Introduction to Probability & Statistics

Prof. Abhay Gopal Bhatt
Department of Statistics
Indian Statistical Institute (Delhi)

Week - 3
Lecture-9
Independent Events

Probability theory me ek bahut mahatvapurna concept hai independence—yaani swatantra ghatnaayein. Conditional probability se humne dekha ki agar koi event B ghat jaaye to hum uski jaankari ka istemaal karke event A ki probability ka anumaan badal dete hain. Lekin kabhi-kabhi aisa hota hai ki conditional probability P(A|B) bilkul hi original, unconditional probability P(A) ke barabar hoti hai. Matlab event B ke hone ka A ki probability par koi asar nahi padta. Tab hum kehte hain ki A aur B independent events hain. Isko motivate karne ke liye ek fair die ka example lete hain jisme outcomes 1,2,3,4,5,6 sabhi equally likely hote hain, isliye har outcome ki probability 1/6 hai. Hum teen events define karte hain: A={1,2,3}, B={2,4,5}, C={2,3,4,5}. Probability of A aur B dono hi 1/2 hain aur C ki probability 2/3 hai. Ab conditional probabilities dekhte hain: P(A|C) = P(A∩C)/P(C) = (2/6)/(4/6) = 1/2 jo ki P(A) ke hi barabar hai, isliye A aur C independent hain. Isi tarah P(B|C) bhi 1/2 hi aata hai, to B aur C bhi independent hain. Lekin P(A|B) = (1/6)/(1/2) = 1/3 aata hai jo P(A)=1/2 se alag hai, isliye A aur B independent nahi hain. Yaani A–C independent, B–C independent, par A–B independent nahi. Independence ko hum reverse direction me bhi dekh sakte hain, kyunki Bayes ke mutabik agar P(A|B)=P(A) hai to P(B|A)=P(B) bhi hoga. Iska symmetric form hai: A aur B independent tab hote hain jab P(A∩B)=P(A)P(B). Yeh rule hamesha sahi nahi hota; yeh sirf tabhi valid hai jab events vaastav me independent ho. Ek important remark yeh hai ki mutually exclusive events kabhi independent nahi hote (agar unki individual probabilities zero na ho), kyunki mutual exclusivity ka matlab hota hai ki agar ek ghat gaya to doosra kabhi nahi ghat sakta—matlab strong negative information milti hai, jo independence ke ulta hai. Independence ka sahi intuition yeh hai ke ek event ke hone se doosre ke hone ki jankari me koi farq nahi padna chahiye. Jaise ki Delhi me aaj barish hui ya nahi, iska 15 din baad ke India–England Test match ke result par koi asar nahi; dono bilkul independent hain. Kuch exercises bhi milti hain: agar A aur B independent hain to A aur B-complement bhi independent hote hain; A-complement aur B bhi independent hote hain; aur A-complement aur B-complement bhi independent hote hain. Independence ka istemaal hum fair coin toss jaisi situations me karte hain, jaise P(HH)=P(H1)P(H2)=1/2×1/2=1/4. Ab multiple events ke liye mutual independence define hota hai

$$for\ every\ k, 2 \leq k \leq n,$$

$$P(A_{i1} \cap A_{i2} \cap \ldots . \cap A_{ik}) = P(A_{i1})P(A_{i2}) \ldots \ldots P(A_{in})$$

$$for\ every\ subset\ \{i_{1,}i_{2,}\ldots i_{k\}}\ of\ indicao\{1,2,3\ \ldots.n\}$$

Simple case n=3 me condition hoti hai: $P(A1\cap A2)=P(A1)P(A2)$, $P(A1\cap A3)=P(A1)P(A3)$, $P(A2\cap A3)=P(A2)P(A3)$, aur saath hi $P(A1\cap A2\cap A3)=P(A1)P(A2)P(A3)$. Tabhi hum kahte hain ki A1, A2, A3 mutually independent hain.