Matrix Computation and its applications

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Lecture - 08
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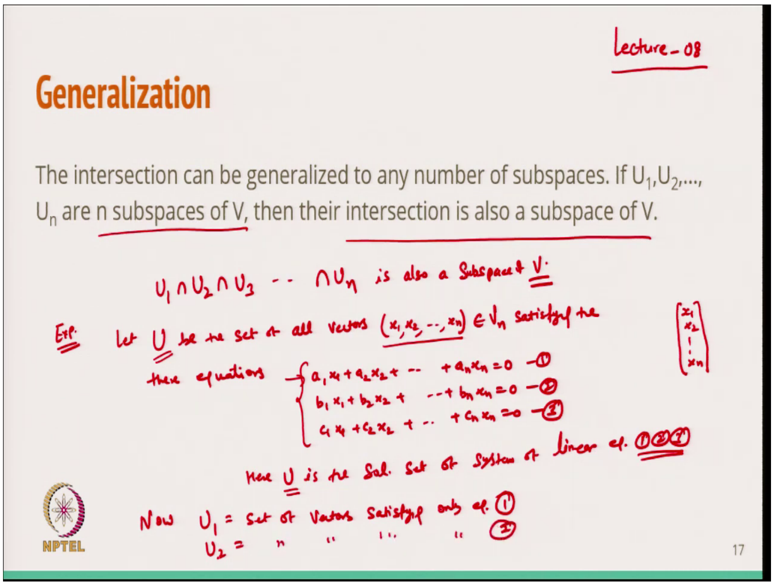

Hello viewers. So, welcome back to the course on Matrix Computation and its Application. So, today we are going to discuss the lecture 8, and we will going to start with the Generalization of the Intersection of the Vectors, Intersection of the Subspaces.

So, in the previous lecture we have discussed that we have a 2 subspaces U and W, and then we have showed that these subspaces, if I take the intersection then this is also the subspace of V. That we have discussed.

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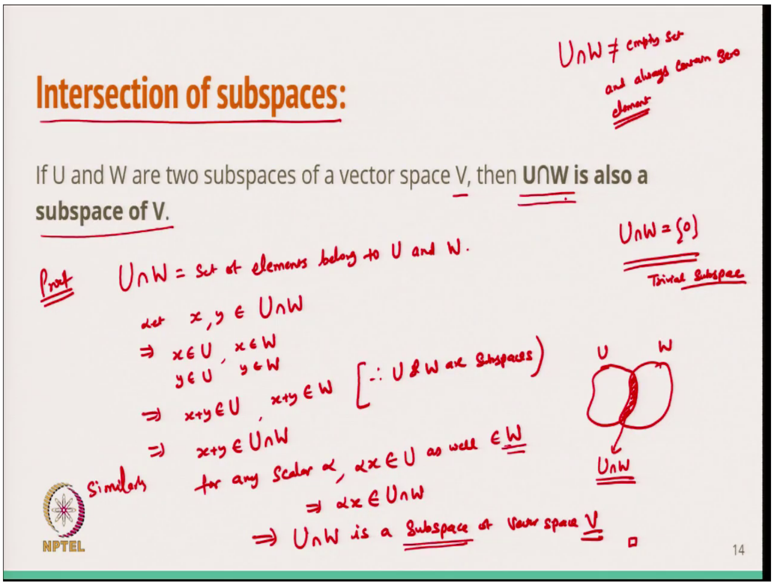

So, today we are going to discuss the generalization, that what is going to happen if I have a n number of subspaces. So, in this case also we have the U 1, U 2, up to U n, the n subspaces of V, then their intersection is also a subspace of V. So, from here I can say that U 1 intersection U 2 intersection U 3, U n.

So, that is the I am taking the intersection of all the n number of subspaces. So, it shows that this is also a subspace of V. And proof is similar that you just take the one element, two elements and then show their vector addition and scalar multiplication belongs to this one. The same way we have done and then we can show that there is a subspace of V.

So, now I want to just want to give you example that how it look like. Suppose, I take, let, I take a subspace U be the set of all vectors. So, I am taking the set of all vectors showing as x 1, x 2, x n that belongs to V n. So, this is the vector I am showing. So, sometime we also show the vector like this one x 2, x 1, x 2, x n as a column vector. So, this is just a column vector, and here I am showing as a row vector, but it is a vector.

So, let U be the set of all vectors that belongs to the satisfying the, suppose I satisfying the 3 equations. So, let I take the equation as a 1 x 1, a 2 x 2, a n x n that is equal to 0. So, this equation I just take the equation number 1. Then I take b 1 x 1, b 2 x 2, b n x n that is equal to

0, second equation. And suppose I taken one more equation c 1 x 1 plus c 2 x 2, c n x n that equal to 0, equation number 3.

So, basically I am taking all the sets of vectors which is a solution of this linear equation. It is just the linear equation. And I will solve this linear equation and I will get the solution. So, I am taking the U as a solution set of this equations the system of equation, ok. So, this is I can call that here U is, so U is the solution set of system of linear equation 1, 2, and 3.

So, it is a system of linear equation. And we know that it is n number of vectors are there, n number of variable and 3 equations, so infinite many solution are going to happen. So, in this case, we are taking that and this is homogenous part, so it is always going to have a solution. So, that set I take as a U.

Now, what I am doing now is I will take U 1, so U 1 I am take that is the space is a set of vectors satisfying only equation 1. So, I will choose only that elements which satisfying only first equation, that is it. Similarly, I choose U 2 the set of vector satisfying equation number 2. And from U 3 set of solution satisfying equation 3.

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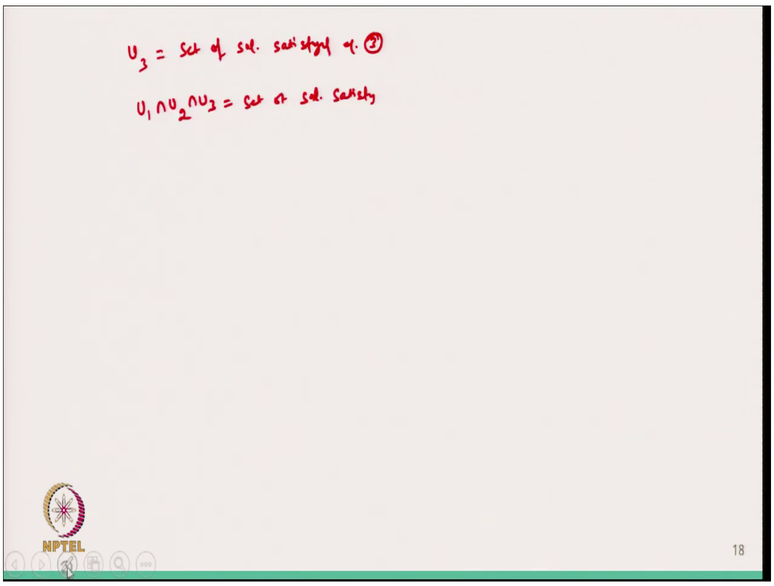

Now, I can find what is U 1 intersection U 2 intersection U 3. So, in this case, you can see that; so, in this case I am getting, I am defining the set of solution or set of vectors, not the solution, I can just write the vectors here. This is a vector, set of vectors satisfying the equation 3, set of vectors satisfying equation 1, 2 and 3 together, ok.

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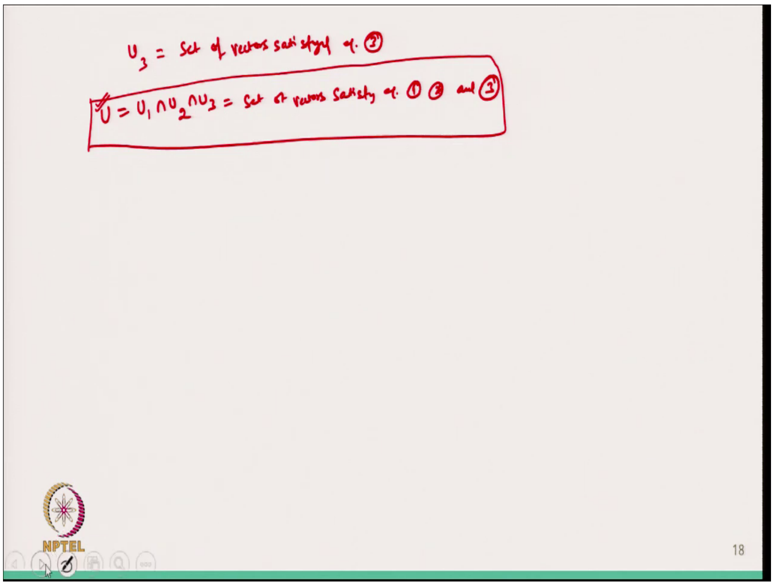

So, and this is you can check that this is equal to U. So, this way we can define the intersection of the sets. So, and I know that U 1 is a subspace, U 2 is also subspace, U 3 also subspace, I am just taking the intersection that satisfy all this condition and that is equal to my U. So, this way we can define that how the intersection is going to play role here.

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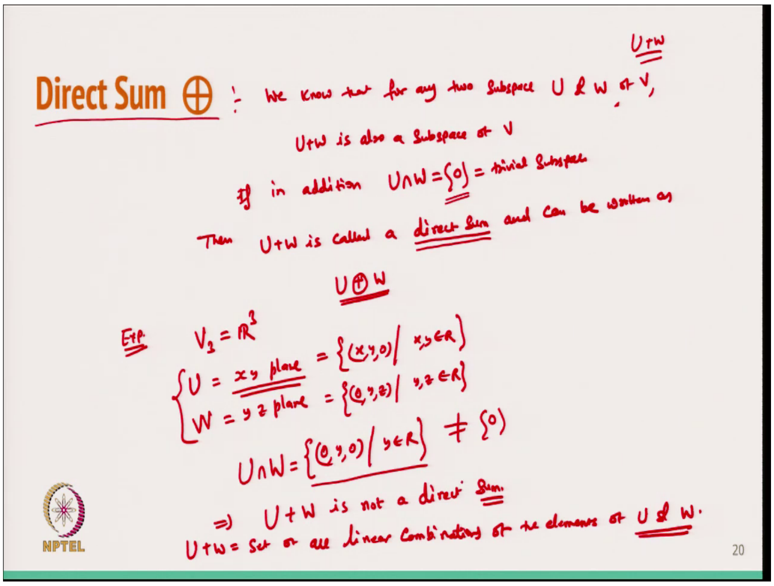

So, after this one we are going to define the another thing that is we call it the direct sum because just now we have seen the U plus W and the sum of two vector spaces, then just I want to go little bit further and I want to show what is the direct sum. So, in this case that if. So, what is that? We know that for any two subspaces suppose let U and W, U plus W is also a subspace of V, that we already seen.

Now, if in addition I find U intersection W because we also know that the U intersection W is again the subspace. So, in addition, if we found U intersection W and that contains only the 0 element that is the trivial subspace, then U plus W is called; so, in that case we can call it a direct sum.

And can be written as U W. So, it is just the summation and putting the circle and this is the notation of the direct sum, ok. So, whenever we write the direct sum, it means we are able to write the summation and the intersection between these is only the 0 element, not any other element. So, let us take one example.

So, let I take V 3 ok, that is equal to R 3. So, from here I just choose U, U I take as x y plane, it means set of elements x, y, 0. So, that x and y are the real number. So, I am choosing U as this one. Then, W I am choosing, that I choose it may be y z plane or maybe I just remove this x y plane and the y z plane. So, y z plane is a set of all the elements 0, y, z belongs to R.

Now, in this case you can see that if I discuss about U intersection W, then I have the x y plane that is just x y plane, and then we have a y z plane. So, y z plane and whenever y z plane and x y plane cuts together then it is going to happen. So, in this case we have only y axis because we get a line and that line is the y axis only.

So, x y plane and y z plane wherever it is intersecting, it is intersecting at the y axis. So, we will get only y. So, it is a space and it is not equal to only the 0 space. So, it in this case from here I can say that if I define U plus W, then is going to be is not a direct sum, ok, because the U plus W will contain set of all, set of all linear, set of all linear combinations of the elements of U and W, ok. So, and this is not going to be the direct sum.

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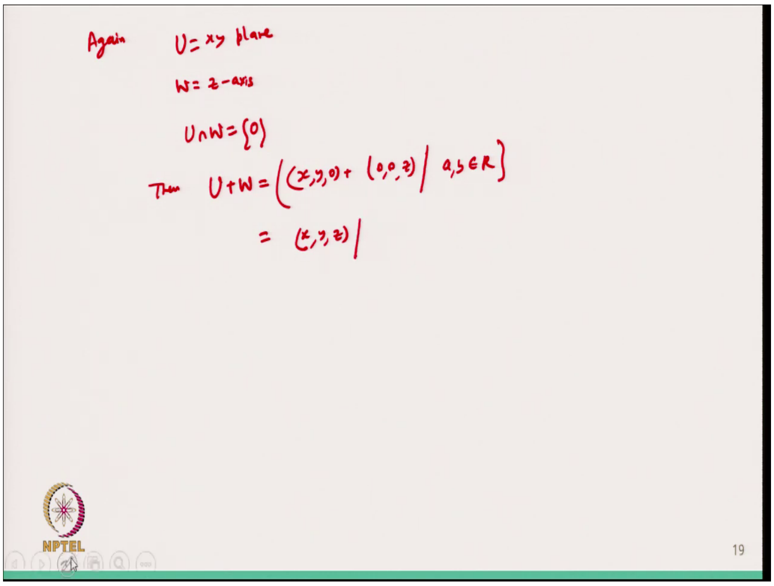

Now, what is going to happen? If I take; so, again if I take U as x y plane and W as z axis. So, in this case, if I take U intersection W, then you know that x y plane and z axis they cut only at the element 0, 0, not any other element.

So, in this case I can say that from here then U plus W is just it will combination x, y, 0, a plus b, 0, 0, z, where a and b are the scalars. And this will be equal to I can write as ax or maybe I can take any element from here taking the linear combination. So, I just remove this a plus this, so I can write this as a x, y, z.

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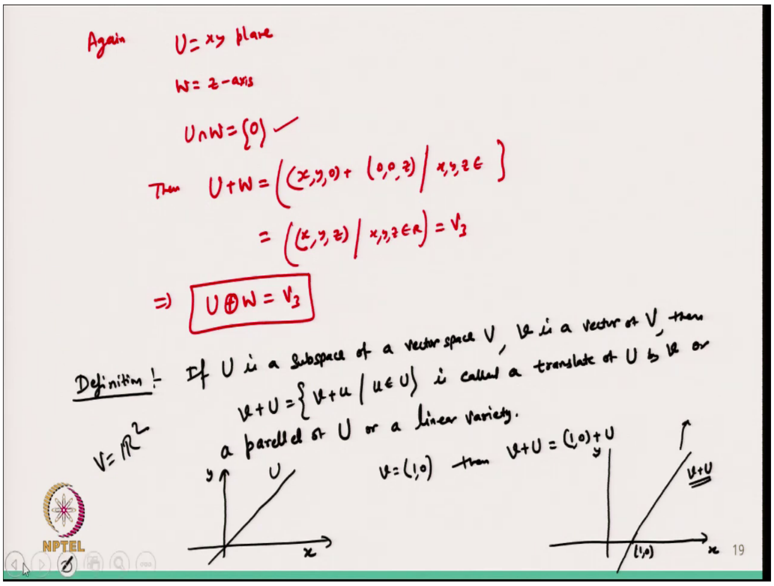

So, if I take x, y, z, where x, y, z belongs to R, belongs to R, ok. So, this is I am getting this one x, y, z belongs to R and this is equal to V 3 because it is containing all the elements of V 3. So, from here I can say that U plus W is the direct sum and this is equal to V 3.

So, we have splitted the V 3 into two parts, in the two subspaces, one is x y plane U and another is W. So, this is called the direct sum. We are dividing the whole vector space into the two subspaces and these two subspaces has nothing common except the 0 element. So, this is called the direct sum.

Now, after this we just want to define one more term, one more definition. Just let if U is a subspace of a for vector space V, I just take a U is a subspace of the vector space V and let small v I am taking is a vector of V, then we are defining this v plus u. And it is set of all the elements v plus u, where u belongs to set of all the elements I am taking. So, I take all the elements ok, and this is called; so, then this is called a translate of the subspace U and the subspace U by v or it can also be written as a parallel of U or it also called a linear variety.

So, basically what we are applying here? Suppose, we have some subspace U. So, let us I am taking a subspace, I know that the subspace always passing through the 0 element, suppose this is the my subspace, ok, and suppose I am taking this V I am just defining R 2, and U I am taking a line passing through the origin. So, this is my x and this is my y.

Now, I take a element, suppose I take a element as 1, 0, then v plus U basically we are getting 1, 0 plus U. So, if I am going to represent that one. So, this is x, and this is y and suppose this is the point 1, 0, so this line will shift here and it will pass from here. So, this is the element 1, 0.

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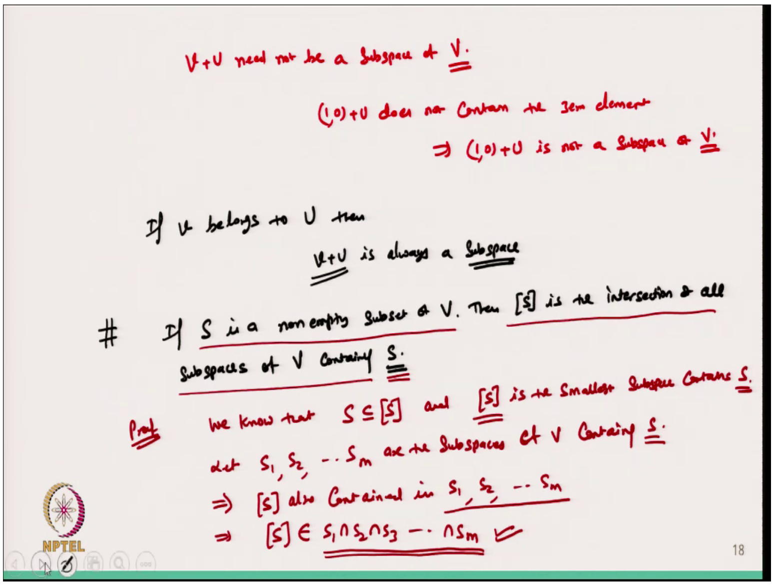

Now, from here you can see that this is my translation of U by this point, this one. And so, this is definitely its parallel to this line U. But from here you can see that, now from here I can say that this is, from here I can say that v plus U need not be a subspace of V because we can see that in this case 1, 0 plus U does not contain the 0 element and for the subspace which definitely should contain the 0 element.

So, does not contain the 0 element. So, which implies that 1, 0 plus U is not a subspace of V. Then, how we can say that it is a subspace of V? So, from here I can write that if v if the v belongs to, if V belongs to U, then v plus U is always a subspace, ok. So, in that case we can say that this is always subspace, otherwise v plus U that is the shifting is not the subspace of V. So, this is also one of the important property we wanted to discuss.

Now, so after this one, now we just want to discuss one more term is that a very important property that if S is a non- empty subset of V, then span of S is the intersection of all subspaces of V containing S. So, this one is just the extension of that because in this case what I was saying that if S is a non- empty subset of V, then this is a belongs to the intersection of all the subspaces of V, because just now we have discussed the intersection property.

So, now it is a I can prove it very because we know that we know that S always belongs to span S, that we already know. And we also know that this span S is the smallest is the smallest subspace contain S, ok. So, whenever the S is there, this span of S is also there in that subspace, ok. So, this is yeah.

So, now, let I have subspaces suppose S 1, S 2, S m, these are the subspaces are the subspaces of V containing S, which implies that, so if I am taking this one S 1, S 2, S m are the m subspaces of V that contains S which implies that span of S also is also contained or belongs to S 1, S 2, S m.

So, that is also containing S 1, S 2, of S m. And from here, I can say that the span S will also belongs to S 1 intersection S 2 intersection S 3 intersection S m. So, this will be contained in all the intersection of this one. So, that is just we wanted to show.

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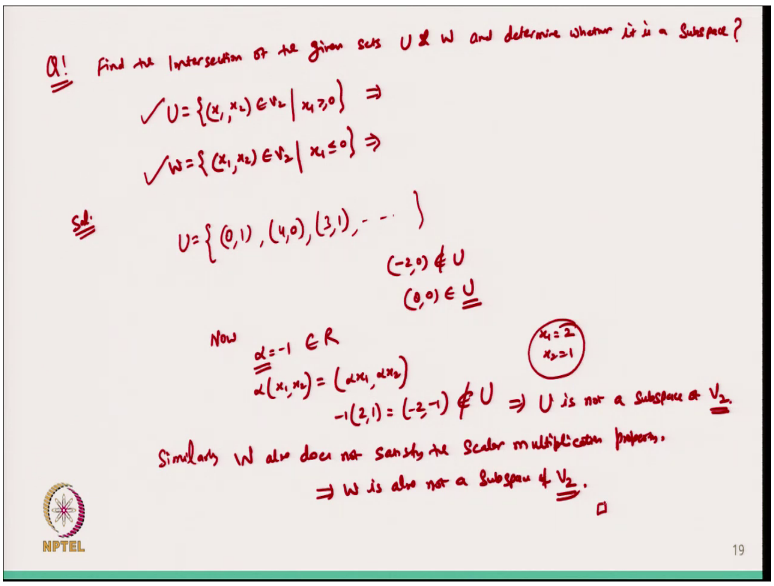

Now, let us do one example. Find the intersection of the given set U and W. So, U is given to me x 1 and x 2 that belongs to v 2. So, this is I am taking one of the subspace of v 2. So, I need to check whether it is subspace or not. So, find the intersection of the given set U and W and determine whether it is a subspace or not. So, I am taking U here and W is I am just choosing x 1 and x 2 that also belongs to v 2 such that x 1 is less than equal to 0. Now, let us do this one.

Now, in this case my U is a set of vectors x 1 and x 2 belong to this one, where x 1 is always greater than equal to 0. So, that type of U I have taken, and this is we have taken where x 1 is always less than or equal to 0. Now, let us check. So, it means U will contain all the elements. So, U will contain this these type of elements like 0, 1 and maybe 4, 0; 3, 1 like this one. But if I have the element minus 2, 0, so that does not belongs to U. This we have to keep in mind, ok. Also, 0, 0 belongs to U, that is there.

So, first we have to check that whether U is a subspace or not. So, in this case, I just take minus 5, if I take any element alpha and that is suppose I take minus 1 and I multiply by any element from U, so alpha into one element I choose maybe x 1 and x 2, ok. Then, it will be equal to alpha x 1 alpha x 2 and suppose my x 1 is 2 and x 2 is 1, so that belongs to U in this case.

Now, if I take this element then definitely minus 1 into 2, 1, it will be minus 2, minus 1 and it does not belongs to U, because I am choosing my x 1 is always greater than equal to 0. So, from here I can say that U is not a subspace of V 2.

Similarly, because this alpha is coming from the real line. So, it can be anything. Similarly, W also, so in this case also it is less than equal to 0 I am taking here. So, I if I multiply by negative sign it will be positive in that case. Also, does not satisfy the scalar multiplication property which implies that W is also not a subspace of V 2. So, in this case neither U is a subspace nor W the subspace. So, we cannot discuss their intersection even. So, this is the example we had done. So, let me stop here.

So, today we have discussed some properties like a direct sum, and then we have discussed about that translation of the vector of a subspace by a vector and then we have discussed few example. So, before, from this we will continue from this in the next lecture.

Thanks for watching. Thanks very much.