Matrix Computation and its applications

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Lecture - 58
Matlab/Octave code for Householder transformation

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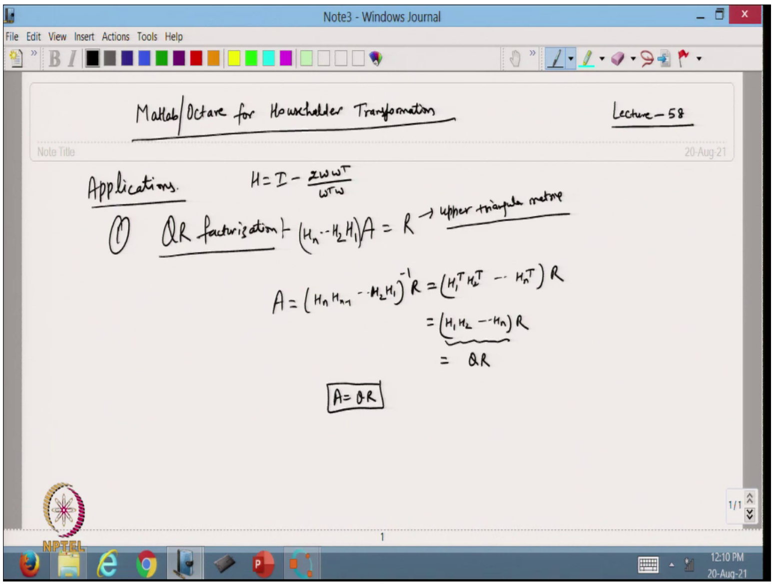

Hello viewers, welcome back to the course Matrix Computation and its application. So, this is the last lecture of our course. In this lecture we are going to solve some examples using the Householder transformation, to find out the QR factorization and the tri-diagonalization of a symmetric matrix with the help of Octave software.

Now we are going to use the application and the first one is the QR factorization. As we have already done the QR factorization with the Gram Schmidt process. So, in this case also we have a matrix A and multiplying by some Householder transformation and on the right hand side I will get R. So, this R is an upper triangular matrix. So, in this case what we get $H_{1,}\, H_2$ orthogonal matrix.

$$H = I - \frac{2ww^t}{w^t w}$$

So, I can find from here then, this can be written as $A = (H_n . H_{n-1} ... H_2 . H_1)^{-1} R$. So, inverse means this is equal to transpose i.e $A = \left(H_n^t . \ H_{n-1}^t .... H_2^t . \ H_1^t\right) R$ and we know that H_n is a symmetric matrix. So, it becomes H_1, H_2, H_n and these are orthogonal matrices so the product is also orthogonal. So, I will get $A = QR$, so this is like QR factorization of the given matrix.

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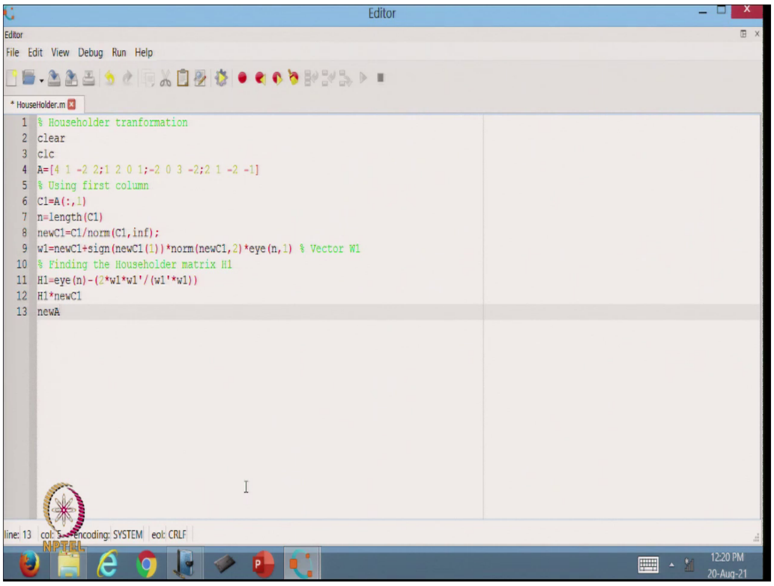

So, let us try this one in the octave and now suppose I take the matrix A. So, let us take the example of this one.

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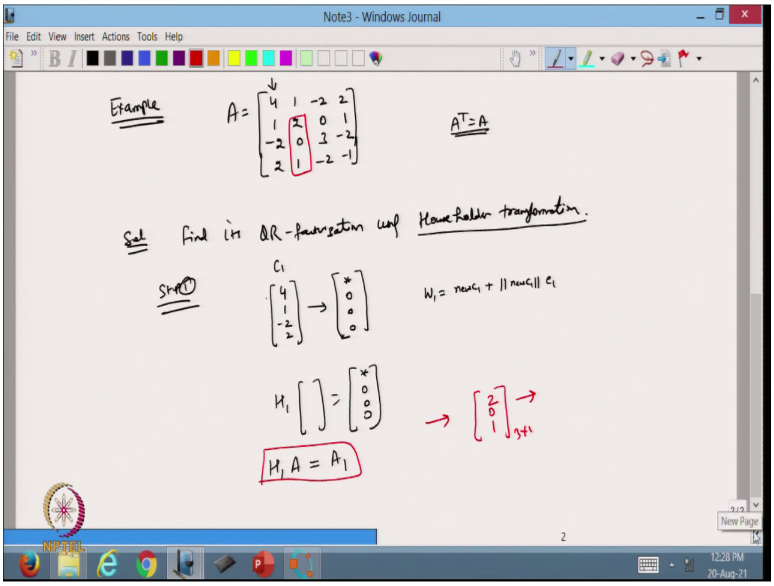

So, I will just take same example and we will define the matrix A as:

$$A = [4\ 1\ \ -2\ \ 2\ 1\ \ 2\ \ 0\ 1\ \ -2\ 0\ 3\ -2\ 2\ 1\ -2\ \ -1]$$

Here we have $A^t = A$. So, I have taken a symmetric matrix and we are going to use this. We find its QR factorization using Householder transformation. So, this is what we are going to do. So, let us start with the octave. I have saved a one script file with the word octave. We start with **clear** and **clc** commands to clear the command window and variable form workspace in Octave/Matlab. I should write here about householder transformation.

So, after defining this matrix now I want to start my Householder transformation. So, I can say that using first column, so I am using the first column C_1 as

$$C_1 = (4\ \ 1\ \ -2\ \ 2)$$

so this one I am going to use. And from here I will make this vector transform value here $(0\ 0\ 0\ 0)^t$. So, this is what I am going to do with the help of this one.

So, now I am going to use the first column, so let us write this one, so let us take column write it C_1. So, in this case I am writing C_1 first columns all the rows and the first columns, so this is my C_1. Now I define my n as the the length of C_1. Now I know that I need to define the norm also. So, in this case what I am going to do I am going to defined $newC_1$ because I am dividing the C_1 by infinite norm of C_1

$$newC_1 = \frac{C_1}{norm(C_1, infinte)}$$

So, norm I will take of what C_1 and I want to take the maximum norm or the infinity norm, so I will write inf. So, this norm I am defined this 1. So, this is my $newC_1$ I will get. So, once I will get the $newC_1$, now I want to define w_1.

$$w_1 = newC_1 + \|newC_1\| C_1$$

So, that is what we are going to see the sign of the first component star multiply this one, what I need to do? I need to find a norm for the length of this. So, I am defining the norm of $newC_1$, so this is my new C_1 $newC_1$. So, that is my new C_1 and I am taking the 2 norms of this one. And then I will multiply by the vector identity vector. So, I will write eye, so this is the command, but I want the vector of length.

So, n should be the number of rows and only one column should be there. So, this is basically we are getting the column vector. So, this is what we are going to find w_1 and now once I find the w_1, I will get the matrix H. So, you can write from here in this case. So, this is you can write here as vector w and now this one I can write finding the Householder matrix H_1.

Why we writing this H_1? Because first I am dealing with this one, so I will find the matrix. So, from using this one I found my w_1 and then I will find my H_1 and then multiply the H_1 with this vector and I will get some value here $(0, 0, 0)^t$ and then I will find out H_1 and my A the matrix and I will get the new matrix that is A_1. So, this is what we are going to do.

So, now I am finding my H_1, so H_1 is equal to, so I need to find out because you will see that H_1 is going to be a 4 by 4 matrix. So, I need to define my identity matrix. So, identity matrix is eye(n) of order n because it is n is the number of length. So, this is what I am going to take now. So, this is the value we are going to take sorry maybe I will define this one divided by w_1 transpose multiply by w_1. So, this is what we have defined, so that is my H_1.

$$H_1 = eye(n) - \frac{2*w*w^t}{w^t*w}$$

Now, I want to see that what will happen if I take H_1 and I will multiply this one with a $newC_1$ as $H_1 * newC_1$ So, maybe I can define it just my $newA = H_1 * A$.

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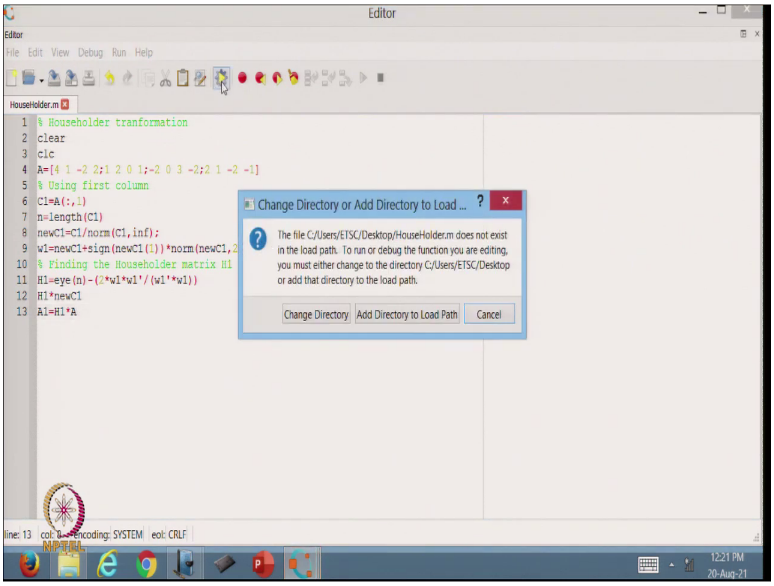

So, $newA$ can be write this as my A_1. So, A_1 I will get by multiplying H_1 with my matrix A.

So, let us see whether it is going to work till here or not. So, first I will save this value and then I will run this one. I will change the directory and let us see if any mistake is there, so ok.

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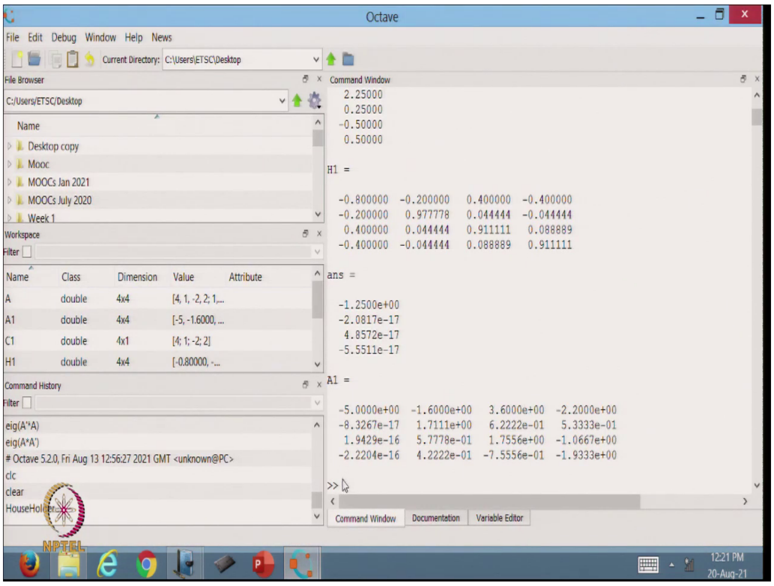

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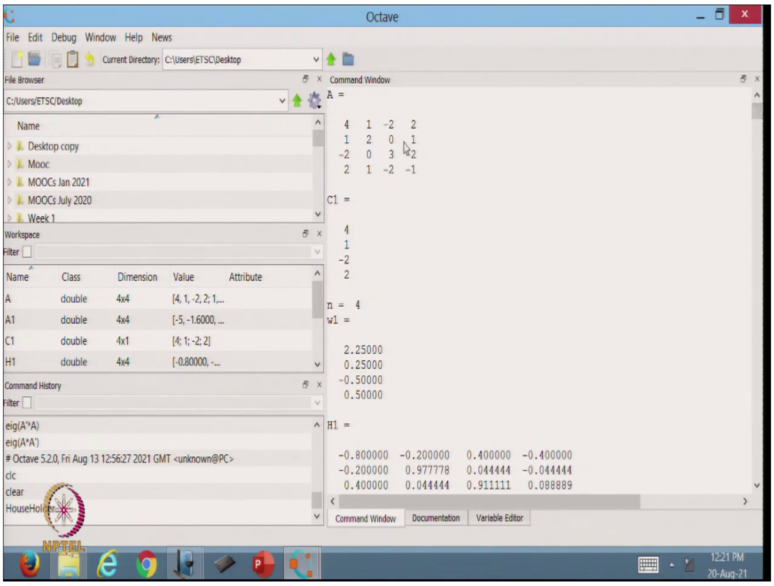

So, no mistake is there first I have taken this matrix. So, this matrix is

$$A = [4\ 1\ \ -2\ \ 2\ 1\ \ 2\ \ 0\ 1\ \ -2\ 0\ 3\ -2\ \ 2\ 1\ -2\ -1]$$

So, 1 divided by 4 is 0.25 . And after this I get my w_1. So, w_1 we have taken that new C_1 plus. So, sign of this is positive, so we have taken the positive sign the magnitude of this one we have taken and then e 1. So, we got this w_1 and after doing this w_1, so w_1 if you see this is my I call it C_1. So, and then I will convert this into the new C_1, so it is new C_1 plus sign of this is positive. So, I am taking the new C_1 norm and then e_1, so this is what we have done.

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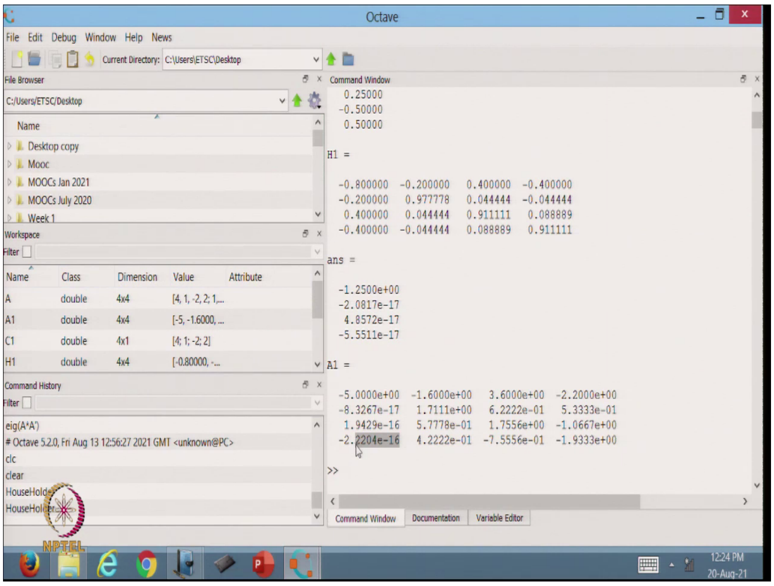

And after that my H_1 become this one. So, once I take this H_1 and applying on the vector, you see that this value becomes -1.25 and this value 0 0 and 0. So, both are all the 3 will become 0 now and I applied this on the A_1 A matrix. So, A_1 is a new matrix with the first column a new vector and then here it is 0 0 0. So, this is what we got. So, my value of this A_1 is this value ok. So, now let us see, so up to here we are able to find out the Householder matrix and its application transformation on the new C_1, so this is what we are able to finish.

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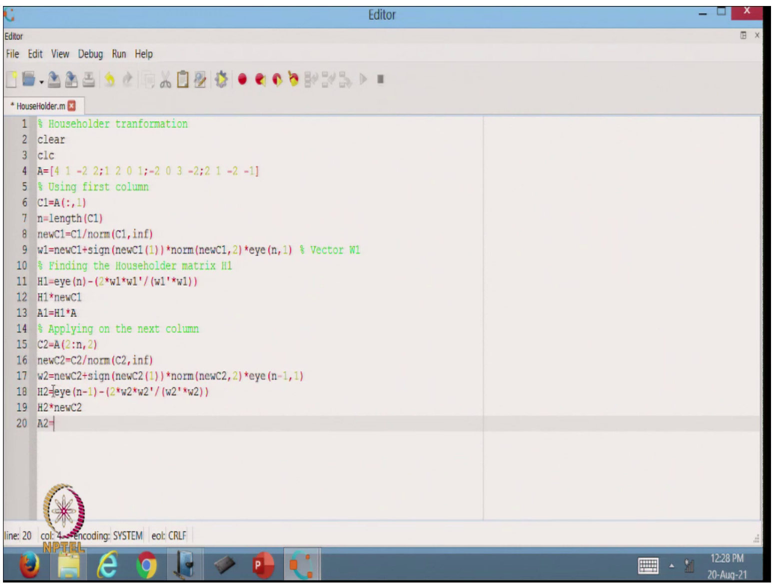
Now, let us do that one, now applying on the next column.

$$C_2 = A(2:n, 2)$$



So, it is my C_2 and now I am going to calculate my $newC_2$ as follows

$$newC_2 = \frac{C_2}{norm(C_2, infinte)}$$

Next one, so I need to find now w_2,

$$w_2 = newC_2 + sign(newC_2(1) * norm(newC_2, 2) * eye(n - 1, 1))$$

And now after this one we will find the Householder matrix

$$H_2 = eye(n - 1) - \frac{2 * w_2 * w_2^t}{w_2^t * w_2}$$

Now we calculate A_2 matrix as:

$$A_2 = newA_1 = H_2 * A_1$$

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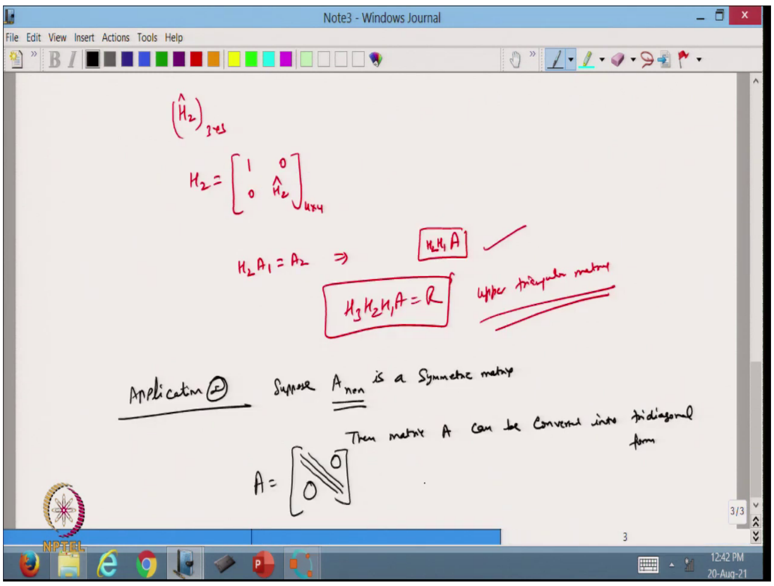

So we have $(\hat{H}_2)_{3*3}$ and we define as

$$H_2 = \begin{pmatrix} 1 & 0 \\ 0 & \hat{H}_2 \end{pmatrix}_{4*4}$$

and

$$H_2A_1 = A_2$$

Implies:

$$H_3H_2H_1A = R$$

Which is an upper triangular matrix.

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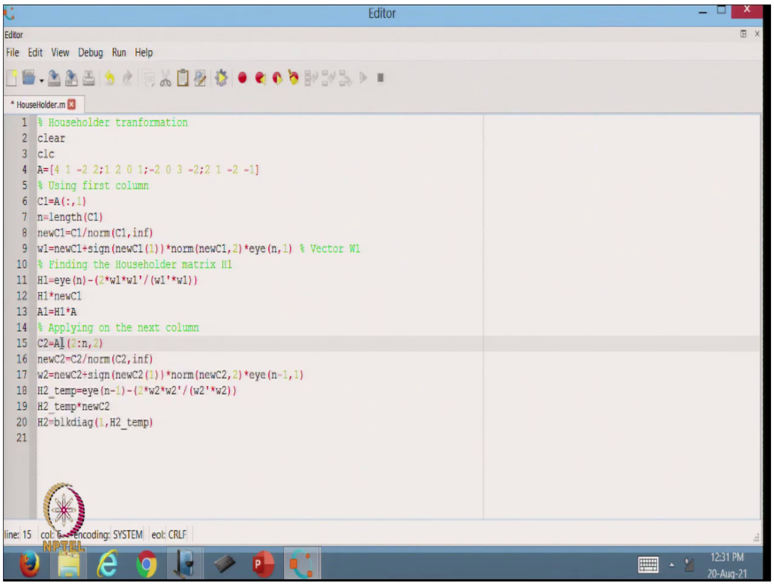

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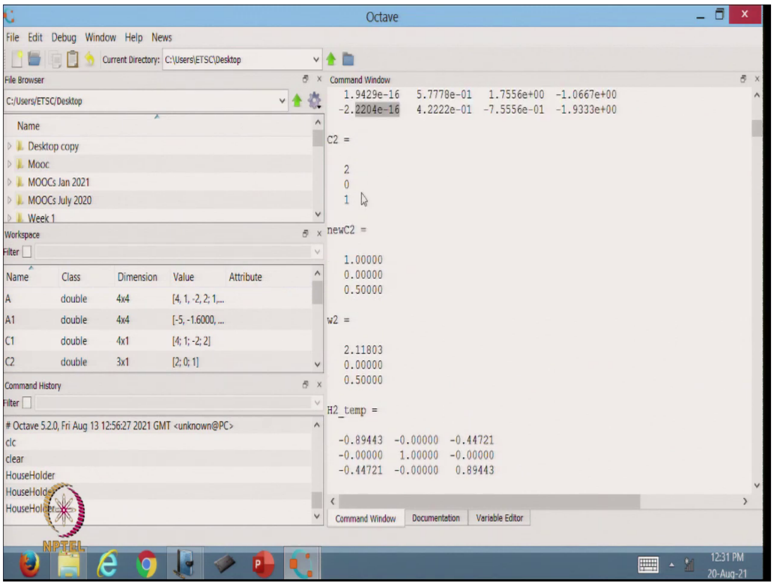

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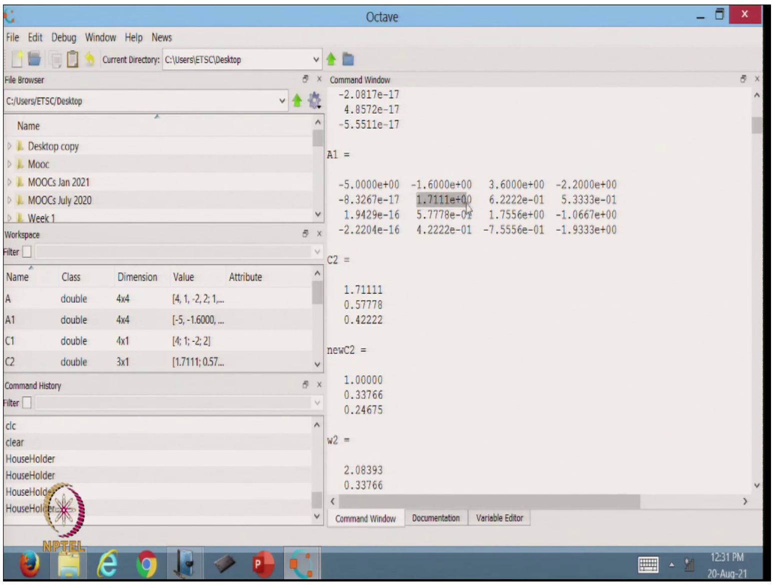

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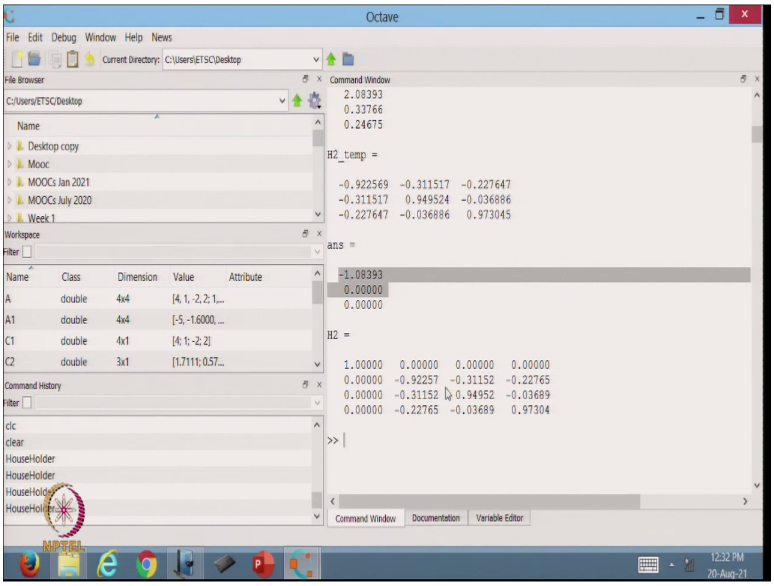

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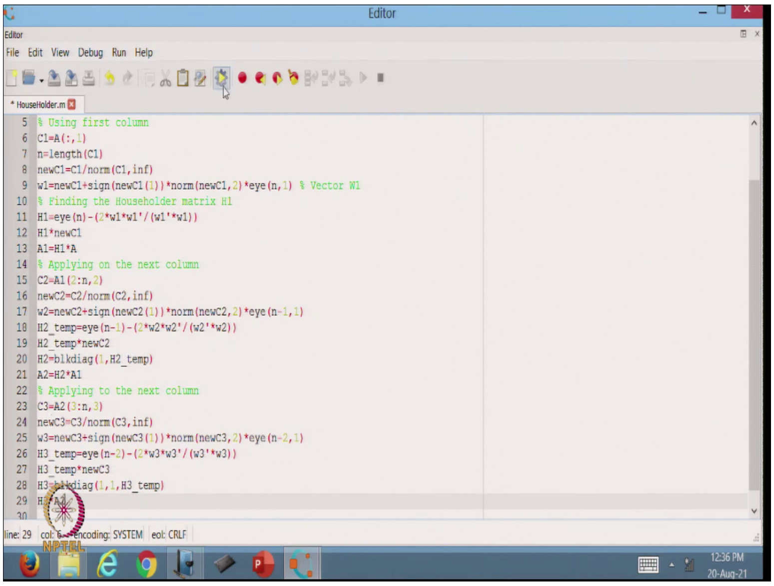

Again using the same technique the value of C_3, H_3 *and* A_3 is calculated.

Now, let us do that one, now applying on the next column.

$$C_3 = A(3:n,3)$$

So, it is my C_3 and now I am going to calculate my $newC_3$ as follows

$$newC_3 = \frac{C_3}{norm(C_3, infinte)}$$

Next one, so I need to find now w_3,

$$w_3 = newC_3 + sign(newC_3(1) * norm\left(newC_3, 2\right) * eye(n - 2, 1))$$

And now after this one we will find the Householder matrix

$$H_3 = eye(n - 3) - \frac{2 * w_3 * w_3^t}{w_3^t * w_3}$$

Now we calculate A_3 matrix as:

$$A_3 = newA_3 = H_3 * A_2$$

Now, we have obtained w_3, C_3, H_3 and A_3 .

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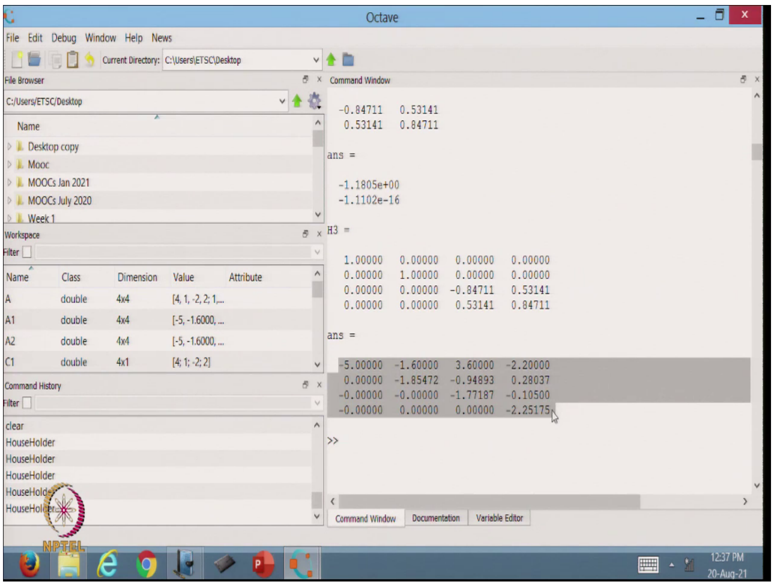

So I put this matrix H_3, here and this is just the block diagonal matrix we got. And now we applying this on the A_3, so you can see that this became my new matrix and that is the upper triangular matrix. So, after doing this one I got my upper triangular matrix, so this is my upper triangular matrix.

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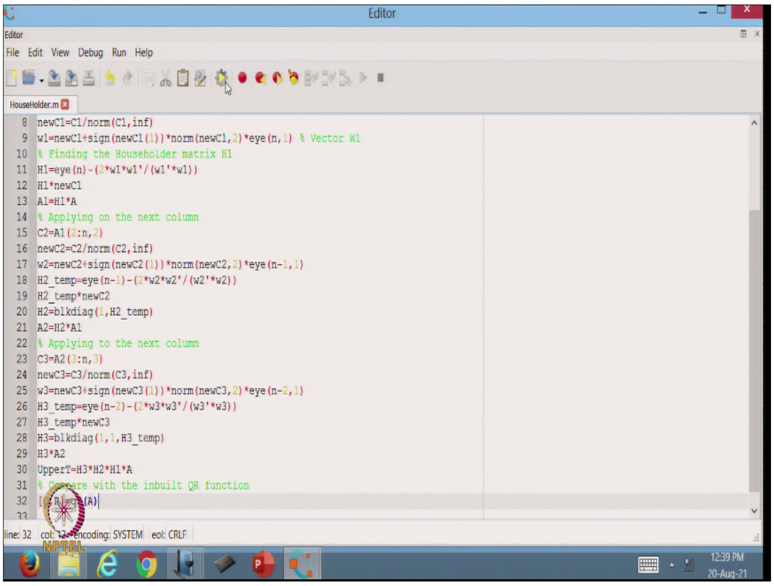

And from here I can find the final upper triangular matrix. So, I can write R a upper triangular matrix as

$$UpperT = H_3 * H_2 * H_1 * A$$

We are going to compare our obtained matrix with the inbuilt QR function. So, that is we write it QR, so this is what we get from here and I will write QR of matrix A and then we can check, so this is let us see.

$$[Q, R] = qr(A)$$

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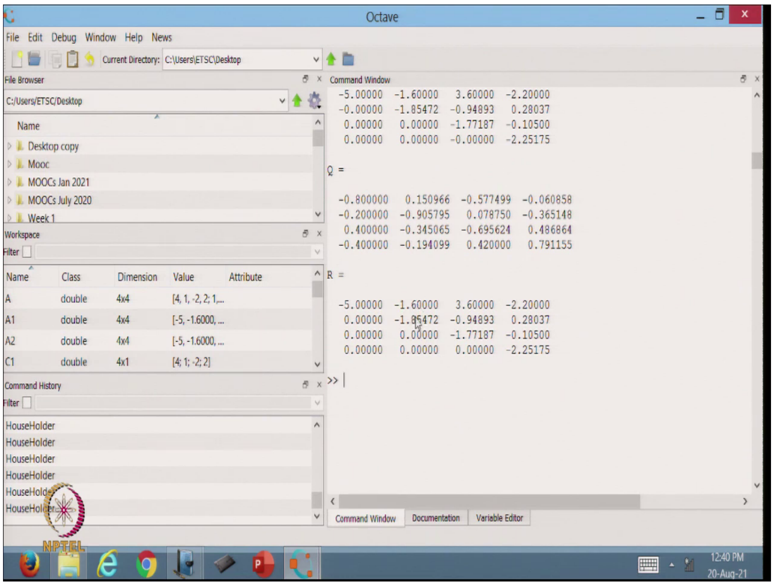

So, we have obtained R by one in the steps and this is the value we got now we have applied the QR factorization inbuilt function. So, it means that we are able to find our transformation.

So, whatever the transformation we are going to us the upper triangular matrix is going to be the same in all this case. So, we can say that we are able to convert $H_3H_2H_1A$ into R, so it is an upper triangular matrix.

Now suppose matrix A is a $n * n$ symmetric matrix, then I can convert matrix A into tridiagonal form. So, the tridiagonal form means suppose I have the matrix A and that is symmetric. So, I will get the values only at the main diagonal one diagonal below this one diagonal above this and all other value are 0. So, we can transform this matrix into that tridiagonal form.

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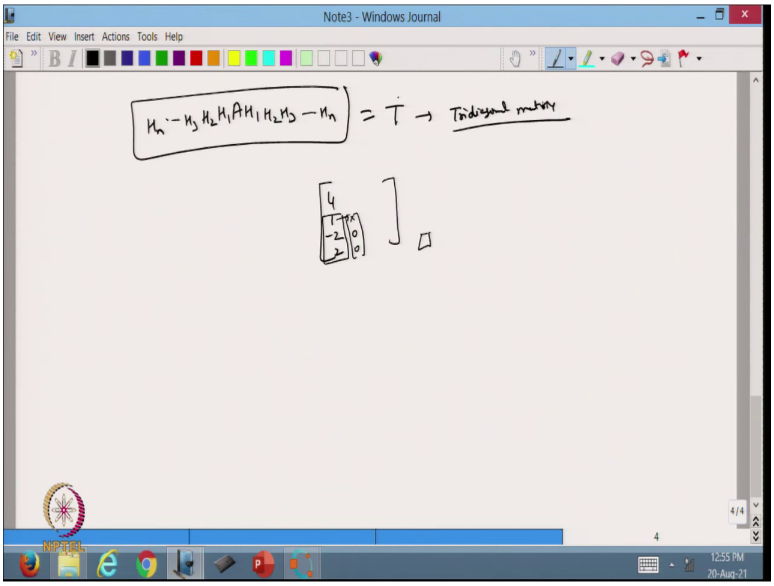

So, these things we can do here, let us suppose I have matrix A, I can apply H_1 left side and the right hand side H_2 H_3 H_4 and suppose it is going up to H_n. So, after doing this, whatever the matrix we get, that is maybe I can write it R or I should write it T. So, this is a tridiagonal matrix.

$$H_n \ldots H_3 H_2 H_1 A\, H_1 H_2 H_3 \ldots H_n = T$$

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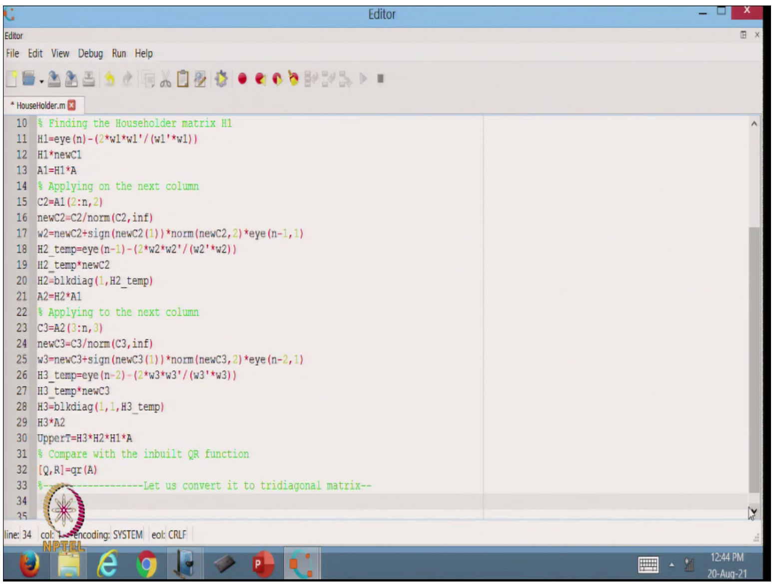

So, now I have chosen matrix A to be the symmetric matrix. So, the same way I can define in here. Let us convert it into a tridiagonal matrix form. So, we have nothing to do in this case only thing we have to do is to choose the first vector of the same matrix i.e $(4\ 1\ -\ 2\ 2)^t$. So, in this case we can apply the same matrix.

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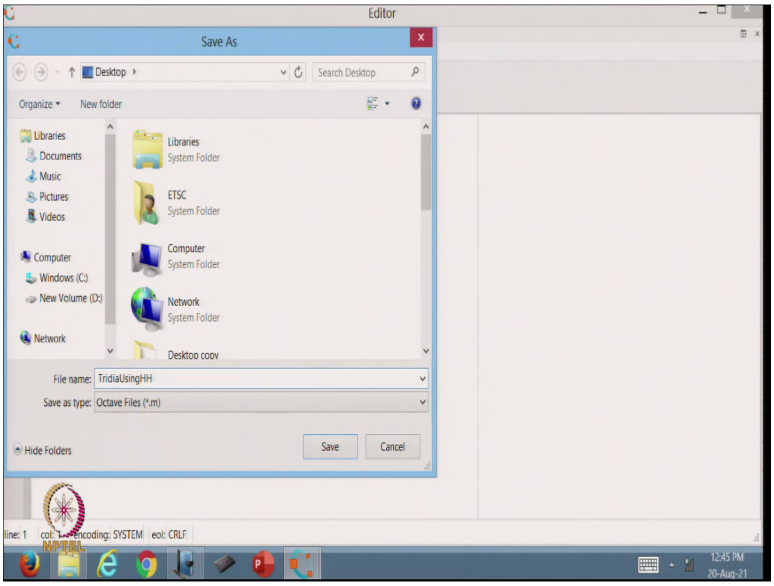

I will call this file a tridiagonal form using Householder. So, I will write it and save it on this one. So, what I am going to do, I will copy this file (control c). and we will put it here by using the shortcut key (control v) for pasting files.

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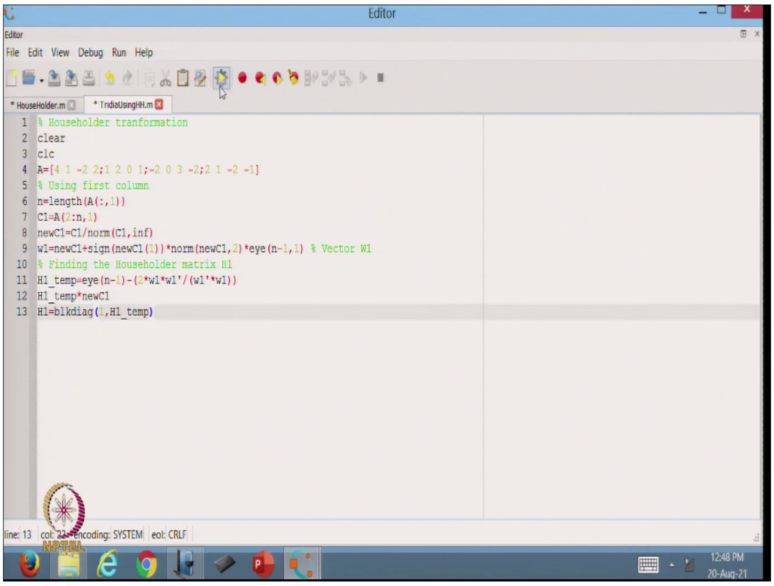

Now, first column C_1 I have choose here is from $A(2:n,1)$. So, this is what we need to do we need to find this value here. So, we are going to find the $newC_1$ and w_1 same way.

So, H_1 would be n-1. So, I will call it as temporary H_{1temp}.

applying on the next column.

$$C_1 = A(2:n,1)$$

So, it is my C_1 and now I am going to calculate my $newC_1$ as follows

$$newC_1 = \frac{C_1}{norm(C_1,infinte)}$$

Next one, so I need to find now w_1,

$$w_1 = newC_1 + sign(newC_1(1) * norm\left(newC_1, 1\right) * eye(n-1,1))$$

And now after this one we will find the Householder matrix

$$H_{1temp} = eye(n-1) - \frac{2*w_1*w_1^t}{w_1^t*w_1}$$

Now we calculate A_1 matrix as:

$$A_1 = newA_1 = H_{1temp} * A$$

$$H_1 = blkdiag(1, H_{1temp})$$

Here H_1 is a block diagonal matrix.

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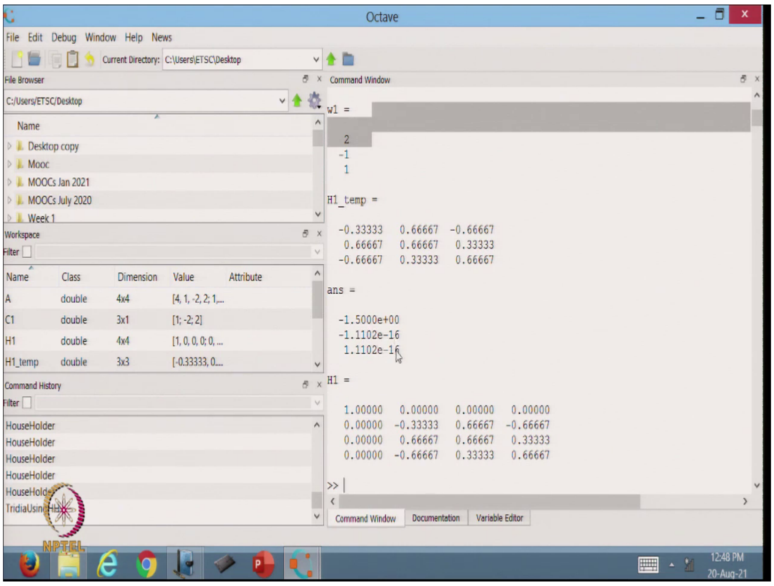

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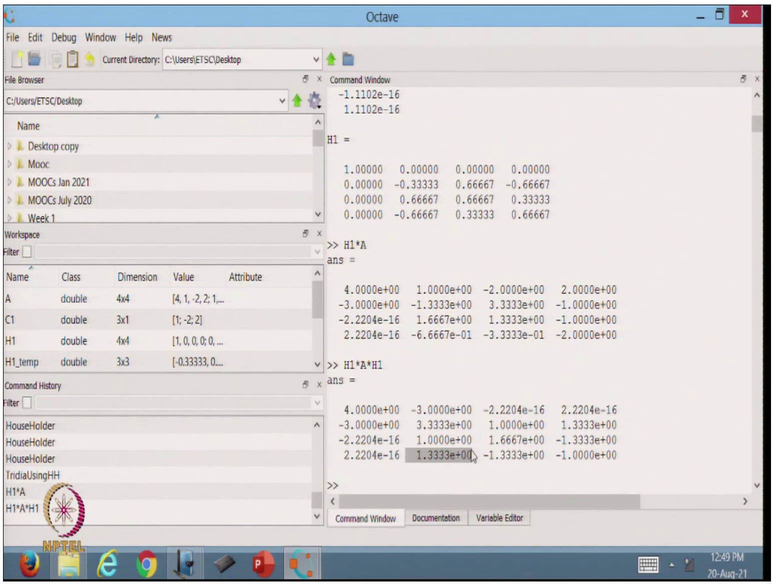

H_1 is a 4*4 matrix, and now we calculate the product of H_1 and A matrix. Then we calculate $H_1 * A * H_1$.

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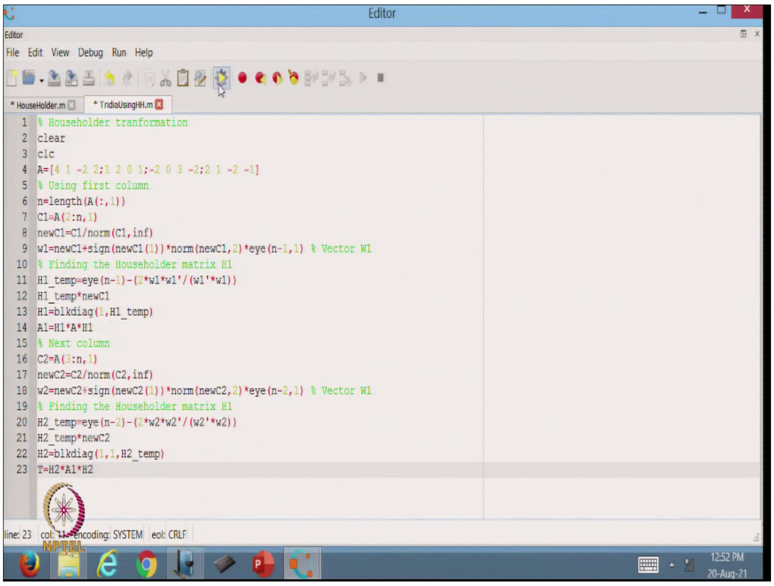

Now from here we got our A_1 matrix that is obtained as $A_1 = H_1 * A * H_1$. Similaryly we find the value of C_2, $newC_2$ and w_2.

Again with the help of C_2, $newC_2$ and w_2 we obtained the value of H_{2temp} and H_2 which is a block diagonal matrix.

And now from here I can find my tridiagonal matrix T. So, T is my tridiagonal matrix, $T = H_2 * A_1 * H_2$. So, you will see that this is going to be our tridiagonal matrix, so let us run this one.

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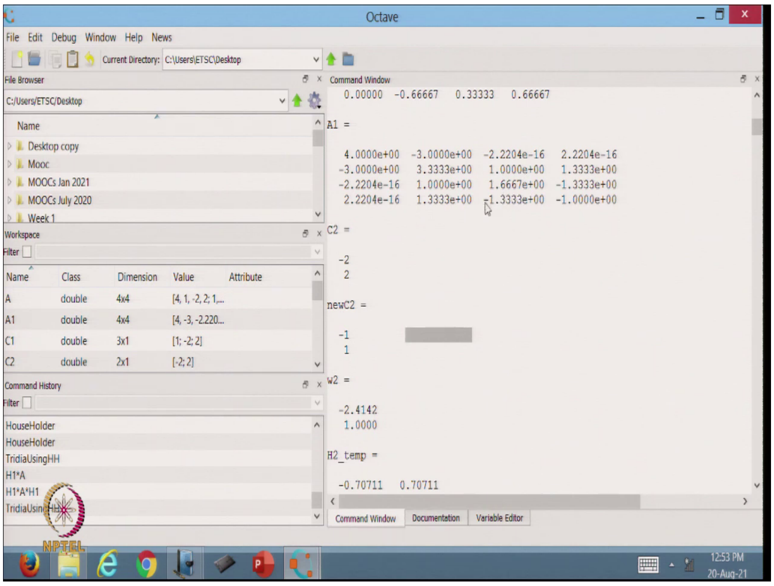

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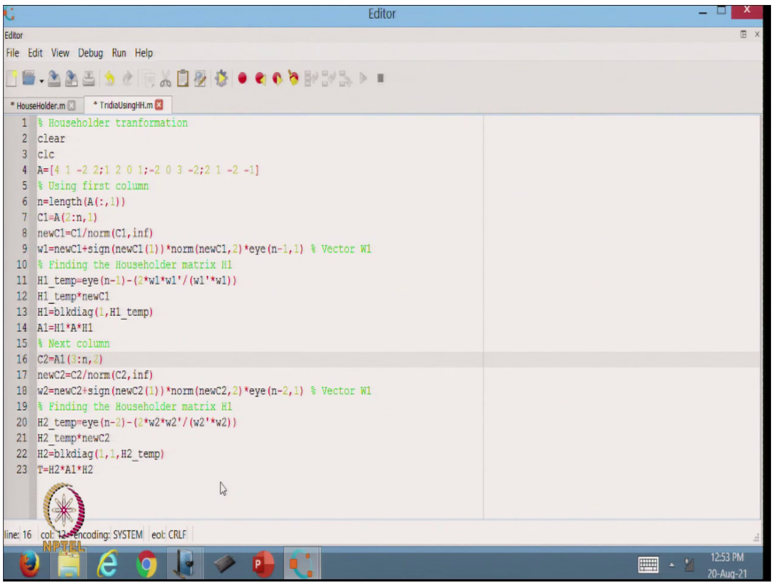

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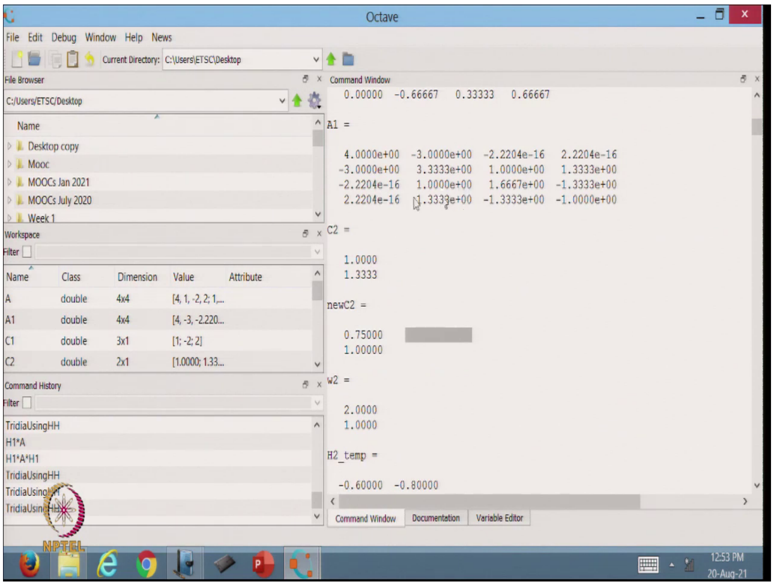

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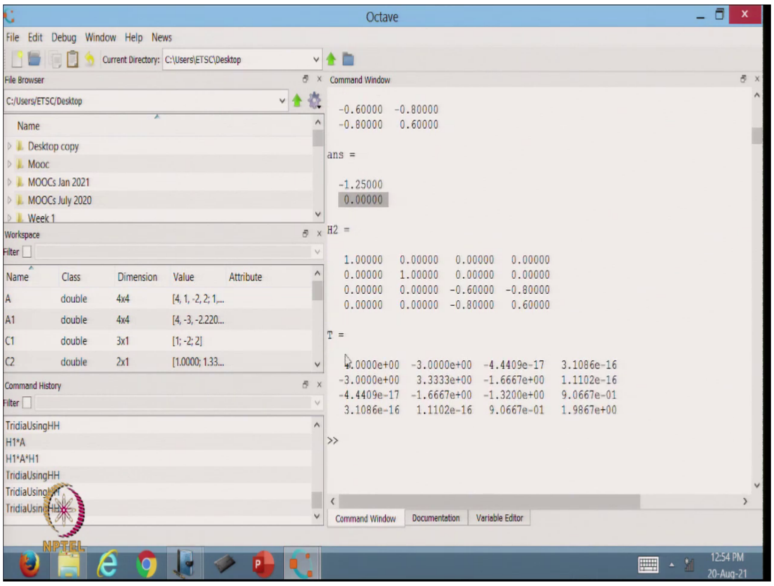

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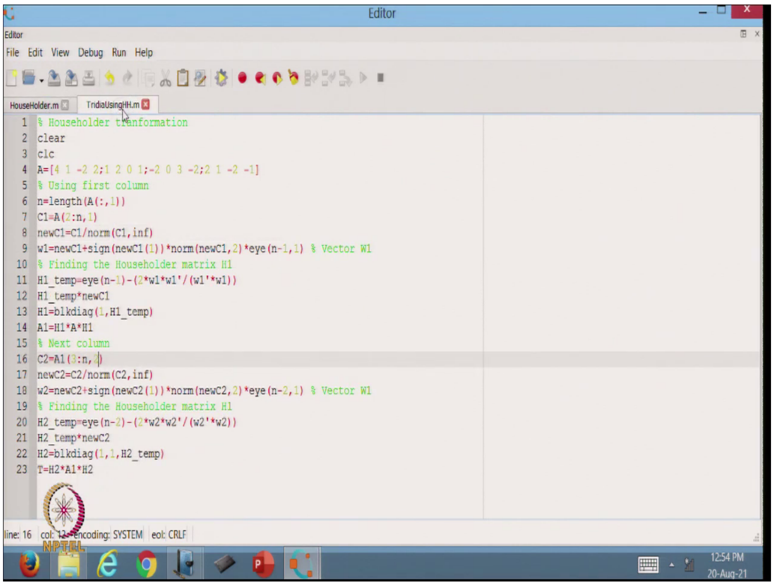

So, by this way in the computer programs you know that it is quite easy once we write the program, then we can just change the matrices and the same program can be run for various type of matrices. So, that is the benefit of writing codes to do the numerics. I think now we should stop and with the help of this code you can run and write your own codes and then you can verify or you can try these things for different type of matrices.

So, in this lecture we have discussed the application of the Householder transformation, first we have shown QR factorization. And then we have shown another application in this lecture that if the matrix is a symmetric matrix, then we can convert this one into the tridiagonal form using the Householder transformation. For computation purposes we have used the Octave software.

I hope that you have enjoyed this course, the complete course on matrix computation and learned a lot of things from here that will be useful for your further studies. So, thanks for taking this course and completing this course.

Thanks very much.