

Introduction to Probability & Statistics
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Week - 1
Lecture - 3
Determining Probabilities

Ab hum ek udaharan dekhte hain jisse ki hum yeh jo properties hain unko kaise istemaal kareng, woh dekhte hain. To ek samajh lijiye electric system hai jismein do ghataak hain. Main ek schematic diagram draw kar raha hoon jismein do ghataak hain ghataak 1 aur 2. Dono identical components hain, components ya ghataak hain. Aur samajh lijiye ki yeh bayi taraf se electric current jaayegi, dāyī taraf jaa rahi hai, electric current ki direction is taraf hai. Aur yeh jo har ghataak hai, woh foolproof nahi hai, woh kabhikabhi bekaar ho sakte hain, defect ho sakte hain, fail ho sakte hain. To ab so, yeh soch maine abhi jo kaha main likh leta hoon.

Two identical components dono ghataak ek saman hain connected in series, ek shankla mein connect kiye hue hain, jaise ki iss diagram mein dikhaya gaya hai. Har component, har ghataak, bekaar ho sakta hai, fail ho sakta hai. Is event ko hum F se darshaate hain component fails. Aur component fail hone ki probability maan lijiye 0.1 hai. To agar fail ho gayi component 1, to component 1 se current pass nahi hogi, to yeh jo pranāli hai, system hai, woh fail hogi, kyunki current pass nahi hogi. Lekin aisa bhi ho sakta hai ki component number 1 kaam kar raha hai, lekin component number 2 kaam nahi kar raha hai. To agar woh hai to bhi system fail hogi. Ya ho sakta hai ki dono components 1 aur 2 dono fail ho gaye, tab bhi pranali kaam nahi karegi.

To pranali, matlab current left se right tab jaayegi jab aur sirf jab dono components number 1 aur 2 working hon, fail na hon. To jaise hum probability mein hamesha karte hain F failure ke liye rakhte hain, S success ke liye rakhte hain jo complement event hai. S yaani ki F complement component is working, woh theek kaam kar raha hai. Aur humne abhi kuch der pehle dekha ki complement event ki probability kaise nikalte hain. Hum probability of S equal to probability of F complement equal to 1 minus probability of F. Jaise ki 1 minus 0.1 equal to 0.9. To ek component kisi bhi samay kaam kar raha hai, iska chance, iski probability hogi 0.9 Aur yeh dono ghataak, dono components jo hain, ek saman hain. Matlab dono ke liye failure probability 0.1 hai, aur success probability, kaam karne ki, 0.9 hai. Lekin agar question kisi ko yeh pata karna ho ki yeh poora system kaam kar raha hai, iski probability kya hogi, to hum ek event ka naam dete hain Let A be the event that the system fails Yeh important hota hai ki hum event ko kya define karte hain. Aap me se koi event B define kare ki system works woh bhi theek hai. Lekin jo event humne paribhashit kiya hai, use humein hamesha dhyaan me rakhna hai. To yeh event A hamare liye kaun sa event hai Event A equal to system fail ho gayi hai. Ab system kab fail hogi Hamara notation yeh hai ki humare paas do components hain. Har component fail F ya success S ho sakta hai. Dono components ke liye F aur S ki possibilities

hain. To event A kab ghatiga? Dhyaan rakhiye event A ka matlab hai pranali fail hogayi. Pranali fail tab hogi jab do components me se kam se kam ek component fail ho jaye.

To sankshipt roop me hum likhenge Pehla component fail ho aur doosra kaam kar raha ho F S Pehla component kaam kar raha ho aur doosra fail ho jaye S F Dono components fail ho jayein F F To event A ke teen outcomes hain F S S F F F Yani kam se kam ek component fail ho gaya hai. On the other hand, is example me A complement likhna zyada aasaan hai. A complement event kya hai System theek se kaam kar rahi hai. System tab theek se kaam karegi jab dono components theek kaam kar rahe hon. Isme sirf ek hi outcome hai S S To A complement zyada simple hai, kyunki isme ek hi result hai Yeh jo at least one wale events hote hain kam se kam ek fail ho Inki probability nikalna mushkil ho sakta hai. Lekin unke complement ki probability nikalna aasaan hota hai. of course, hum event A me bhi teenon outcomes ki probability nikal kar result laa sakte hain. Lekin aage chal kar, jab bade examples honge, jisme bahut saare outcomes honge, tab har outcome likhna mushkil ho jayega. Tab compliment event ka method helpful hota hai.

Ab chaliye probability nikalen A complement equal to system working equal to S S Dono components independent hain. Har component ke success ki probability 0.9 hai. To PS S equal to 0.9 0.9 equal to 0.81 Yeh ho gayi PA complement. Agar hume A ki probability chahiye yani system fail hone ki

To hum complementary rule use karenge A complement ka complement equal to A Isliye PA equal to 1 minus PA complement equal to 1 minus 0.81 equal to 0.19 Is tarah hum probability ki properties ka use karke aasani se calculations kar sakte hain.

Property No. 5 Union of Two Events Aage badhte hain property number 5 ki taraf Yeh property probability ki countable additivity ya finite additivity ka general form hai. Agar A aur B koi do events ho, to PA Union B equal to PA Plus PB minus PA intersection B Is statement se ek important baat saamne aati hai Agar A aur B mutually exclusive hain, to PA intersection B equal to 0 Aur formula ban jayega PA Union B equal to PA Plus PB Is tarah finite additivity ek special case ban jaata hai property 5 ka jab A intersection B equal to phi hota hai.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

Yeh samajhne ke liye hum ek baar phir se Venn diagrams ka use karenge, jaise humne pehle ki class me dekha tha. To yeh hamara sample space Omega hai. Hum A union B ko dekh rahe hain. Samajh lijiye yeh yellow region event A hai aur yeh blue region event B hai. Dono ke beech ka jo hara portion hai, woh A intersection B hai. To poora colored portion A union B ko dikhata hai. Ab hum isko teen hisson me baant dete hain sirf yellow wala A ka portion, sirf blue wala B ka portion jo A me nahi hai, aur beech ka hara portion jo A intersection B hai. Yeh red se

shade kiya hua jo right side ka hissa hai woh B to hai, lekin A ka part nahi hai. To mathematically yeh region B intersection A complement hai. A complement intersection B bhi likh sakte hain; dono ka matlab ek hi hai. To ab A union B teen mutually exclusive hisson me baant gaya A ka portion, A intersection B ka hara portion, aur B intersection A complement ka shaded red portion. Kyunki yeh teenon mutually exclusive hain, to inke union ki probability hum finite additivity se nikaal sakte hain. To hum A union B ko likh sakte hain A union A complement intersection B. Dono mutually exclusive events hain. Uski probability hogi $P(A \cup A^c \cap B)$. Ab is expression me hum A intersection B ko add aur subtract kar dete hain $P(A \cup A^c \cap B) + P(A \cap B) - P(A \cap B)$. Jis quantity ko humne add kiya, usi ko subtract bhi kar diya, isliye value same rahegi. Ab beech ke do terms A complement intersection B aur A intersection B donon mutually exclusive hain. Inka union poora B ban jaata hai. To un donon ki total probability $P(B)$ ho jaati hai. To humara expression ban gaya $P(A \cup A^c \cap B) + P(B) - P(A \cap B)$. Is tarah humne prove kar diya ki kisi bhi do events A aur B ke liye $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Ab ek example dekhte hain. Samajhiye ek locality me 60% gharon me internet connection hai, 80% gharon me telephone connection hai, aur 50% gharon me dono connections hain. To event A equal to randomly chosen household has internet connection; event B equal to household has telephone connection. Diya gaya hai $P(A) = 0.6$, $P(B) = 0.8$, $P(A \cap B) = 0.5$. Ab hume probability nikaalni hai ki randomly chosen ghar me at least one connection ho. At least one ka matlab A union B hota hai. To property ke according $P(A \cup B) = 0.6 + 0.8 - 0.5 = 0.9$. Yani 90% chance hai ki randomly chosen ghar me kam se kam ek connection ho. Next question hai exactly one connection ho. Exactly one ka matlab A union B me to hona chahiye, lekin A intersection B me nahi hona chahiye. Venn diagram ki bhaasha me yeh hoga A intersection B complement union A complement intersection B. Yani sirf internet wala aur sirf telephone wala. Inki probability hogi $P(A \cap B^c) + P(A^c \cap B) = 0.6 - 0.5 + 0.8 - 0.5 = 0.4$. Yani 40% gharon me sirf ek hi connection hai. Lekin ab hum is event ko C se darshate hain. Yani hume jis event ki probability nikalni hai, use hum event C kahenge. Ab C aur A intersection B yeh dono events mutually exclusive hain, aur inka union hi A union B hota hai, jaise humne thodi der pehle Venn diagram me dekha tha. Isliye hum seedhe finite additivity ka use kar sakte hain. Left side me jo do events hain C aur A intersection B dono mutually exclusive hain, to unke union ki probability hogi probability of C plus probability of A intersection B. Aur yeh union event hi A union B ke barabar hai. Isliye ek equation ban jaati hai $P(A \cup B) = P(C) + P(A \cap B)$. Is example me $P(A \cup B) = 0.9$ mila tha. Aur $P(A \cap B) = 0.5$. To hume $P(C)$ nikalna hai $P(C) + 0.5 = 0.9$ $P(C) = 0.9 - 0.5 = 0.4$. To humne dekh liya ki kaise alagalag probability properties ka use karke hum aasani se probability calculations kar sakte hain. Ab aage badhte hain Probability of union for 3 events. Do events ke union ka formula humne dekha $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Ab agar teen events A, B, C ho, to sirf teeno ki individual probabilities ko jodna kaafi nahi hota, kyunki kuch portions do-do baar add ho jaate hain. Isliye ek general formula hota hai jise inclusion-exclusion rule kehte hain. $P(A \cup B \cup C)$ ke liye pattern yeh hota hai $P(A) + P(B) + P(C)$

pehle sab individual probabilities add fir subtract karna padega unke pairwise intersections $P(A \cap B) + P(A \cap C) + P(B \cap C)$ Lekin isme teenon ka intersection $P(A \cap B \cap C)$ zyada baar subtract ho gaya hota hai, isliye use ek baar add karna padta hai $P(A \cap B \cap C)$ To final formula hota hai $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$ Aap chahe to Venn diagram bana kar verify kar sakte hain clearly dikhai dega ki beech wala common region kai baar add/subtract ho raha hai, isliye use last me ek baar add karna zaroori hai. Yahi pattern aage bhi chalta hai Agar 4 events ho to pehle sab individual probabilities add, fir sab 2-2 ke intersections subtract, fir sab 3-3 ke intersections add, fir sab 4-4 ke intersections subtract, and so on. Is pattern ko hum inclusion exclusion principle kehte hain, aur aage chal kar hum is property ka use karenge.