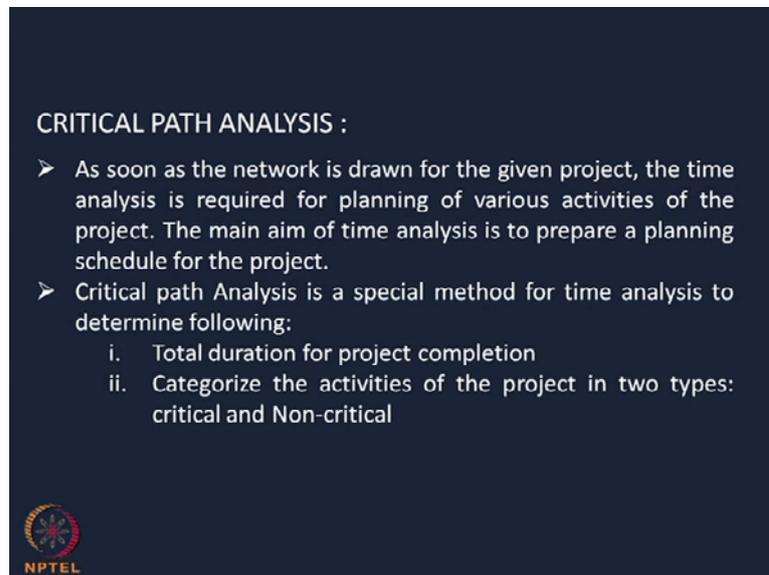**Optimization**
Prof. A. Goswami
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture - 17
Critical Path Analysis

So, in the last class, we have started the project management, where we have seen that, whenever we are having a project, the project is being divided into several sub components, which we are telling as activities or events. And we want to see the flow of the activities using some diagram, what we call as the network model. And so, for the project, we are drawing the network model and from the network model, we will see the flow.

Now, to find out the duration of the project, the cost of the project, how the duration of a project can be reduced for that. As I discussed in the last class, there are two mechanisms, one is critical path analysis, another one is the part.

(Refer Slide Time: 01:06)

So, today at first we will start with the CPM that is, Critical Path Analysis, how to find out the solution of the critical path analysis. If you see here, as soon as the network is drawn for the given project, the time analysis is required for planning of various activities of the project. So, the main aim of time analysis is to prepare a planning of schedule for the project.

So, what critical path analysis done, it is a special method for time analysis to determine basically two things as I was telling, total duration of the project completion and the number 2 is categorize the activities of the project into two parts, one is critical, another one is non critical. Or in other sense, we want to say that, we want to find out the total duration of the project and whenever you are having the activities, from the activities of the nodes, we want to find what are the critical activities, what are the non critical activities.

Now, critical activities means, which we have to closely watch, for non critical activities we do not want to watch closely.

(Refer Slide Time: 02:16)
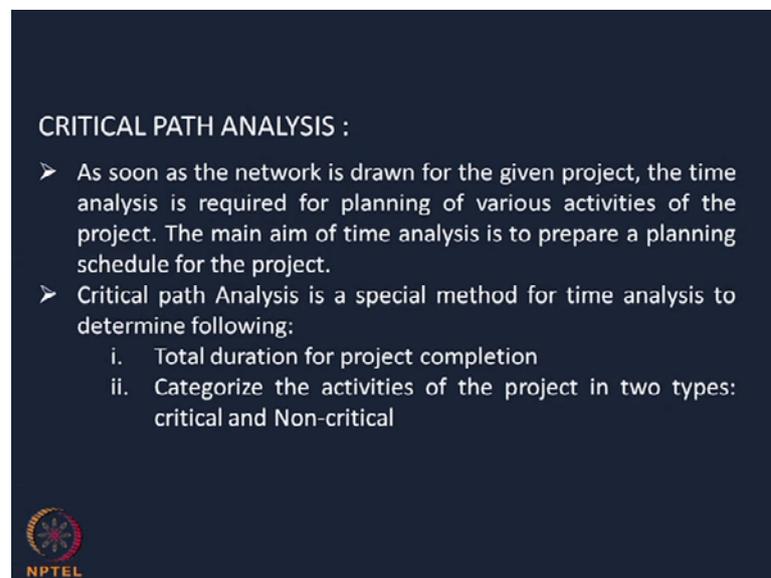
CRITICAL PATH ANALYSIS :

➤ As soon as the network is drawn for the given project, the time analysis is required for planning of various activities of the project. The main aim of time analysis is to prepare a planning schedule for the project.

➤ Critical path Analysis is a special method for time analysis to determine following:
   i. Total duration for project completion
   ii. Categorize the activities of the project in two types: critical and Non-critical

First let us see the definition of these two, critical activity, an activity in network diagram whose delay in the beginning will further delay the project completion time. You see, this one whose delay, critical activity is the activity whose delay in the beginning will further delay the completion of the project or project completion time.

(Refer Slide Time: 02:40)
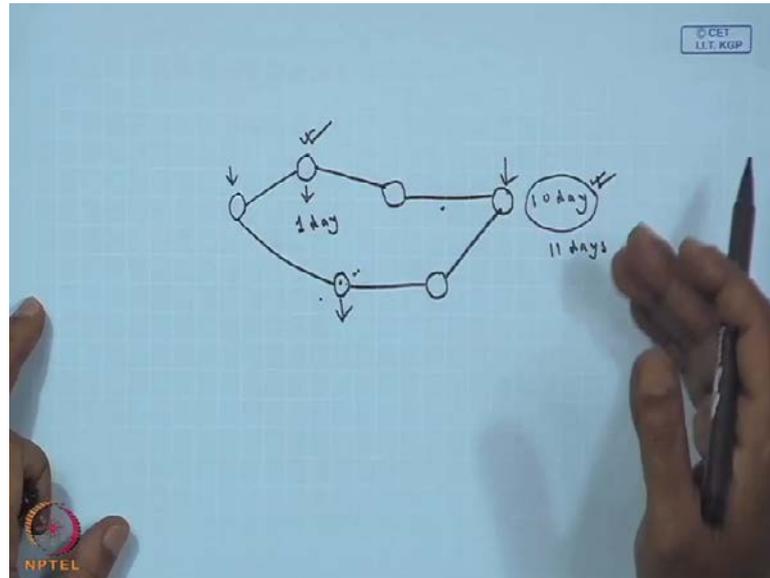
Or in other sense, if you see, if I have drawn a network something like this, it is going like this way and so, it is going to end, so this is your starting node, this is your ending node. Now, these activity if you consider, these activity I will tell as critical activity, if I start it late then, the project completion time also will be late. Suppose, project completion time is 10 days and if I start this activity one day later then, this will affect the project completion time, say it will become 11 days.
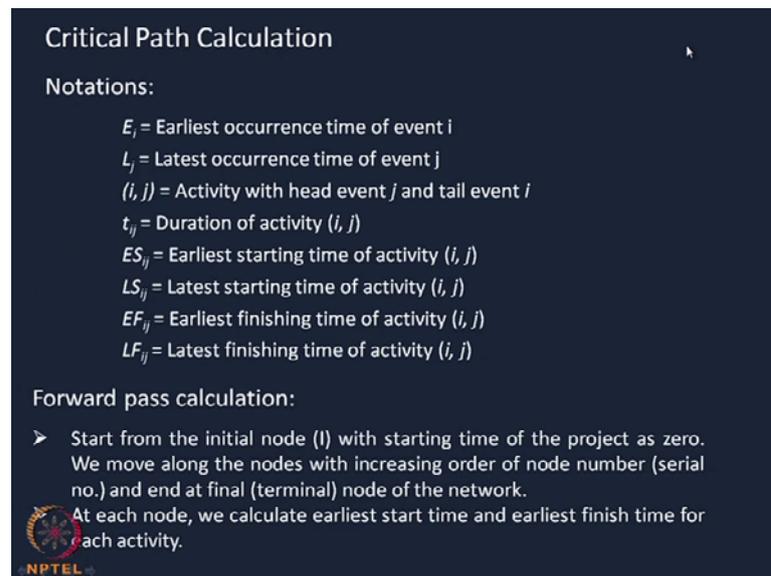
So, basically, if I delay the process of starting of one activity, which causes the completion of the project also in the delay further, that activity we are calling as the critical activity. So, what is non critical activity, the non critical activity is an activity which allows some scheduled slack so that, the start time of the activity may be delayed or advanced within some range without affecting the completion time of the entire project.

You see this one, here you can allow some scheduling slack so that, the starting time of the activity may be delayed or advanced that is, if you want you can delay it little bit, if you want you can advance it little bit, but that will not affect the project completion time. If you see in the figure, suppose I have drawn something like this, along with this, this is again the end node. For this particular node, this will be a non critical activity, if I start it at the beginning one day earlier or if I start it one day later, this will not affect the project completion time.

Or in other sense, if this activity starts before hand or if it starts after one day, the project completion time remain 10 days only then, the activity is known as non critical activity. So, I hope it is clear, critical activity means, the activity if you delay the beginning of that activity, this will affect the completion of the project. Whereas, for non critical activity, whether you start the activity before hand or after hand within some range of course, it will not affect the project completion time then, that one we call it as the non critical activity.

So, basically we have to watch closely that, we are starting the critical activities at the very pin pointed time, otherwise you cannot complete the project in the scheduled duration. So, you have to note this one that, at first I have to find out by some mechanism, from this which activities are the critical activities and which are non critical activities. So, that afterwards if we wish, we can change the values of the critical activities and we can check, what is the affect on the project completion time.

(Refer Slide Time: 05:50)

Now, how to find out these things, for this, for calculation of the critical path, we will use some notation, just see this one E i, capital E i is the earliest occurrence of time, occurrence time of event i. L j is the latest occurrence time of event j that is, earliest when it will occur or latest when it will occur, that we are denoting by E i and L j, i comma j activity, one activity is denoted by this with head as j and tail as i. If you see

here ((Refer Time: 06:26)), here i to j, so this is the head basically, this node is the head, this node is the tail.

So, i comma j means, head is this and tail is this one, so i comma j denotes like this, the next one is the t i j, duration of the activity i j. So, as I was telling, duration of the activity i j means, t i j is this one, this t i j will be this thing, the duration of the activity or the completion of this activity takes how much time or what is the cost, whatever way you want to take, so this we call t i j, duration of the activity i comma j. Next one is ES i j, earliest staring time of activity i j that is, one activity i j at the earliest when I can start that is, ES i j.

Similarly, LS i j is the latest starting time of the activity i j, latest when you can start that is, one activity can start at the beginning utmost when and utmost on the other part with. What is the range, this range we are calling as ES i j, earlier starting activity i j and the latest starting time of activity i j. Similarly, for the finishing also we have two notations, EF i j, earliest finishing time of activity i j and LF i j that is, latest finishing time of activity i j, so these are the notations we will use, you will see afterwards.

Now, let us see the, whenever we want to calculate this, we use basically the two methods, one we call as the forward pass method and one as the backward pass method. Let us see first the forward pass method what we do, in the forward pass method we start with the initial node.

(Refer Slide Time: 08:21)
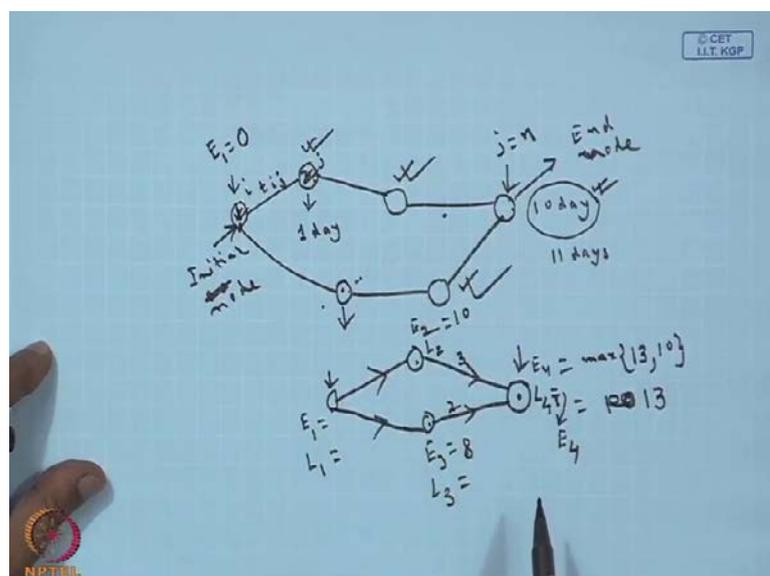
Just like in the diagram, in this diagram, this we call as the initial node, this one is your initial node and this node we call it as the end node that is, this is the end of the diagram. So, at first we have to start with the initial node and then, with starting time of the project as 0, whenever your starting time obviously, that starting time we should assume as 0. We move along the nodes with increasing order of node number, increasing order of node number means, the serial number 1 2 3 like this way and we have to end at the final node of the network.

At each node, we have to calculate the earliest start time and earliest finishing time that is, the E i and L j, these two we have to calculate. At each node, we will calculate the earliest start time and the earliest finishing time. Now, what would be the algorithm for this one.

(Refer Slide Time: 09:27)

Algorithm for forward pass:

Step 1: Initialize $i=1$, $E_1=0$ (First node)

Step 2: Calculate earliest start time for each activity that begins at node i; as

$$ES_{ij} = E_i \text{ for all } (i, j) \text{ with starting node } i$$

Step 3: Compute earliest finishing time for each activity that begins at node i; as

$$EF_{ij} = ES_{ij} + t_{ij} = E_i + t_{ij}$$

where t_{ij}=duration of (i, j).

Step 4: Proceed to next node, say node j ($j>i$) and compute the earliest occurrence foe node j using

$$E_j = max_i\{EF_{ij}\} = max_i\{E_i + t_{ij}\}$$

for all immediate predecessor j.

Step 5: If $j=n$ (final node), then the earliest finish time for the project is given by:

$$E_n = max\{EF_{ij}\} = max\{E_{n-1} + t_{ij}\}$$

If you see, the algorithm for forward pass, so step 1 initialize i equals 1 and E 1 equals 0. So, in the figure if you see, you will take say this one as E 1, this is E 1 equals 0, so this is your starting one, if this node is 1 say, this node is 2 something like this, we will see in the figures, so at first I will make E 1 this is equals to 0. Next step is, we have to calculate the earliest start time for each activity that begins at node i and earliest start time for each activity, that begins at node i, ES i j which is equals E i that is, for all i j with starting node at i.

Then, step 3, compute the earliest finishing time of each activity at node i, this we have denoted as EF i j. The value will be ES i j plus t i j, ES i j already you have seen the initial value that is, E i, so it will be E i plus t i j, where t i j is the duration of i j. Whenever we are going through the problem, you will see how we are calculating, the mathematical part let us finish first. Step 4 is, you have to proceed to the next node, say node j and we have to complete the earliest occurrence time for node j.

First, the earliest occurrence time if you see, the formula we have given E j equals maximum over i EF i j, this is equals maximum over i E i plus t i j. So, that means what, why we are telling the maximum, maximum means suppose, you are having something like this, you are having this, from these two you are getting another one that is, this one, the flow is something like this way. So, whenever you are trying to compute for this one E i that is, earliest occurrence time for this one, it depends on both these.

I have to calculate what was the value of here, so this is E 2, this is E 3 and this is say E 4. So, what I will do that, this activity t i j, that the duration is say, it is 3, it is 2, here E 2 is 10 and E 3 is 8 suppose. So, what will be E 4, whenever you are calculating E 4, you calculate E 2 plus t i j that is, you are getting 13 maximum of i, 13 here you are getting, 13 comma, here it is for this one, for this link, it is 8 plus 2 that is, 10. Maximum of these two whatever you get that will be the value of this, that is E 4 will become not 10, but 13.

So, like this way, we are telling that the maximum value will be, from these two node it may be, whenever the node is joining from more than one cases, one activities then, we will take the maximum value, so we are calculating in step 4, the earliest occurrence time. Step 5 is the earliest finishing time, the earliest finishing time will be similarly your, if j equals n, if you see you are going on, whenever you are going to the j equals n, the last node that is, this whenever j equals n.

In that case, your formula will be E n equals maximum over EF i j, earlier finishing time i j that is, you just see the PPT. In the PPT, it is E n equals maximum over EF i j and which is equals EF i j you can write it as, E n minus 1 that is, the earlier node plus t i j. Now, n minus 1 node means, this it may be this, it may be this one also, so therefore, for the last node, it is equals to maximum of E n minus 1 plus t I j, so this is your forward pass calculation.

So, forward pass calculation basically what is happening, if you see, this is your starting node, this is your ending node. If it is this, you are calculating E 1 first then, E 2 then, E 3 then, E 4 like this way, forwardly one by one. By sequence wise, you are calculating the earliest occurrence time and earliest finishing time, this is the basic idea of the forward pass.

(Refer Slide Time: 14:07)

Backward pass calculation:

➤ Here, we begin from terminal (last) node of the network proceed through the network visiting nodes in the decreasing order of node numbers and end at the initial node. At each node we calculate the least finish time for each activity.

Step 1: Initialize $L_n = E_n$ for $j=n$.

Step2: Set the latest finishing time for each activity (i, j) that end at node j.

$$LF_{ij} = L_j$$

Step 3: Calculate latest starting time for each (i, j)

$$LS_{ij} = LF_{ij} - t_{ij}$$

Step 4: Proceed backward to the node in the sequence that decrease j by 1. Latest occurrence time of node i $(i<j)$

$$L_i = min_j \{LS_{ij}\} = min_j \{L_j - t_{ij}\}$$

Step5: If $j=1$ (beginning node)

$$L_1 = min \{LS_{ij}\} = min \{L_2 - t_{ij}\}$$

Now, let us see the other one, that is I told you, the backward pass calculation. In the backward pass calculation, basically it is just the opposite of the forward pass calculation that is, we begin from the terminal node of the network proceed through the network, visiting the nodes in the decreasing order of the node number and end at the initial node. At each node, we calculate the least finishing time for each activity that is, for this one, basically I will start the least time from the end node.

From L 4 I will start, what will be the value of L 4, value of L 4 will be whatever we got for E 4 that is, the earliest completion time. So, L 4 will be initialized with this then, you will calculate L 3, we will calculate L 2, we will calculate L 1, like this way you are proceeding. So, basically from backward or from the end node you are starting, end activity was starting and you are proceeding, ultimately you are coming to the beginning one.

So, this is the basic process of the backward pass calculation, let us see the steps, in step 1 as I told, initialize L n equals E n for j equals n. Already through the backward pass

calculations, you know what is E n, so L n equals E n. Step 2, set the finishing time for the activity i j at the end node of j as LF j equals L j. Step 3 calculate the least starting time as LS i j equals LF i j minus t i j, here you note one thing for forward pass, we were calculating LF i j plus t i j, here we have to make the minus, because we are going to the backward side.

And similarly, in step 4, you proceed backward to the node in the sequence decreasing by j by 1 and the formula will be here L i equals minimum over j LS i j and LS i j is nothing but, basically L j minus t i j, so minimum over j L j minus t i j. For forward pass, we were doing the maximum over i L i plus t i j, here it is minimum over j L j minus t i j. And whenever step 5, when j is equals to 1, in that case L 1 will become the beginning node that is, L 1 equals minimum of LS i j and LS i j means, LS 1 2.

So, it will be minimum of L 2 minus t i j, whatever calculation you are getting, so this is for the backward pass.

(Refer Slide Time: 16:54)
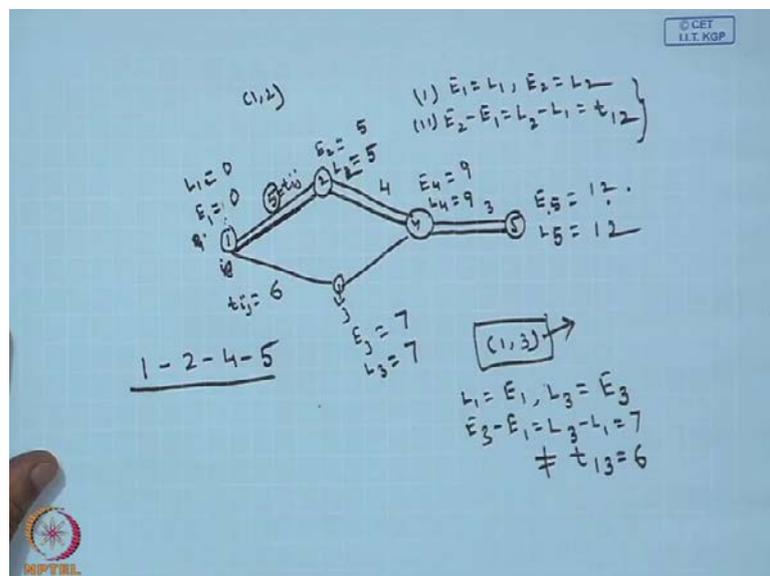
So, once I have calculated using forward pass and using the backward pass, you will have the figure something like this, I am just imaginarily I am drawing one figure like this, say these activity is 1, this is 2, this activity is 3, this is 4, this is 5. So, at first what we are doing, we are calculating E 1, E 2, E 3, what is E 4 we are calculating then, we are calculating E 4. Once I am getting E 5 then, again for the backward pass, I am

calculating L 5, L 5 is nothing, but E 5, from here I can calculate L 4, from here I can calculate L 3, from here L 2 and at last L 1.

So, L 1 I am getting, so using forward pass, you are getting the values of E 1, E 2, E 3, E 4 ,E 5. Using backward pass, you are getting L 5, L 4, L 3, L 2 and L 1, so now, what are the critical activities, how to find out, let us see what are the critical activities.

(Refer Slide Time: 17:53)

The mathematical expression for the critical activity, first I am telling mathematically then, I will explain through the figure. Based on the forward pass and the backward pass calculation, an activity i j will be critical if it satisfies the following condition. What is the condition, number 1 E i equals L i and E j equals L j that is, for one activity, i comma j, you have the activity i comma j over here, say this is your i th node or if I say 1 2. For the activity 1 2, what happens let us see, for activity 1 2 what happens.

If I told E i equals L i that is, E i equals L i means, if E 1 value L 1 value same and E 2 value and L 2 value same, this is say 0 and this is 5, if both are same and there is second point. So, first point is, if E 1 L 1 are same and E 2 L 2 are same for the activity 1 2, this was point number 1. Point number 2 is, if E j minus E i equals L j minus L i equals t i j, suppose for this activity, t i j this is equals 5, what is E j minus E i that is, E 2 minus E 1 that is, 5 minus 0, 5 then, L 2 minus L 1 that is, L j minus L i, that is also 5 minus 0 5, that is equals t i j, t i j here is 5.

So, if these values are matching then, this part we call it as the critical activity then, this activity i j or 1 2 we call it as the critical activity. So, I think it is clear, after calculation I will check, whether the values of L 1 E 1 same or not, L 2 and E 2 are same or not and the difference between E 2 minus E 1 that is, this one E 2 minus E 1 should be equals to L 2 minus L 1 and this should be equals to t 1 2, this is the second one.

And first one is E 1 equals L 1 and E 2 equals L 2, if for the activity 1 2, if these two conditions are satisfied, E 1 equals L 1, E 2 equals L 2 and E 2 minus E 1 equals L 2 minus L 1 equals t 1 2, in that case we say that, the activity is the critical activity. But, suppose, instead of 5, suppose this one is you consider now this part, L 1 0 E 1 0 after calculation, I got E 3 7, suppose L 3 7 and the value of t i j this is equals 6. Then, here if you see, t i j equals 6, so here for the activity 1 3 what happens now please check.

For the activity 1 3, your L 1 equals E 1 this is valid, L 3 equals E 3 this is valid and E 3 minus E 1 this is equals L 3 minus L 1, this is also valid equals same, but this is not equals t 1 3, because t 1 3 is 6. So therefore, this activity 1 3 is not a critical activity, because these two conditions are not being satisfied, therefore this is not a critical activity. So, I think, whenever I will calculate the activity, how to check, whether the activity is critical or non critical, we have to follow these two.

For the activity 1 2, L 1 should be equals to E 1, L 2 should be equals to E 2 and your E 2 minus E 1 equals L 2 minus L 1 equals, it should be the duration of the activity that is, t 1 2 then, the activity 1 2 will be known as the critical activity, otherwise it will be known as non critical activity. So, once I am observing that, pin pointing what are the critical activities, my next thing is what is the critical path. The sequence of critical activities in a network is called a critical path, sequence of critical activities known as the critical path.

It is the longest path in the network from starting event to ending event and determines the minimum time for completion of the project. So, basically what is happening, suppose I am going something like this, this is the L 2 and here again it is say 9, it is 9, so I am telling it as 4 and then, it is 3 and 12. So, if you see, the critical activities I can assume as 1 2, 2 4 and 4 5, because they are satisfying my conditions, all these values are same over here.

So, the critical path will be 1 2 4 and 5, this one, so the project completion time, minimum project completion time is becoming now this 12, may be the time of E 5 and L 5. So, this is the activity, I will show through examples after some time, how we denote it, we denote it by this double line that is, to show the critical path, afterwards we make it something like this.

So, the double line tells that, this is the critical path and through this path, the activities can be further down. So, the last point if you see, we have retrun to specify critical path on the network, double lines are used to distinguish from non critical path. So, whenever I have double line, those are critical path and if I have single line that means, that path is the non critical path.

(Refer Slide Time: 24:03)

Float and Slack times:

➤ After labeling the network diagram and performing the forward and backward pass computations one has to determine float and slack times.
  Float → used for activities
  Slack → used for events.

Slack of an event: (Also known as Event float)

➤ The slack of an event is difference between its latest time (L_i) and its earliest time (E_i). That is
$$Event\ Slack = L_i\text{-}E_i.$$
➤ An event with zero slack time is called critical event.

Float of an activity: (Activity Float)

➤ There are four types of Activity floats, which determine activity's time-estimates.
  i. Total float
  ii. Free float
  iii. Independent float
  iv. Inference float

The next one is the float and the slack times, so after I have labeling the network diagram and performing the forward and backward pass computations, now I have to determine the float and the slack time. What do you mean by float, float time is used for the activities, whereas slack time is used for the events. What is slack of an event, opposite one also known as event float the same thing, the slack of an event is difference between the latest time L i and it is earliest time that is, I know what is your latest time L i and what is the earliest time E i.

So, L i minus E i whatever the difference, that we call as the event slack, so an event if we have a zero slack time then, we call it as the critical event. Whenever as I was telling

you, for critical activity E i should be equals to L i. So, E i equals L i means, event slack should be equals to 0, so event slack is L i minus E i, so an event with zero slack time, we call it as the critical event. Now, what is the float of an activity, the activity float basically there are 4 types if you see as I have written, total float, free float, independent float and the inference float.

(Refer Slide Time: 25:31)

(i) Total float:
➤ Amount of time by which an activity can be delayed without delay in project completion date. It implies the free time associated with the activity which can be used before, during or after completion of this activity.

Mathematically,
Total Float= Latest Finish time-Earliest finish time
Or   = Latest start time – Earliest start time
i.e. $TF_{ij}=LS_{ij}-ES_{ij}$
➤ An activity with zero total float is called critical activity.

(ii) Free Float:
➤ A portion of the total float within which an activity can be manipulated without affecting the float of the subsequent activities.

Mathematically,
Free Float of activity (i, j) $FF_{ij}=(E_j-E_i)-t_{ij}$
Note that, $0 \leq FF_{ij} \leq TF_{ij}$
Free float is used to reschedule activities with minimum disruption of earlier plans.

What is your total float, total float is the amount of time by which an activity can be delayed without delay in the project completion date. It implies that, the free time associated with the activity, which can be used before, during or after completion of this activity. Please note this one, amount of time by which an activity can be delayed without delay in the project completion date. That is, project completion time I am not compromising, but I am delaying the activity, starting of the activity can be delayed, but this will not affect the total project completion time.

You will see in the part, which we call it as the crash time also, so that free time for which the activity which can be used before, during or after completion of this activity. Mathematically if I have to say, total float TF i j this equals to the latest finish time minus earliest finish time or you can say, latest start time minus earliest start time that is, TF i j equals LS i j minus ES i j. Please note that, an activity which has a zero total float then, we call it as the critical activity.

So, to find out the critical activity, we can calculate the total float also, so total float is equals to latest finish time minus earliest finish time or latest start time minus the earliest start time. So, the next one is your free float, what is the free float, free float is a portion of the total float, within which an activity can be manipulated without affecting the float of the subsequent activities. Please note this one, a portion of the total float within which an activity can be manipulated without affecting the float of the subsequent activities.

Subsequent activities will not be affected and a portion of the total float that is, free float is a subset or less than equals the total float. Mathematically if I have to say then, we will say that, free float of an activity F i j, which we denote by FF i j that is, free float equals to E j minus E i minus t i j, EF E j minus E i minus t i j. And please note that, 0 less than equals FF i j less than equals t f i j that is, free float should be less than equals 0 and it is must be less than equals the total float.

So, what is the use of free float, free float is basically used to reschedule the activities with minimum disruption of the earlier plan. That is, say suppose in this figure, without disrupting this earlier plans, if I can change something on this activity, on this node if I can change without changing the earlier plan. That is, suddenly something I have to introduce, I will introduce a new thing or new activity somewhere, but I will try to introduce it in such a fashion that, it does not affect the earlier plans whatever I have done, so for this, we use the concept of the free float.

(Refer Slide Time: 29:02)

(iii) Independent Float:

➤ A portion of total float within which an activity can be delayed for start without affecting floats of preceding activities.

Mathematically,

$$IF_{ij} = (E_j - L_i) - t_{ij}$$

➤ Negative independent float is assumed zero.

(iv) Inference Float:

➤ A portion of total float which causes reduction in the float of the successor activities.

$$INF_{ij} = TF_{ij} - FF_{ij}$$

➤ Note the relation:

Independent Float ≤ free Float ≤ Total Float

The next one is the independent float, a portion of the total float again, the independent float is also a portion of the total float, within which an activity can be delayed for starting without affecting the floats of the preceding activities. The earlier one was subsequent activities and this one is the preceding activities that means, afterwards. So, the mathematically if I have to say IF j, which we denote as a independent float we denote as IF i j, this equals E j minus L i minus t i j.

So, IF i j equals E j minus L i minus t i j, please note that negative independent float, if there is any negative independent float, that we will assume as 0, we will ignore it. So, basically, the independent float is just the opposite one, opposite in the sense that, in the free float, we were using the preceding activities, whereas in the independent float, we are using the successive activities.

The last one is the inference float, in the inference float, a portion of the total float which causes reduction in the float of the successor activities that is, INF i j this equals to TF i j minus FF i j. Please note the relation, that independent float is less than equals free float and is less than equals the total float.

(Refer Slide Time: 30:40)

Draw the network for the following project. Compute the latest and earliest time for each node and also find critical path.

Activity	Immediate Predecessor	Time
1-2	-	5
1-3	-	4
2-4	1-2	6
3-4	1-3	2
4-5	2-4, 3-4	1
4-6	2-4, 3-4	7
5-7	4-5	8
6-7	4-6	4
7-8	6-7, 5-7	3

Now, let us see some example, how we can calculate the critical path, let us see this example. We want to draw the network for the following project and also we want to compute the latest and earliest time for each of the nodes and also find the critical path of this one. So, if you see the activity 1 2, activities are there, immediate predecessor is

there and what is the time it is given. If you see 1 2 and 1 3, there is no immediate predecessor and their time is 5 4.

(Refer Slide Time: 31:16)
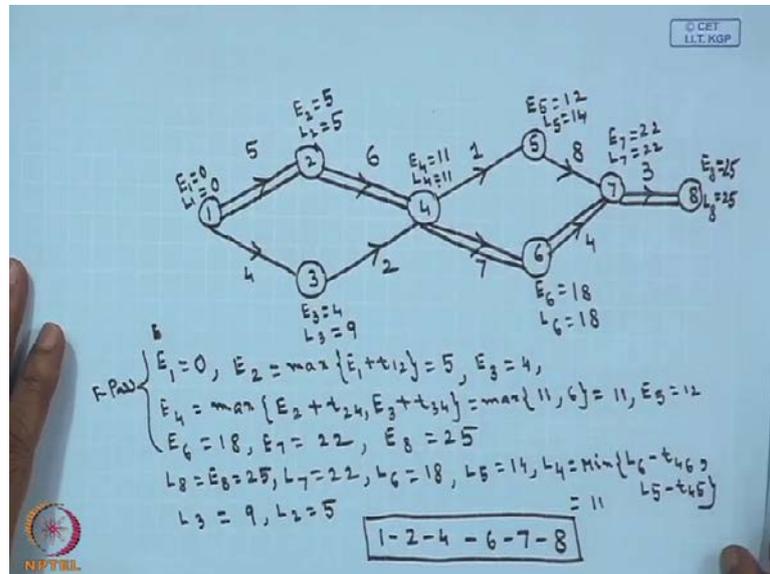
So, if you calculate, you have the activity 1, what we have told, activity 1 and activity 2 has no immediate predecessor. That is, from activity 1, basically it is the starting point and you are starting this, what is the time for activity 1 2, time for activity 1 2 is, if you see 5. And similarly, time for activity 1 3 is 4 and activity 1 3 also, there is no immediate predecessor. So, we are writing activity 1 2 time is, t i j is this, now for 3, activity 1 3 also you draw it and there is no immediate predecessor, the time taken is 4.

What is the next one, the next one is if you see in the table, activity 2 4 if you see, immediate predecessor is 1 2 and it is time taken is 6. That means, once activity 1 2 has been completed then only, you can perform the activity 2 4, so4 should come over here. So that, activity 2 4 is this thing, time taken is 6. The next one is, if I draw it in the little bit different way, let me just draw not this one, from the beginning let me draw it. This is the activity 1, this is your activity 2, time taken is 5, this is your activity 3, time taken is 4, you had the activity 4, now 2 to 4 the time taken we have given 6.

The next one in that table is activity 3 to 4 and immediate predecessor is 1 3 and it is starting time taken is 2. So, for 3 4, immediate predecessor is 1 3, so you see in the figure, after 1 3, activity 3 4 will be done. So now, I can draw a join these activity 3 4, the time taken is 4 then, after that, activity 4 5 if you see, activity 4 5 can be started after

completion of both 2 4 and your 3 4, both you can start. So, you see in the figure, after completion of activity 2 4, after completion of activity 3 4, you can start the activity 5, so am drawing the activity 5, the figure is this one, time taken is 1.

Then, next one is activity 4 6, that again starts from activity 2 4 and activity 3 4, time taken is 7. So therefore, after finishing activity 2 4, after finishing activity 3 4, your activity 6 can start, so I am writing this one, so activity 6 is coming over here. Next you see, the activity 5 7 can start after activity 5, so I am drawing this activity 7 after completion of 4 5, I can start it, so you are drawing it, from the table, time taken is 8. The next one in the table is activity 6 7, which can start after activity 4 6 and the time taken is 4.

So therefore, after activity 4 6, I can start the 6 7, so this one and the time taken is 4 and the last one is activity 7 8, which can start after completion of the activity 6 7 and 5 7, and the time taken is 3. Completion of activity 5 7 and 6 7 then only, you can start activity 8, so I can write down this, the time taken is becoming 3, I am drawing this thing, afterwards directly I will draw. For the first problem we have drawn it, so this is your network table, network diagram you have done.

First what I have to do now, I have to calculate the forward pass, using forward pass I have to calculate what is E_1, I am just first writing this then, I will write on the figures. Your E_1, initially your E_1 is 0, what is your E_2, from the formula E_2 I can tell, so I am writing E_1 as 0 over here, simultaneously I am writing your E_2 will be maximum over E_2 means, this is your E_2, what will be your E_2, E_2 will be maximum of E_1 plus t_{12}.

E_1 plus t_{12}, E_1 is 0, because from this node, only one node is going, from here only one node is attaching, so E_1 plus t_{12}, t_{12} what is the length, length is 5, so basically it is 5 only, there is only one element, so your E_2 you are getting as 5. Now, in the next the sequence is E_3, you have to calculate E_3, similarly E_3 will be maximum of E_1 plus t_{13}, what is t_{13}, t_{13} is 4 and E_1 is 0, so 0 plus 4, so E_3 will be equals to 4, so E_3 is 4.

Let me calculate now the E_4, let me write down E_4, I am writing this, afterwards I will not write for the next problems, maximum of E_2 plus E_4, it is coming from node 2, node 3 also. So, one path will be E_2 plus t_{24}, the another one will be E_3 plus t_{34} this length, so if you calculate maximum of E_2 plus t_{24} that is, 6 plus 5, E_2 plus t_{24} that

is, 11 and this one is E 3 plus t 3 4, so 4 plus 2, 6, so the value becomes 11, I hope the concept is clear now.

So, once I am calculating E 4, now I have to calculate E 5, E 5 is coming from only one node that is, E 4 plus this length, so E 5 will be equals to 12. Once I am getting E 5, you are getting E 5 equals 12 then, you have to calculate E 6, 4 to 6, 4 to 6, it is 7, the duration is 7. You have to calculate E 6 now, for E 6 your E 4 is 11, plus this time length that is, 7, so it will become 18, E 6 is 18. Your E 7, E 7 is approaching from two nodes, node 5 and node 6, so you have to calculate the maximum of these two, maximum of E 5 plus t 5 7 and E 6 plus t 6 7. So, here it is 12 plus 8 20, here it is 18 plus 4, 22.

So therefore, that maximum is 22 and 20 is 22, therefore your E 7 you will get as 22, so E 7 is 22 and E 8 the last one that is, this one is E 8, this E 8 you will get obtain as 22 plus 3 that is equals 25. So, you are getting 25, so this calculations your E 8 is 25, this calculations you are getting using forward pass. So, once I obtain by forward pass this one, now I have to go through the same fashion backward pass also. Your backward pass your L 8, L 8 will be 25 that is, E 8, so your L 8 is equals to E 8, this is equals 25.

So, once I have obtained L 8 25, now I can calculate L 7, what is your L 7, L 7 will be equals to the, what is the earlier one, earlier one is equals to L 8 minus this length that is, 25 minus 3, so it will be 22, L 7 this will be 22. So, E 7 and L 7, L 7 is becoming 22, now once I am getting the L 7 then, I can calculate L 6, your L 6 it is coming from here, 22 minus 4, so it will be 18, so L 6 is 18. What is your L 5, L 5 you will get from here, 22 minus 8 that is, 14.

So, you are coming from the backward side, backward side and the duration you are subtracting, so your L 5 this is equals to 14 and then, your L 4, what is your L 4, L 4 you see, L 4 is coming from two sides. If you see, L 4 is coming from node 5, node 6, so L 4 you can write down, minimum of L 6 minus t 4 6 comma L 5 minus t 4 5, L 6 minus t 4 6 that is, 11 you will get and here L 5 minus this, that is 13, so minimum of 11 and 13 that is, 11. So therefore, your L 4, this will become 11, so I think it is clear, L 4 is 11, because minimum of this 14 minus 1, 13 and 18 minus 7 11, minimum of 13 and 11 is this.

So, similarly you can calculate now your L 3, your L 3 will be 11 minus 2, this is equals 9, your L 3 is becoming 9. Your L 2 is coming over here, 11 minus 6 that is, L 2 will be

5, your L 2 will become 5 and the last one is L 1, your L 1 again is coming from both, here it is 0, 5 minus 5 0, here it is 9 minus 4, 5, so minimum of 0 and 5, this is equals 0. So, by this way, you have calculated the next one that is, L i L j's also, now what will be the critical path, critical activities then, E 1 0, L 1 0.

So, one activity is 1 2, because it is satisfying both the conditions over here if you know, so 1 2 it is satisfying then, it will satisfying this one, 2 4, E 2 E 4, 11 minus 6, it is 5. And after that, it is satisfying the activity 4 6, 11 E 4 L 4 same, E 6 L 6 same and their difference is also same. Then, E 7 L 7, 6 7 activity, here also E 7 L 7 same and from here, from 7 you are going to this. So, I can draw the critical path by double line as I told, from activity 1 2 then, from activity 2 4, after that from activity 2 4 to activity 4 6 and activity 4 6 to 6 7, from 6 7 to 7 8.

So therefore, my critical path is, I can say 1 2 4 6 7 and 8, this is my critical path, I think the problem is clear now, 1 2 4 6 and 7 8. And what is the critical duration, the critical duration or the critical time is, path is this and the time taken will be obviously the maximum value that is, 25. So, I hope it is clear now that, how to find out the solution of this thing, how we are drawing first the network, from the network we are calculating the E i's and L j's then, we are finding the critical path.

(Refer Slide Time: 43:56)

A building construction project has following time schedule:

| Activity	Times (in month)
1-2	2
1-3	2
1-4	1
2-5	4
3-6	8
3-7	5
4-6	3
5-8	1
6-9	5
7-8	4
8-9	3

a) Construct the network diagram and
b) Compute total float for each activity.
c) Find the critical path and its duration.

Let us see the next problem, in the next problem, I will do it little faster, let us see this problem. A building construction company has the project, has the following schedules,

activities and corresponding times are being given, 1 2 2 3 like this way. And we want to construct the network diagram and then, what is the critical path and here the added thing is, we want to compute the total path also.

(Refer Slide Time: 44:30)
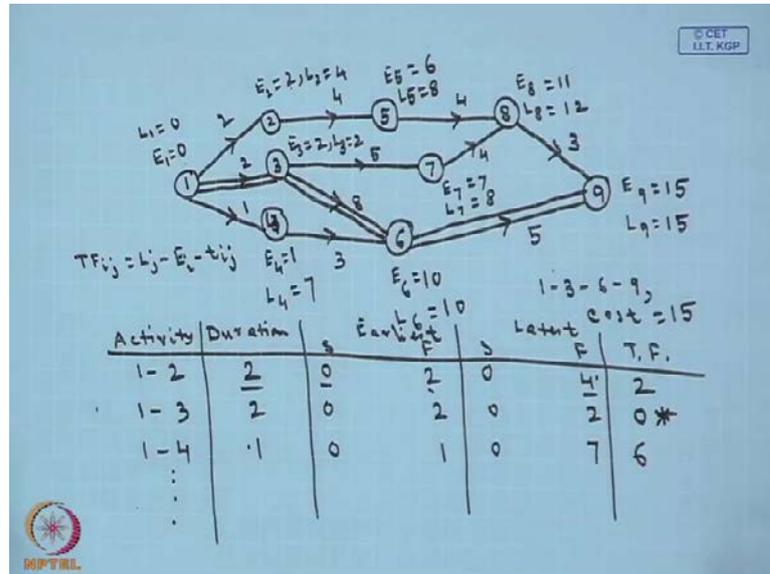
So, for this particular activities, I am just drawing first the diagram, 1 2 you can match it with the problem, 1 2 this is your, let me make it 3, let this one be 4, this is 2, this is 1, I am not explaining now, because this is same as we have explained in the first problem. From 3 to 6 it will come, 3 6 is there, so from 3 6, time is 8 then, we are having 3 7, 3 7 time duration is 5 then, we are having 4 6 that is, 4 6 is 3, after 4 6 we are having 5 8, so say 8 I am drawing here, 5 8, 5 8 is 4 and then, after 5 8, you are having 6 9.

So, from 6, there is say 6, 6 to 9 it will go, 6 9 duration is 5 then, you are having 7 8, so 7 8 is this one, time duration is 4, the last one is 8 to 9 and the time duration is 3. So, you are drawing the network diagram, on the same way now you can calculate, what will be your E 1. Initially E 1 is 0, your E 2 will be, I am just drawing this afterwards, I will write down directly, E 2 will be E 1 plus 2, so 2 E 3, E 3 will be equals to again 2, because E 1 plus this length E 4, E 4 will be equals to 1.

What will be your E 5, E 5 is E 2 plus 4, that is 6, what will be your E 6, your E 6 will be here 8 plus 2 10, here 3 plus 1 4, so 10 is maximum, so E 6 is this. Your E 7 is 5 plus 2 7, your E 8 this is equals to 6 plus 4 10, 7 plus 4 11, maximum is 11 and this one E 9, this is 11 plus 3 14, 10 plus 5 15, maximum is 15, so this is your E 9 that is, the earliest one.

From forward pass, from backward pass, now simply you can calculate, L 9 will be 15, L 8 this is equals 15 minus 3, that is equals this then, your L 7, L 7 12 minus 4, that is equals this.

Your L 6 will be 15 minus 5 that is equals 10, your L 5 now you will get for 12 minus 4, that is equals this, L 4 will be 10 minus 3 7. Your L 3 is how much, your L 3 you are getting here 8 minus 5 that is, 3 and here it is 10 minus 8, 2, so minimum is 2, so you got 2, your L 2 is equals to 8 minus 4, that is equals 4 and your L 1 this is equals 0, so I got this one. If you see, the critical activities are equal at 1 3 then, of course 6 and after that, 9.

So, if I wish, I can draw the critical path like this, 1 3 6 and 9, so what is your critical path, 1 3 6 9 and your total cost is, cost in terms of time, this is equals 15. Now, we want to calculate, what is the TF i j, your TF i j is what, TF i j if you would remember, TF i j the formula total float is L j minus E i minus t i j. So now, you see, you are having the activity, just let me draw, you are having the duration, you are having the earliest start time, earliest finishing time, you are having the latest start time, latest finishing time and what is the value of the total float.

Your activity is 1 2, from the table if you see, the duration is 2, what is the earliest start time, what is the earliest finishing time. You can see, earliest start time and finishing time is 0 and 2 and the latest start time, latest finishing time is 0 and 4, L 2 is 4, so latest. So, from this you can get it, earliest start time is 0, earliest finishing time is 2, so 0 2 earliest finishing start time is 0, L 2 is 4, so I have written 4. So, your total float, basically this minus this, please note this one, L j minus E i minus t i j. so it will be 4 minus 2 minus 0, so this will be equals to 2.

On the same way, you can calculate for 1 3, for 1 3 whenever you are writing then, I am just now I am writing this is 0, this is 2, because for 1 3, it is E 1 is 0, E 3 2, similarly L 1 0, L 3 2, so this is 2. If you calculate again, 2 minus 0 minus 2, so it becomes 0, so total float just like this way, you can calculate the next one is 1 4. For 1 4, I am just doing the last one, 1 4 the total duration for 1 4 is 1, the duration is starting 0 1, this is starting 0 7, so 7 minus 1, 6.

So, like this way for all others, you can calculate of your own, if you see, 1 3 is an critical activity. For any critical activity, total float must be equals to 0, please note this thing that, for the total float, the critical activity must be equals to 0.

(Refer Slide Time: 51:34)

Following table gives activities in construction project and time duration:

Activity	Preceding activity	Normal time (days)
1-2	-	20
1-3	-	25
2-3	1-2	10
2-4	1-2	12
3-4	1-3, 2-3	5
4-5	2-4, 3-4	10

a) Draw activity network of project.
b) Find total float and free float
c) Determine the critical path and project duration.

NPTEL

Now, let us see one more problem, this problem you just see, you can solve it of your own in the same way. Here, we have calculated the free float also, you can calculate the free float of your own using the formula, whatever we have given earlier.
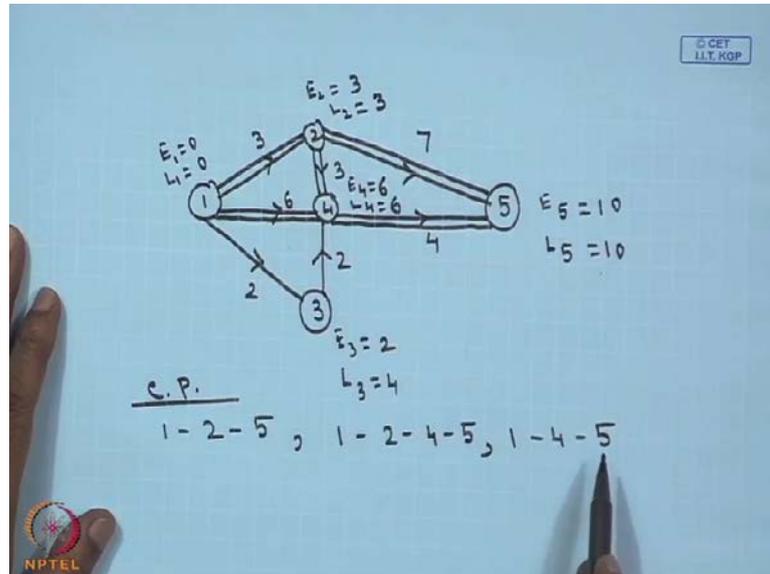
(Refer Slide Time: 51:49)

Example:
Consider a problem with the following data.

Activity	Times (in month)
1-2	3
1-3	2
1-4	6
2-4	3
2-5	7
3-4	2
4-5	4

NPTEL

Now, let us come to the next problem, there is one particular case is there. For this particular problem, you see the activities 1 2 3, 1 3 2, like this way. Let me draw it, for 1 2 it is 3, for 1 3 it is 2.

(Refer Slide Time: 52:05)

So, if you see, 1 here you are having the activity 2, you are having the activity 3, for 1 2 it is time taken is 3, for 1 3 the time taken is 2. Now, next one is 1 4, 1 4 the time taken is 6, after 1 4, it is 2 4, 2 to 4 that is this one, 2 to 4 it is 3. Next one is from 2 to 5, that is say 5 is here, so 2 to 5 the time taken is 7, next one is 3 to 4 that is, in this direction, from 3 to 4 time taken is this and the last one is 4 to 5, for 4 to 5, the time taken is 4. Now, from here, you can calculate what are the values of E 1.

E 1 is 0, your E 2 it will be 3 plus 0, 3 then, E 3 is equals to 2, E 4 this is equals to, now E 4 will be, this is your 6, this is your 4, this is your 6. So, E 4 is maximum value is 6 and your E 5, E 5 is again 6 plus 4 this is 10, 7 plus 3 10, so E 5 is 10. Next one is L 5, L 5 is 10, you are starting then. your L 4, what is your L 4, L 4 will be 10 minus 4, so it is equals 6. What is your L 3, L 3 you can calculate from here, so L 3 is your 6 minus 2, so this is 4.

What is your L 2, your L 2 is 10 minus 7 3, this is 6 minus 3, so this is also 3 and L 1 this is equals 3 minus 3, so it is 0. So, if you see 1 2 4 6, there is a peculiar thing that is, critical path is more than 1, one path I can say, it is 1 to 2 and 2 to 5. So, one path you can write as 1 2 and 5, one critical path I am writing the critical path, so the critical path

one is 1 2 and 5, other one you can say, other critical path is 1 2, I can go in this direction 2 to 4 and then, from 4 to 6.

So, the other critical path is 1 2 4 and 5 this is another critical path, the other critical path is also there, because E 1 L 1 and E 4 L 4 is 0 that is, another critical path is 1 4 and 5. So, you see, for this particular problem, basically we are having three critical paths, but for all critical paths, the total project duration is same that is, 10. So therefore, you have to remember one thing that, the project cost or the critical path is not unique, it may not be unique I will say.

It may not be unique, there may have more than one critical path, but always the project cost or the duration will be fixed if you have multiple critical paths. So, using the CPM method, basically we have tried to find out, what should be the critical path, what should be the critical activities and what are the values of total float and free float so that, if I try to readjust certain activities, what would be the effect. Now, if you see, we have taken in the CPM, whatever times we have taken, that is deterministic or constant times.

In the next class, we will talk about the part, which basically deals with the probabilistic time. And also, whenever if we can reduce the time that is, if I can crash the time, what will be the effect, that we will see in the next class.

Thank you.