Linear Algebra
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Lecture – 11
Elementary Matrices

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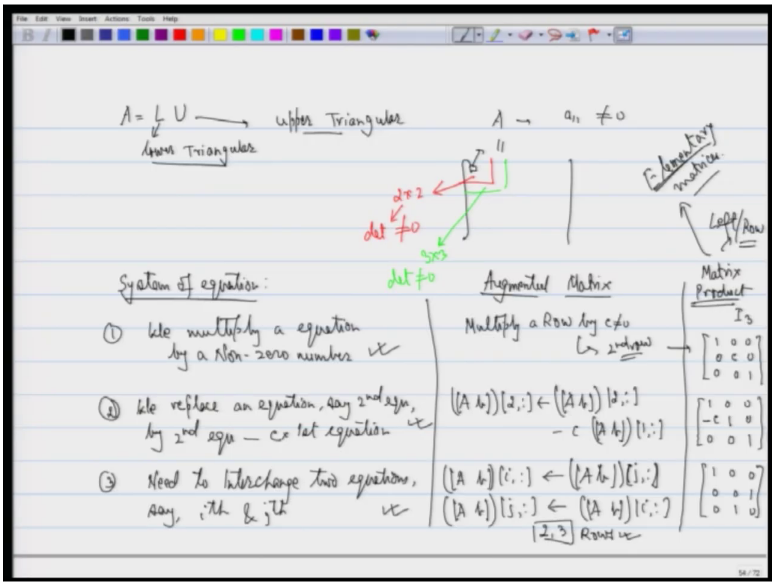

So, in the last lecture we learnt how to write the matrix A as LU, where L was lower triangular and U was upper triangular fine. And there was a condition for that, in the sense that we needed that if I look at A, then first thing that we needed was that the a 11 should be nonzero, then what we had was that look at if A was say this matrix, then the first entry was supposed to be nonzero alright. At the next stage, we wanted that this 2 cross 2 matrix, look at the determinant of that, so that determinant should be nonzero.

At the next stage, we require that we had a 3 by 3 matrix here, whose determinant should be nonzero. So, somehow when I am trying to solve a system of equation, these determinants are coming into play and we had learn that in our school days, how to compute the determinant till the level 3 cross 3.

We will not worry about the determinant at this stage, but what we will see is that these determinants lead to what are called non singular matrices and this is what you saw that the matrices with which I was trying to multiply on the left which I had obtained from the identity matrix, where invertible matrix.

So, I would like to proceed with those directions and then, come out with things. So, let us go back to the system of equations. So, if you go back to the system of equations, system of equations alright, then what are the operations we may do?

So, generally what we do is the first thing we do is that, we multiply a equation by a nonzero number, that is the first thing that we do. Second thing, we do is that we replace one equation; we replace an equation say 2nd equation by 2nd equation minus c times for some c's the first equation. This is what we do.

And we do similar work in the sense that we try to make some entry 0 in the 1st row sorry in the first column, then the second column and so on and proceed. Sometimes if you do not have a pivot at the right place, then we need to interchange two equations alright.

So, we need to interchange two equations alright; say ith and jth alright. So, these are three things that we do when we look at system of equations. What we saw is that all these equations, they get translated into the language of augmented matrix.

So, then we have the notion of what is called augmented matrix and we can do similar thing in the augmented matrix. So, what we do? We can multiply a row by a nonzero number. So, multiply a row by c which is not 0, I can do that fine. Similarly, I can replace this equation.

So, I can replace the second equation. So, if I have the augmented matrix as A and b, this is augmented matrix; then, I need to replace the second equation, the 2nd row, I want to replace it by 2nd row minus c times the 1st row alright.

This is what we need to do. And similarly, if I want to interchange, then what happens is that if I look at this augmented matrix the ith row here. So, ith row is replaced by the jth row alright and the jth row is replaced by the ith row alright. So, these are the equations. So, whatever we do for the system of equation, we need to do the similar thing for augmented matrix fine.

Now, the next thing is that how do I do it from the matrix perspective alright? So, from the matrix product, how do I go about it? So, I will look at matrix product alright. So, let us do it for only by 3 by 3 matrices.

So, you start with the identity matrix I 3 alright and suppose, I want to multiply the 2nd row. So, 2nd row, I want to multiply by c. Then, the matrix that I need here is I am not doing anything to the first row. I am not doing anything to the 3rd row. I am only multiplying the 2nd row by c.

So, multiply the 2nd row of identity by c. So, just write it here 0 c 0 alright. So, we are multiplying 0 1 0 by c fine. So, that is the corresponding matrix that I need to look at, when I look at matrix multiplication I am multiplying from the left or what is called row multiplication alright.

The second one is replacing here. So, what we do is again here if I note here there is no change in the 1st row, there is no change in the second in the 3rd row and the 2nd row is being replaced by 2nd row alright, 2nd row minus c times the 1st row alright.

So, we write this matrix. So, minus t c corresponds to the first column here fine. Now, here if I want to look at in place of i and j, suppose I want to interchange the 2nd and 3rd rows alright. So, if I want to interchange the 2nd and 3rd row, I again go back to identity for me alright. First row there is nothing, no change; 2nd row is being replaced by the 3rd row. So, this is the 3rd row of identity and the 2nd row of identity is this alright.

So, these matrices that I get here which relate to the system of equation whatever we do for the system of equation, they are called elementary matrices. So, these matrices are called elementary matrices alright. So, what are elementary matrices? It is the set of those matrices which can be obtained from the identity matrix by applying exactly one row transform here. Is that ok?

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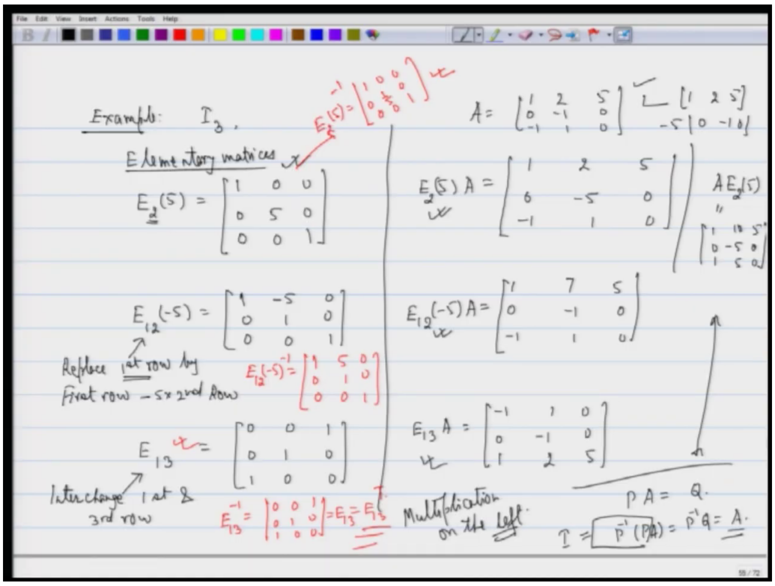

So, again so let us look at some examples. So, that better understanding example. So, I have identity with me say I 4 or let it be I 3 itself for us alright and I want to look at what are called elementary matrices. So, there are three types elementary matrices that we saw. So, let me right say E 2 of 5. So, E 2 of 5 will be second row is being multiplied by 5.

So, 1st row there is no change, 2nd row is being multiplied by 5 and 3rd row also remains the same fine. Then, I have what is called E 12 of say minus 5 alright. So, it says that replace 1st equation of 1st row by minus 5 times by first row minus 5 times 2nd row. So, I am replacing the first row. So, 1st row, I have to take care of this. The 1st row is this and then, I am replacing by 1st row minus this. So, minus 5 times a 2nd row, so this is my thing is. There is no change in the 2nd row, there is no change in the 3rd row alright.

Then, I have the last one what is called say 1 3 of 1 3 which is interchange of; so, interchange 1st and 3rd row. So, 1st row is being interchanged by the 3rd row. There is no change in the second and the first becomes the third one alright. So, let us try to understand this for an example A.

So, let me take the matrix A as 1 2 5 0 minus 1 0 and say minus 1 1 0. Suppose, I have this matrix A, I want to look at what is E 2 5 times A. So, what we are doing is we are multiplying this to the first one. So, 1st row is replaced by as it is. We are multiplying by 5 to the 2nd row.

So, just multiply by 5, you get 0 minus 5 0 and the 3rd row again remains the same. So, we get this alright. I can look at what is E 12 of minus 5 times A. I would like you to see that this is nothing but the 1st row is being replaced, second and third remain the same. So, the second remains the same, third remains the same alright. First is this minus 5 times this. So, since it is 5 times this, I just have to add it. So, it is 1, 5 plus 2 is 7, I hope; finally 5 here is that ok.

So, I am writing 1 2 5 and then, minus 5 times 0 minus 1 0 which is 1, 5 plus 2 is 7 and 5 this what we get. Then, we have E 13 of A will be equal to minus 1 1 0 0 minus 1 0 1 2 5 alright. So, this is the way it is, but the important thing here is we are multiplying these elementary matrices on the left. Multiplication on the left and why on the left? Because we are looking at system of equation, we are trying to solve the system of equation. So, we are manipulating the equations or we are manipulating the rows alright.

If I do it for example, if I want to look at what is A times E 2 of 5 alright, then it is nothing but we are multiplying the second column alright. So, this becomes first column, there is no change; second gets multiplied by 5.

So, it will become 10 minus 5 and 5. There is no change for the third one, it will remain 5 0 0 alright. So, similarly, you have to look at the rest of it and compute the thing yourself. So, these are called elementary matrices. What is important is for these matrices, we also have an inverse.

So, let us look at the inverse of E 2 5. So, the inverse of this will be equal to E 2 of 5 will be inverse of this is 1 0 0, 0 1 upon 5 0, 0 0 1. So, why 1 by upon 5? Because we multiplied by 5, so we need to divide by 5 alright. What about the inverse here, E 12 minus 5 inverse? Here, the inverse has to be here. What we had done was we had added minus 5 times the 2nd row to the 1st row.

So, I have to get it back. So, get it back means I have to add 5 times. So, I have to look at 1 5 0 0 1 0 0 0 1. So, this is the inverse and it turns out that for these matrices which are interchange, the inverse is itself; there is no change as such. So, this inverse will be equal to 0 0 1 0 1 0 1 0 0 itself which you can see is same as E 13 and this is also equal to E 13 transpose, alright this is important that you get transpose here.

So, the inverse is same as transpose that is very very important here; fine. So, what we are seeing is that the notion of invertibility is basically saying that I have a matrix A, I multiply by a matrix and I get result fine. So, I have a matrix A, I multiply by some matrix P and get a Q alright.

Now, I need to find inverse in such a way that P inverse, we multiply to PA which is same as P inverse of Q. I should get back identity, I should get back A. So, in the sense that this P inverse P should be same as identity. Is that ok?

So, these are the ideas that all these elementary matrices are invertible matrices and they are the ones which we had used in the gauss elimination method or to get the LU decomposition alright. So, we will proceed on this further in the next class. That is all for now.

Thank you.