Linear Programming and its Extensions

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Lecture No. # 10

Big-M method Graphical Solutions Adjacent Extreme PTS and adjacent BFS

In the last lecture, we talked about method to find out a starting basic feasible solution by linear programming problem. And problems are large; you cannot always find them starting basic feasible solution by inspection.

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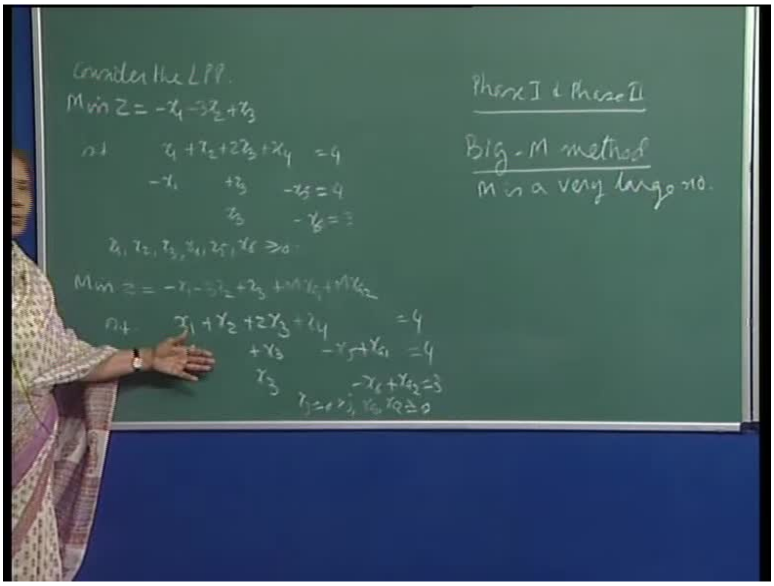

So, I give you phase 1 and phase 2, phase 1 and phase 2 method. So, essentially, phase 1 did the job of finding feasible solution. If there was one basic feasible solution or I told you where there was no basic feasible solution to this system, and hence, the problem was infeasible. So, once phase 1 was over and you found a basic feasible solution, then we went on to phase 2 to get an optimal linear programming solution, the optimal basic feasible solution. I will give you another alternate method, when, when you do not have starting feasible solution right away, and so, that is known as the… So, I am going to talk about Big-M method. Here also we make use of artificial variables and let us see how the method works.

So, if you have a linear programming problem given by this, you have plus x 4 when you have minus x 5 minus x 6. So, therefore, a starting feasible solution is not right away available. So, we will use the trick of again adding artificial variables essentially to the second and the third constraint, because I already have a unit vector here, and you see, because the idea is that we want to have a starting basis as an identity matrix, because then, you do not have to do with the calculation side in the beginning and later on you can of course, by updating or pivoting, you can keep getting the new quantities that you need.

So, here, therefore, I will add the artificial variables and I will come back to the objective functions. So, this will be x 1 plus x 2 plus 2 x 3 plus x 4 is equal to 4, this constraint remains an altered, then minus x 1 plus x 3 minus x 5 plus x a 1. With this, I will add an artificial variable. Here, to start off, then the third constraint is x 3 minus x 6 plus x a 2 is equal to 3 and all the variables x js are greater than 0 for all j and x a 1 x a 2 are greater than or equal to 0.

Then, what I have done here is - added a cost coefficient of the artificial variables is M. So, this is a very large number. So, let we say here. That is why we call big-M method; M is a very large number. The idea is that in case there is a feasible solution to the original problem, then because these cost coefficients are very large and you are minimizing. So, therefore, M is a very large positive number I should of say; that is very important - plus number, positive number.

So, because of that for, if there is a feasible solution, then since the linear programming problem always goes to a better of solution will be no incentive for the algorithm to select x a 1, x a 2 as a basic variable. Initially, of course I am starting with the basis which consists of x 4 x a 1 and x a 2 is the basic variables, but later on, because the process of simplex algorithm improves the value of the objective function at each iteration, at least does not make it worse; it may remains the same in case of degeneracy, but otherwise, it improves the value. So, hopefully, if there is a feasible solution to the original problem, these artificial variables should be driven out of the basis.

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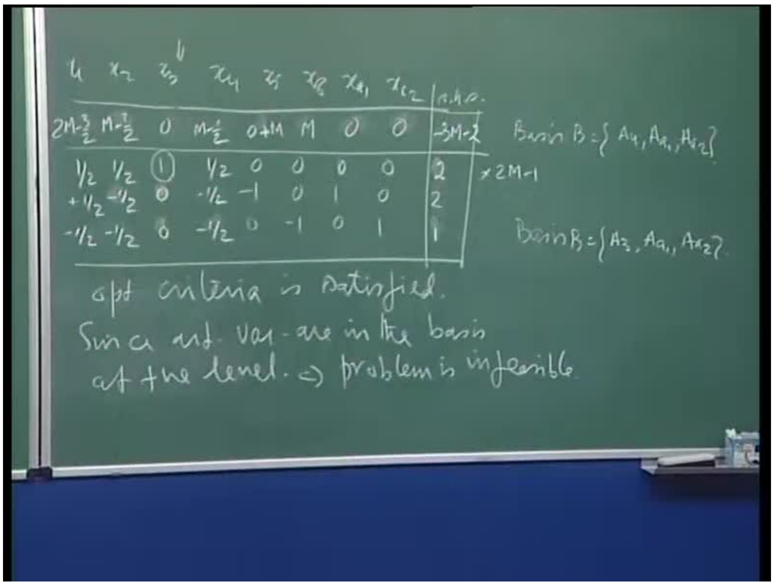

So, let us see. We will start the algorithm here, so, quickly x 1 x 2 x 3 x 4 x 5 x 6 x c 1 x c 2 and this is your hand side, right hand side. So then, the objective function this thing is minus 1 minus 3 1 0 0 0; this is M M, and here, right now this is 0. I am writing this as a constraint and 1 1 2 1 0 0 0 0 and this is 4. I am trying to solve as many problems as possible sothat you get very familiar with the simplex algorithm. Second constraint will be minus 1 0 1, minus 1 0 1, then 0 minus 1, 0 minus 1, and then, you have this 0 1 0 and this is also 4 and the last constraint is 0 0 1 and then you have 0 0 minus 1 0 1 and this is 3.

So, this is your starting tableau, but it is not actually a starting tableau, because my basis is, my basis consist of, your basis B consists of the columns A 4, A A 1, A A 2. So, I need to make these two coefficients here zeroes so that my top row will tell me the C j minus Z j corresponding to this basis. So, multiply by this row by M; this row by m and subtract. So, I will make that computation here because we are not doing any other.

So, m times your adding; so, this becomes M and this does not change. When 2 M, so, 1 minus 2 M, this does not change; this becomes plus m. So, let me rewrite it here plus M. This also becomes plus M and these 2 become 0 and this is 7 4 m plus 3 m, so, minus 7 M.

So, right now the objective function value is very high and this term is your starting tableau. Since M is very large, this is the first negative entry in your C j minus Z j row, and so, I will make x 3 of a basic variable (( )). So, here, this is 4 by 2 or 4 by 1 and 3 by 1; so, no tie in the minimum ratio. This will be my variable to, the variable to leave the basis will be x 4.

So, since the table is large, I will try to make the computations here. So, this becomes 1 by 2 1 by 2; this is 1 1 by 2 and this is 2. Then, the idea is that you have to actually multiply this by 1 minus 2 M and then subtract. So, let us quickly do it, is going to take up time, but so, essentially you are multiplying this row by 2 M minus 1. You are multiplying this by 2 M minus 1 and adding. So, what will this coefficient be? I will do the computations here. So, this is M minus 1 plus half times 2 M minus 1. So, this may comes M minus 1 plus M minus half; so, 2 M minus 3 by 2, 2 M minus 3 by 2.

Then, this is also the same thing. You have this and minus 3. So, here this is minus 3. So, m, and so, this will be M minus is minus 7 by 2. This becomes 0 and this is again M minus half, because minus 2 M minus 1, I am taking minus of that. Then, this does not change; this does not change, nothing changes here. Your objective function value becomes minus 7 M plus, see remember, we multiplied this by 2 M minus 1, so, 4 M minus 2.

So, the value is now minus 3 M minus 2, minus 3 M minus 2, so, the value has improved. Then, you want to make 0 here quickly. So, just subtract from this here, so, this becomes plus half; this is minus half 0 minus half and then this is 2. You must keep checking that you are, if you are pivot choice is right, then this side should also remain non-negative, because you are going from one basic feasible solution to another.

Now, again, I subtract this row from here, so, this becomes minus 1 by 2 0 minus 1 by 2, nothing changes. Here, this becomes 1. So, this is your new basic feasible solution. Your new basis is basis B now is consisting of A 3, A A 1, A A 2. This is a new basis and look at the optimality criteria; everything is, so, why is it away.

So, M is a very large number and you see that optimality criteria satisfied. So, optimality criteria satisfied. We cannot proceed with the simplex algorithm, but the value of the objective function, the, your artificial variables are in the basis. Therefore, since artificial variables are in the basis at positive level. So, this implies problem is in feasible. As I told you in the beginning, I will repeat it again that in case there was a basic feasible solution to the original problem, the artificial variables would have been driven out; they would have become to 0 level. So, this tells you that the, problem, starting problem is in feasible; so, there is no point going ahead with it.

Now, I want to show you another case that can happen. And in the meantime, try to think if there is any disadvantage of using these big numbers, because M has to be quite large. Here, of course I cannot take examples to, but you see, depending on how large your m is, how large other numbers are; correspondingly, your this thing will be there.

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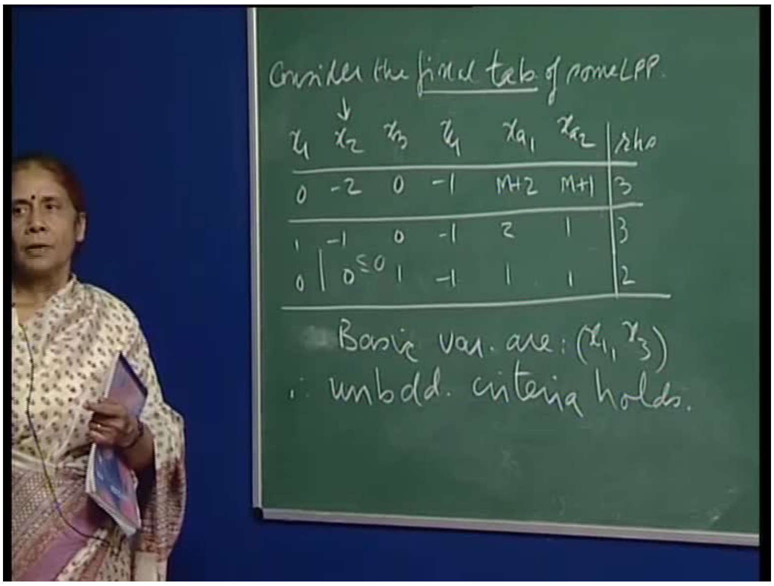

So, M has to be correspondingly larger. Let me take the, take another example for you. Consider the final tableau of some LPP. So, final tableau I am talking of the big-M method. Suppose it shows you x 1 x 2 x 3 x 4, then x a 1 x a 2 right hand side. Now, the top row is 0 minus 2 0 minus 1 M plus 2 M plus 1 3 and it is 1 0 minus 1 0 0 1 minus 1 minus 1 2 1 1 1 and this is 3 2. So, you see the final tableau shows you that optimality criteria is not satisfied but artificial variables are at 0 level. Your basis consists of, basic variables are, basic variables are, variables are x 1 and x 3. Basic variables are x 1 and x 3; x 3 is at 3; x 3 is at level 2 and this is a candidate for entering the basis because the corresponding C j minus Z j is negative, but we see that all entries here are less than or equal to 0. Therefore, unboundedness criteria hold.

And since the artificial variables are out of the basis, the original problem is unbounded. So, everything is applied in a same way. Here also so, here we could detect infeasibility; here we can detect unboundedness. Now, if artificial variables were in the basis at positive level and you had the unboundedness criteria, then you would conclude that the problem is infeasible.

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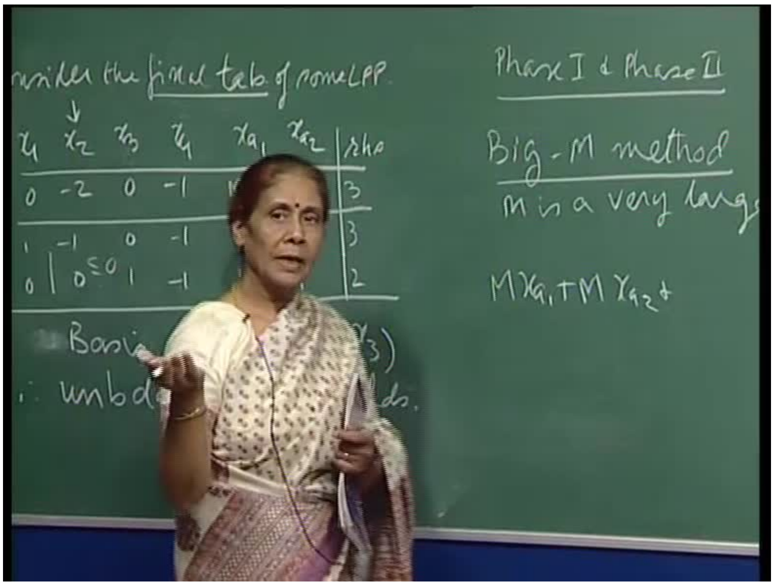

In this case, the problem is un bounded, because the artificial variables are not in the basis, but in case, see even though you have positive coefficients for artificial variables and whatever the number of artificial variables, even though the unboundedness criteria can still hold, yes, you may detect the unboundedness criteria when the artificial variables are in the basis, because the other variables can take very large values, then therefore, go beyond the M values.

So, you can encounter at unboundedness criteria in big-M method with artificial variables being in the basis at positive level. In that case, you will conclude that your problem is infeasible. But if you encounter the unboundedness criteria and the artificial variables are out of the basis, then you will say that the problem, original problem is unbounded. And in this case of course, your, you come to the optimal solution with artificial variables being in the basis, then you conclude that the problem is infeasible.

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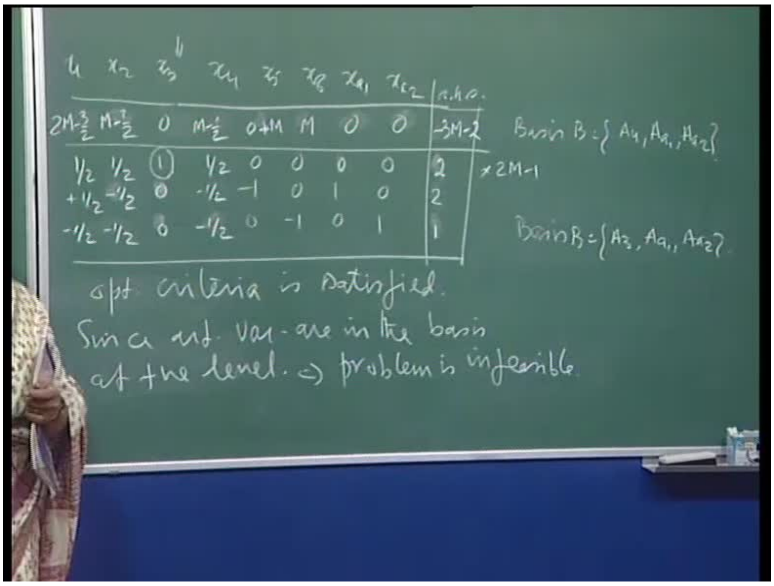

So, that takes, so this, now, the only disadvantage is I told you to think it is that n has to be chosen very very large compare to the other numbers so that you do not ever mistake the sign of the C j minus Z j number and so on, because if these numbers, for example, here, if this number or this number is very large, then m has to be correspondingly large so that here you are saying that this C j minus Z j is non-negative is positive. So, it should not happen that if we choose M, if you do not choose M sufficiently large, this number may be big, and so, you will say that the optimality the C j minus x j is negative, whereas, actually it is required to be positive.

So, all these problems are there, and so, when you have one single entry to start off, when you have few entries which are very large compare to the other data, then computations of error or errors of computation can come in; so, one has to be careful. And therefore, mostly as I told you, the computer programs are written for phase 1 phase 2; we seldom use, but sometimes this can be a good device to take care, because certainly this does the job in 1 phase, whereas, phase 1 phase 2 you often first an obtain a starting feasible solution and then you start optimizing.

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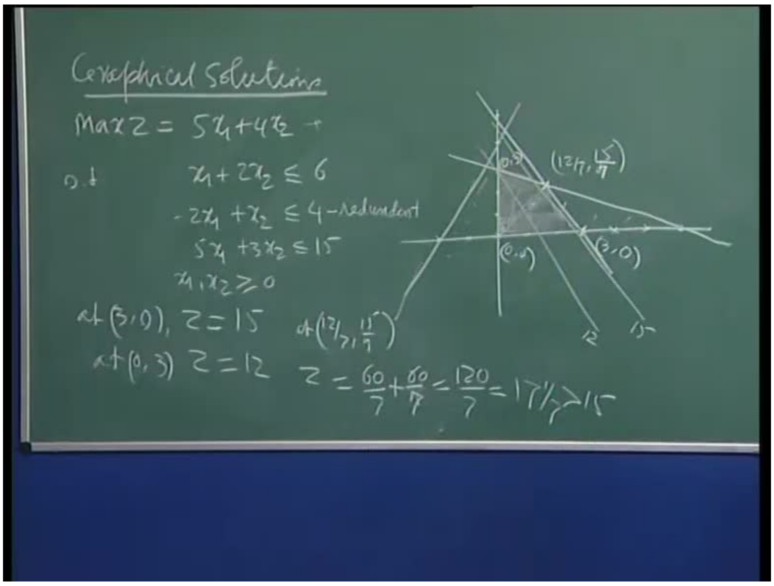

So, that is the thing. So, that takes care of you this thing. Now, we want to look at some other geometrical aspects of the simplex algorithm that makes it, you can, you can visualize what is actually happening. So, to do that, I will first give you a graphical method of looking at the algorithm. So, graphical solutions I would like to give you. Let me consider; so, we can, of course I can draw it for you in two dimensions also small problem.

Suppose and I am choosing just for a change a maximization problem is whereof minimization. So, this is 5 x 1 plus 4 x 2 subject to x 1 plus 2 x 2 less than or equal to 6; minus 2 x 1 plus x 2 less than or equal to 4; 5 x 1 plus 3 x 2 less than or equal to 15; x 1 x 2 greater than or equal to 0. So, let us draw it. We can, we can actually draw the feasible region in a plane and I will treat this as a equality first. So, when you take it equal to 6, when you put x 2 equal to 0, x 1 has to take the value 6, so, 1 2 3 4 5 6. This is 1 point when I put x 1 equal to 0, x 2 is equal to 3; so, 1, 2 and 3. So, this is the thing and your this side is the feasible region because 0 is on this side less than.

Now, for this one, when you put x 2 equal to 0, this gives you x 1 equal to minus 2. So, this is another 1 point. When you put this equal to 0, x 2 equal to 4. So, this is, and here, again this is this side. And the third constraint, when you put this equal to 0, x 1 is 3, and when you put x 1 equal to 0, x 2 is going to be 5.

So, you see, and x 1 x 2 non-negative, so, your feasible region is actually this. This is your feasible region; we can quickly brought out the extreme points here. This is 0 0; this is 1 2 3, 3 is 0, and at this point is 0 3, and the point intersection, so, you see that this constraint which was your second constraint. Minus 2 x 1 plus 2 less than or equal to 4 is actually redundant; it does not play a role in defining your feasible region, because it is outside your feasible region and this particular point I found out is 12 by 7 and 15 by 7.

So, just solve the 2, just solve the 2 equations - this and this and the intersecting point is this. Now, just see, the value of the objective function at the point 3 0 for example, at 3 0, your Z is equal to what? Fifteen, and at 0 3, your z is 12. So, and of course, at 0, it is 0, at 0. So, the values increasing in this direction; that means something like this and then I am not write down. So, this line will pass through this. So, this corresponds to 12 and this corresponds to 15, so, the values actually increasing. So, you see that it will, when it hits this point 12 by 7 and 15 by 7, so, let us compute at 12 by 7 15 by 7 your value of Z is equal to 12 fives are 60 by 7 plus 60 again by 7.

So, this is 120 by 7 which is 7 ones are 7 7 7 and 1 by 7, so, which is more than 50. So, the value of the objective function is actually increasing in this direction, and when it hits the point this, the drawing is not very clear, but any way, you can see that this plane is moving parallelly to itself, and at this point, it will be the maximum point.

So, the value of the, so, this is the whole idea that in a simplex algorithm, see actually you, what you will see the, to start with this as a feasible solution, then it will either move here or here, because we have following blank rules; so, it is not necessary that it moves here; it may move here, and then, from there, it will come to this extreme. So, this is what will happen.

So, the simplex algorithm and you start, because see this is written it will come out to be, so, anyway, you will start with x 3 x 4 and x 5. You will add the slack variables. All these are non-negative, so, you will have a starting feasible solution consisting of a identity column, and then, you can begin with your simplex algorithm, and then, you will see that you will either move here or here, and then, from there, you will come to this point.

So, in two dimensions, it was, it looks so nice and simple, but think you can become very difficult, because when dimensions are very large, you cannot see the picture, but with the idea, this is the idea to show you how this simplex algorithm was thought of and how it proceeds.

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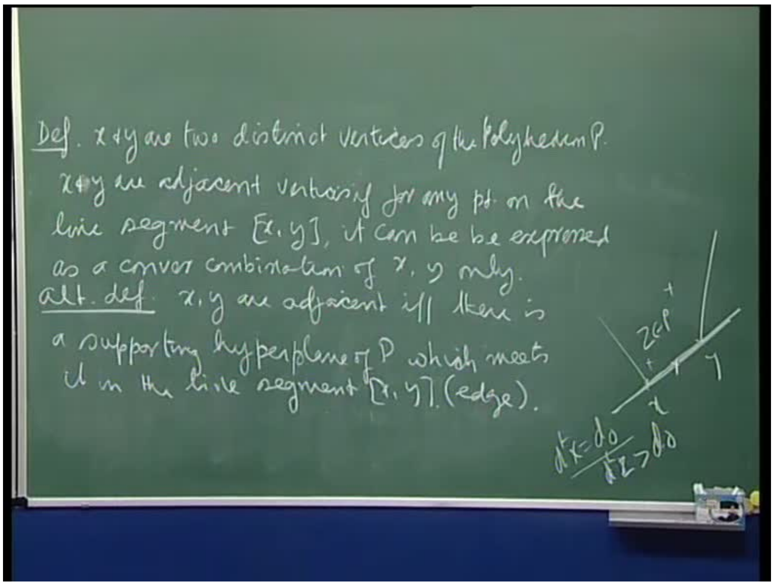

So, to give you a little more insight into the geometry behind the algorithm, I will make a few definitions now - concept of adjacency. We say that so few definitions. So, see the thing is that x and y are two distinct vertices are extreme points. I will be using vertices are extreme points synonymously. Vertices of the polyhedral P it is understood that I am always referring to the standard LPP. So, therefore, the constraints are equality constraints and you have the corresponding polyhedral.

So, x and y are two distinct vertices of the polyhedral P. We say that x and y are adjacent vertices if for any point on the line segment x, y. So, I will refer to the line segment by this x comma y. So, you have two distinct vertices; you have joined them, and because the polyhedral is a convex set. Remember, the line will be in the set. So, the line segment x by 4 any point on the line segment x, y, it can be expressed as a convex combination of x y only, x y only.

This is in the context that we said that for any feasible for a convex set, you and for a polyhedral, we said that would any point in the polyhedral can be expressed as a convex combination of its extreme points and directions if it is unbounded. Now, what I am saying very specifically here is that if x and y are adjacent vertices or extreme points, then they will, for any point on the line joining x and y, I can only express the point as a convex combination of x and y; no other direction or extreme point will play any role, it is just that this. Another way of looking at it is, at showed you that when you have a hyperplanes and you have supporting hyperplanes, we had talk about it.

So, supporting hyperplane if you have a convex set like this, then I can have a supporting hyperplane which meets it at one point. I can have a supporting hyperplane which meets it in this. I can show you only two particular phases of a polyhedral here, but when you had dimensions, you have many dimensional phases.

So, a supporting hyperplane will always intersect the polyhedral in a phase of the polyhedral, and what that means is that the whole polyhedral will lie when the supporting hyperplane, the whole polyhedral lies one side of the planes hyperplane. So, this is the idea. So, here also you can look at it this way that to define two extreme points to be adjacent, we say that the alternate definition.

Alternate definition is x and y are adjacent, so, I am not repeating all the other things, are adjacent if and only, if there is supporting, there is a supporting hyperplane of P which meets it, which meets it in the, this is line segment x y. Remember, we were also calling this is an edge; this is an edge. So, if the supporting hyperplane meets it in your edge, then these are adjacent and vice versa and you can see very clearly that the two definitions are really equivalent, because what we are saying is maybe I will just show you geometrically that you have this. So, I have a supporting hyperplane here. I am saying that this is x and this is y.

Now, all are the points of the, of the, polyhedral lie on the other side of the, that means for any other, if you take any z belonging to P, then if my hyperplane, if the supporting hyperplane, this is something like let us say t transpose x P transpose x equal to some d naught.

So, this is the hyperplane. Now, for Z which is of the other side of this d transpose Z will be greater than d naught, let say all less, let me just me take it to the this thing. This is the idea, because it is on the other side. So, any two point, any point lying here, because it is a convex combination of x and y. It will also satisfy this as equality because it is only hyperplane, the supporting hyperplane, but so, it cannot be expressed to the convex combination of any other two points of P, because for both these points, this will be strict inequality greater than d naught. So, how can a convex combination of two numbers which have both greater than d naught P equal to d naught? That is not possible. So, the two definitions are actually the same; so, I will use both the definitions.

And now, let me also introduce the concept of adjacent basic feasible solutions, and then, we want to show you that actually again as I in some time ago, I showed you that the concept of extreme point and basic feasible solution the two things are the same. Actually there is 1 1 of course correspondence in the absence of degeneracy. Now, I will show you that this concept of adjacent fease also translates to the corresponding basic feasible solutions being adjacent.

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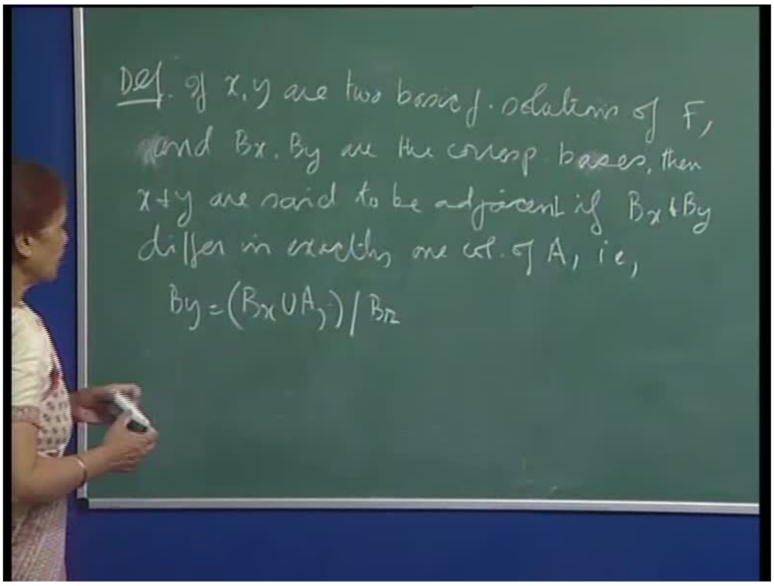

So, another definition, we say that, so, I would remember we have been using the same notation even though some text might use different notations for the vertex and the corresponding basic feasible solution. I have been doing the same just because to want to make it make the presentation simple.

So, I will say that if x and y are two basic feasible solutions of f, and remember, we calling the corresponding feasible set as f. So, the polyhedral is P and the feasible set is F. So, if x and y are two basic feasible solutions of F, then and B x B y are the corresponding basis, then x and y are said to be adjacent if B x and B y differ in exactly 1 column of A, that is, B y is B x union. Let us say a j slash we have been, I will use the notation that we have been minus B r.

So, remember, we replace a column here and then replace it by, so, the B r column goes out of the basis and a j comes instead. So, therefore, the basic feasible, basis obtain by removing one column from b x and adding another one that makes it adjacent basis, adjacent basis. So, two basic feasible solutions are set to be adjacent if one is obtained from the other by removing, by replacing one column of the basis.

So then, now, the theorem that I want to prove to you is that for two distinct extreme points, what is this, of the, polyhedral P. If the two vertices are adjacent, then the corresponding basic feasible solutions are also adjacent and vice-versa. So, after defining adjacent basic feasible solutions and adjacent as spring points, now, let me show you the correspondence between the two. So, the result that we want to prove is that for two distinct extreme points, two distinct adjacent extreme points, again I do not need to use the word distinct when they are calling they are adjacent, because if they are adjacent, they are distinct. So, two adjacent extreme points and the corresponding basis will also be adjacent.

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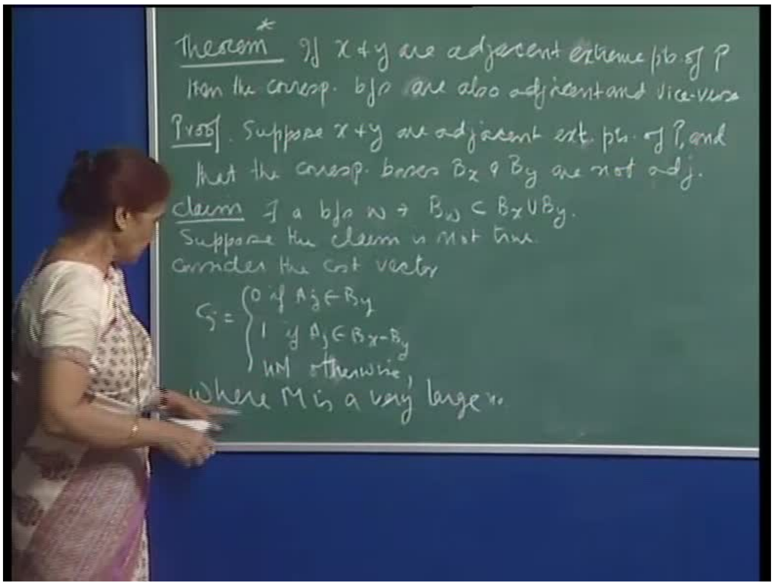

So, therefore, the corresponding basic feasible solutions will also be adjacent. So, this is what I want to prove now. Proof is a little I would say difficult, but if we go slowly, we can surely understand it, and then, maybe you can come back to it and read it again and again to, but the idea is beautiful because it is a constructive proof and it shows you again the working of the simplex algorithm. So, the theorem and I will put a star to it so that, you know, it is a level of difficult is there. So, we say that for if x and y are adjacent vertices or let me extreme points of p, then the corresponding basic feasible solutions are also adjacent and vice versa.

So, two extreme points are adjacent. The corresponding basic feasible solutions are adjacent if two basic feasible solutions are adjacent and the corresponding basic the corresponding extreme points will also be adjacent. So, let us give us the proof. So, we start with the assumption that suppose x and y are adjacent extreme points of P, and suppose, this and that the corresponding basis B x and B y are not adjacent; that means at least 2 column, that means a B x and B y differ in at least two columns, because remember the adjacency concept said that B x and B y must differ only in one column that the corresponding basis B x and B y are not adjacent.

Now, claim and it will be clear why I am saying it claim there exists a basic feasible solution w such that B w is a subset of B x union B y. What we are saying is that if b x and B y are not adjacent basis, then I should be able to find a basic feasible solution w consisting of columns which make up, which are made up from the columns of B x and B y, because remember, this number is at least m plus two, because b x has m columns b y has m columns and they differ by two at least. So then, this has m plus two columns that is all we are assuming. So then, you can be able, you will be able to find m columns from here such that the corresponding set of columns is linearly independent and the basic feasible solution and it gives your basic feasible solution.

So, we are saying that claim. So, suppose the claim is not true, suppose the claim is not true, then I have to contradicted; I have to show you that it can be easily contradicted. What does it mean? Suppose the claim is not true, consider the cost vector, consider the cost vector C j is equal to 0 if a j belongs to B y is equal to 1 if a j belongs to B x minus B y; that means B x and B y have some columns in common, but the once which are not there in B y but are in B x, for those they cost this one. For all columns which are in B y the cost is 0 and is n m otherwise, sorry, NM otherwise - where m is a large number, is a very large number.

Now, with this cost vector, you can immediately see that for the feasible solution y, the cost is 0. For the feasible solution x, the cost is not 0, and for any other feasible solution, which does not have the columns from B x and B y, the cost is very large.

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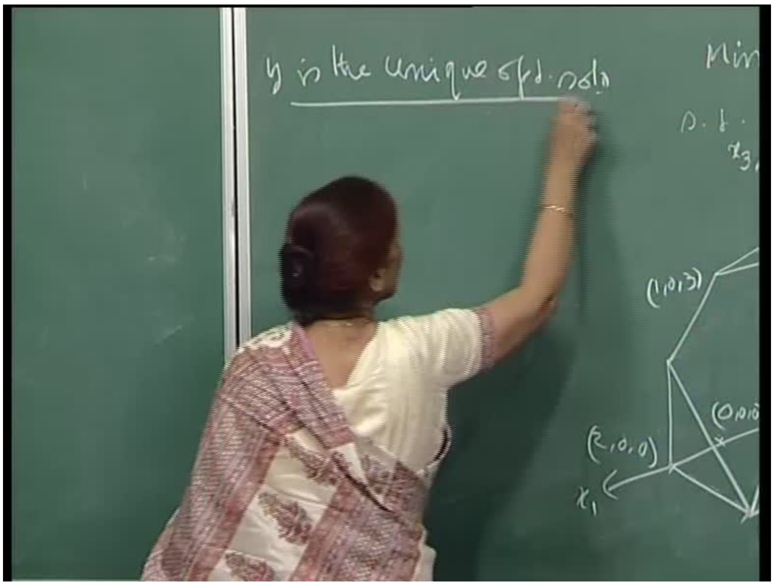

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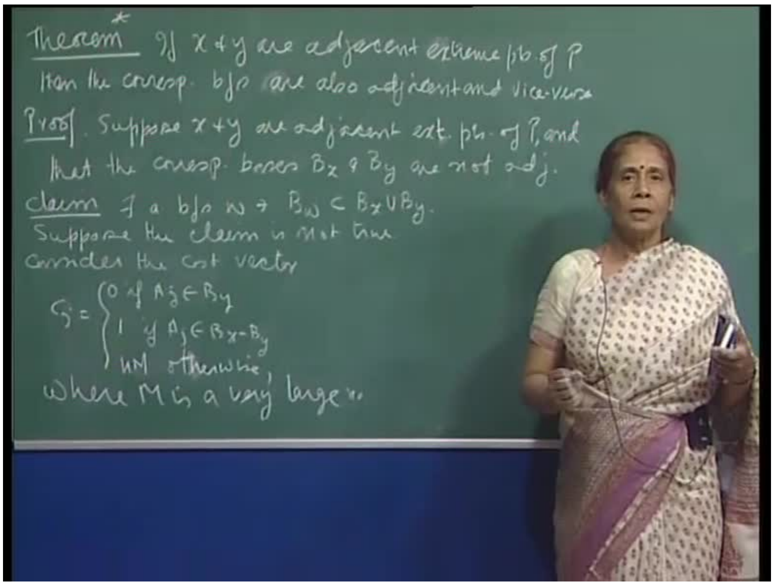

So, that means that y is the unique optimal solution, yes, y is the unique optimal solution. And if we say that there is no basic feasible solution w consisting of the columns here, you see that if I start my simplex algorithm for solution x from the basic feasible solution x, then if there is nothing in the neighborhood because B x union B y the columns do not contain any other basic feasible solution. What is it mean? When I have a feasible solution, starting feasible solution x, there is some cost. Any other solution which does not have all its columns here, the cost will be very high, because remember, the corresponding C j values n m - where m is very large number.

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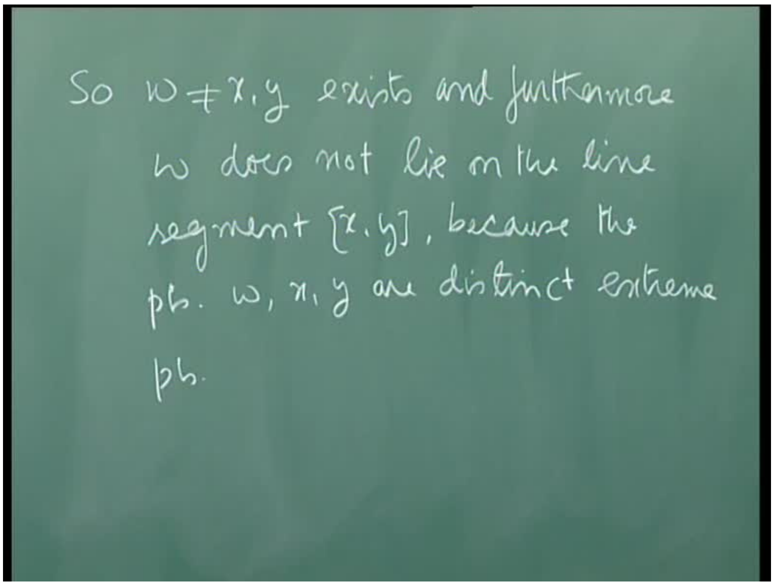

So, that means the simplex algorithm when started from x will not be able to reach y but that is absurd, because the simplex algorithm will always find an optimal solution, I showed it to you. Therefore, they claim the w that the claim that exists a basic feasible solution w. So, that it is columns are within B x and B y must hold. It is true; the claim is true – so, where m is a very large number. So, let me write it down why the unique optimal solution and the simplex algorithm when started from x will not be able to able to proceed to the unique optimal solution. So, this is absurd, but this is absurd; therefore, the claim is true. So, there is a basic feasible solution w consisting of column (( )) B x union B y.

So, in the proof, we have to also emphasis though it has been set, but I want to emphasis this again that w is not equal to x, y exist. We have shown this. just now and further more we have also shown that w does not lie on the line segment x, y, because it points w x comma y are distinct extreme points. When the proof we have shown that from x, we will go to w and then to y when we apply the simplex algorithm and began it with from x construing the cost function, that is given to us. So, this needs to be emphasized, and then, we will finally show that this is not possible, because x and y are also, the basis are also, the corresponding bases are also adjacent.

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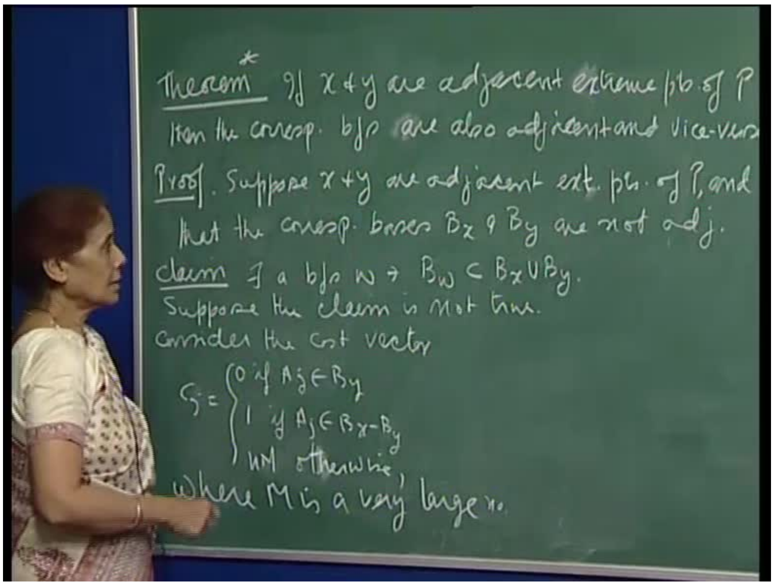

Once I have this, this, now what we do is – yes, I have to construct; I have to actually show you what do I have to contradict. I have to contradict the fact that x and y are not adjacent, because I am starting with this assumption that B x and B y are not adjacent. So then I will come to the contradiction; come to the conclusion that x and y are not adjacent which contradicts our starting assumptions. So, therefore, this is not correct; B x and B y must be adjacent phases.

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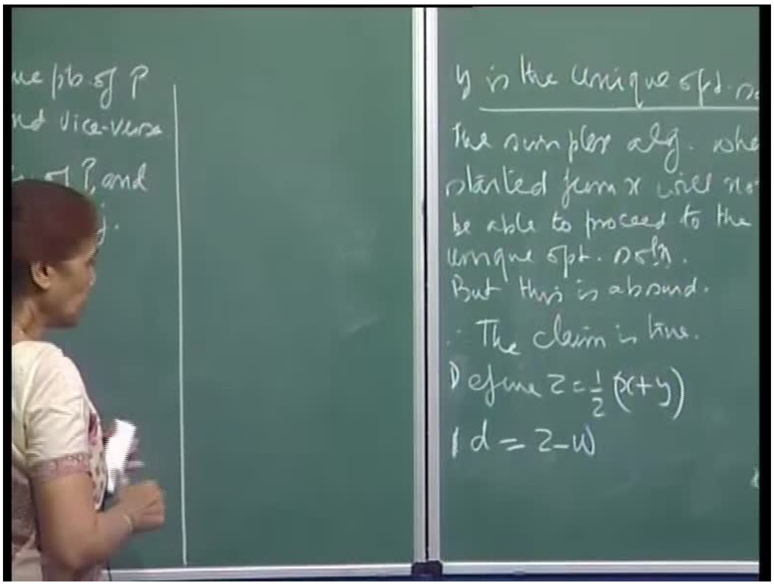

So, the claim is true. Now, let us define. So, let us define z equal to half x plus y. Now, you see whatever the components here is z as either the x component is 0, then the corresponding y component is positive. So, anyway, this has the number of this. We are assuming that is more than m plus one, then all those components here are positive whether here or here; z is this, and then, define this and a vector d which is z minus w. So, define a vector d which is z minus w. So, here, you see w also has all its columns come; that means the non-negative components of w are corresponding to the subset of column from B x union B y.

And z has all components non-negative from the columns B x union B y, because it is x plus y solution, and here, this is this, but it is possible that some component here may be, the, this thing may be some components of d may be negative; then some will be positive, it is possible, and some may be 0; so, d is this.

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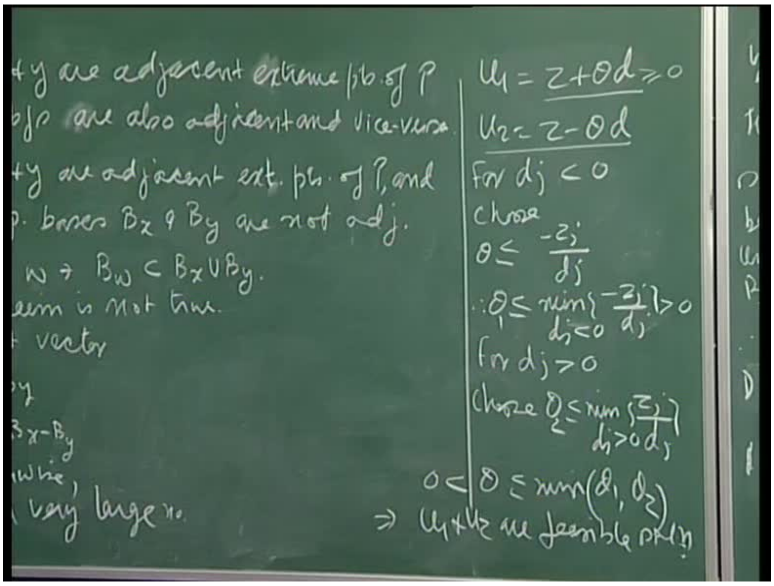

Now, a simple observations you see that for component of d which is negative, let say for d j less than 0, or now, I define two solutions - u one as, u one as z plus theta d and u two as z minus theta d where I have to choose theta judicially so that both these are feasible solutions. Once they have feasible solutions, you can see that when I add them and divide by 2, I will get z; that means if these two are both feasible, that means they are in my P, then the convex combination I have another convex combination for this thing, for z.

Because this is one convex complex combination for z and this of course is a convex combination on the edge connecting x and y. Now, I will be able to show you that this another this is whole idea and the construction is not difficult, because you see that here, the only problem theta of course, I will choose to be a positive number and I will show you how we choose theta. Z has all components non-negative. Now, here, if there is a negative component, that can create problems because by theta should be chosen such that the corresponding component does not become negative.

So, that means for d j less than 0 choose theta such that it is less than or equal to, see I want this to be greater than 0, that means I want z j plus theta d j to be greater than or equal to 0. So, this should be minus z j upon d j and d j is negative. So, the ratio is a positive number, and this, you, therefore, theta should be less than or equal to minimum of minus z j upon d j, for all d j which are less than 0 and this number is going to be positive. So, therefore, I can have, I can choose theta to be some positive number but satisfying these conditions. Similarly, for u 2, you have to take care when whenever component of d is positive. Then, again, my choice of theta should be such that the corresponding component here does not become negative.

So, for d j greater than 0, choose in the same way, choose theta to be less than or equal to minimum of, yes, in this case, it will be z j upon d j and d j is greater than 0. So, this is what you do, and then, you choose theta finally to be something which is less than or equal to. So, 0 greater than theta and minimum of maybe I can call here this as theta 1 and this as theta 2. So, minimum of theta 1 and theta 2, or in other words, I am choosing a theta with satisfies both these in equalities less than this quantity and it is less than this quantity, then I have my theta here, and then, this will imply that u one and u two are feasible solutions. So, they are feasible solutions. You have another, and so, let me come back to, I will go here.

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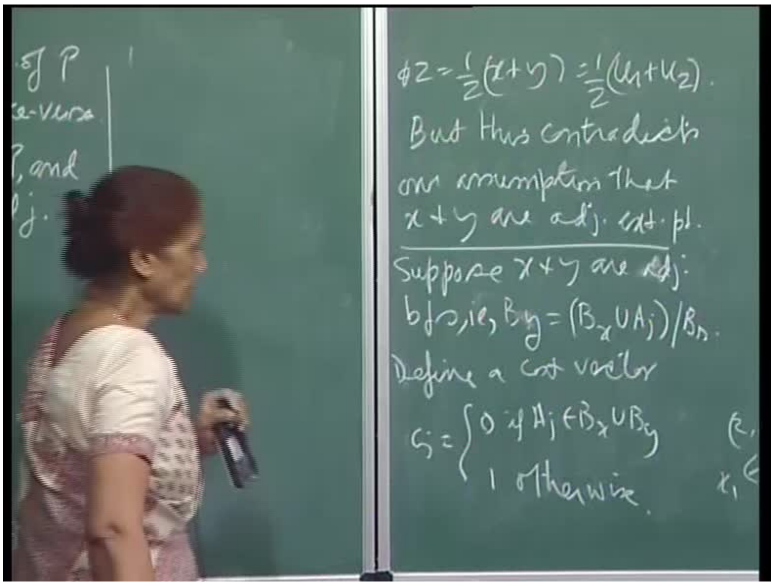

So, you have another representation are both feasible and z which is half x plus y is also half u 1 plus u 2, but this contradicts the fact, this contradicts our assumption that x and y are adjacent extreme points. So, that shows you and goes through the proof again slowly to understand what we are actually doing; so, this is it. Now, we want to proof the other way.

So, now, I want to say that suppose x and y are adjacent basic feasible solutions. Now, suppose, suppose x and y are adjacent basic feasible solutions and, yes, so, suppose x and y adjacent basic feasible solutions, that is, B y I should said this, that is, B y is B x union A j slash B r.

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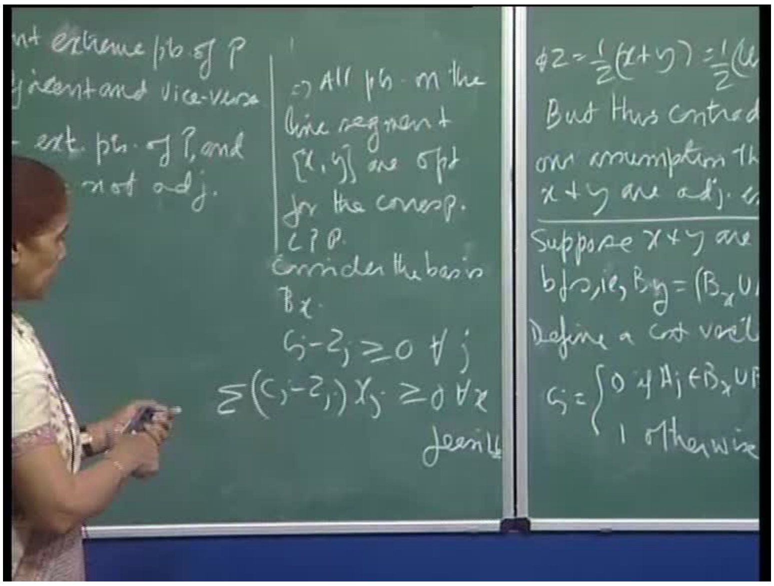

Now, we want to show you that the corresponding extreme points are adjacent. So, choose define a cost vector C j is equal to 0 if a j belongs to B x union B y and one otherwise. So then, see immediately, you can immediately see what I am driving at. What I am saying is that because the cost vector cost components are 0 for all columns in B x union B y; it means that, yes, so, what it means is that the objective function value is optimal is 0 for all points lying on x on the line segment. So, this implies that all points on the line segment x y are optimal for the corresponding LPP. With this objective function, all points here are optimal fine. Which implies what?

So, if you take, let consider the basis, consider the basis B x, so, corresponding to this basis, you have the condition that C j minus Z j are greater than or equal to 0 for all j, because x is an optimal solution. So, optimality criteria must be satisfied, and therefore, C j minus Z j is greater than or equal to 0 for all j and this implies that C j minus Z j times x j summation this is greater than or equal to 0 for all x feasible, because a components of x are non-negative for any feasible x; which means that this is greater than or equal to 0. And you have your hyperplane. You see that this hyperplane that means for all points on the line segment x y, this would be satisfied as 0, because remember, the cost coefficients here are all zeros; the Z j's will be zeros, and so, this will be equal to 0 for all points here.

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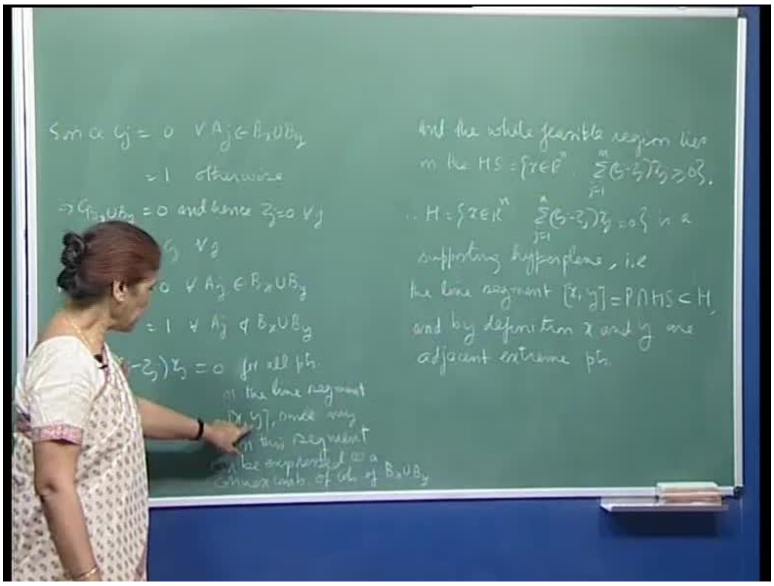

But for all other points in the polyhedral all feasible points this will be greater than 0. So, we were looking at c for all columns in B x union B y, your c js are zeros and cost coefficients are one otherwise.

So, that means for all basic variables, the corresponding cos has 0 and hence you have Z j will be 0. Remember, Z j is computed as C B B inverse A j. So, since C Bs are all zeros, therefore, your z js will be all zero, and hence, your C j minus Z js are c js for all j, but then, for all basic variables since c js are also 0, so then, C j minus Z j 0 is 0 for all j for all columns in the, in the basic set B x union B y and this is one for all j, for all a j which are not the basic columns in either B x or B y.

And therefore, this expression is 0 for all points on the line segment x, y. Now, on the line segment x comma y every point will be a convex combination of the columns. So, I have written here convex combination of columns of B x union B y, and since for all columns in B x union B y, this quantity is 0. Therefore, C j minus Z j x j is 0 for all points on the, and the x js are 0 for points which are coefficient, which are not on the line segment.

So, therefore, this expression will be 0 for all points on the line segment x, y. I hope this is clear because see the components of points on the line segment x, y, only those components will be non-negative or positive corresponding to the columns in b x union

B y; other components will be 0, but for components for the columns which are in B x union B y, the corresponding C j minus Z j is 0. Hence, this sum is 0 for all points on the line segment x, y.

And therefore, we have already shown here that the whole feasible region because the optimality criteria satisfied. Therefore, the whole feasible region lies in this half space where C j minus Z j x j summation this varying from this should be 1 to n greater than or equal to 0. So, this is the half space and h is a hyperplane, because you have this one-dimensional equation satisfy by all the points here. So, this is the hyperplane. So, and but this becomes a hyperplane, because you see that from your working out here that this line segment is actually the intersection of the polyhedral. The feasible region polyhedral P intersection this half space and this is a subset of this hyperplane h.

Therefore, h becomes supporting hyperplane, and by our definition, x and y are therefore adjacent extreme points. We started out with x and y as a extreme points, and now, because this is a part of the hyperplane, supportive hyperplane. This part of the polyhedral meets this, the hyperplane, this hyperplane meets the hyper, the polyhedral P in the line segment x y. Therefore, the line the points x, y, x and y are adjacent extreme points. This is what you wanted to show.