Introduction to Probability & Statistics
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Week - 1
Lecture - 1
Random Experiments

Aaj hum ek bahut hi interesting aur important topic start karne ja rahe hain Introduction to Probability and Statistics. Sabse pehle samajhte hain ki Probability kya hoti hai aur Statistics ka matlab kya hai. Hamari daily life mein kuch cheezein aisi hoti hain jinke results hamesha certain hote hain, jaise agar hum pen chhodte hain to wo neeche girta hi hai, ya agar ice ko dhoop mein rakhein to wo pighal hi jaata hai — inhe hum deterministic statements kehte hain. Lekin kai baar aise situations aate hain jahan result uncertain hota hai, jahan humein pehle se nahi pata hota ki kya hoga, wahi se Probability ka concept start hota hai. Jaise agar hum ek coin uchhalte hain, to ye to sure hai ki wo neeche girega, lekin head aayega ya tail, ye pehle nahi pata hota; do possible outcomes hote hain H aur T. Isi tarah agar hum dice roll karein, to uske 6 faces hote hain aur koi bhi number 1 se 6 tak aa sakta hai, par pehle se decide nahi hota kaunsa aayega. Agar dobara roll karein to result same bhi ho sakta hai ya different bhi koi guarantee nahi. Yehi uncertainty Probability ka base hai. Isi tarah ek deck of cards mein 52 cards hote hain, aur agar hum ek card randomly nikaalein to kaunsa aayega, humein pehle nahi pata hota. Factory mein banne wale components defective ya non-defective ho sakte hain, aur check karne se pehle humein nahi pata hota kaunsa kya niklega. Hospital mein jab baby born hota hai to wo male ya female ho sakta hai, lekin birth se pehle hum sure nahi ho sakte. In sab examples mein ek common cheez hai result uncertain hota hai jab tak experiment perform na ho jaaye. Yahi uncertainty ko hum Probability ke through study karte hain, jisse hum samajh sakein ki kisi event ke hone ki possibility kitni hai aur isi ke saath hum Statistics use karte hain taaki data ko analyse karke meaningful conclusions nikal sakein. Ab hum is topic ko thoda systematically padhenge, aur isi ko hum Probability Theory yaani Sambhavyata Siddhant kehte hain. Jo humne abhi 4–5 examples liye jaise coin toss, dice, cards, factory components, ya baby ka gender ye sab actually random experiments ke examples hain. Hum inhe "random" isliye kehte hain kyunki experiment karne se pehle humein pata nahi hota ki result kya aayega. Humein ye to pata hota hai ki possible outcomes kya-kya ho sakte hain, lekin unme se actual result kaunsa aayega, ye pehle se decide nahi hota. Aur agar hum experiment ko repeat karein, to har baar result same bhi ho sakta hai ya different bhi koi guarantee nahi hoti. Ye sabhi examples random experiments kehi jaate hain, aur inhi par based hoti hai Probability Theory jo basically uncertainty aur randomness ka systematic study hai.

Probability Theory mein hum in random experiments ko mathematically samajhte hain, analyze karte hain aur unke results ke chance ko quantify karte hain. Is course ke pehle half mein hum isi Probability Theory ko detail mein padhenge kis tarah events bante hain, unke outcomes kya hote hain, aur unhe measure kaise karte hain.

Fir second half mein hum Statistics padhenge. Statistics shabd aapne pehle bhi suna hoga iska matlab hota hai "data ka adhyayan". Jab humare paas bahut saara data hota hai jaise measurements, scores, ya observations to us data ko samajhna, summarize karna aur analyze karna Statistics ke through kiya jaata hai. Simple shabdon mein, Statistics wo science hai jo random data ka analysis karti hai, aur us analysis ke liye hum Probability Theory ke models ka use karte hain.

Jaise agar hum cricket ka example lein to hum batting average ki baat karte hain. Har batsman ke alag-alag innings ke scores hote hain, aur un sab data ko ek saath dekhna mushkil hota hai. Isliye hum us data ko mathematically summarize karte hain jaise average nikalte hain taaki ek single number se uske performance ka idea mil jaaye. Isi tarah, climate data mein hum daily temperature dekhte hain, agricultural yield mein production data dekhte hain ye sab Statistics ke examples hain.

To short mein, Probability Theory humein uncertainty ko samajhne ka systematic tareeka deti hai, aur Statistics humein real-life data ka analysis karne ka method sikhati hai. Ye dono subjects ek doosre se jude hue hain Probability theory se hum models banate hain, aur Statistics un models ko real-world data par apply karta hai.

In sab data ko government aur researchers kai saalon se collect karte aaye hain. Un data ka analysis karke hum future predictions karte hain jaise agle saal kitni agricultural yield (krishi utpaadan) hogi, ya temperature trends kya honge. Ye sab cheezein Statistics ke part hain. Ab hum thoda Probability Theory ko systematically samajhte hain.

Jaise humne pehle kaha, Probability Theory ka base hai random experiment ya yaadritchhik prayog. Random experiment ka matlab hota hai aisa experiment jiska result pehle se certain nahi hota yaani, outcome uncertain hota hai. Lekin jaise humne examples mein dekha, humein ye zarur pata hota hai ki possible outcomes kya-kya ho sakte hain. Matlab, outcome in known possibilities mein se hi ek hoga, lekin kaunsa hoga, ye humein experiment karne se pehle nahi pata. Aur agar hum experiment baar-baar repeat karein, to result har baar same bhi ho sakta hai, different bhi koi guarantee nahi. Aise experiments ko hi hum Random Experiments kehte hain.

Ab random experiment mein ek bahut important concept aata hai Sample Space, jise hum Greek letter Ω (Omega) se denote karte hain. Sample Space ek set (ya samuchay) hota hai jo kisi experiment ke sabhi possible outcomes ka collection hota hai. Matlab, kisi bhi random experiment ke jitne bhi results aa sakte hain, un sabka set hi us experiment ka sample space hota hai. Isko hum generally likhte hain "Ω = set of all possible outcomes."

Ab examples ke through samajhte hain

Example 1: Coin Toss
Agar hum ek coin uchhalte hain, to do possible outcomes hote hain — Head (H) aur Tail (T).
To sample space Ω = {H, T}.

Example 2: Rolling a Die
Ek die ke 6 faces hote hain — 1, 2, 3, 4, 5, 6.
To sample space Ω = {1, 2, 3, 4, 5, 6}.

Example 3: Drawing a Card from a Deck
Ek standard deck mein 52 cards hote hain — Hearts, Spades, Clubs, Diamonds, har ek suit mein 13 cards.
Agar hum randomly ek card pick karte hain, to possible outcomes 52 hain.
To sample space Ω = {all 52 distinct cards}. Itna bada set likhna mushkil hota hai, to hum generally dots ya symbols se represent karte hain.

Example 4: Tossing Two Coins Together
Ab agar hum ek coin ki jagah do coins uchhalte hain, to har coin ke liye 2 possibilities hain — Head (H) ya Tail (T).
To total possible outcomes honge $2 \times 2 = 4$.
Sample space Ω = {HH, HT, TH, TT}.

Yani dono coins ke saath milkar 4 results ho sakte hain.

Example 5: Time Waiting at a Traffic Signal
Maan lijiye main office jaate waqt ek traffic signal par rukta hoon. Kabhi green light milti hai to main seedha nikal jaata hoon (rukhne ka time 0 second), aur kabhi red light milti hai to mujhe poore 120 seconds (2 minutes) rukna padta hai. Iske beech koi bhi waiting time possible hai — jaise 5 seconds, 20 seconds, 45 seconds, etc.
To sample space Ω = set of all x such that $0 \le x \le 120$.
Yani Ω = {x | $0 \le x \le 120$}.

Yeh ek continuous sample space ka example hai, kyunki outcomes discrete nahi hain, balki ek poore range mein aa sakte hain.

Is tarah se hum dekhte hain ki sample space ko represent karne ke kai tarike hote hain kabhi finite (jaise coin toss), kabhi infinite ya continuous (jaise time, temperature, etc.). Probability Theory mein hum inhi sample spaces ke outcomes ke upar kaam karte hain, aur yeh study karta hai ki kisi particular event ke hone ki sambhavana (chance) kitni hai.

Abhi hum probability theory ke concepts ko systematically samajhne ja rahe hain. Jaise humne pehle random experiments aur unke sample space ke examples dekhe the, waise hi ab hum events ke baare me baat karenge. Random experiment wo hota hai jisme result uncertain hota hai, par possible outcomes hume pata hote hain. Inhi outcomes ka jo collection hota hai, usse hum sample space ya Omega (Ω) kehte hain. Ab kabhi-kabhi hume kisi specific outcome ya kuch outcomes me interest hota hai, aur un outcomes ka jo collection hota hai, usko hum event kehte hain. Matlab mathematically event ek subset hota hai sample space ka yaani event $A \subseteq \Omega$. Jaise agar hum ek dice roll karte hain to Ω = {1,2,3,4,5,6}. Ab agar hum event E define karein "outcome is even", to E = {2,4,6}. Isi tarah agar event F ho "outcome is prime", to F = {2,3,5}. Event A agar ho "outcome divisible by 4", to A = {4}. Aur event B agar ho "outcome divisible by 3", to B = {3,6}. Har event basically ek set hai jo Ω ke kuch outcomes ko represent karta hai.

Isi tarah hum aur bhi examples le sakte hain aur dekh sakte hain kaise random experiments badalne se sample space aur events dono badal jaate hain. Is chapter me aage hum isi tarah ke aur examples aur definitions samjhenge jaise sample space, event ke types, aur unke relationships taaki probability theory ki foundation strong ho jaye.

Ab hum events ke aur concepts samajh rahe hain, jaise ki event ko represent karne ke alag-alag tareeke hum aage jaake use karenge. Maan lijiye C ek event hai jisme result 10 se chhota hai. Agar hum ek dice roll kar rahe hain jisme outcomes 1, 2, 3, 4, 5, 6 hain, to in sabhi numbers me se saare hi 10 se chhote hain. Matlab event C me saare outcomes aa jaate hain, yani C = Ω. Iska matlab hai event C aur sample space Ω same hain. Isi tarah agar hum D event define karein "outcome greater than 10", to Ω ke kisi bhi outcome me aisa result nahi hai jo 10 se bada ho, to event D ek empty set ban jaata hai yani D = ∅. Yeh bhi ek valid event hai, jisme koi outcome nahi hai. Ab hum events ke types dekhte hain simple event aur compound event. Simple event wo hota hai jisme sirf ek hi outcome hota hai, jaise A = {4}. Compound event wo hota hai jisme ek se zyada outcomes hote hain, jaise E = {2,4,6} ya F = {2,3,5}. Ab agar hum saare examples A, B, C, D, E, F dekhein, to unme se A ek simple event hai, aur baaki sab compound events hain. Ab maan lijiye humne ek dice roll kiya aur result 3 aaya. To hum dekhte hain 3 kis event me belong karta hai 3 F me hai (prime numbers), B me hai (divisible by 3), aur C me hai (less than 10). To hum kahenge ki events F, B, aur C hain. Lekin A (divisible by 4), E (even), aur D (greater than 10) nahi ghati hain. To hum kehte hain ki koi event tab ghatti hai jab experiment ka result us event ke andar aata hai. Yehi concept probability theory me fundamental hai — jab bhi koi result kisi event ke andar hota hai, tab hum kehte hain ki wo event occur hua hai.