Introduction to Probability & Statistics
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Week - 3
Lecture - 12
Probability Mass Function

Ek definition se shuruaat karte hain, aur usse pehle yeh samajhna zaroori hai ki humne dekha ki har sample space par hum ek ya kai random variables define kar sakte hain. Random variable ek aisa niyam (rule) hota hai jo har outcome ke liye ek real number assign karta hai. Pichhli classes me humne dekha ki har random variable ke saath judi probabilities kaise calculate karte hain. Ab jo definition aa rahi hai woh isi process se sambandhit hai probability distribution ya probability mass function (PMF) ki paribhasha. Random variable X ke liye hum PMF ko $P_x(x)$ likhte hain yeh ek function hota hai jo poori real line R par define hota hai. Har real number x ke liye $P_x(x)$ ki value hoti hai P(X = x) yaani woh probability ki random variable X ka value small x ho. Yahaan X = x likhna ek short form hai us event ka, jismein sabhi outcomes ω include hote hain jinpar x(ω) = x hota hai. PMF sirf discrete random variables ke liye define hota hai yaani un random variables ke liye jinki values ka set countable ho (finite ya countably infinite). PMF ek function hai real numbers par, jismein har x ke liye P(X = x) ek real number hota hai jo 0 aur 1 ke beech hota hai. Kyonki X = x ek event hota hai aur probability hamesha 0 se 1 ke beech hoti hai.

$$P_x(x) = p(x) = p(X = x) = P(\{\omega: X(\omega) = x\})$$

Doosri important property yeh hai ki sabhi possible x ke liye P(X = x) ka sum 1 hota hai, kyonki yeh mutually exclusive events hote hain, aur unka union poora sample space deti hai jiska probability 1 hota hai.

$$0 \leq p(x) \leq 1$$

$$\sum\nolimits_x p(x) = \sum_x P(X = x) = p\left(\bigcup_x \{X = x\}) = p(\Omega\right) = 1$$

Ek example lete hain fair die wala example: Ω = {1,2,3,4,5,6} aur random variable X(i) = i. Pichhle lecture me humein mila tha ki P(X = i) = 1/6. To PMF hai P(x) = 1/6 for x = 1,2,3,4,5,6. Lekin PMF real line par define hota hai, isliye humein un values ka bhi sochna hoga jo X kabhi nahi le sakta, jaise P(7). Par X kabhi 7 nahi leta, to event X = 7 ek empty set hai jiski probability 0 hoti hai. Isi wajah se P(x) = 0 for all other real numbers jise X nahi le sakta. Isi tarah P(-3.3) bhi 0 hoga. To poora PMF is tarah likh sakte hain: P(x) = 1/6 for x = 16, aur P(x) = 0 for sabhi doosre real values. Isi tarah har discrete random variable ka PMF hum define kar sakte hain.$P(x) = 1/6$ hota hai jab x ki value 1, 2, 3, 4, 5, ya 6 ho, aur baaki sabhi x values ke liye $P(x) = 0$ hota hai. PMF likhne ka matlab hota hai ki har real number x ke liye $P(X = x)$ ki value batani. Ab ek example lete hain: toss a coin repeatedly till the first head appears.” Har toss me heads aane ki probability p hai. Is experiment ka sample space countably infinite hai jaise H, TH, TTH, TTTH, Is par random variable X define kiya gaya hai: X = total number of tosses taken till the first head appears. Agar pehle toss me hi H aa jaye, to $X = 1$. Agar outcome TH ho, to $X = 2$. Agar TTH ho, to $X = 3$. Agar outcome me k tails aur last me ek head ho, to $X = k + 1$ hoga. To X ki possible values positive integers 1, 2, 3 hain. Ab PMF nikalte hain. $P(X = 1) = p$, kyunki pehle toss me H aana hai. $P(X = 2) = (1–p)p$ kyunki pehle T aur phir H. $P(X = 3) = (1–p)^2 p$, aur general form me $P(X = k) = (1–p)^{(k–1)} p$ hota hai. To PMF likh sakte hain: $P(X = x) = (1 – p)^{(x – 1)} p$ for x = 1, 2, 3, … aur $P(X = x) = 0$ for all other real x. Ab ek doosra example: ek dukaan me 10 baje se 10:10 tak kitne customers aate hain. Random variable X ki possible values 0, 1, 2, 3 di hui hain, aur unki probabilities table me di gai hain: P(0)=0.3, P(1)=0.2, P(2)=0.4, P(3)=0.1. Inka sum 1 hota hai, to PMF valid hai. Tabular form me ya functional form me PMF dono tarah se likh sakte hain. Ab isi PMF ko use karke events ki probabilities nikal sakte hain for example $P(X \leq 0) = 0.3$; $P(X \leq 1) = P(0) + P(1) = 0.3 + 0.2 = 0.5$; $P(X \leq 2) = 0.3 + 0.2 + 0.4 = 0.9$; aur $P(X \leq 3) = 1$. Isi tarah $P(1 \leq X \leq 2) = 0.2 + 0.4 = 0.6$. Ek aur example: kisi restaurant me randomly chosen customer chai ya coffee prefer karta hai. Random variable X define hai: X = 1 agar chai prefer ho, X = 0 agar coffee prefer ho. Diya hai P(0)=0.3, to P(1)=0.7. Aise random variables jo sirf 0 aur 1 values lete hain, unko Bernoulli random variable kehte hain. Agar $P(X = 1) = p$ ho, to $P(X = 0) = 1 – p$ hoga. Hum likhte hain: X Bernoulli(p). Isme p ko parameter (praachal) kehte hain, aur p ki har value ek alag PMF degi. Pure PMFs ka group Bernoulli family kehlata hai. PMF ko table, function form, line graph, ya histogram se bhi show kiya ja sakta hai. Maan lo X values 0, 1, aur 3 leta hai probabilities 0.5, 0.4 aur 0.1 ke saath. To function form hoga: P(0)=0.5, P(1)=0.4, P(3)=0.1, aur baaki sab x ke liye P(x)=0. Line graph me x-axis par 0,1,3 aur unki heights 0.5, 0.4, 0.1 hongi. Histogram me rectangle banate hain jinke base par 0,1,3 honge aur heights PMF ki values hongi. esi tarah PMF ka har graphical form ek hi baat batata hai random variable ka distribution.