Introduction to Probability & Statistics
Prof. Abhay Gopal Bhatt
Department of Statistics
Indian Statistical Institute Delhi
Week - 5
Lecture - 18
Hypergeometric and Poisson Distribution

Aur ek standard distribution hum padhenge hypergeometric distribution ya Hindi me hypergeometric bantan ek tarah se yeh bhi binomial trials se juda hua hai binomial trials me har trial me do parinaam S ya F aur success ki probability har trial me same hoti hai lekin hypergeometric me thoda antar hai: yeh situation hoti hai sampling without replacement from a finite population yani hum ek seemit jan-sankhya finite population se ek ke baad ek namoone chunte hain lekin jo sample choose ho gaya usko hum wapas population me nahi daalte without replacement isliye probability change hoti rehti hai pehle define karte hain population jahan se sample liya jaa raha hai usme capital N individuals objects hote hain aur har individual ya to S hota hai ya F population me exactly M individuals S hain aur N minus M individuals F hain ab hum ek ke baad ek randomly small n samples draw karte hain in a manner that all sets of possible samples of size n are equally likely random variable X define karte hain: X equal to number of successes S in the sample of size n yeh hypergeometric experiment hai jisme 3 parameters hote hain capital N population size capital M number of S in population aur small n sample size ab isko ek udaharan se samajhte hain maan lijiye ek company ke IT cell me 20 service complaints aayi printer ke liye jinme se 12 inkjet printers ke liye thi aur 8 laser printers ke liye in 20 printers ko service ke baad company unme se randomly 5 printers ko customers ke feedback ke liye choose karegi selection completely random hoga prashn: what is the probability that exactly X of the selected 5 printers were inkjet printers Yani population size capital N equal to 20 population me number of S inkjet capital M equal to 12 number of failures N minus M equal to 8 laser aur sample size small n equal to 5 agar inkjet ko S define karein to hume PX equal to x nikaalna hai exactly x selected printers inkjet nikle yeh hypergeometric distribution ka direct case hai.ki jo sample chuna gaya hai usme se kitne successes hain to yeh bilkul hypergeometric situation hai jismein X ek hypergeometric random variable hai ismein jo teen parameters the woh the capital N population size equal to 20 capital M number of successes in population equal to 12 kyunki yahan inkjet printers ko S mana hai aur sample size small n equal to 5 humein pata karna hai ki X agar value 0 1 2 3 4 ya 5 leta hai to uski probability kya hogi X equal to 2 ke case me example se samajhte hain: event hai 2 inkjet printers out of 5 selected aur baaki 3 laser printers number of inkjet printers in population equal to 12 to 12 me se 2 choose karne ke tarikay equal to 12 choose 2 number of laser printers in population equal to 8 to 8 me se 3 choose karne ke tarikay equal to 8 choose 3 dono selection ek saath hone ke total tarikay equal to 12 choose 28 choose 3 denominator hoga total number of ways of selecting 5 printers from population of 20 equal to 20 choose 5 selection random hai to har sample equally likely hai therefore probability Xequal to2 equal to 12C2 multiply 8C3 by 20C5 generalizing: probability

Xequal tox equal to M choose xN minus M choose n minus x by N choose n jahaan X ki valid values hoti hain 0 se lekar minn M tak lekin constraint yeh bhi hai ki n minus x less than or eual to N minus M ho warna combination meaningless is tarah hypergeometric PMF define hota hai jab X counts number of successes in a sample of size n drawn without replacement from population N with M successes and N minus M failures expected value hoti hai EX equal to n multiply M by N aur variance hota hai Var X equal to n multiply MbyN1 minus M by NN minus n by N minus 1 yeh formula shayad complex lage lekin agar population size N sample size n se bahut zyada bada ho to factor N minus n by N minus 1 equal 1 ho jata hai aur distribution lagbhag binomial jaisa ho jata hai jismein expected value hoti hai np aur variance hoti hai np multiply 1 minus p jahaan p equal to M by N yani hypergeometric distribution ko binomial distribution ki approximate form maana ja sakta hai jab population bahut badi ho aur sampling without replacement ka effect negligible ho.

$$E(X) = \left(\frac{M}{N}\right)n$$

$$V(X) = n\left(\frac{M}{N}\right)(1 - (\frac{M}{N})(\frac{N-n}{N-1})$$

$$if\ P = \frac{M}{N}$$

$$E(X) = P * n$$

$$V(X) = n * p * (1-p)(\frac{N-n}{N-1})$$