Introduction to Probability & Statistics
Prof. Abhay Gopal Bhatt
Department of Statistics
Indian Statistical Institute (Delhi)
Week - 2
Lecture - 5
Equally Likely Events

Hum continue karte hain ki probability ko systematically kaise jaach kar lein. Hamara sample space Omega ya to finite hota hai ya countably infinite, aur Omega1, Omega2, … har ek outcome ko darshate hain. Har outcome ka ek simple event hota hai — EI, jismein sirf aur sirf ek hi outcome hota hai, yani EI = {omega_i}. Is tarah har outcome ke liye ek simple event ban jaata hai. Systematically probability determine karne ka pehla step yeh hota hai ki har simple event ki probability P(EI) nikaal lein. Hume pata hota hai ki probability kabhi negative nahi hoti aur sab simple events mutually exclusive hote hain, kyunki EI me omega_i hota hai aur EJ me omega_j, aur agar i ≠ j, to omega_i ≠ omega_j, isliye EI ∩ EJ = ∅. Ab countable additivity ke hisaab se, sab simple events ki probabilities ka sum hamesha 1 hota hai kyunki unka union poora Omega banata hai, aur P(Omega)=1. Ab koi bhi event A sample space ka subset hota hai, aur A tab ghatega jab usme maujood outcomes me se koi outcome aa jaye. To P(A) nikaalne ka systematic tareeka yeh hai: woh saare i jinke omega_i event A me hain, un sab P(EI) ko jod dein. Yani P(A) = sum of P(EI) over all i such that omega_i ∈ A.

$$\Omega = (\omega_1 + \omega_2 + \cdots + \omega_n)$$

$$P(E_i) >= 0$$

$$\Omega = \bigcup_{i=0}^{\infty} E_i$$

$$\Omega = \sum_{i=1}^{\infty} P(E_i) = \bigcup_{i=0}^{\infty} E_i = P(\Omega) = 1$$

Ek example dekhte hain: Metro/train me 3 compartments hain. Random traveler kis compartment me chadhta hai, yeh hamara experiment hai. Omega = {1, 2, 3}. Simple events: E1 = {1}, E2 = {2}, E3 = {3}. Let Pi = P(Ei). Hume diya hai ki traveler ke middle compartment me chadne ka chance end compartments se do guna hai. To P2 = 2P1 aur P2 = 2P3. Iska matlab P1 = P3. Aur total probability P1 + P2 + P3 = 1. Ab P2 = 2P1 aur P3 = P1 rakhkar equation banegi: P1 + 2P1 + P1 = 1, yani 4P1 = 1. To P1 = 1/4. Phir P2 = 2 × 1/4 = 1/2 aur P3 = 1/4. Is tarah hum given information ka use karke simple events ki probabilities nikal sakte hain, aur ek baar P1, P2, P3 mil gaye to kisi bhi event ki probability aasani se nikaal sakte hain. Jaise ki kya chance hai ki woh pravasi do end waale dibbo me jayega? Ya to pehle dibbe me ya teesre dibbe me. To probability hogi P1 + P3, yani 1/4 + 1/4 = 1/2. Isi tarah hum probabilities ka use karte hain. Ab hum ek special case dekhte hain jisme jo information di jaati hai, woh aapas me simple events ke sambandh batati hai, jaise P2 = 2P1. Lekin bahut se examples me hum dekhte hain ki har outcome ka chance dusre outcome ke chance ke barabar hota hai. Isi topic ko hum "equally likely outcomes" kehte hain — matlab sample space ke saare outcomes bilkul same possibility se hote hain. Yeh tabhi ho sakta hai jab sample space finite ho. Agar sample space me total N outcomes ho aur saare equally likely ho, to har outcome ki probability 1/N hogi. Yani P(Ei) = 1/N for all i. Aur kisi bhi event A ke liye probability hamesha usme maujood outcomes ki ginti divided by total outcomes hoti hai: P(A) = NA / N. Yahan NA = event A me maujood outcomes ki sankhya. Yeh formula istamal karne ke liye hume counting techniques chahiye hoti hain. Simple examples me counting aasaan hoti hai, par kai baar counting mushkil bhi ho sakti hai. Jaise coin ko 2 baar uchhalne me outcomes HH, HT, TH, TT — total 4 outcomes milte hain. 3 baar uchhalne par har outcome ke saath do choices hoti hain, to outcomes 8 ho jaate hain. Dice ke case me 6 outcomes hote hain; do dice uchaalne par total outcomes 6 × 6 = 36 hote hain. Ek coin aur ek dice saath uchaalne par sample space pairs (i, j) se banta hai — jahan i dice ka outcome (6 choices) aur j coin ka outcome (2 choices) hota hai — to total 12 outcomes. Yahan se ek general rule nikalta hai jise product rule ya rule for ordered k-tuples kehte hain. Agar k steps me experiment hota hai aur pehle step me N1 choices, doosre step me N2, teesre me N3 … aur kth step me Nk choices hon, to total outcomes N1 × N2 × N3 × … × Nk honge. Isi se hum complex counting problems solve kar sakte hain. Jaise ek ghar banane ke liye hume ek plumber, ek electrician, aur ek civil contractor chahiye. Agar 4 plumbers, 7 electricians aur 3 contractors available ho, to total distinct configurations honge 4 × 7 × 3 — kyunki har plumber ke saath 7 electricians aur un dono ke saath 3 contractors ki choices hoti hain. Aise hi product rule ko use karke hum probability aur counting dono ko systematic tareeke se samajh sakte hain. Teen choice toh aasaan hai. Ab product rule ko direct use karke dekhte hain ki kitne bhinn tareekon se yeh choices ki ja sakti hain. Wo 4 × 7 × 3 hai, jiska maan hota hai 84. Isi tarah product rule ko use kar sakte hain. Ab ek aur example dekhte hain: ek class me 5 students hain, unhe kisi tareeke se rank karna hai — unka kram tay karna hai, permutation banana hai. Samajh lijiye students ke naam A, B, C, D, E hain. Rank alag criteria se decide ho sakta hai — height, weight, marks, ghar kitna door hai, etc. Har criteria se alag permutation aa sakti hai. To sawal hai: kitne alag-alag sambhavit rank orders possible hain? Yeh bhi product rule ka example hai, lekin thoda advanced. Pehla rank kaun lega? Paanchon me se koi bhi. To 5 choices. Pehla rank decide karne ke baad doosre rank ke liye 4 students bachte hain, to 4 choices. Doosra rank de dene ke baad teesre rank ke liye 3 choices, phir chauthe ke liye 2, aur paanchve ke liye 1 choice. To product rule ke hisaab se total possible rankings hongi:

5 × 4 × 3 × 2 × 1 = 120.

Is number ko ek special notation se likhte hain: 5! (5 factorial). General form me, n factorial ka matlab hota hai:

$$n! = n \times (n - 1) \times (n - 2) \times ... \times 2 \times 1$$

Yani 1 se n tak saare positive integers ka gunakar. Aur jaise humne dekha:

$5! = 120$.