

Introduction to Probability & Statistics
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Week - 6
Lecture - 20
Continuous Random Variable

ki aaj hum continuous random variables ke baare me padhenge ab tak humne jo random variables dekhe the unhe discrete random variables kaha jata tha yaani unke possible outcomes ki sankhya gananiy countable thi continuous random variable me iske vipreet, range of values ek santat samuchchay hota hai jaise ek interval pehle example me 0 se 1 ke beech ka number randomly choose karne ka random experiment liya: isme sample space hota hai omega equal to 0,1 omega me outcomes ki sankhya agananiya uncountable hoti hai agar hum xomega equal to omega define karein, to x ek random variable hai jiske outcomes agananiya many values lete hain isi tarah koi measurement length, weight, PH value, temperature ye sab continuous hota hai continuous random variable ko define karne ke liye do baatein essential hoti hain: 1 range of values continuous ho ek interval ya intervalon ka disjoint union 2 kisi bhi exact real number C ke equal x honi ki probability 0 hoti hai yani $P(X \text{ equal to } C) = 0$ for all C continuous random variable ke saath jo function jura hota hai use kehte hain probability density function PDF praiykta ghanva phalan agar x continuous random variable hai to koi function

$$P(a \leq X \leq b) = \int_a^b f(x)dx \text{ for any } a < b$$

$$P(X \in [a, b]) = \int_a^b f(x)dx$$

continuous case me probability integral se milti hai kyunki $P(X \text{ equal to } a) = 0$ aur $P(X \text{ equal to } b) = 0$ hota hai, isliye $P(a \leq X \leq b) = P(a < X < b)$ aur $P(a \leq X < b) = P(a < X \leq b)$ aur $P(a < X \leq b) = P(a < X < b)$

$$\begin{aligned} P(a \leq X \leq b) &= P(a < X \leq b) = P(a \leq X < b) \\ &= P(a < X < b) \\ &= \int_a^b f(x)dx \end{aligned}$$

$$P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x)dx = 1$$

yani end point include kare ya na kare koi fark nahi padta ye continuous random variable aur uske PDF ki basic intuition aur properties hain. to tab bhi yeh same hi rahegi probability iska kaaran hai ki X continuous random variable hai, aur definition ke anusaar continuous random variable ke liye X ke kisi bhi exact particular value A ya B ko lene ki probability hamesha zero hoti hai isliye jab hum inequalities likhte hain chahe strict ho ya non-strict probability me koi farq nahi padta continuous case me ye charon probabilities same hoti hain aur unhe integral ke dwara calculate kiya ja sakta hai ab PDF ke kuch aur points dekh lein: doosra remark fx yani probability density function hamesha shunya ya usse adhik non-negative hota hai, for all x agar hum fx ka graph banayenge to kyunki yeh non-negative hai, iska graph x axis ke upar hi rahega aur agar hum do numbers A less than B lein to integration from A to B of fx dx is area ko represent karta hai jo x axis, fx ke graph, aur x equal to A aur x equal to B ki vertical lines ke beech me trapped hota hai shaded region ke roop me teesra property probability that X lies between minus infinity to infinity yaani poore sample space me uska integral hamesha 1 hota hai kyunki woh Pomega hai yeh sab properties PDF ki hain.

Ab is concept ko example se samjhate hain: maan lijiye ek golakar vastu hai jaise tyre aur uski boundary par kahin ek defect impurity ho sakta hai reference line se clockwise direction me us defect ka angle measure karein to wo angle ek random variable x hoga jo 0 se 360 degrees ke beech value le sakta hai iske liye ek PDF define karte hain: fx equal to 1 by 360 if 0 less than or equal to x less than or equal to 360, aur fxequal to 0 otherwise iska graph ek flat horizontal line hai 1 by 360 par from 0 to 360 baaki jagah zero clearly fx greater than or equal to 0 hamesha aur integration from minus infinity to infinity fx dx equal to integration 0 to 360 1 by 360 dx equal to 360 by 360 equal to 1 so yeh valid PDF hai agar hume probability chahiye ki angle 90 degree se 180 degree ke beech ho to P90 less than or equal to X less than or equal to 180 equal to integration 90 to 180 1 by 360 dx equal to 180 to 90 by 360 equal to 1 by 4 equal to 0.25 agar pucha jaye ki angle reference line se 90 degree ke radius ke andar ho within 90 degrees without specifying direction, to wo hoga do intervals ka union: 0 to 90 aur 270 to 360 to P equal integration 0 to 90 1 by 360 dx plus intergration 270 to 360 1 by 360 dx equal to 1 by 4 plus 1 by 4 equal to 1 by 2 equal to 0.5.

Aisa PDF jahan function kisi interval me constant ho aur baahar zero us distribution ko kehte hain Uniform Distribution agar random variable X uniform on interval A, B hai to uska PDF hota hai fx equal to 1 by B minus A for A less than or equal to x less than or equal to B aur fxequal to 0 otherwise pehle example me jo random number choose kar rahe the interval 0,1 se wo X Uniform 0,1 hota hai tab probability X greater than or equal to 2 by 3 equal to intergration 2 by 3 to 1 1 dx equal to 1 2 by 3 equal to 1 by 3.

Discrete random variables me hum usually random experiment se start karte the continuous random variables me aksar hum direct PDF ki definition se start karte hain ab ek new PDF ka example dekhte hain: $f(y) = y$ for $y < 1$ aur $f(y) = 0$ otherwise iska matlab PDF 1 se plus 1 tak defined hai minus 1 se 0 tak $f(y) = -y$ hota hai aur 0 se 1 tak $f(y) = y$ hota hai baaki jagah zero isliye random variable Y ki values hamesha minus 1 aur plus 1 ke beech rahengi agar probability nikalni ho $P(-1/3 \leq Y \leq 2/3)$ to integral do parts me to denge: integrate from minus 1 by 3 to 0 of minus y dy aur 0 to 2 by 3 of y dy result hota hai $5/18$ equal to 0.277.