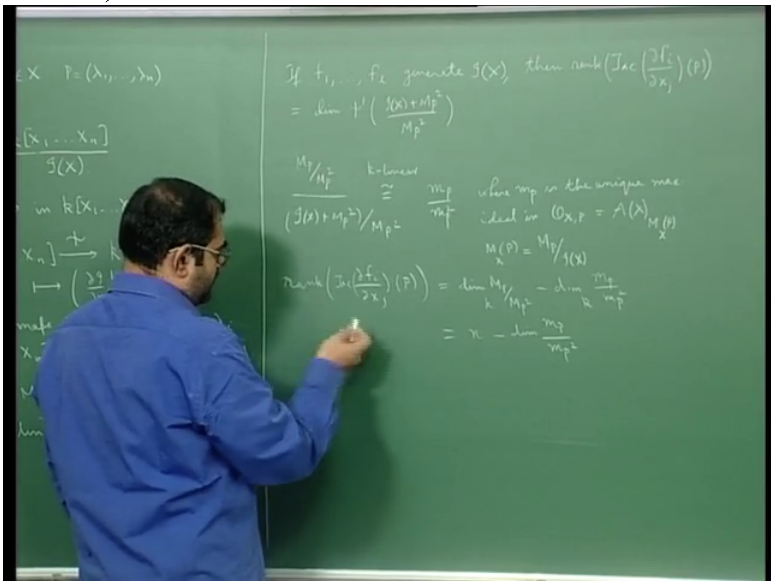

P is non-singular, P is non-singular, P is a non-singular point of X if and only if the definition is, well

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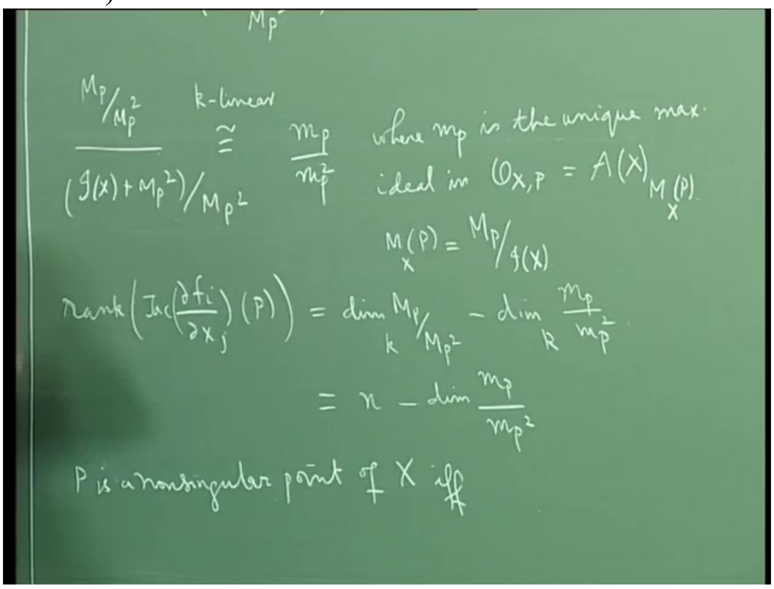

if you calculate the rank of this Jacobian, the rank of the Jacobian of the set of generators for the ideal of X at the point P should give you the co-dimension of X.

So this is true if and only if, this is co-dimension of X. So you get co-dimension of X in A n, co-dimension of X in A n is equal to n minus dimension over k of M P mod M P squared and that leads, but co-dimension of X in A n is dimension of A n minus dimension of X.

So

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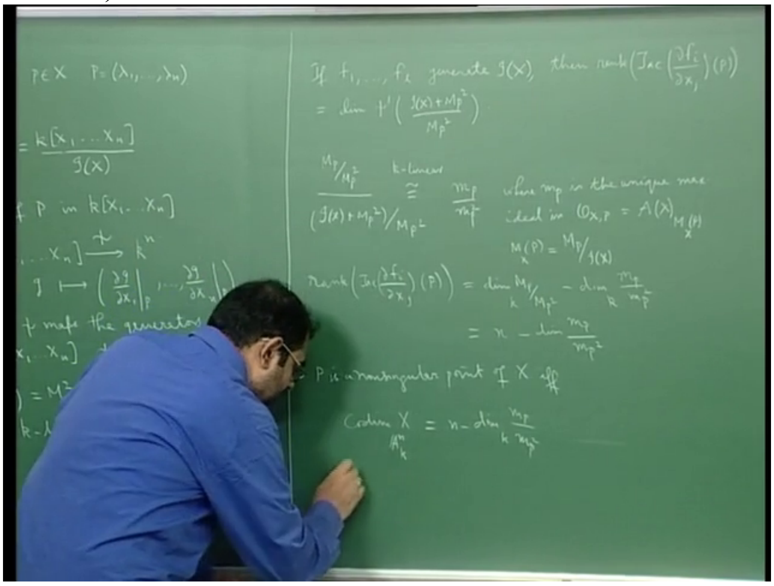

that condition translates to just saying that the dimension of X is equal to 0:14:03.2 the dimension of M P mod M P square. And, and mind you that

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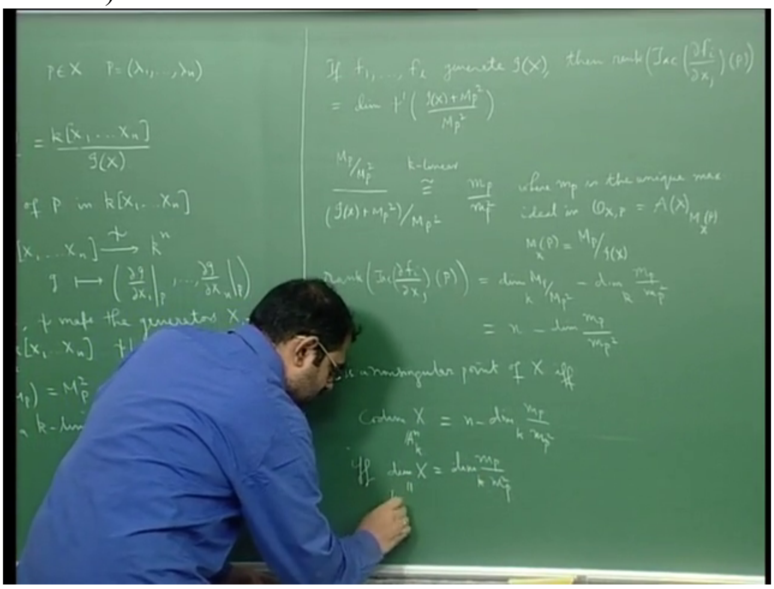

dimension of X is always given also by the dimension of the local ring of X at any of its points, Ok.

Therefore and of course here when I say dimension of X is topological dimension, when I say dimension of the ring

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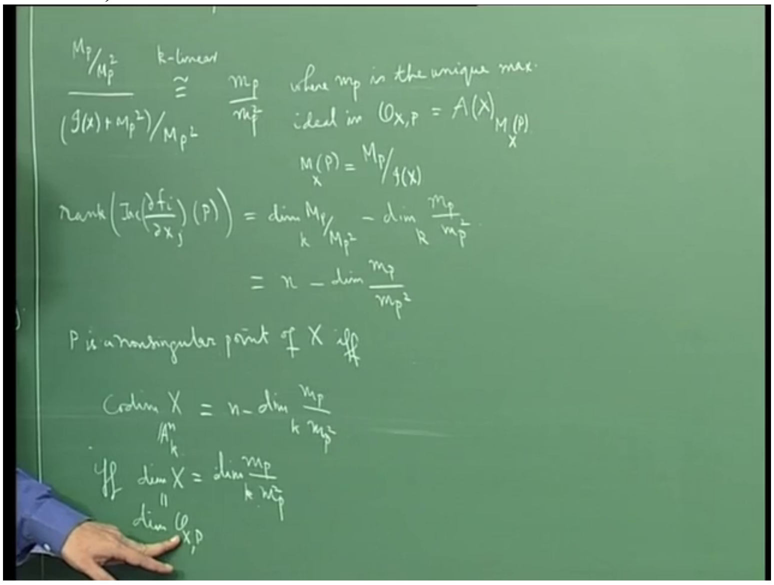

here is the Krull dimension, alright. This is the topological dimension. This is Krull dimension. This is dimensions of vector space, Ok.

So the moral of the story is that this, the point

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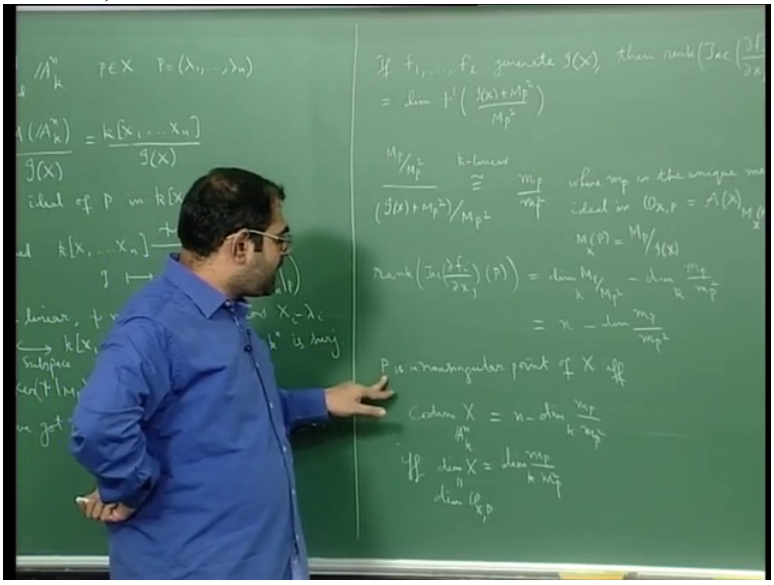

P of X is, you know a smooth point,

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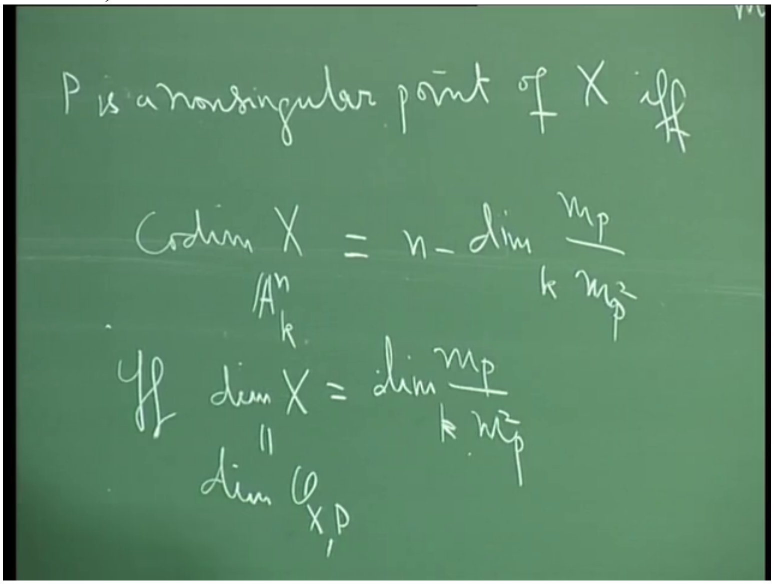

a non-singular point if and only if the dimension of M mod M square at that point, Ok where M is the maximal ideal of the local ring at that point, is equal to dimension of X, which is same as the dimension of the local ring.

And of course this tells us that really you know, finally you see, this n,

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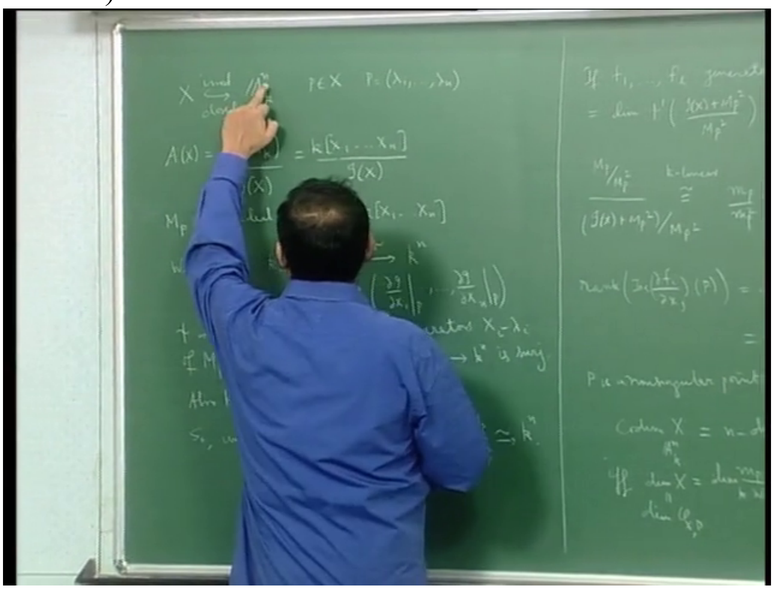

this is dimension of affine space in which X was embedded, that has finally gone out of the final statement, Ok. So if I had, I could had, this n did not come in the final picture,

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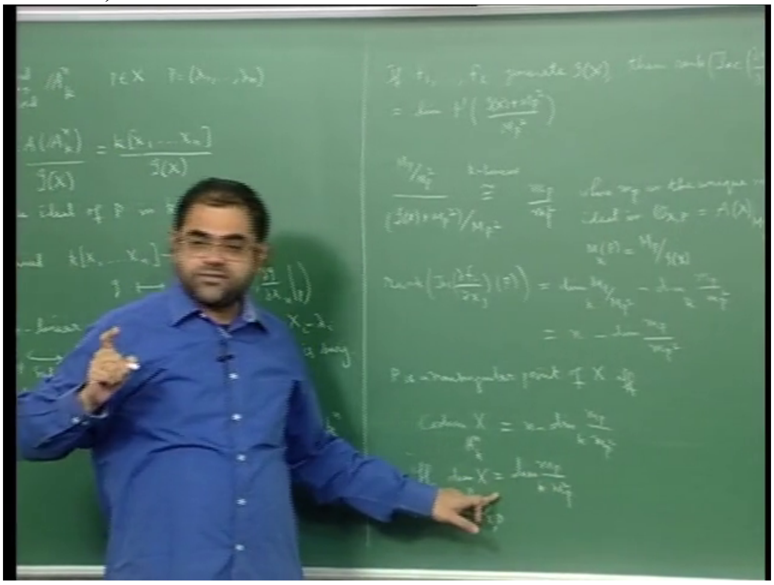

Ok. I could have, instead of this n it could have been some other n, Ok but that would not have affected this

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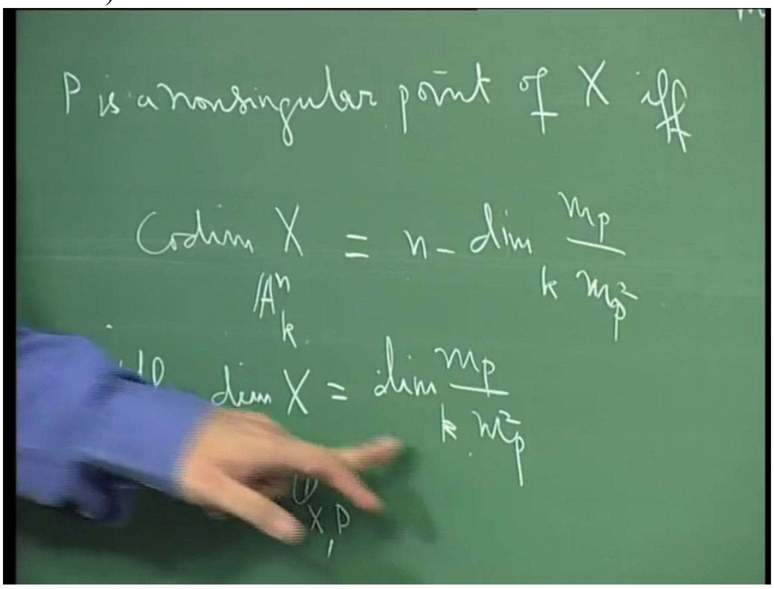

final statement, alright.

So you see, one it really did not depend on the embedding of X in affine space and you also see that after you have fixed, after you have fixed one embedding of X in affine space as irreducible subset, it also did not depend on number of generators.

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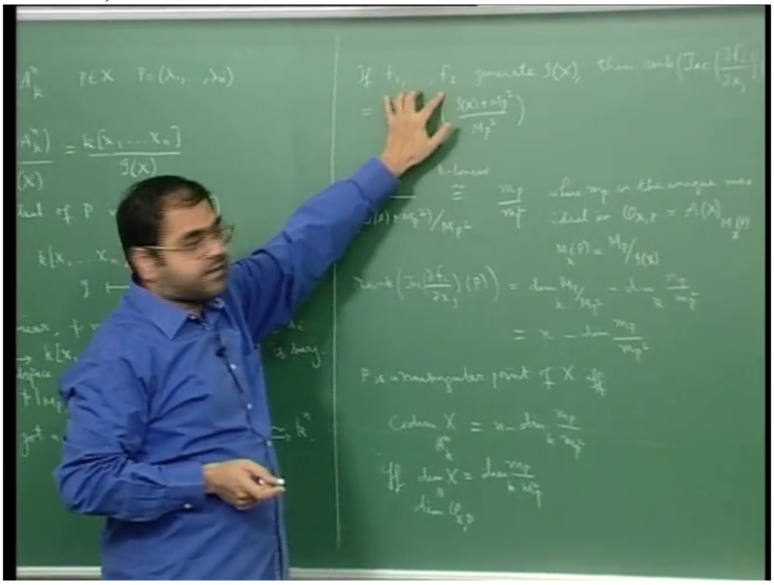

This l also could have been arbitrary. I could have taken any other set of generators but still I will give the same conclusion.

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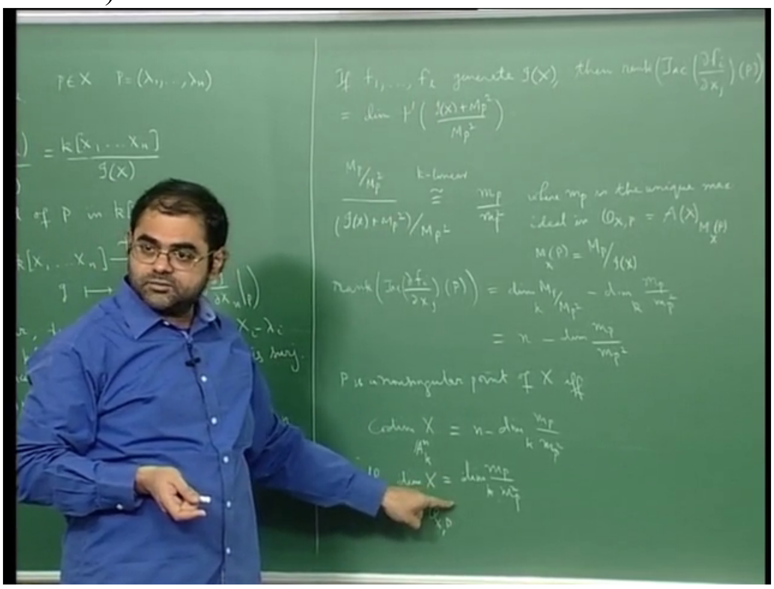

So this fact tells you that the definition of non-singularity which is given by this, by choosing an embedding

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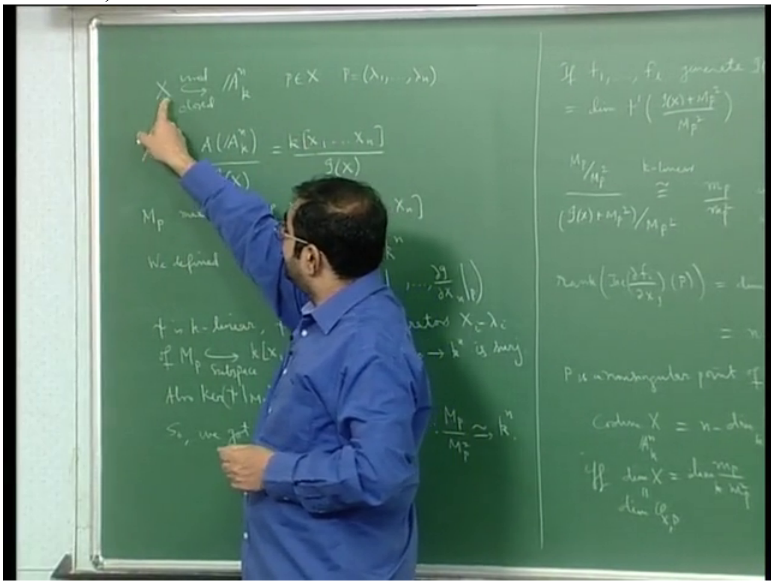

in affine space and then choosing a set of generators and calculating the rank of the Jacobian at that point for these generators, that, though it seems to depend on the embedding in affine space and on the choice of generators for ideal actually does not depend on these things.

The definition is, therefore

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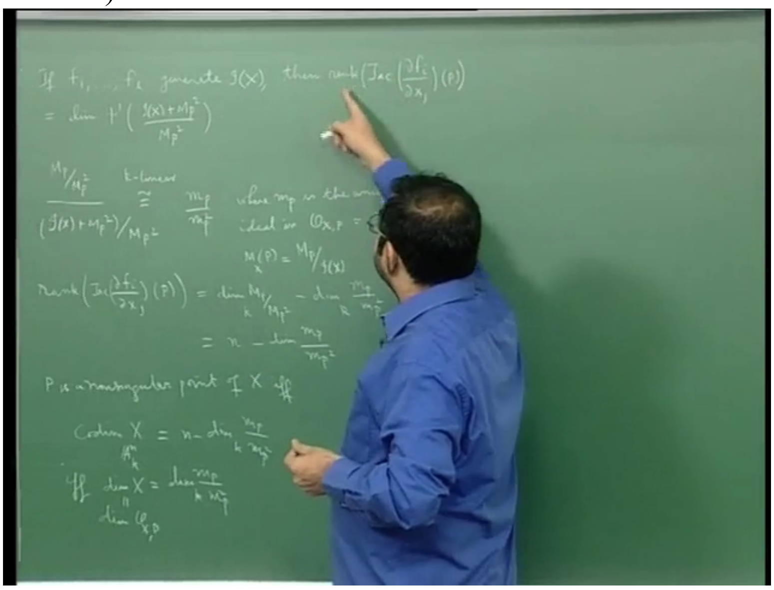

in this, the definition that the point P is smooth point if and only if this rank turns out to be the co-dimension of X, Ok. That is independent of all these choices. And finally the condition that the point P is

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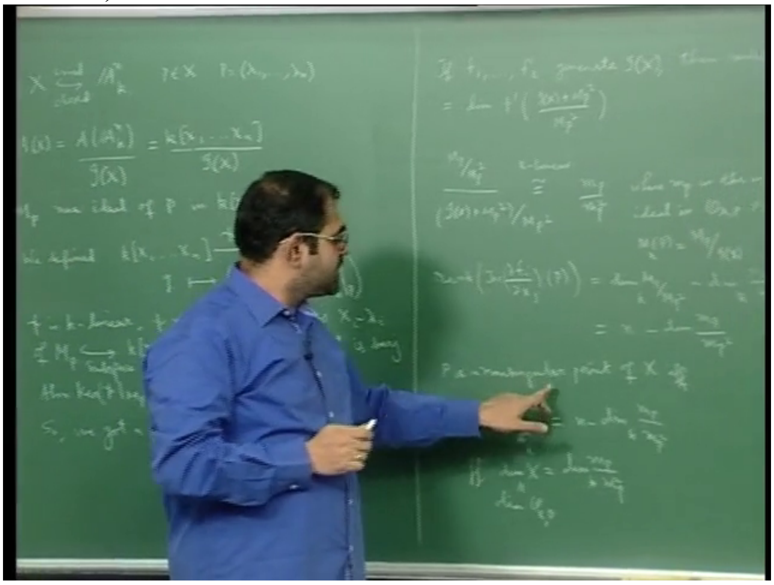

a smooth point or a non-singular point is a very nice condition. It does not depend on any of these things. It is very intrinsic.

It is, the condition is just that the, dimension of M P

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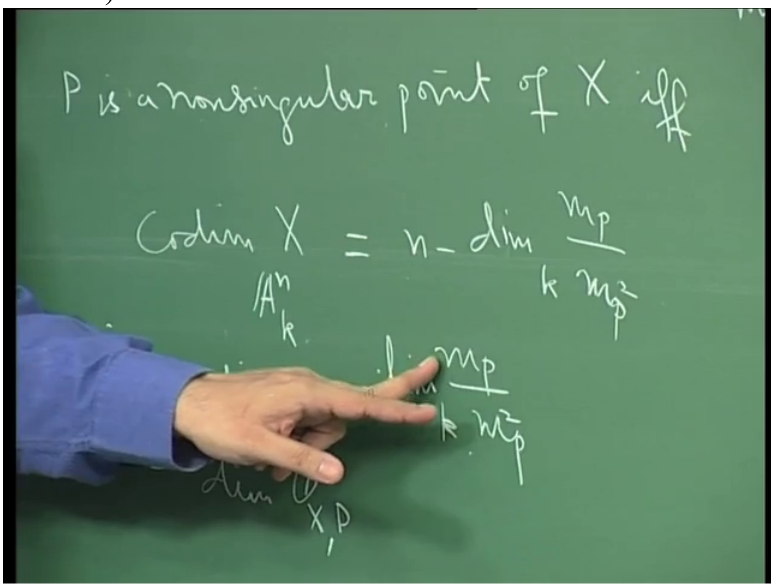

where M P squared over k, as vector space over k should be the same as the topological dimension of X which is the same as the dimension of the local ring at that point, Ok.

So, but perhaps, so this, this

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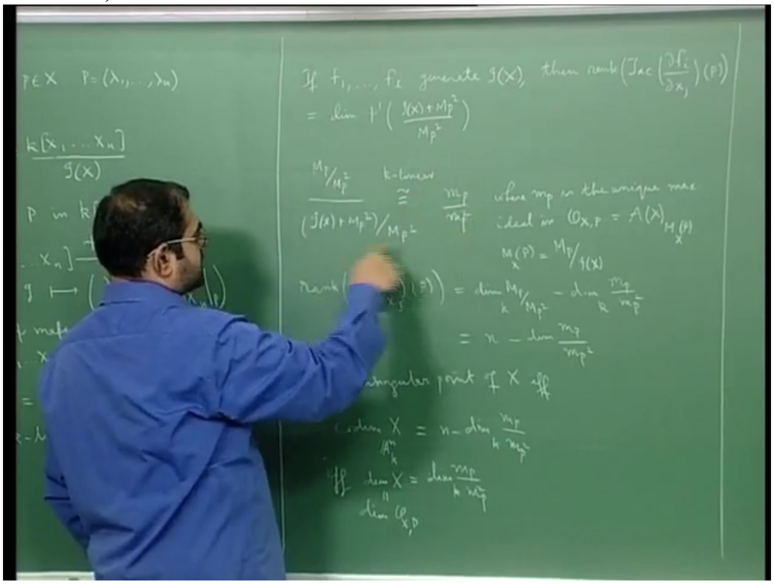

isomorphism is something that needs to be, this needs to be checked, Ok. This isomorphism needs to be checked. But it is kind of pretty easy, because you know, so as a remark let me, let me tell you that, as a remark, yeah you see that I X plus M P squared sitting inside, this is sitting inside M P as subspace alright and you well, on the other hand you have, you know, even if you take, if you take I X, I X sits inside M P

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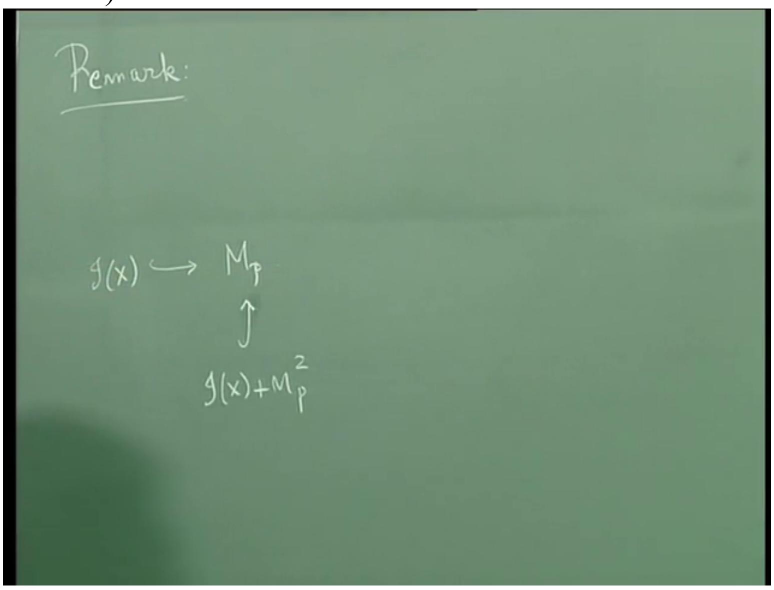

and it also sits inside this.

Ok so you know if you take the quotient, that is you go mod, M P mod I X what you will get is the short exact sequence, Ok, namely f, this modulo which is isomorphic to this,

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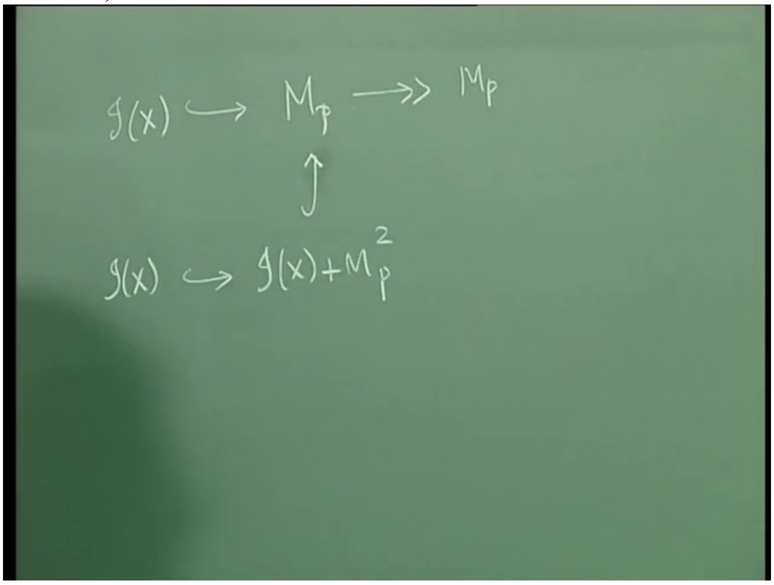

Ok so you get M P mod I X. But M P mod I X is actually

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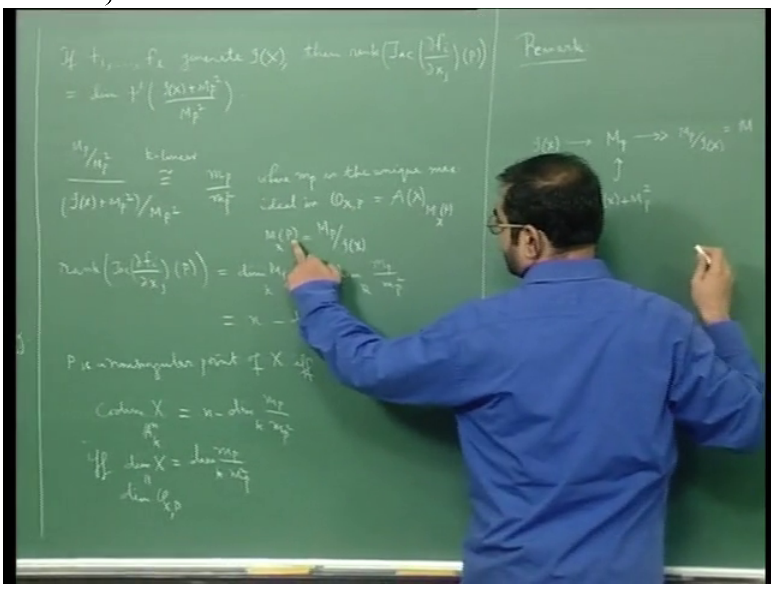

M P sub x, Ok and

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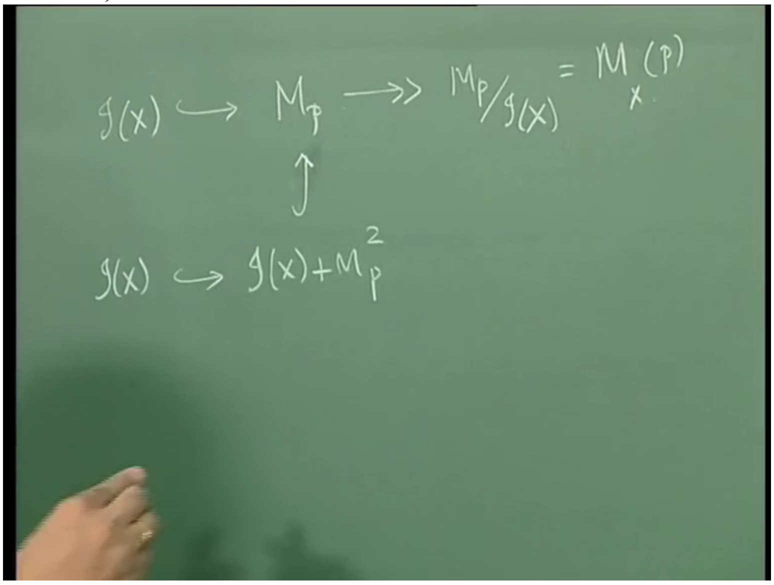

well if you take the quotient here what you will get is I X plus M P squared by x is just simply going to be

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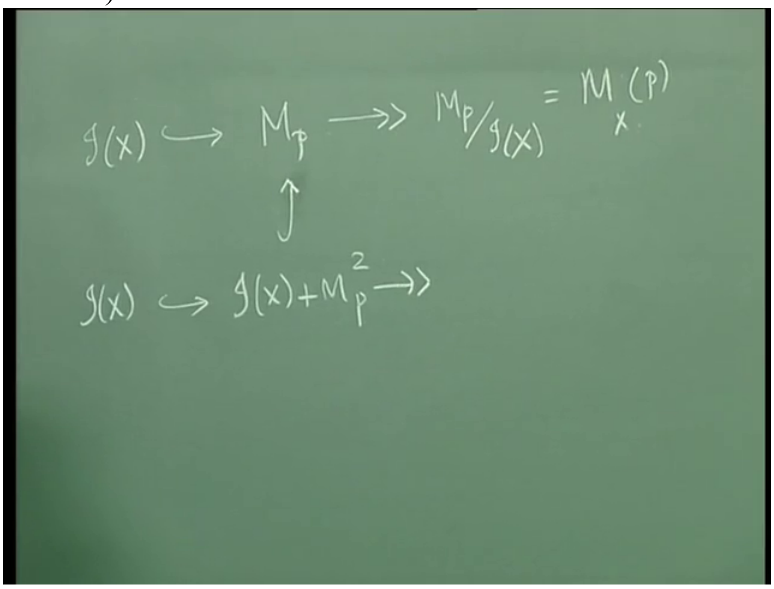

M P mod I X the whole squared, you will get.

So here you will get M P mod I X whole squared, Ok which is just

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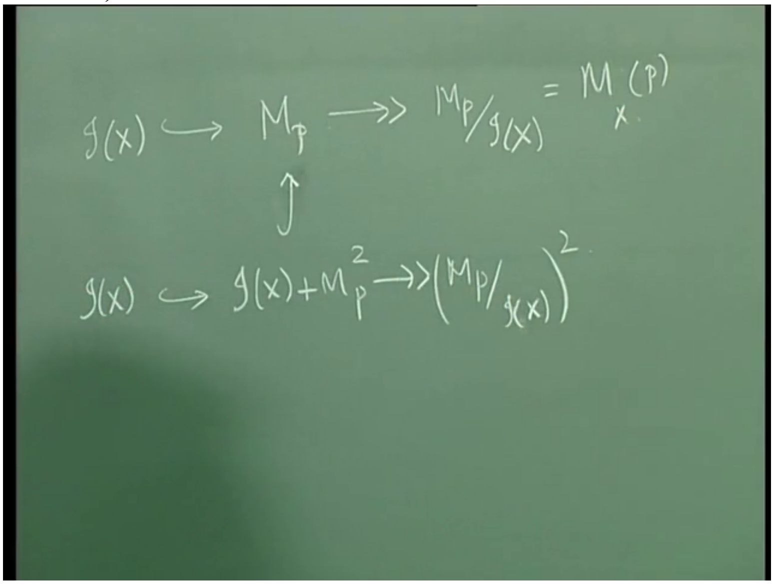

M P sub x squared, this is what you get, alright? And

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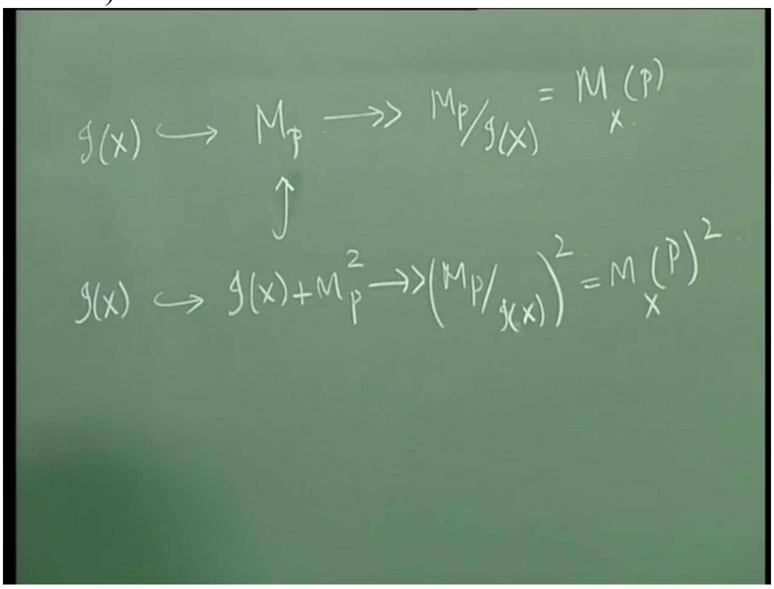

well, you see, so if you calculate, so this is again a, so when I divide by I X that is the same as dividing M P squared by I X but I claim that there is dividing

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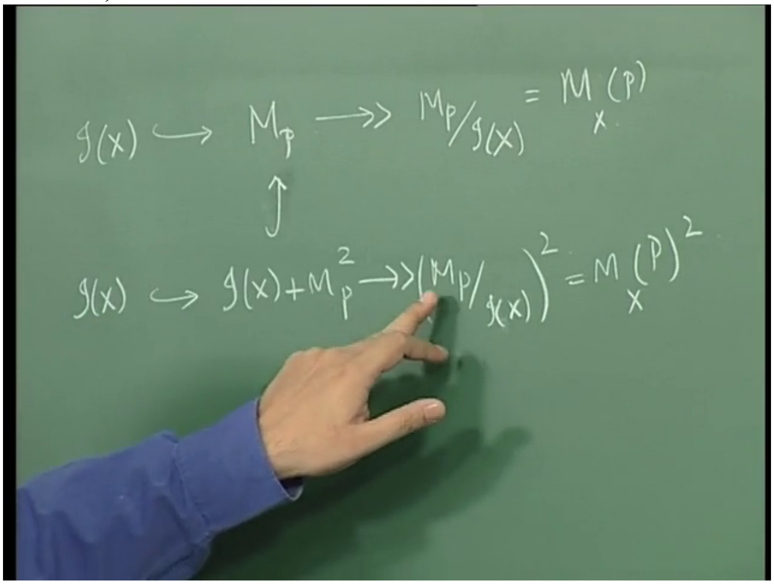

the same, there is the same as dividing M P by I X and then taking squared, Ok. This is something that you can easily check, right?

And, and you see, you know if you take this inside this and now take the quotient here, I get, here I will get M P mod I X plus M P, I X plus M P squared, Ok and here

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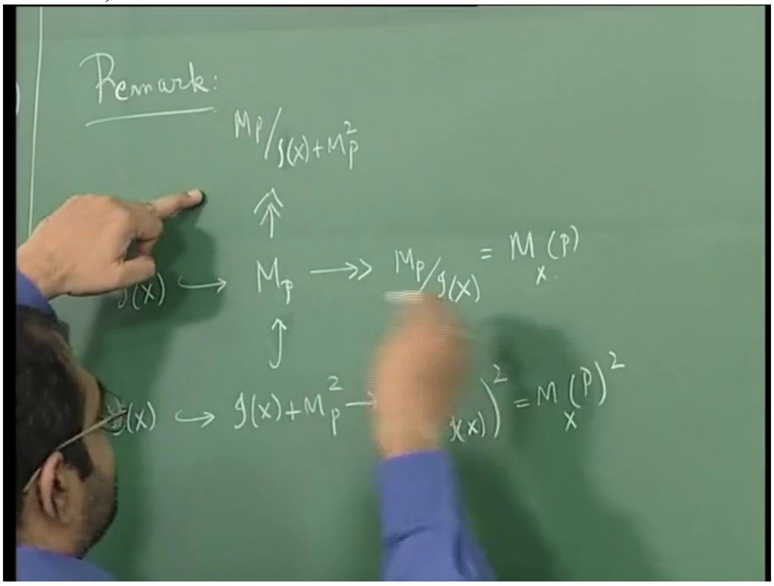

you know, here what I am going to get, here if I again do this I am simply going to get M P M x P mod M x P squared, Ok, I will get this.

And the fact is that this is an isomorphism, right. This will be an isomorphism. You can check that,

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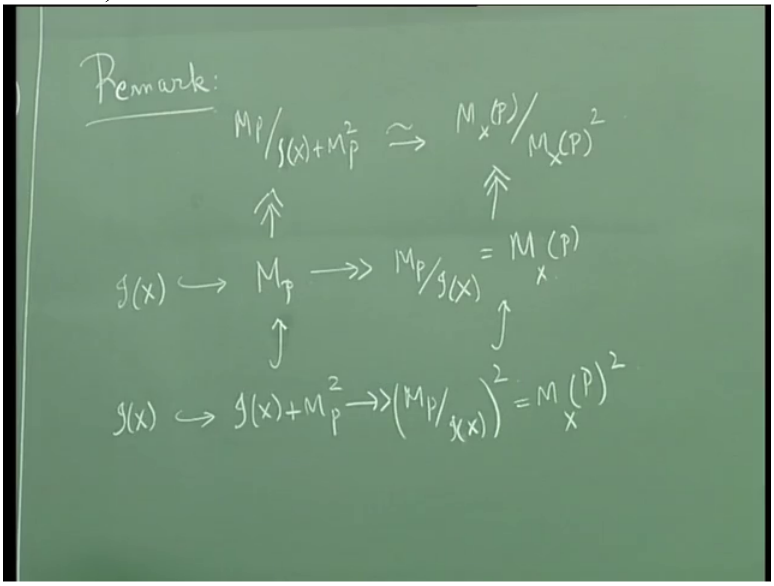

alright. And further Ok, so this will be an isomorphism and mind you, what I have written on the left side is exactly M P mod M P squared by

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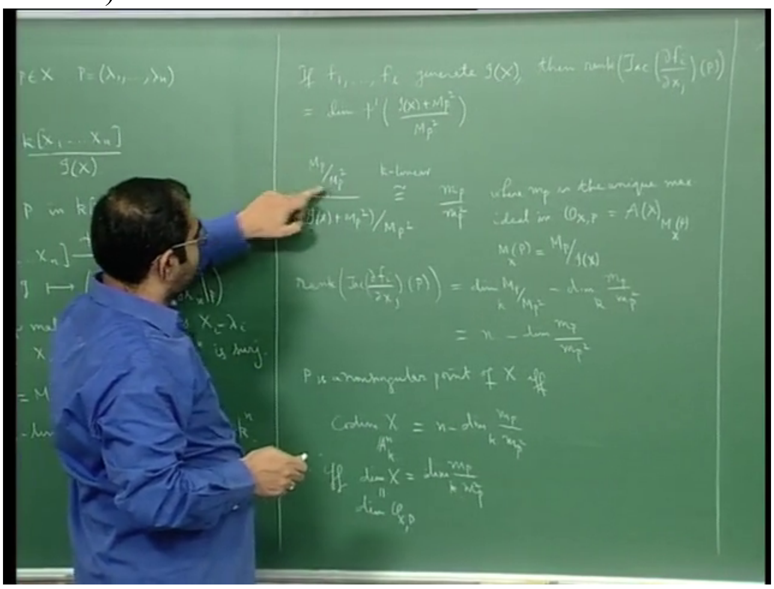

x plus M P squared mod M P squared , this is also M P mod I X plus M P the whole square, alright?

So and I want to show that statement this is, this is linearly isomorphic to this, alright. Now what I want to tell you is once you, once you check, once you check this isomorphism, of course this is 0:21:15.0, Ok and

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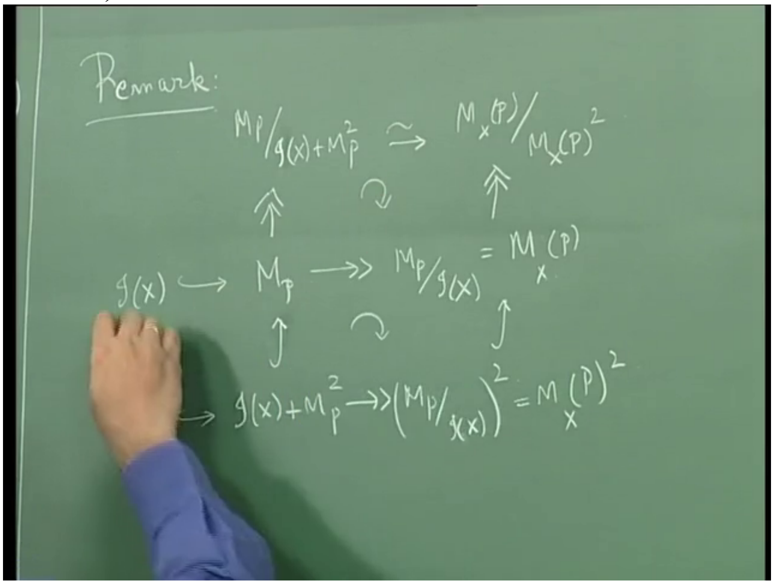

this is identity map,

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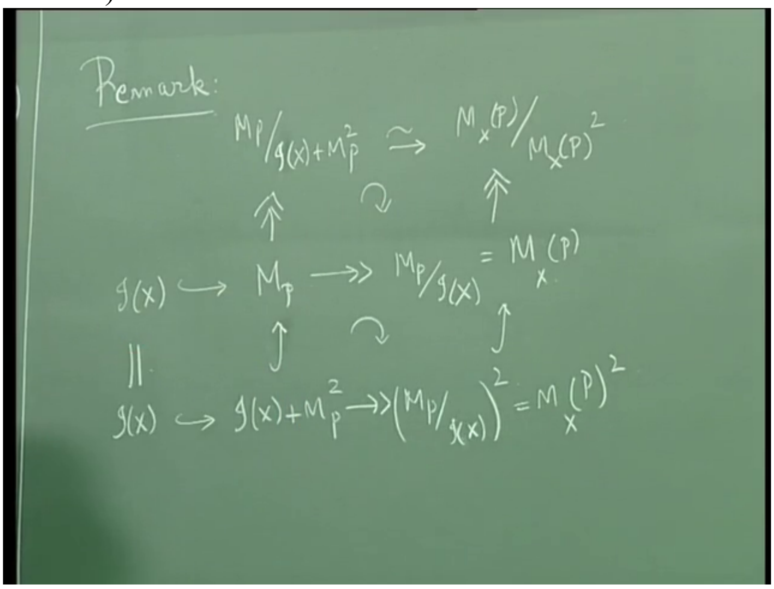

alright. If you want, you know, I can even put zero here because I X mod I X is zero, right? And in fact

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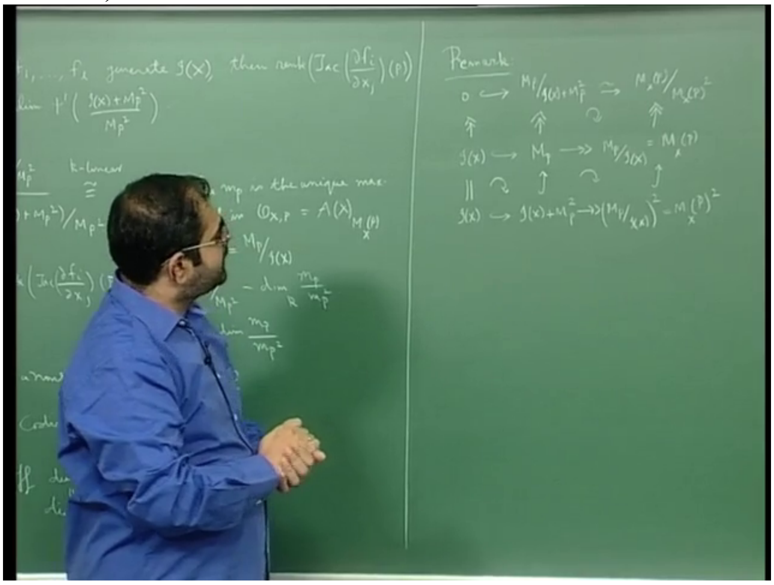

now what you must understand is, that you know, you have, now you have the short exact sequence, zero I mean you have M x P squared sitting inside M x P and then you have the quotient M sub x mod M sub x P squared, Ok, I have this short exact sequence,

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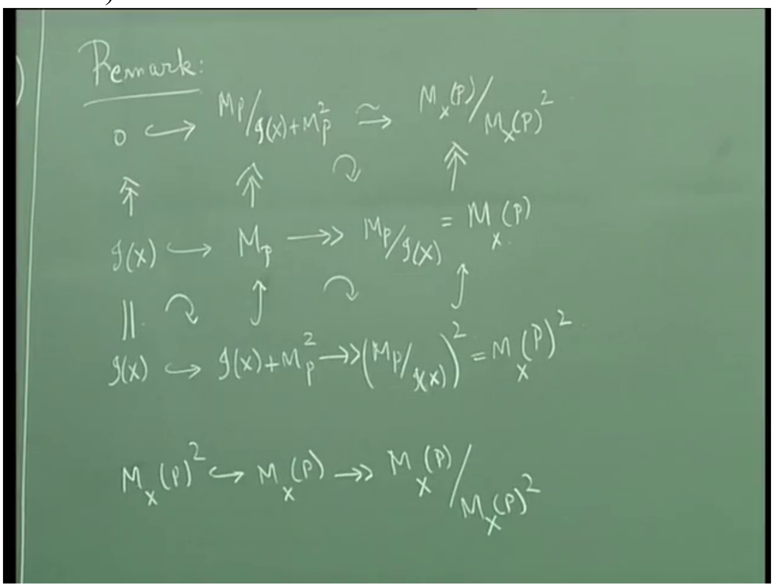

alright.

Now what you do is you just tensor this with, with the local ring A x localized at M x P over A x. This is a short exact sequence of A x modules Ok, because these are all ideals. These two,

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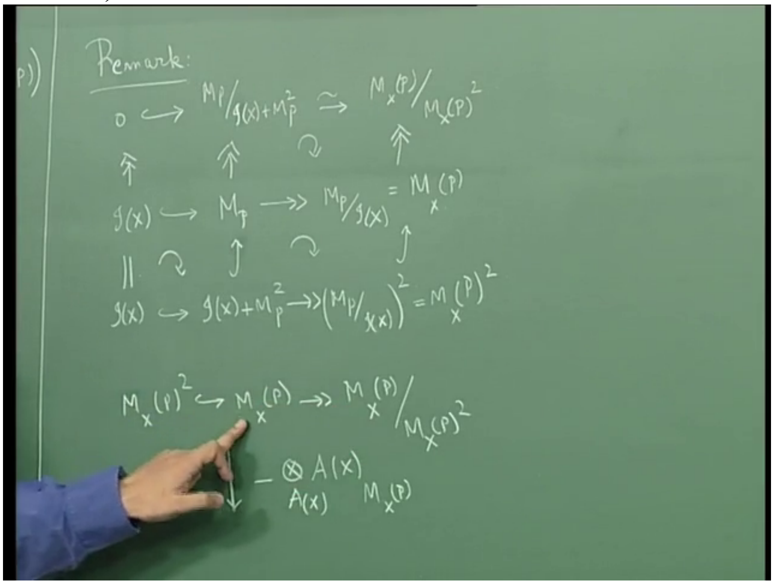

this is an ideal in A x, Ok, this is the maximal ideal of point P in A x. And this is the square of the ideal. And the quotient, the quotient will not be an ideal but it will be a module. Because ideals of a ring are just sub-modules, Ok. So this quotient will not be a ideal but it will be a module, it will be A x module.

So this is a short exact sequence of A x modules and then if you tensor it with A x over, tensor it over A x with A x localized at M x P, Ok, you know that if you have seen in the first course of commutative algebra, you would have learnt that tensoring with the local ring, tensoring with the localization is the same as, tensoring with localization as, tensoring sequence of modules with the localization of the ring is the same as applying localization, applying localization to the sequence of modules, Ok.

In other words localization is an exact functor, Ok. Therefore this sequence when you tensor with A x, with this local ring, Ok, then I will again get an exact sequence but after you tensor it, if you tensor

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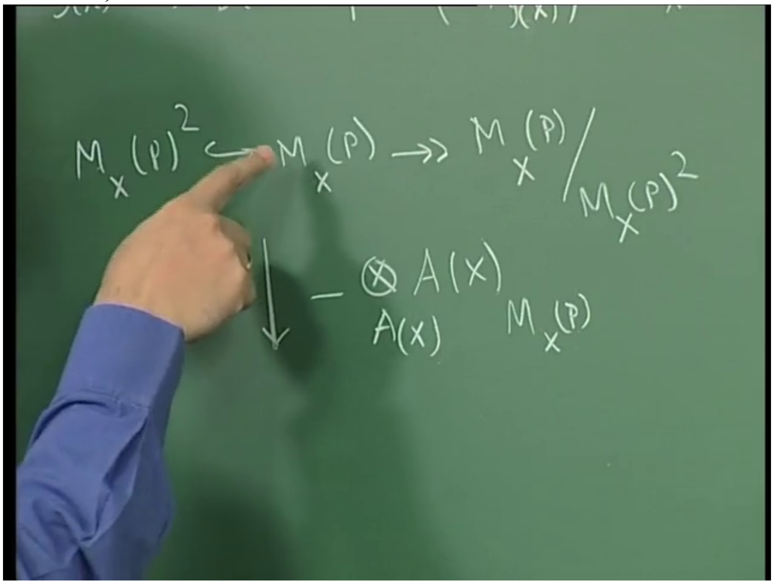

the maximal ideal with this, with this local ring what you will get is you will get the maximal ideal of that local ring, Ok.

So when you tensor M x, M P x with this,

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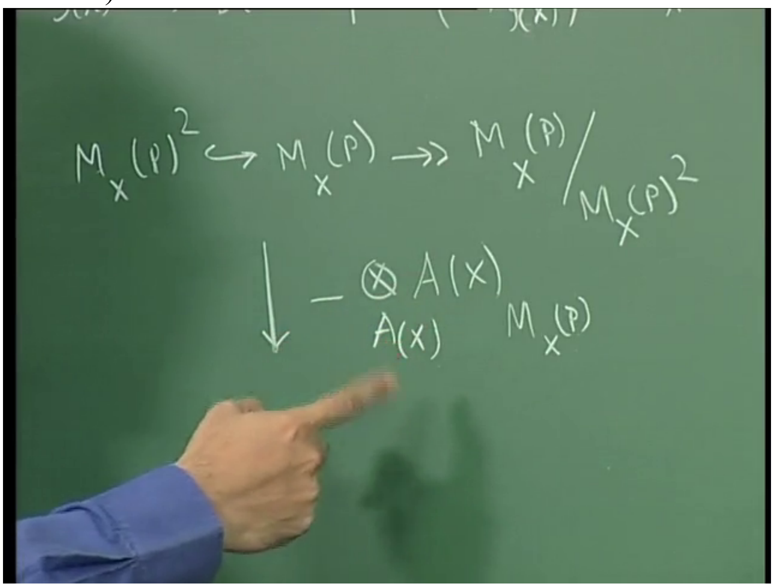

what you get is the maximal ideal of this local ring which is small m p. So after tensoring what you will get is this, exactly,

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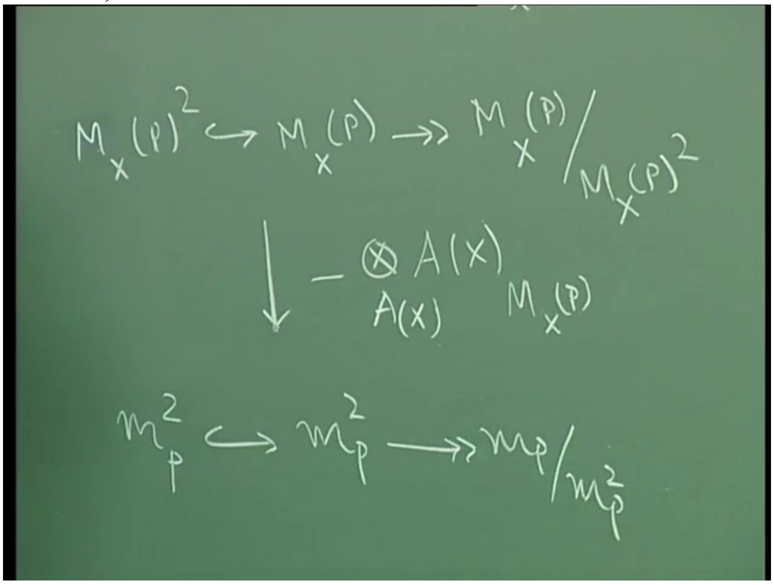

Ok. And so you know, yeah, so the point I want to make is that these two have the same dimension, Ok. This has the same

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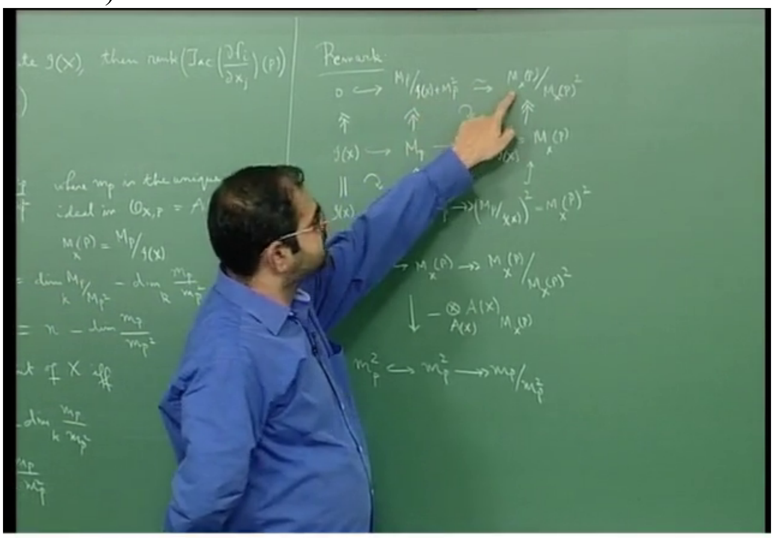

middle term of last line should be m_P and not m_P^2

dimension as this, alright.

But then whatever its dimension is, Ok, see the same dimension you will get if you tensor it over this. because the residue field is still k. The residue field of

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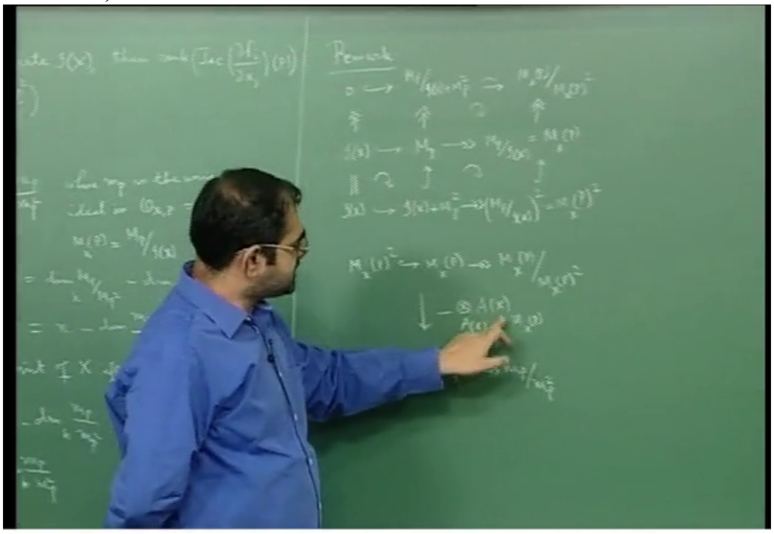

middle term of last line should be m_P and not m_P^2

this local ring, Ok, mainly this local ring if you go modulo that maximal ideal, you will still get the same residue field k. Therefore if you,

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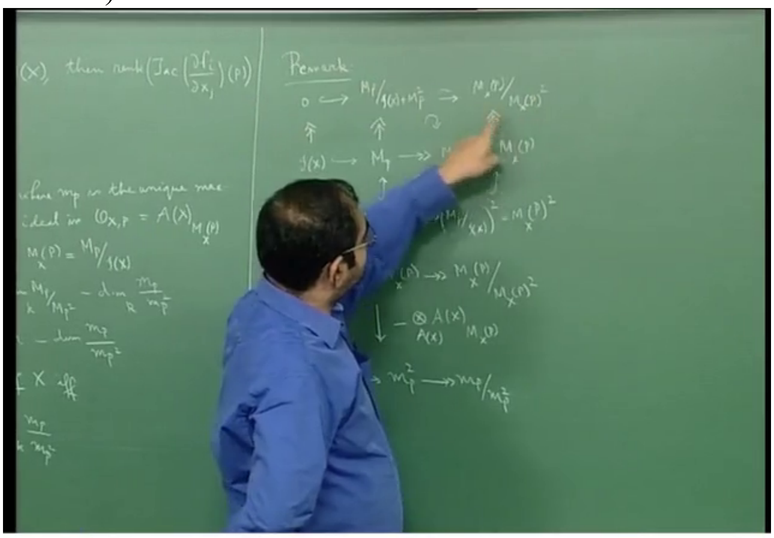

middle term of last line should be m_P and not m_P^2

whatever this dimension is as the k vector space, you will get the same dimension

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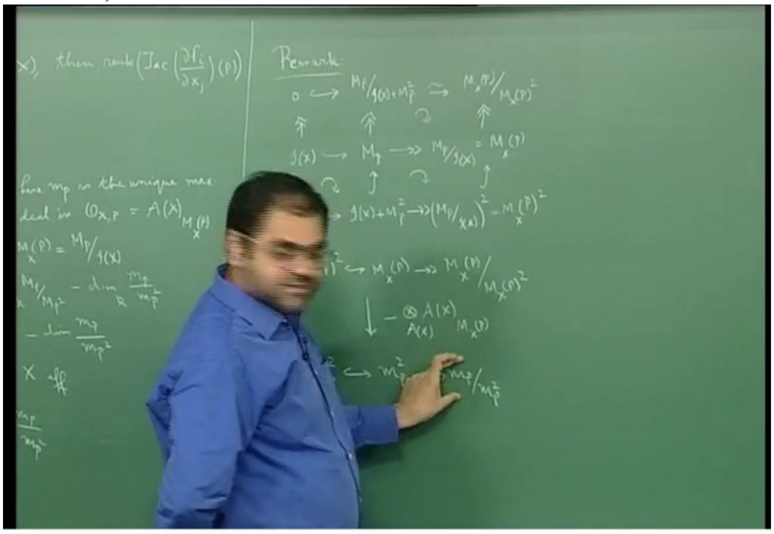

middle term of last line should be m_P and not m_P^2

for this. After tensoring over this local ring, for the simple reason the residue field of that local ring is the same k. Ok.

That is why you will still get the dimension of this over k is same as the dimension of this over k but then these two are isomorphic, therefore you get the dimension of this over k the same as the dimension of this over k. And that is what we want to show. Ok. So that is, that is something that you have to check. I mean this is an outline of what you have to check, right? So this is one thing.

Then the other thing I want to say is

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in connection with this proof. Ok, several things in connection with this proof. So the first thing I want to say is let Sing x be the set of all points of X such that P, x is singular at P. Look at the

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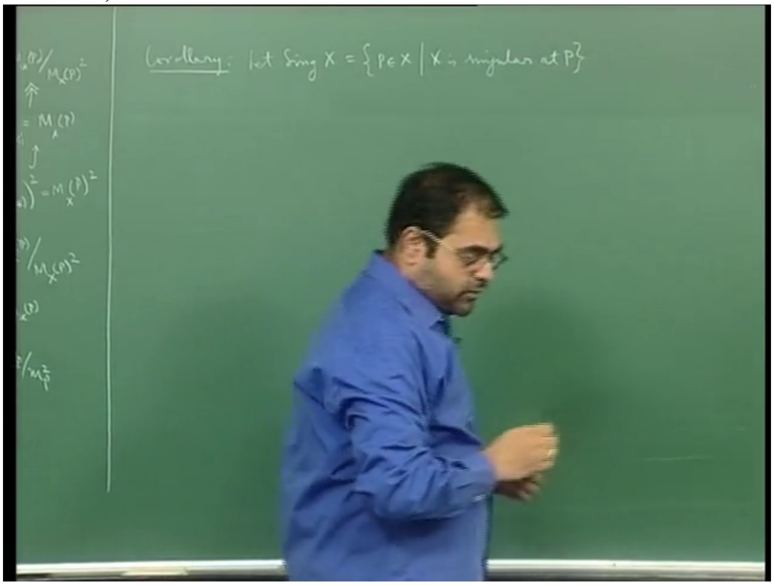

set of singular points of a variety. Ok, so x is a variety. And look at all those points of X which are not non-singular. Points which are not non-singular are called singular, Ok. Look at those points.

Now the fact is that, the fact that I want to make is that the set of singular points is a closed set and it is a proper closed set. So here is the, here is theorem, then so the first corollary is that the set of singular points is a closed subset of X, Ok. So here from now on X is any variety, need not be affine.

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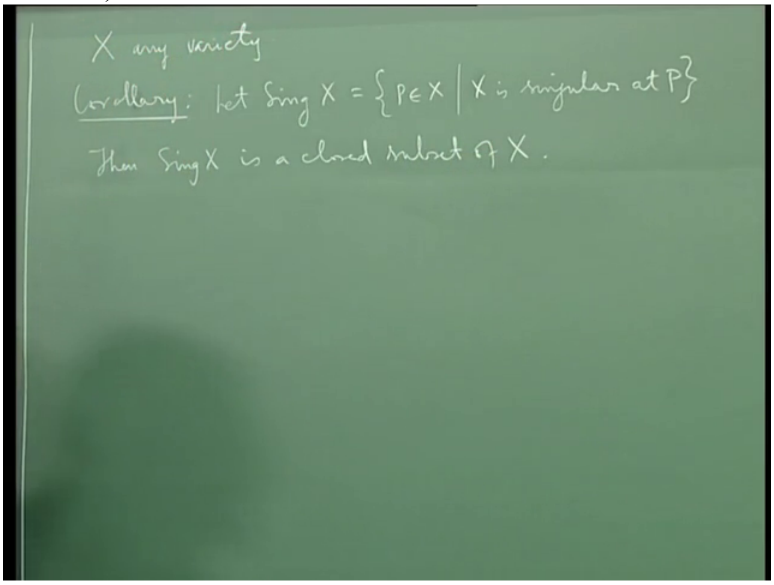

x is any variety.

Mind you, for any variety of point is called a non-singular point if you know, you take that point as a point of an affine open sub variety and there it should be non-singular, Ok. Because whether the point is smooth or not is a kind of condition that is checked only in the neighborhood of that point, Ok. It is something that you should take in a local neighborhood of that point. Locally at that point you have to check it. Ok.

So if you have any point of any variety, how do you check, how do you check whether it is smooth, whether it is non-singular, what you are supposed to do is you take that point, that point will certainly be contained in affine open subset. Because you know any variety is contained in finitely many affine open sets, mainly it is covered by finitely many open subsets, each of which is isomorphic to an affine variety, Ok.

So if you want to check that point is smooth or not, well you can, you have two ways now. If you want you can check it intrinsically,

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you calculate dimension M mod M squared and check whether it is, you get the same thing as dimension of X. If you get the same thing as dimension of X, then it is a smooth point.

But otherwise computative algebra, a little bit more computative algebra will tell you that you might end up getting more dimension. Because dimension of M over M squared gives you the dimension of the tangent space at that point. Ok. And in general, you could, at a bad point you could get more tangent vectors. Dimension of tangent space could be more than the dimension of the object, of the variety. So this could be bigger than this.

If this is strictly bigger than this, then it is bad point, singular point. If this is equal to this, then it is smooth point. So this is one way of checking. The other way of checking is of course, you take

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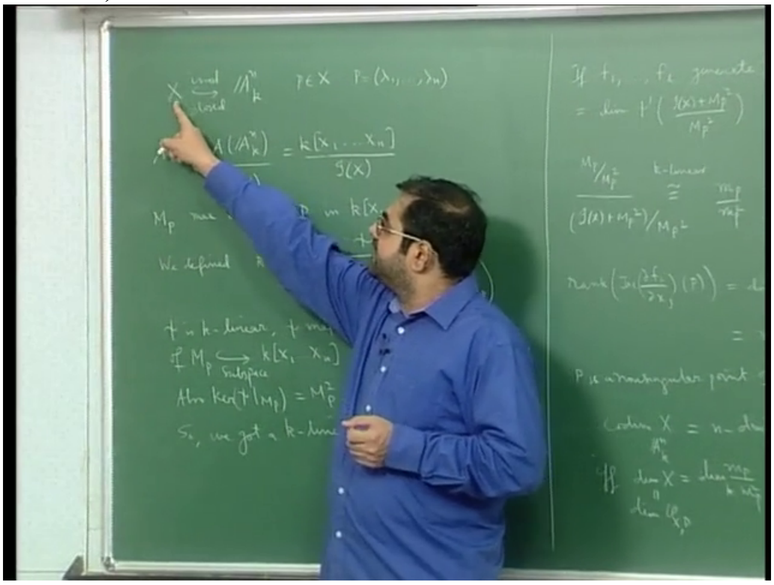

an open, you take an open affine sub variety, Ok and you embed it is affine space. Then you take the set of generators for the

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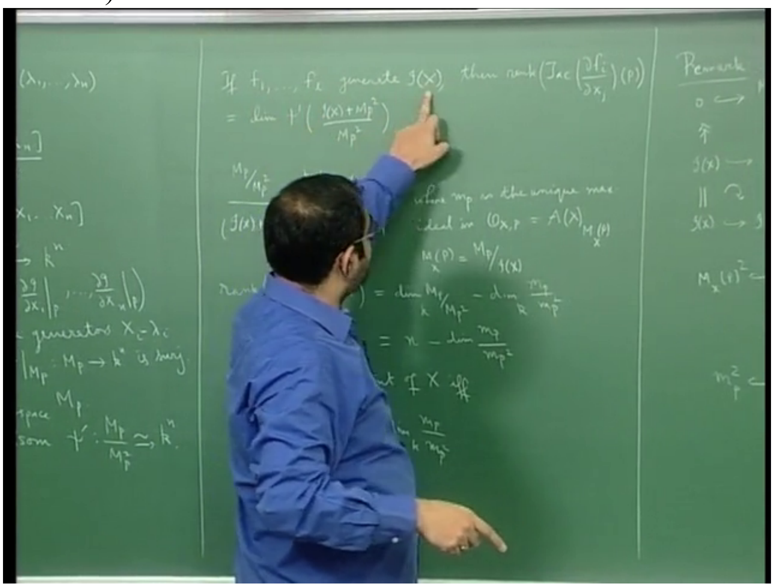

ideal, calculate the rank of the Jacobian and checking whether you get the co-dimension. Mainly whether you get the dimension of the ambient affine space minus the dimension of variety, I mean the open affine subset that you have embedded, Ok. That is another way of checking.

But you know which one is good and which one is not good depends on the situation. If you want an abstract theorem, if you want to prove an abstract theorem about points, some non-singular points, then it is better to use this.

On the other hand if you have a specific situation where you know the equations of the variety, Ok then that means you know the set of generators for the ideal, Ok then you can use this. Ok. So what you have to use is depends on the situation, alright.

Now, but anyway what I want to say is the set of points which are, which are bad, what kind of the set do they form? They form a closed set, alright and why is that a corollary of this. It is a corollary of this, it is a corollary of this because you see what you are doing is when I define this map psi,

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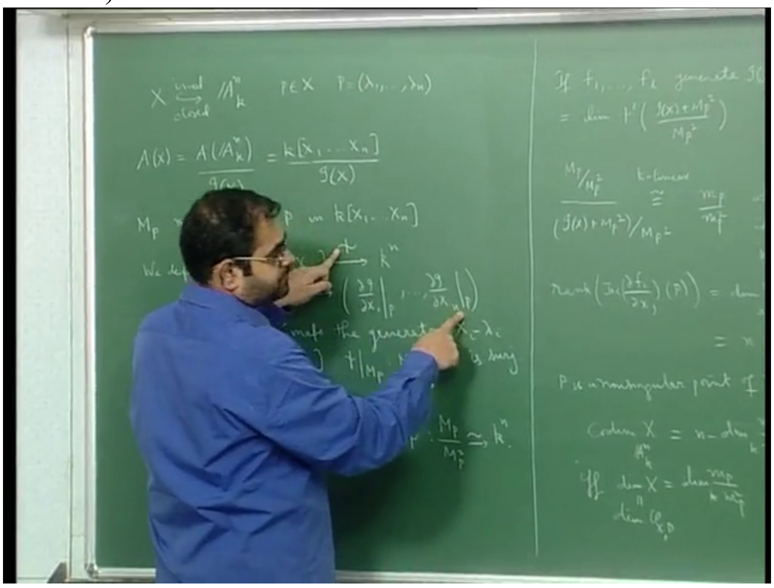

Ok, you know I could have, you can think of, see all these things I had fixed this point P. I fixed this point P in x. Ok.

But you think of the point P as a variable point. Think of a point P as a variable point. If you think of point P as a variable point, Ok then you know what are going to be the points P which are singular, they are all those points such that when you calculate the rank of this

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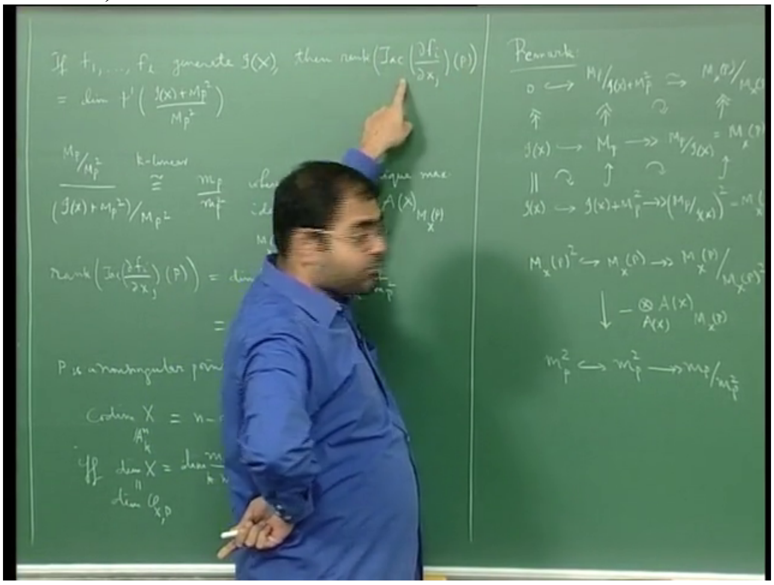

Jacobian, you see, the rank of this, rank of this matrix, you know this should not be equal to the co-dimension,

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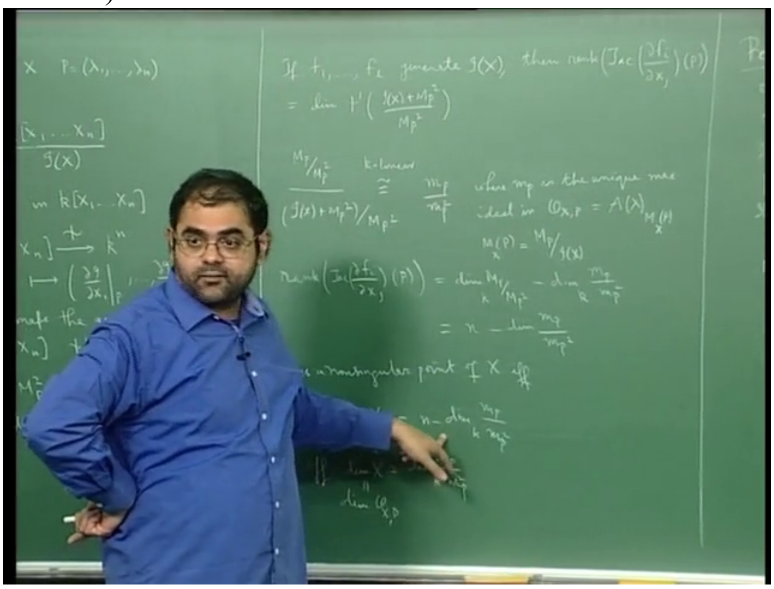

alright. So it will be, it will be lesser than that. Ok.

So as I told you, you see at a bad point, this is going to give you the dimension of the tangent space, Ok. At a bad point you will have more tangent vectors so the tangent space, this is the dimension of the tangent space, this is going to go up. If this goes up, this is going to go down. But this is exactly the, this is exactly the rank of the Jacobian matrix at that point.

So what are the bad points

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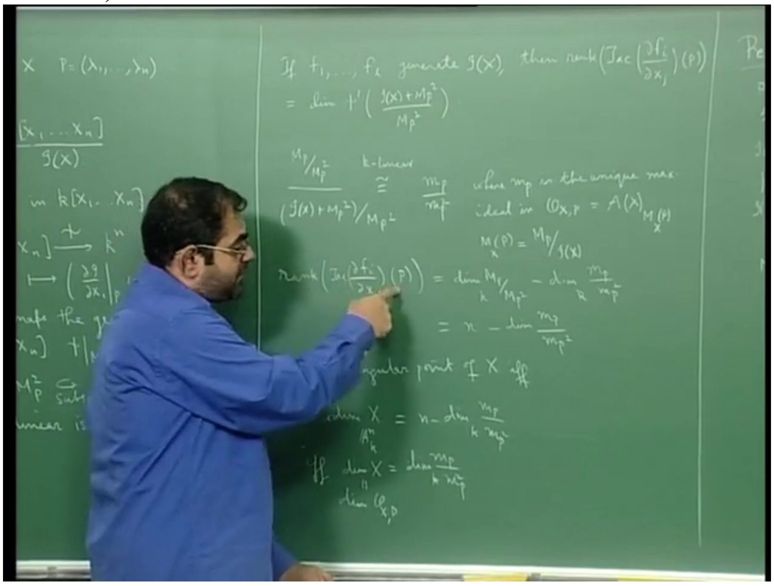

P? Bad points P are those points for which the rank goes down, Ok and what are the points P for which the rank goes down? Therefore they are P defined by the, they are those points which satisfy the polynomial equations given by taking all the,

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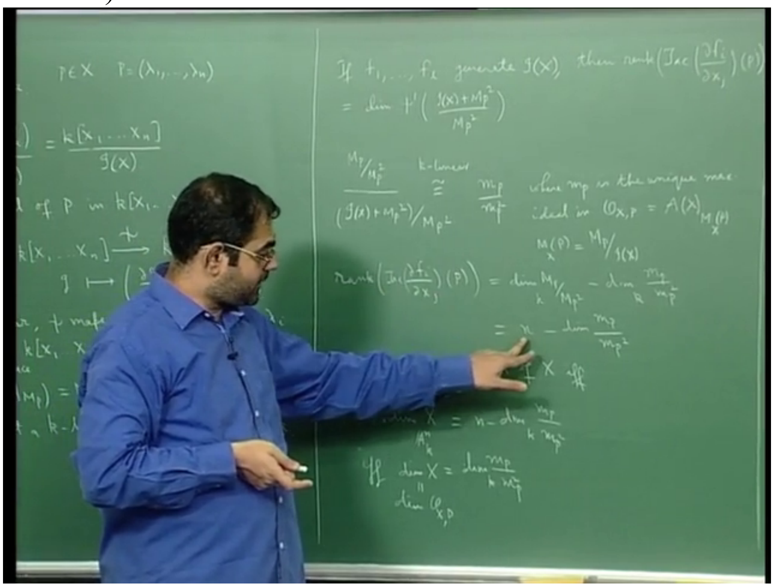

you know all the minors, Ok, you take the minor determinants of this matrix.

But you take not the biggest but you know, or you take the biggest, you take the largest possible minors and they all should match. Ok. If, when does a rank of the matrix, when is a matrix of full rank? When the largest possibly, when at least one of the largest possible minors, minor determinants does not match, Ok

So when is it going to be of lesser rank? It will be of lesser rank if you take all possible maximal minor determinants, they should all vanish. So the condition that the rank falls is a vanishing condition. It is given by vanishing of some minor determinants and therefore you know the points P which are singular, they correspond to the points which, they correspond to the points of X at which if you take the maximal minors, determinants of this which are again by the way, polynomials, Ok.

Those polynomials given by the maximal minor determinants, they should also vanish, Ok. And that is of course a closed set. Ok, so it is, so because of that observation what happens is that if x is an

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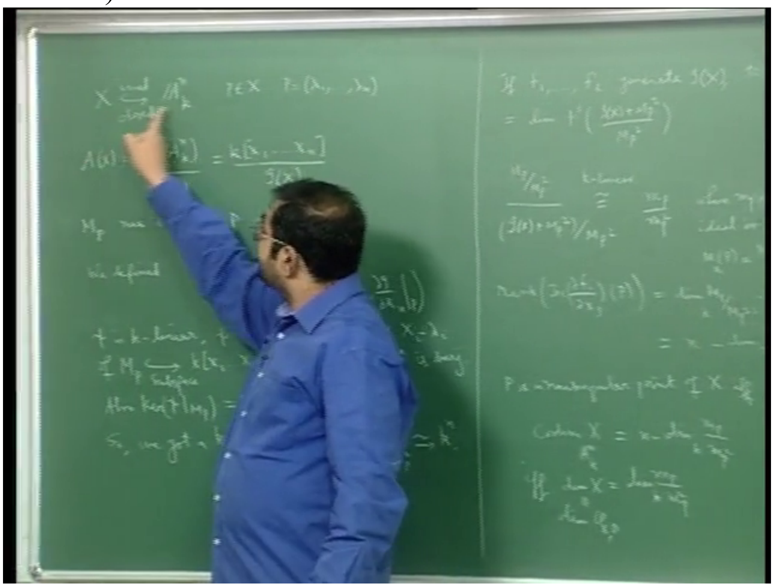

affine variety in A n then you get a singular point of X is a closed set. Ok, and but topologically the set being closed is something which you can check on a cover, Ok

And therefore the same is

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true for any variety because any variety admits a cover by open sets is affine variety. So the moral of the story is that a set of the singular points is a closed. Ok, so what is ideal for this? I mean x is affine, an ideal for

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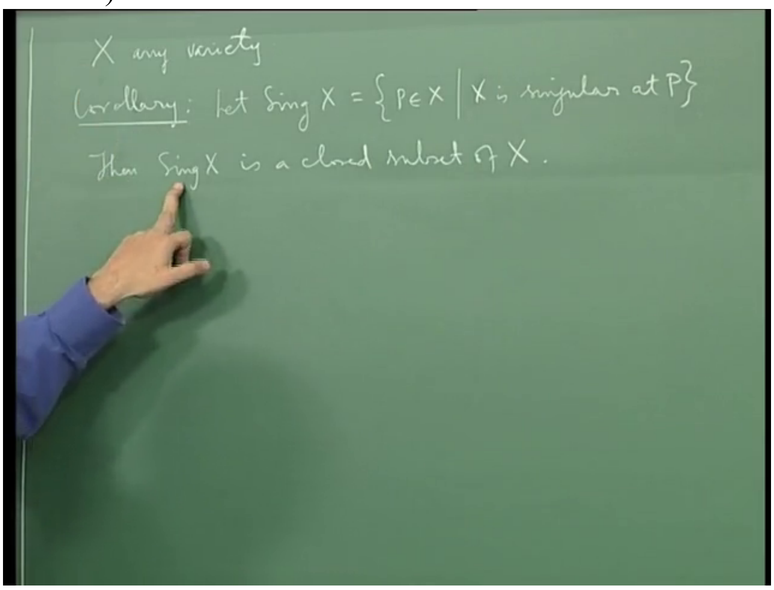

this is just given by, you take the ideal of X, Ok and you also take, you are adding also these

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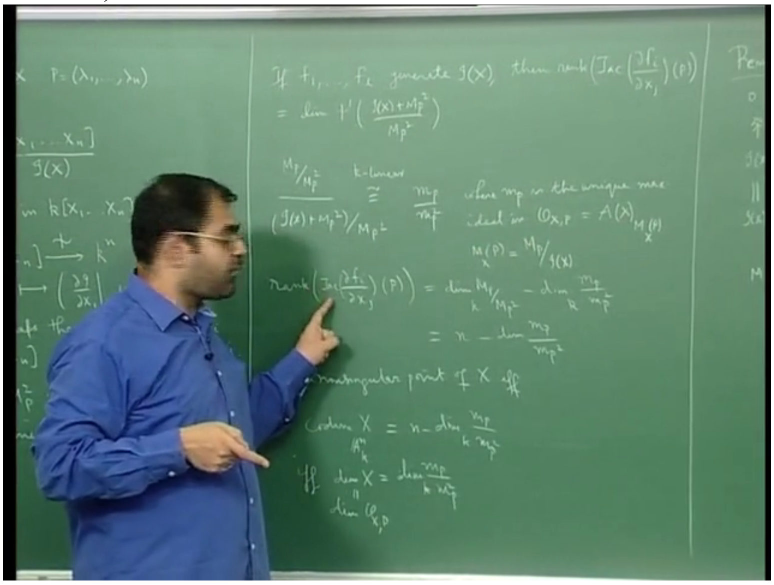

polynomials. Ok, without the P.

You take all the maximal minor determinants. They will also be some more polynomials, finitely many polynomials. You add them also to the ideal of, to take them together with the ideal of X and the zero set of this is going to be exactly the set of singular points. Therefore the zero set of, it is an algebraic set. Therefore it is a closed subset. Ok. Therefore this is always a closed subset, right.

Now what is more serious, what is more serious is that, and this is very, very important. What is more serious is that the bad situation that all points are singular, does not occur. In other words, this closed

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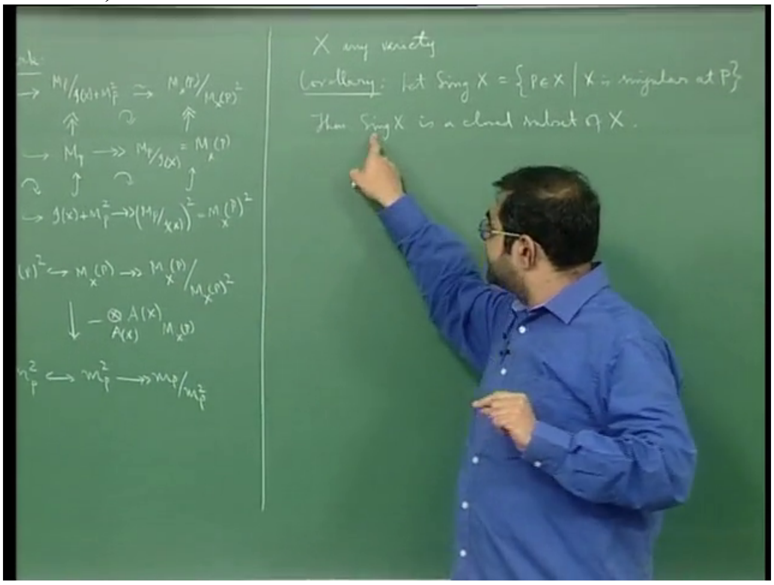

set is a smaller portion. It is not the whole space. Ok. What it means is the complement is a non-empty open set and you know non-empty open set is irreducible and dense Ok

So which means there is a huge open subset of points which are all good points. They are all non-singular. The set of bad points is only smaller closed set. Ok it is a small closed set.

So here is more serious theorem, here is a theorem; theorem is that singular points of X is a proper closed set, subset of X. Ok. This is an important theorem. And what it implies is that,

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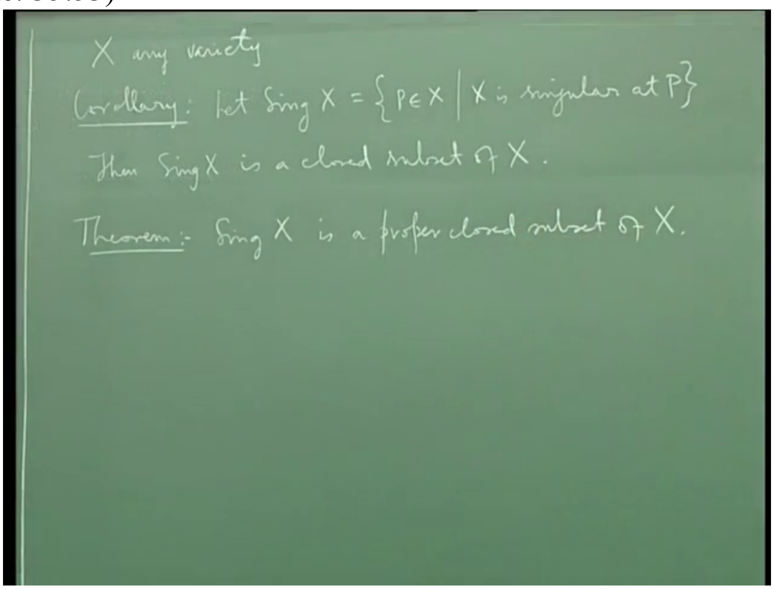

I mean what it says, you know from the viewpoint of, if you think of it with respect to; you know usual analysis, where we study, where we think of, you know smooth geometric objects being given by manifolds.

So what it says that every variety has a huge open set which looks like a manifold, Ok its complement of that open set is the boundary of that open set; that is the one which contains the non-manifold points. Ok. The manifold points are the smooth points, the non-singular points. And that is a huge open set, Ok.

And the non-manifold points are the non-smooth points. They are the singular points. Though these are not non-singular. And that forms a proper closed set. Ok. So you, if you want to think of it, if you want to think in terms of analysis, you must think of variety as a, as a nice manifold with a boundary consisting of points which are not manifold points.

And of course the singular points may be empty also, Ok. We have already seen last time that, you know, if you take any affine space, then that is, that is smooth which means it is non-singular. Every point of affine space is a smooth point, it is a non-singular point. That is because, that is basically because of this isomorphism,

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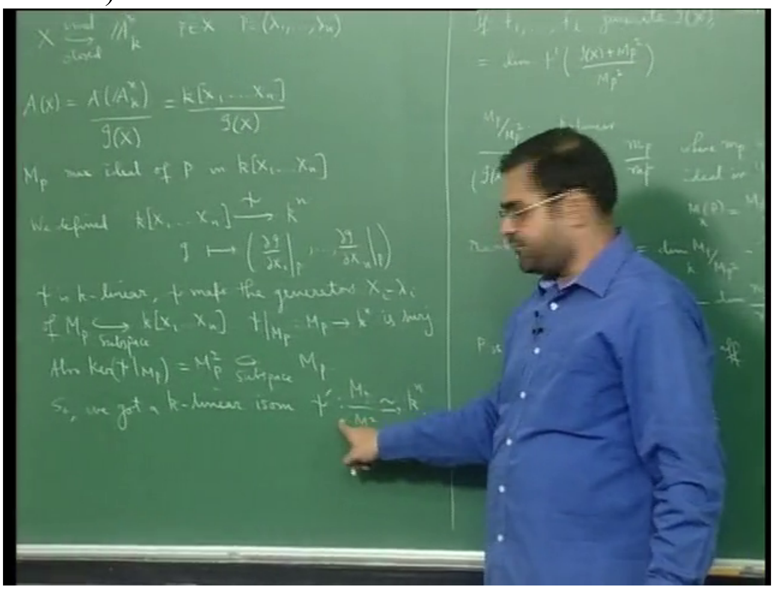

Ok. That is basically given by this isomorphism.

So this isomorphism will tell you that the dimension of, if you take the local ring of the point P in the affine space then M mod M squared is just k n. Ok. And therefore the dimension of the local ring is equal to n is equal to dimension of affine space. So this calculation itself tells you, after you accept this theorem, that calculation, that first calculation itself tells you that every point of affine space is smooth.

So affine space, if you take x C equal to affine space, then

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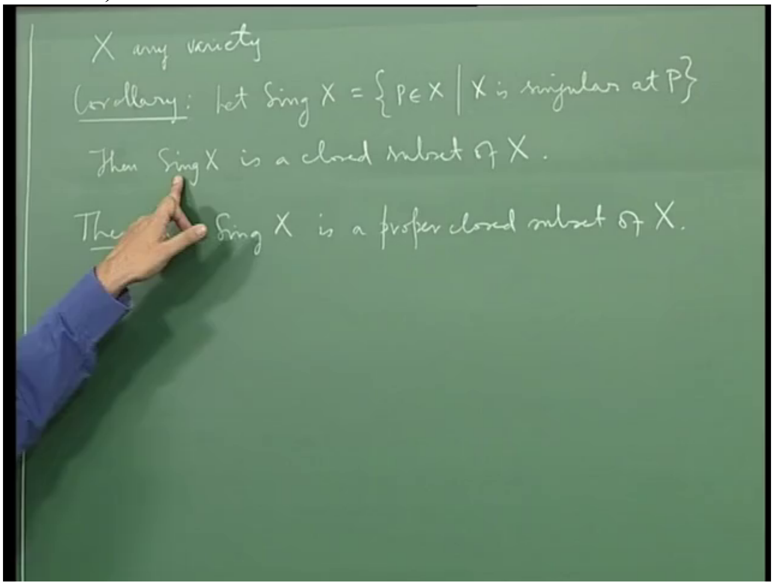

it is empty. The singular point of X is empty. So the singular points can be empty, Ok, in which case we say the variety is a smooth variety or non-singular variety.

If the singular points are not empty, tells you life is not so bad. They, there the set of singular points is only a small set. See you have huge set of good points.

So now to prove this theorem, one needs to go into a small detour which involves little bit of Field Theory and, and, you know which come up because of concepts of bi/ birationality, issues of birationality.