Introduction to Probability & Statistics
Prof. Abhay Gopal Bhatt
Department of Statistics
Indian Statistical Institute (Delhi)
Week - 2
Lecture 06
Equally Likely Events

Yeh jo example humne abhi dekha jisme paanch students the, ya general case me n objects hote hain aur unhe re-arrange karna hota hai, unhe alag-alag kramon me rakhna hota hai, isko ek vishisht naam se jana jata hai permutations. Permutation ka matlab hotahai karm anusar badlav karna, yaani items ko alag-alag order me rakhna. Agar 1,2,3 ka ek order hai to doosra order 2,1,3 ho sakta hai, teesra 3,1,2 ho sakta hai, aur aise hi. Number of permutations of n objects hota hai n factorial. Product rule ko use karke hum dekhte hai ki pahle stathan k liye n choices hoti hain, phir n−1 choices, phir n−2, aur last me 1 choice iska product hota hai n!. Ab ek thoda alag example dekhte hain: 4 log hain, aur ek committee ke liye ek chairman aur ek secretary chunna hai. Chairman ke liye 4 choices, chairman chunne ke baad secretary ke liye 3 choices. To total $4 \times 3 = 12$ alag tareeke. Isko factorial form me hum 4! / 2! likh sakte hain, kyunki $4 \times 3 \times 2 \times 1$ ko 2×1 se divide karne par 4×3 hi bachta hai. Isi se ek general formula milta hai: Number of permutations of size k chosen from n objects = $P(k, n) = n! / (n − k)!$.

Is notation ko $P(k, n)$ ya kabhi kabhi nPk bhi likha jata hai. Jab order important hota hai tab permutations use hote hai. Lekin agar order important nahi hai — jaise ki 4 logon me se ek committee banani hai jisme bas do member honge, koi chairman ya secretary nahi — to fir sirf combination matter karta hai. Agar committee ke member B aur C ho, to B pehle choose karein ya C pehle choose karein, committee same hi rahegi. Yaani order matter nahi karta. Is case me hum permutations ko k! se divide kar dete hai, kyunki ek hi group ke k! alag permutations hote hai jo sab ek hi set ko represent karte hai. Isliye number of combinations ka formula hota hai:

$C(k, n) = n! / (k! (n − k)!)$.

Isko "n choose k" padhenge. Ab cricket example dekhte hain: ek touring team me 15 players hain aur final 11 choose karne hain. Order matter nahi karta, isliye total possible teams hongi: $C(11, 15) = 15! / (11! 4!)$. Isko simplify karne par milta hai 1365. Ab ek realistic team selection consider karein: 15 me se 6 batsman, 2 wicket-keepers, aur 7 bowlers hain. Hume final 11 ki team banana hai jisme 5 batsman, 1 wicket-keeper aur 5 bowler hon. To pehle 6 me se 5 batsman choose karne ke 6 choose 5 = 6 tareeke, phir 2 wicket-keepers me se 1 choose karne ke 2 tareeke, aur 7 bowlers me se 5 choose karne ke 7 choose 5 = 21 tareeke. Total distinct teams hongi: $6 \times 2 \times 21 = 252$. Ab maan lijiye ek team humne chuni jisme total 11 players hain. Is 11-player team ke batting order banane ke kitne tareeke hain? Yahan order important hota है — kaun pehle aayega, kaun baad me. To batting orders ki sankhya hogi 11!, kyunki ye 11 players ko

arrange karne ka number of permutations hai. To is poore question ka final answer pehle team selection ke liye 252 aur batting order ke liye 11 factorial hoga.