Introduction to Probability & Statistics
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Week - 3
Lecture - 11
Discrete Random Variables

Ab hum random variables ke baare me padhte hain, jise Hindi me yaadru ChChik char kahte hain. Random experiment ka matlab hota hai koi bhi aisa prayog jisme kai sambhavit parinaamon me se ek parinaam nishchit roop se aayega, lekin pehle se yeh nirdharit nahin hota ki kaunsa parinaam aayega. Random experiment ka sample space hota hai sabhi parinaamon ka samu. Har event, sample space ka ek up-samu hota hai, aur har event ke liye probability define hoti hai jo 0 aur 1 ke beech hoti hai. Parantu aksar hum parinaam ko seedha use karne ki jagah us par kisi maap (measurement) ka prayog karte hain jaise kisi student ki height, age, marks, ya phir kisi office me ek tube-light kitne ghante chalegi. Har outcome ke saath agar hum ek vaastavik sankhya (real number) jod dete hain, toh is process ko hum random variable kehte hain. Random variable asal me ek real-valued function hota hai jo omega (sample space) ke har outcome ko ek real number assign karta hai. Udaharan ke liye, ek class ke students ka sample space lein. Har student ek outcome hai. Hum unki height, age, ya marks ko measurement ke roop me assign kar sakte hain—ye sab alag-alag random variables honge. Isi tarah office ki 100 tube-lights ka sample space ho sakta hai, aur har tube-light kitne ghante me fuse hogi, yeh ek random variable ka example hai. Ab hum sabse basic experiment coin toss ka lete hain. Sample space hai $\Omega = \{H, T\}$. Is par hum kai random variables define kar sakte hain. Pehla example: $X(H) = 1$ aur $X(T) = 0$. Dusra random variable Y, jo alag function hai, jisme $Y(H) = 0$ aur $Y(T) = 1$. Teesra example Z, jisme $Z(H) = +1$ aur $Z(T) = -1$. Ye sab alag-alag functions hain aur isi wajah se alag random variables hain. Ek hi omega par hum anek random variables define kar sakte hain, kyunki random variable ki paribhasha bas itni hai ki outcome se real number assign karne ka ek niyam ho. Isi tarah hum aage padhenge ki random variables ka analysis kaise hota hai, unke distributions kaise define hote hain, aur probability theory me unka prayog kaise kiya jaata hai.तो Ab hum aur random variable ke examples dekhte hain. Coin toss se aage badhte hue hum dice ke uchhal ka example lete hain. Isme sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$ hota hai. Yahan har outcome pehle se hi ek real number hai, isliye random variable define karna aur aasaan ho jata hai. Pehla random variable hum define karein: $X(i) = i$ for all i in Ω. Matlab $x(1) = 1$, $x(2) = 2$, ..., $x(6) = 6$. Doosra random variable y define kar sakte hain: $Y(i) = 6 - i$; jaise $i = 2$ ho to $y(2) = 4$. Teesra random variable $Z(i) = i^2$, jisme values hongi 1, 4, 9, 16, 25, 36. Ek aur random variable T define karte hain jo even–odd par adharit hai: agar i odd ho to $T(i) = 0$, aur agar i even ho to $T(i) = 1$. Isko long form me bhi likh sakte hain: $T(1) = T(3) = T(5) = 0$ aur $T(2) = T(4) = T(6) = 1$. Pichhle example me humne dekha ki Omega me 6 outcomes the, par random variable T sirf do values leta hai 0 ya 1. Ye hamesha possible hai ki random variable ki range, Omega ke outcomes se kam ho. Aage badhte hue ek aur situation lete hain jahan Omega ek infinite interval hota hai:

Omega = [0, 1], yaani 0 aur 1 ke beech ke sab real numbers. Yahan random variable define karna bhi seedha hai: $X(\omega) = \omega$; yaani jo number choose hua wahi X ki value hogi. Doosra example Y define kar sakte hain: $Y(\omega) = \sin(\omega)$. Ye dono random variables uncountably many values lete hain. Lekin isi Omega par hum ek aisa random variable bhi define kar sakte hain jo sirf finitely many values leta hai. Udaharan ke liye: interval [0,1] ko n equal parts me baat diya aur define kiya $Z(\omega) = k/n$ jab ω interval $[k/n, (k+1)/n]$ me ho. Yaani Z ki range sirf n values hogi: 0, 1/n, 2/n, ..., $(n-1)/n$. Yahan observe karne layak baat yeh hai ki Omega aganeya (uncountable) hone ke bawajood random variable Z ki range parimit (finite ya countable) ho sakti hai. Isi tarah X aur Y ki range continuous hai, lekin Z ki range discrete finite rationals tak simat jaati hai. Ab hum agle definition ki taraf badhenge.Ismein hum do tareeke dekhte hain jinke zariye random variables ko classify kiya jaata hai. Pehla class hota hai discrete random variables, jinki range countable hoti hai yaani ya to finite hoti hai ya countably infinite. Discrete random variable ko Hindi me asatata ya asantat ya asantat random variable bhi bol sakte hain. Upar jis example me $Z(\omega)$ sirf n values leta tha, woh clearly ek discrete random variable tha. Aage badhne se pehle ek summary: hamare paas sample space Ω hota hai sabhi outcomes ka samuchay. Events, jaise A, B, C, hamesha Ω ke subsets hote hain. Outcomes real numbers hone zaroori nahi jaise coin toss me outcomes H aur T hote hain. Lekin random variable ek real-valued function hota hai jo har outcome ko ek real number assign karta hai. Yaani random variable ki values real numbers ka ek subset hoti hain. Ab hum random variables ke saath judi probabilities dekhte hain.

Le ek simple example coin toss ka. $\Omega = \{H, T\}$. Humne random variable X define kiya tha: $X(H) = 1$ aur $X(T) = 0$. Ab hum puchte hain—X ka value 1 hone ki probability kya hai? Yaani $P(X = 1)$. Lekin probability hum events ki nikalte hain; isliye “X = 1” ek event hai. Uska matlab hai: sabhi ω in Ω jinke liye $X(\omega) = 1$. Is example me sirf H aisa outcome hai. Yaani event {H}. To $P(X = 1) = P(H)$. Fair coin me ye hota hai 1/2. Isi tarah $P(X = 0) = P(T) = 1/2$. Agar coin fair na ho aur $P(H) = p$ ho, to $P(X = 1) = p$ aur $P(X = 0) = 1 - p$ hoga. Dice example me $\Omega = \{1,2,3,4,5,6\}$ aur random variable $X(i) = i$ tha, to $P(X = i)$ simple event {i} ki probability hui jo 1/6 hai. Dusre random variable T me $T(i) = 0$ tha agar i odd ho aur $T(i) = 1$ agar i even ho. To $P(T = 0) = P(\{1,3,5\}) = 3/6 = 1/2$, aur $P(T = 1) = P(\{2,4,6\}) = 3/6 = 1/2$. Isi tarah har random variable ke liye hum uske events (X= value) nikal kar unki probability calculate kar sakte hain.