Introduction to Probability & Statistics
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Week - 10
Lecture - 37
Method o Moments

hum alag–alag estimators aur unki preferable properties dekh rahe the, jaise unbiasedness ya kam variance, lekin ab tak hamare paas koi systematic method nahi tha jisse hum ek accha estimator nikal saken; isliye ab hum do algorithmic methods dekhne wale hain pehla hai method of moments; iske liye pehle hum moments define karte hain: let x1, x2, …, xn be a random sample from a population (jo discrete bhi ho sakta hai ya continuous), aur population ke PMF/PDF ko hum fx se denote karenge; phir kth population moment define hota hai E[X^k] aur kth sample moment define hota hai (1/n)·Σ(x_i^k); first population moment = mean, E[X]; second population moment = $E[X^2] = \sigma^2 + \mu^2$; first sample moment = x-bar; second sample moment = (1/n) $\Sigma(x_i^2)$; in general population moments unknown parameters θ1, θ2, …, θm ke functions hote hain, aur sample moments observed data se compute kiye ja sakte hain; ab definition—suppose population fx depends on unknown parameters θ1 … θm; tab method of moments estimator θ1-hat, θ2-hat, … θm-hat obtain hote hain by writing m equations formed by equating first m sample moments to first m population moments, aur in m equations ko solve karke hume estimators milte hain; ab example: x1, x2, …, xn iid exponential distribution with parameter λ>0, jo service times represent kar sakte hain randomly chosen customers ke liye yahin se hum method of moments apply karenge. to hamare paas kitne unknown parameters hain? sirf ek — lambda; to pehle hum population moment dekhenge: exponential distribution ke liye $E[X_1] = 1/\lambda$; first sample moment hoga $\bar{x}$; ab kyunki humare paas ek hi unknown parameter hai, to hum first sample moment ko first population moment ke barabar set karenge: $\bar{x} = 1/\lambda$; isko solve karne par milta hai λ-hat = $1/\bar{x}$; ab lambda-hat ek random variable hua aur kisi actual sample ke liye $\bar{x}$ compute karke hum estimate nikal sakte hain; isi tarah udaharan: sample gamma distribution se, jisme do parameters α aur β unknown; to do moments use karenge: $E[X] = \alpha\beta$ aur $E[X^2] = \alpha\beta^2 + \alpha^2\beta^2 = \alpha(\alpha+1)\beta^2$; inhe sample moments se equate karenge: $\bar{x} = \alpha\beta$ aur $(1/n)\, \Sigma x_i^2 = \alpha\beta^2 + \alpha^2\beta^2$; in equations ko solve karne par milta hai: α-hat = $\bar{x}^2 / [(1/n)\cdot\Sigma x_i^2 - \bar{x}^2]$ and β-hat = $\{[(1/n)\cdot\Sigma x_i^2 - \bar{x}^2]\}/\bar{x}$; ab final standard example: normal population with unknown μ and σ^2; to first moment => μ-hat = $\bar{x}$; second moment => $\mu^2 + \sigma^2 = (1/n)\cdot\Sigma x_i^2$; to yahan se sigma-hat² = $(1/n)\cdot\Sigma x_i^2 - \bar{x}^2$; yahi method of moments estimator hua; dhyan rahe ki normal case me $\bar{x}$ to unbiased estimator hai μ ke liye, aur S^2 (sample variance) unbiased hota hai σ^2 ke liye lekin yeh jo moment estimator ($\hat{\sigma}^2$) mila, yeh unbiased nahi hai; phir bhi overall method of moments ek simple aur direct tarika hai estimator derive karne ka population moments ko sample moments ke barabar rakhkar.