Introduction to Probability & Statistics
Prof Abhay Gopal Bhatt
Department of Statistics
Indian Statistical Institute Delhi
Week - 12
Lecture - 42
Tests of Hypotheses – 1

abhi hum dekhenge test of hypothesis, yah ham kisi daave ke parikshan ke baare mein padhne wale hain; sabse pehle samjhte hain statistical hypothesis kya hota hai ye hota hai koi claim ya dava population ke kisi parameter ke value ke baare mein (jaise population mean ki value kya hai), ya kai parameters ke baare mein, ya kabhi-kabhi distribution ke poore shape ke baare mein; examples: ek dava ho sakta hai population mean $\mu = 3$, ya population proportion $p < 0.1$, ya do parameters μ_1 aur μ_2 ek jaise hain, matlab $\mu_1 - \mu_2 = 0$; teesra type ka dawa shape ke baare mein bhi ho sakta hai jaise "stopping distance normal distribution follow karta hai"; is course mein hum mainly parameters ke daave ko test karenge; jab hum hypothesis test karte hain, to hum do contradictory hypotheses consider karte hain ek ko dusre ke against test karte hain; jaise $\mu = 3$ ko test karenge $\mu \neq 3$ ke against; ya $p < 0.1$ ko test karenge $p \geq 0.1$ ke against; ya $\sigma^2 = 10$ ko test karenge $\sigma^2 \neq 10$ ke against; note: contradictory ka matlab ye hai ki dono daave ek saath sach nahi ho sakte; inme se ek ko hum "primary" daava manege aur doosre ko uske khilaf rakhenge aur iske liye hum ek legal analogy bhi dekh sakte hain, jahan ek hypothesis ko default maana jata hai aur doosre ko challenge kiya jata hai. so jaise ki in legal matters, samajh lijiye ki kisi ke against koi aarop laga hua hai aur case court me gaya hai, to india ke legal system me (aur duniya ke kaafi countries me) ek basic principle follow hota hai: accused ko shuru se hi nirdosh mana jata hai ye hamara initial assumption hota hai; aur court usko doshi tab karega jab uske nirdosh hone ke against substantial evidence present ho; accused ko guilty tab hi paya jata hai jab uske khilaf kaafi zordaar saboot ho jo uske dosh ko strongly support kare; sirf "ho sakta hai ye doshi ho" ke basis par court kisi ko guilty nahi thehraata tab tak nahi jab tak uske actions ki koi alternative explanation ho; jab strong evidence hota hai tab hi guilty declare kiya jata hai; bilkul aise hi statistical hypothesis testing me bhi do contradictory hypotheses hote hain: ek basic assumption null hypothesis (H0), jise legal analogy me "innocent" ke barabar samajh sakte hain; doosra hota hai alternate hypothesis (H1), jo claim karta hai ke null hypothesis galat hai ye legal analogy me "accused is guilty" ke barabar hota hai; ab hum H0 ko tabhi reject karenge jab sample data ke form me kaafi strong statistical evidence mile jo H0 ke against ho; to null hypothesis hota hai hamara initial claim, jise hum tab tak reject nahi karte jab tak uske against strong evidence na ho; aur alternate hypothesis tab accept hota hai jab sample data itna strong ho ki null hypothesis ko maintain karna unreasonable ho jaye; hypothesis test ka conclusion hamesha do tarah ka hota hai ya to reject H0 ya do not reject H0; dhyan rahe: "do not reject H0" ka matlab "H0 is true" nahi hota fir se legal analogy dekhe: court me agar evidence weak hai to accused ko free kar diya jata hai lekin court ye nahi bolta: "hum declare karte hain ki ye 100% nirdosh hai", balki bolta hai: "guilty prove nahi hua" ye hi statistical language me hota hai "do not reject H0"; is analogy ko hum ek practical statistics example se samajhte hain ek company jo

ball bearings banati hai aur unki coating test kar rahi hai; current coating ke saath life average 1000 hours hai ye existing knowledge hai, ye banega H0: $\mu = 1000$; new coating ke favour me claim hai: “life badh jayegi” yaani $\mu > 1000$ ye banega H1; company new coating tabhi adopt karegi jab strong statistical evidence mile ke $\mu > 1000$; isliye alternate hypothesis wo claim hota hai jisko support karne ke liye strong evidence chahiye hota hai; summary: null hypothesis current belief hoti hai, alternate hypothesis new claim; test karne ke time null hypothesis ko equality ke form me set kiya jata hai $\theta = \theta_0$ aur alternate hypothesis teen possible form me hota hai: $\theta > \theta_0$, $\theta < \theta_0$, ya $\theta \neq \theta_0$; kaunsa wala H1 use karna hai ye situation par depend karta hai. vo hi darsata hai ki hamara H1 kya hona chahiye H1: μ ki value jo pehle coating me 1000 thi, naye coating me usse behtar yani usse zyada ho gayi; ye situation clearly batata hai ki H1 = $\mu > 1000$ hona chahiye; ab humne to claims dekh liye ab karna kya hai? karna hai test of hypothesis; to test procedure me do main steps aate hain: pehla set H0 (null hypothesis) aur H1 (alternate hypothesis), jaisa humne example me kiya; dusra ek test statistic choose karna; test statistic kya hota hai? wo ek function hota hai sample values ka usme population parameter nahi aata, isliye sample data se uski exact value hum compute kar sakte hain; example ke liye, agar hum mean ke baare me hypothesis test kar rahe hain, to ek natural statistic hota hai $\bar{x}$ (sample mean); agar hum variance test kar rahe hain, to statistic hota hai s^2 (sample variance); yaani test statistic ka form depend karta hai kaunsa parameter test ho raha hai; test procedure ka second part hota hai rejection region set karna ye ek set hota hai un test statistic values ka jinke aane par H0 reject kiya jayega; matlab agar humne $\bar{x}$ statistic liya aur actual sample data se compute kiya hua $\bar{x}$ agar us rejection region me aata hai, to hum H0 reject kar denge; agar us region ke bahar aata hai, to hum H0 reject nahi karenge; ab ek example: manufacturer ke 10% circuit boards defective mil rahe hain; ek engineer naya manufacturing process suggest karta hai jo defective proportion ko reduce kare; hum P define karte hain true proportion of defectives with new process; manufacturer check karna chahta hai kya $P < 0.1$; to null hypothesis H0 = $P = 0.1$, aur H1 = $P < 0.1$; ab test procedure: wo pehle pilot sample lega say $n = 200$ boards; aur x denote karega defective boards ki count; agar H0 true hota hai to $x \sim \text{Binomial}(200, 0.1)$, aur $E[X] = np = 200 \times 0.1 = 20$; agar H1 true hota hai to $E[X] < 20$; lekin agar sample me $x = 19$ milta hai kya ye enough evidence hai? maybe yes maybe no; strong evidence kab? jab x bohot kam aata hai, for example: reject H0 if $x \leq 15$; isse hamara rejection region $R = \{x: x \leq 15\}$; matlab agar observed x is 15 or less, hum reject karenge H0; agar $x \geq 16$, hum H0 reject nahi karenge; is example me humne dekha kaise H0 aur H1 set karte hain, kaise test statistic choose hota hai (x), kaise uska distribution jante hain under H0, aur kaise rejection region decide hota hai; agle steps me hum aur advanced testing concepts dekhenge.