Chaotic Dynamical Systems
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Lecture – 02
Elementary Concepts

Welcome to students. So, today we start our lecture on talking about some elementary concepts of dynamical systems.

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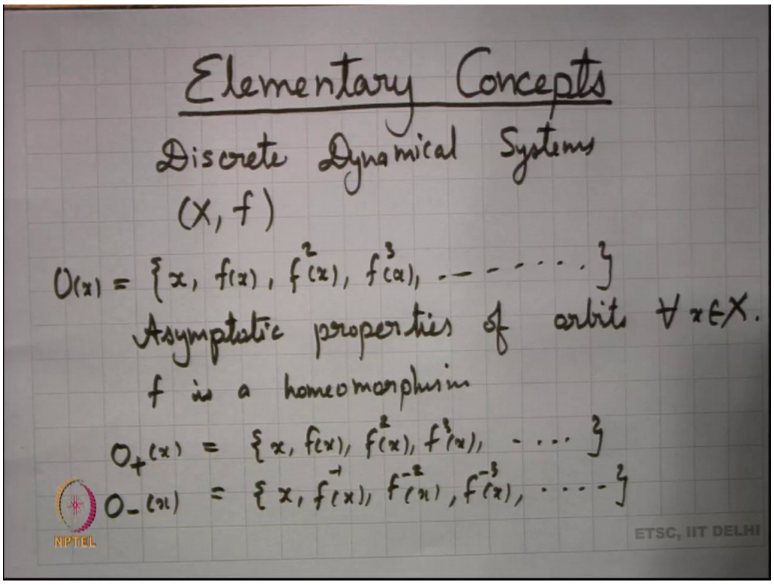

We had looked into what we call as discrete dynamical systems, our discrete dynamical systems are basically a space x and a self map f on x and what do we study in this discrete dynamical system, is basically for a typical point x we studied what is f of x what is f of f of fx and so on.

This basically comprises what is the orbit of x and what we study is we study the Asymptotic properties of orbits for every x and X. Now what happens if f is a homomorphism? So, if f is a homomorphism we have 2 ways of looking into the orbits. So, we look into the forward orbit of x which is again your it goes as x fx f square x f cube x and so on and there is also a possibility of looking into the backward orbit of x. So, the backward orbit is basically your x f inverse x, we start with this basic concepts and we try to look into what sort of dynamics do we want to study for this system.

So, if you look into this fact right we are since we are looking into orbits, the most elementary orbit that comes up is when f of x remains stationary at x.

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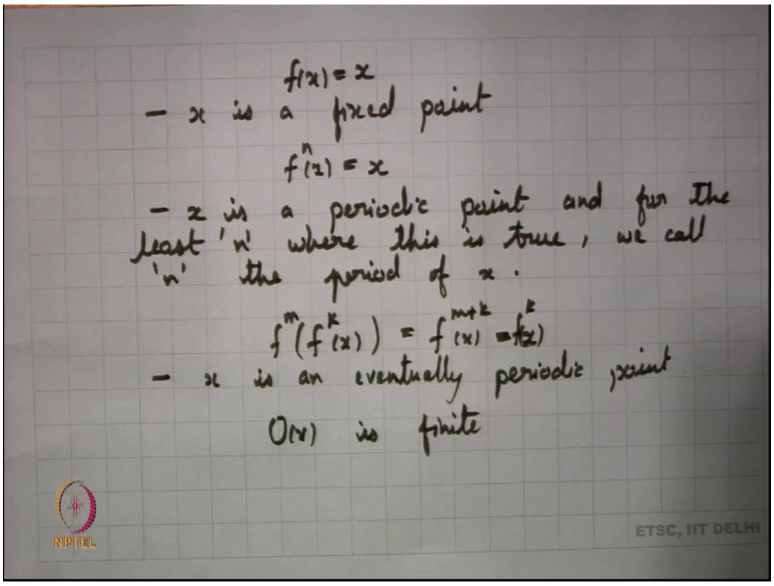

So, x is a stationary point and this is what we called as a fix point. So, x is a fix point in this case, the second elementary observation that 1 can make is what happens when fn of x; that means, after nth step the point written the orbit returns back to x. So, in this case we say that x is a periodic point and for the least n where this is true we call n the period of x. So, x is a periodic point and it has a period n, if n is the first instant where fnx is equal to x.

Now, another elementary observation here is those points right, where you are exactly fnx is not equal to x, but what I find is that I have a k such that if I take fk of x right and then if I apply fn to it; that means, what I get here is if m plus k of x like that returns back to this turns out to be f kx. So, this is not something which is periodic, but what happens is some iterating of it behaves like a periodic point. So, what we call such points as x is an eventually periodic point.

Now in all the 3 cases what we observe is that the orbit of x here is finite and the orbit of x is finite only in these 3 cases. So, when orbit of x is infinite it becomes a totally different case right, which again we shall study what happens in particular for those cases. So, how do we study these orbits or how do we make these elementary observations. So, we find that let us look into some kind of elementary spaces and the best way to look into the dynamics or observe elementary things about dynamics is to study the phase portraits. So, let us now look into what are phase portraits.

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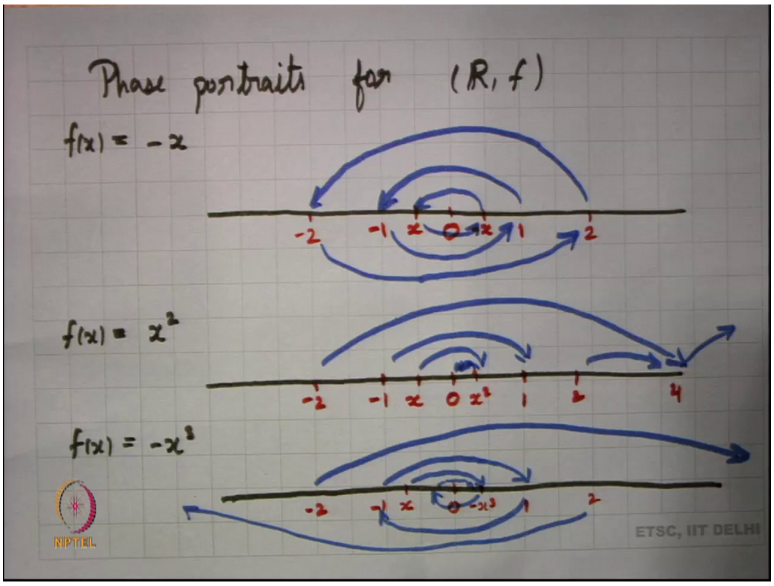

So, we look into the phase portraits say for the system we have the real line and we have f and let us first look into the phase portrait for f x equal to say minus x, now phase portrait we simply look into the real line; now what happens over here? So here we observe that if we consider the point 0 right remains at 0, what happens to minus 1 and what happens to 1. So, we observe here that under minus x right the point 1 is mapped into minus 1 and the point minus 1 is mapped into 1.

What happens if we are going beyond 1 and minus 1, again we can think of 2 write and minus 2. So, what happens here is that the point 2 is map to minus 2, so the point 2 is map 2 minus 2 and the point minus 2 is map to 2. This is also true for any elementary x that I take up over here, if I think of x over here this point x gets map to minus x. So, again we have the system. So, the phase portrait here is very simple right and we can diagnose everything, what happens to all the points the scenario the dynamics of this system is very clear from this particular portrait.

So, what we have is x, we have for this particular system we have 0 is a fix point here and any other point happens to be a periodic point of period 2. So, the dynamics here is very simple for all points on the real line right 0 is a fix points, it is a point of period 1 we can say and all the rest of all the points are periodic points of period 2.

Let us now look into the case of fx equal to x square, what happens in this particular case? So, if I look into this particular case again my point 0 remains as it is, again 1 would remain as it is and I look into minus 1 minus 1 is map to 1. So, we have minus 1 being map to 1; what happens to say minus 2 here? So, we find that minus 2 is map to say 4 here right and we find that minus 2 is being mapped to 4, similarly your point 2 right is also map to 4; what happens to a typical point x which lies between minus 1 and 0. So, if I take this x lying between minus 1 and 0, this typical point x is map to x square, so it is map to a point between 0 and 1, but with a lesser magnitude.

So, this is map to some point x square over here and if you find what happens to x square right, then this point x square is map to a point which has a further less magnitude and ultimately that is being driven towards 0. So, if we look into the phase portrait for this particular system, for this particular system the phase portrait is very simple, all the points on the left of 0 or map to the points on the right of 0, all the points are mapped with a magnitude less than that self; if your value of the realist less than 1, if the real is greater than 1 it is map to the greater magnitude. So, all the points on the right of 0 right, if they are greater than 1 they are drifting towards infinity because, we know that we can see easily see that 4 here is mapped further towards 16 right

So, any point which lies to the right of 0 right the right of 1 basically is going tending towards infinity, any point between 0 and 1 is tending towards 0. So, the phase portrait for this particular system is very simple.

Now, we can have a certain different kind of phase portrait also for example, if we consider the system fx equal to minus x cube, what happens at minus x cube? So, we find here that our system our 0 remains as it is, what happens to the point minus 1? So, the minus 1 under x cube is map to minus 1, but then it is minus x cube. So, it gets map to 1, so your minus 1 is map to 1.

What happens to any point between 0 and 1? So, if I take think of typical x over here, since x itself will be map to x cube right; since it is it with a lesser magnitude but then it is map to minus of that, so it is map to a point with a lesser magnitude minus x cube; ultimately if we try to look into and what happens to a point, which has value greater than minus 1. So, supposing I look into the point 2 over here on I look into the point minus 2 over here, then minus 2 is map to basically minus 8 and minus 8 gets map to 8.

So, the phase portrait here happens to be very simple, if I look into the points which are less than minus 1. So, these are map to a point of a very higher magnitude on the right hand side of negative of that part right, on the right hand side of 1. Now any point which is greater than minus which is greater than 1 right, its map again to a point which is of greater magnitude right and that goes towards the left of minus 1. Your point minus 1 is map to 1 your point 1 is map to minus 1, your point x is map to minus x cube and if I look into minus x cube it is again map to some point which is of lesser magnitude right. But it goes towards left of 0 and it spirals around 0, you can think of this part spirals around 0 rights to basically in the end converge towards 0.

So, the phase portrait for this particular system is very simple. So, you find that this part right to the left of 1, it basically is going towards the with a higher magnitude it is going to the to the right of 1, the higher magnitude is going to the left of minus 1 the left half minus 1 with a higher magnitude is going to the right of 1; what happens between minus 1 and 1? So, between minus 1 and one we find that again it is spiraling around 0 and the magnitude decreases and it ultimately converges to 0. So, the phase portrait is a very simple way of looking into the system.

Now, we can have other systems also and let us look into 1 such example. So, let us look into the example fx equal to 1 minus x square, now it seems to be a little bit not so trivial case of looking into what happens to 1 minus x square, it is not very simple to compute.
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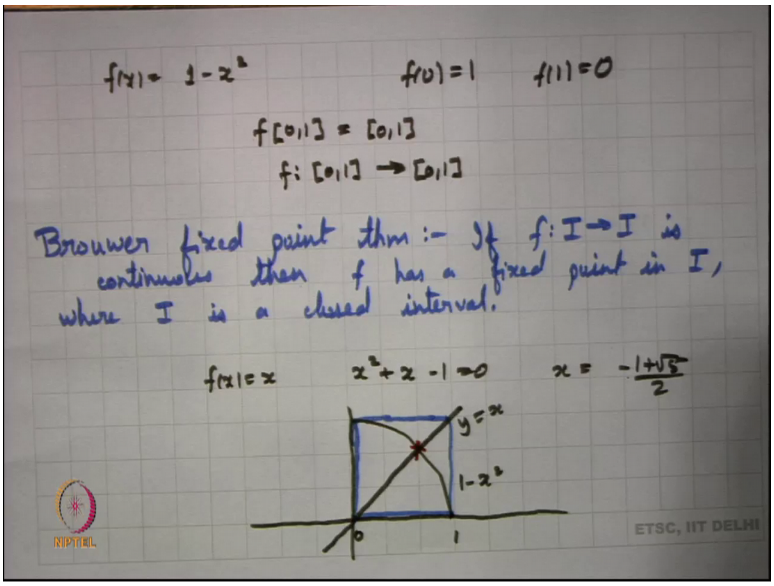

But then if I look into this particular map, I know that f of 0 here is 1 and f of 1 here happens to be equal to 0 and if I look into the unit interval I find that f of 0 1 is same as 0 1.

So, I can think of this particular map as a map from 0 1 to 0 1 right. So, f is basically a map from 0 1 to 0 1, supposing I want to study the dynamics of this map only in the interval 0 1; we need to look into what sort of orbits can we expect from the points over here and ultimately since it is very difficult to see fine I have a periodic 2 I have a periodic point of period 2. So, I have a period 2 orbit over here where 0 goes to 1 and 1 goes to 0, but then what comes to my rescue is we know some results from analysis.

So, we know this Brouwers fix point theorem and this theorem says that if f from I to I is continuous then f has a fix point in I, where my I has to be a closed interval. So, for a closed interval Brouwers fix point theorem guarantees that there has to be a fix point and so if we look into fx equal to 1 minus x square it should have a fix point in the interval 0 1. So, what does this fix point of course, we can look into this fix point by solving the equation fx equal to x or basically this equations would turn out to be x square plus x minus 1 0, which gives me x equal to minus 1 plus root 5 by 2.

So, I do have such 1 fix point here, but re ideally what can I say about the dynamics of this particular system, it is also easy to look into this system by again taking the graph of this particular function and since I am interested only in 0 1. So, I am interested only in 0 1, I find that the graph of the system looks to something to be like this part. So, this is your 1 minus x square and then I can draw the line y equal to x, we know that these 2 lines will only meet where x is same as 1 minus x square and so we get a particular fix point here. So, instead of computing we could easily find out that yes we do have 1 particular fix point in this particular interval.

Then it has 1 fix point it has a periodic point of period 2, for example 0 and 1 they turn out to be periodic point of period 2. So, 0 and 1 is a periodic point of period 2 does it have any other periods. So, it is very easy to guess what happens over here. So, how do we find out whether it does have periodic points of any other period or not? So, the easy way is to look into a graphical analysis of this particular map, and if we try to look into graphical analysis.

It is very simple we have this point as a fix point I look into what happens to 0, 0 is map to 1 and then this goes to 1 right which is again map to 1, which is again map to if I go back to the line y equal to x it is again map to 0 and then 0 takes the value 1. So, if we take if we draw the line back to the graph it is map to 1, does it have any other periodic point. What is basically the nature of the other periodic points? What happens to the other points over here? Do the points remain where they are, do the points converge to some particular point maybe right. Now I am not looking into this we will look into this part once again, but this gives us another way of looking into dynamics and that is we can look into what is called graphical analysis.

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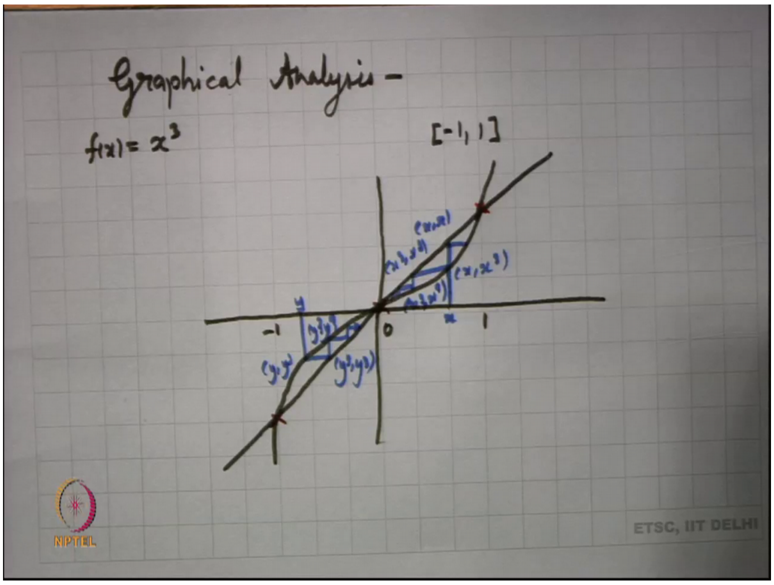

If you try to look into how we can look into the graphical analysis of some points, what do we exactly mean by graphical analysis? So, we try to see maybe for effects equal to say let us look into x key now and we want to look into this right with our domain being minus 1 n 1, what happens over here? So, if you try to look into this particular graph over here, so, the graph goes something like this.

Now, what we find here is that f of x equal to x cube, I have 3 fix points here I have a fix point we just draw the line y equal to x right, extend the line y equal to x and we find that this has 3 fix points it has 1 fix point here at 1, it has another fix point here at 0 and it has a third fix point here at minus 1. So, we find that this has 3 fix points here at 0 at 1 and at minus 1.

What happens to a typical point between 0 and 1. So, we start with a typical point between 0 and 1. So, if I am starting with my x here right this goes to the point which is called x comma x. So, it goes to x x cube and then it goes to the graph this goes to x sorry, this goes to x x cube right and then it goes to x cube x cube, then it again comes back to this phase which is basically my x cube x to the power 9 and it comes back further.

Typical point between minus 1 and 0 right, look into any x here it comes back here to this particular point. So, I would start with this point this was my point x and this is my point y. So, I start with my point y here. So, this particular point goes to y y cube, then it again goes to the point y cube y cube, then it goes to the point y cube y 9 right and we ultimately find that it converges towards 0.

So, the graphical analysis of this particular dynamical system is very simple, what happens here is that 0 remains 0 1 n minus 1 remains fixed, but all the rest of the points right there orbits are converging towards 0, what happens to points beyond that right very easy to guess that the points to the left of minus 1 will be tending to minus infinity, the points to the right of 1 will be tending to infinity. Let us now look into another particular graph say y equal fx equal to 2 x minus x square.

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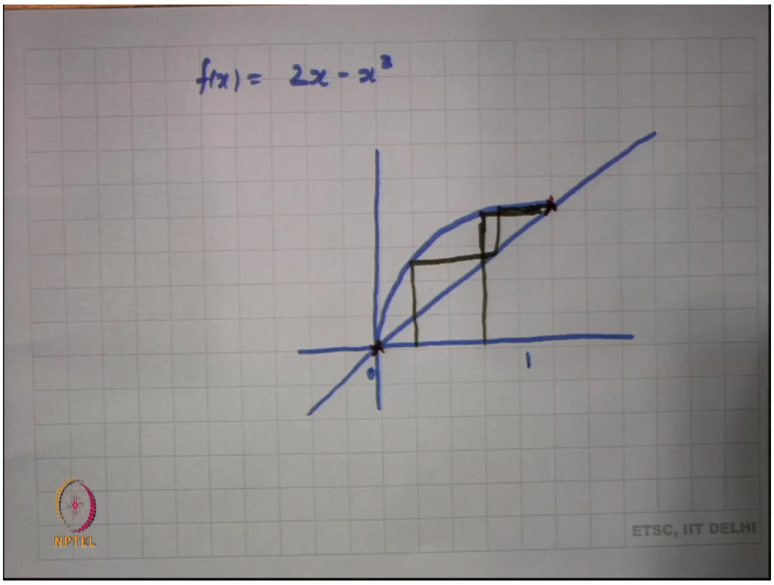

We look into fx equal to 2 x minus x square and now we try to see what happens over here between 0 and 1. So, I am not interested in further I am looking into what happens between 0 and 1 and we find that if I draw this line right, at 0 it takes the value 0 and at 1 it takes the value 1. So, this is something which looks like this between 0 and 1. So, I have 0 I have 1 right it takes this value between 0 and 1. So, I have 2 fix points here I have a fix point at 0 and I have fix point at 1.

Now, what happens to this particular graph as between 0 and 1, what are the orbits of all the points here between 0 and 1? So, we typically look into what happens to the points over here, then we find that any point over here goes towards this graph right. Then it converges to converges to 1, by graphical analysis we find that any point over here right you start with any point over here right, it goes further right and it converges to 1. So, by graphical analysis it is very simple to say that this will not have any other periodic points, right it has only 1 point periodic point that is it has 2 fix point S 0 and 1 and rest of all the points the move towards 1.

So, let us try to see what happens in cases where we cannot draw graphs. So, we can think of them to be simple examples, so let us consider the example of a circle. So, we all know what is a unit circle?

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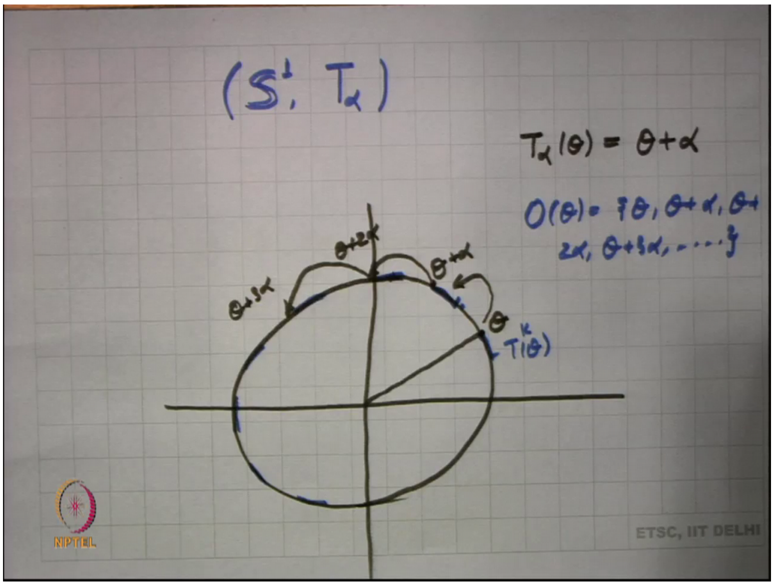

So, I am looking into the unit circle as 1 right and I want to take a particular map T defined by a constant alpha, I am looking into this particular system and this system is very simple when I talk look into my S1 right, my S1 happens to be a circle the unit circle I can think of this right and any point on the circle can be specified by theta. So, it can be parameterized by theta. So, my function happens to be very simple my T alpha of theta right I can say that this is nothing, but theta plus alpha.

So, I start with the point over here I know that this point is my theta right and what it does is its nothing but it jumps to another point which is my theta plus alpha, where does this point theta plus alpha do. So, this theta plus alpha also does nothing, but it jumps again theta plus alpha. So, we get here something called theta plus 2 alpha right, then it goes to something over here which is nothing, but theta plus 3 alpha and so on.

So, my system is very simple over here that the orbit of point theta right, if I look into what is my orbit of point theta. So, the orbit of point theta happens to be nothing, but theta plus alpha, theta plus 2 alphas, theta plus 3 alphas and so on. So, this is basically my orbit of theta, now I want to look into the dynamics over here. So, I am trying to analyze the dynamics over here It is very important to know what is alpha? Because, we know in the circle the circle itself has a period of is periodic period 2 pi right. So, since the circle itself is periodic with period 2 pi It is very important to know what your what is the constraint on alpha.

So, let us try to see we will come back to this again, plus let is let us try to see what happens when alpha is a rational multiple of 2 pi.

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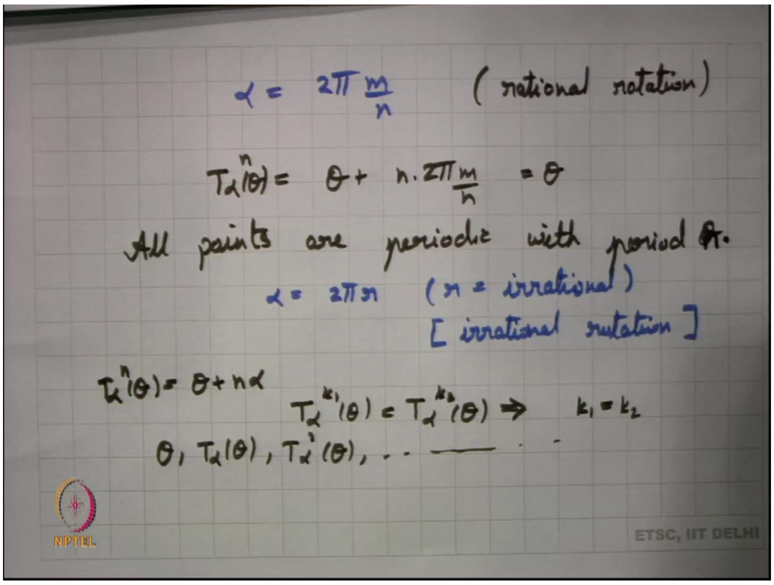

So, supposing my alpha happens to be equal to say 2 pi times m by n, what happens in that particular case? So, if we try with our point theta right, what we find is that say T alpha right to the power n will be nothing but theta plus n times 2 pi m by n right. So, what is it just a translation of theta by a multiple of 2 pi. So, what I get back is just theta.

So, here for given any theta so for given any theta over here, what we find here is that T alpha to the power n of theta goes back to theta. So, the interesting case here is that all points are periodic with period theta, a period n sorry with period and all points are periodic with period n. Now we look into this fact that, so this is basically also something which we call as the rational rotation. So, this is like we are rotating along the circle and this is a rational rotation. So, this is also a rational rotation of the circle and we find that for a rational rotation what we find is that this all points here are periodic with the same period.

Now what happens when alpha is an irrational multiple of 2 pi, because we can have either an rational multiple or an irrational multiple. So, what happens if alpha is an irrational multiple of 2 pi. So, I am taking 2 pi times r right where r is a rational. So, this is basically typically I should say this is an example of an irrational rotation, we have 2 pi r. So, I know that typically if I look into my T alpha to the power n of theta right it is just going to be my theta plus n alpha.

If at any instant if I find that there are 2 e traits which are going to be equal, supposing T alpha k 1 of theta should be same as T alpha k 2 of theta right, then that would imply that k 1 is equal to k 2, because theta plus alpha of k 1 theta plus k 1 times alpha will be same as theta plus k 2 times alpha right. Which would imply k 1 equal to k 2 in the sense because my k 1 minus k 2 should be some kind of a integer right and integer multiple of 2 pi right since my alpha is irrational that would imply k 1 equal to k 2 .

So, if I now look into my typical orbit of theta, I will find that this theta this T alpha theta this T alpha square theta right, these are all distinct. Now these are all distinct think of that let us come back to our circle again, what is our circle here? So, we find that we have the unit circle it is a closed and bounded subset of r square right, it is a compact space right and we very well know that for a compact space, if I have an infinite set that infinite set will always have a limit point.

So, this particular infinite set which is the orbit of theta right, always has a limit point and hence I will have some 2 values right, I have since this has a limit point right. So, there will be some point where it converges where the orbit where a subsequence of thus converges.

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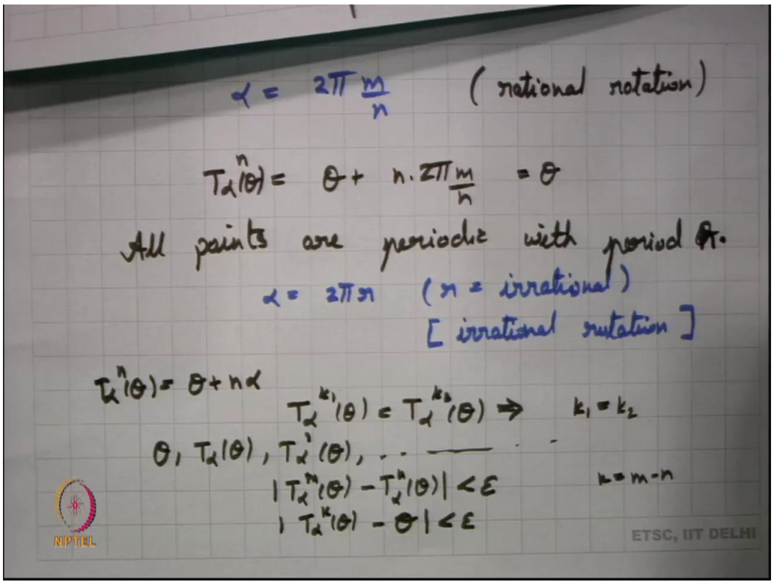

So, I will always have some kind of mn, such that if I look into mod of T alpha m of theta minus T alpha n of theta right, this modulus will be less than some epsilon.

Now, I can assume my k 2 be equal to m minus n, assuming that my m has will be the bigger iterate than n; then what does this imply that my T alpha k theta minus theta is less than epsilon, now I am using the fact here that our function right basically preserves the length here it is an isometric. It preserves the length here because every time it is just moving it is just cutting an angle it is just cutting arc length of alpha. So, every time you have the same arc length there. So, this is basically typically an isometric and I am looking into the isometric property, I can say that if this happens if I assume that, means that T alpha k of theta will be within epsilon range of theta.

Now, what does that give us? So, let us typically look that to our example what does that give us. So, in that case what we get here is that, I have perhaps I am looking into T k theta here. So, I have theta in TK theta over here. So, what happens next here is that I have theta this theta plus alpha and TK theta plus alpha somewhere over here. So, T k plus 1 theta over here and what I find is that every time I have this cutting itself right in range of theta.

Now, I will also know that no point in the orbit can come back to itself. So, when I look into all this arc lengths right, they will never be coming overlapping each they will never be basically superimposed on each other they could be overlapping and then since these skips on moving all the fact what we have done is actually what this does is that in small lengths of less than alpha we are basically covering the whole circle.

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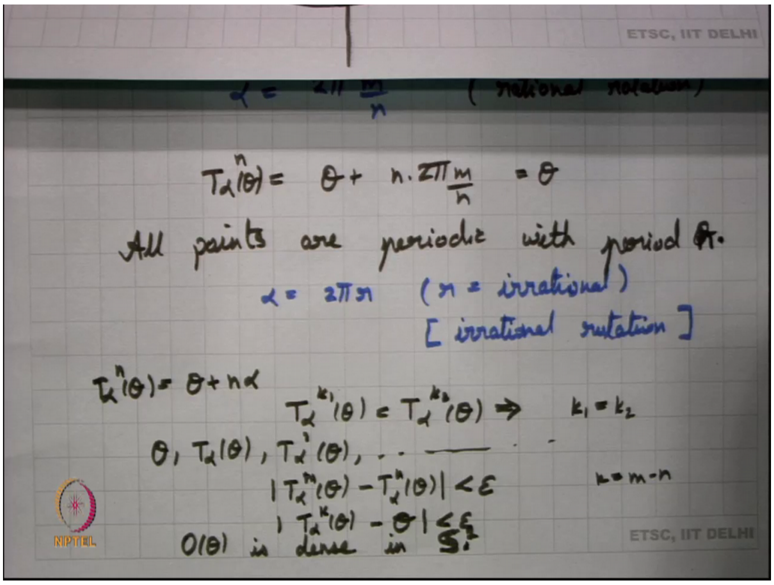

So, the whole unit circle is covered by small units of alpha and that tells me that basically typically everywhere you will find 1 point; you will at least find 1 point in the orbit of theta? Which means that the orbit of theta happens to be dense the orbit of theta is dense in your S1.

So, orbit of theta is dense in S 1 right and you can easily see that for an irrational rotation, the orbits are very simple I would say that we do not have any periodic point here because the look into the entire orbit of any theta right, all the points in the orbit are distinct there is no periodic point there is no eventually periodic point, but it has a better property that if you look into all the orbits right. So, for any theta its orbit densely covers S1. So, the orbit of all the points here is dense. So, we find that for when you have an irrational rotation the orbit here is dense. Now, we try to look again into the circle and this time I am looking into a different example.

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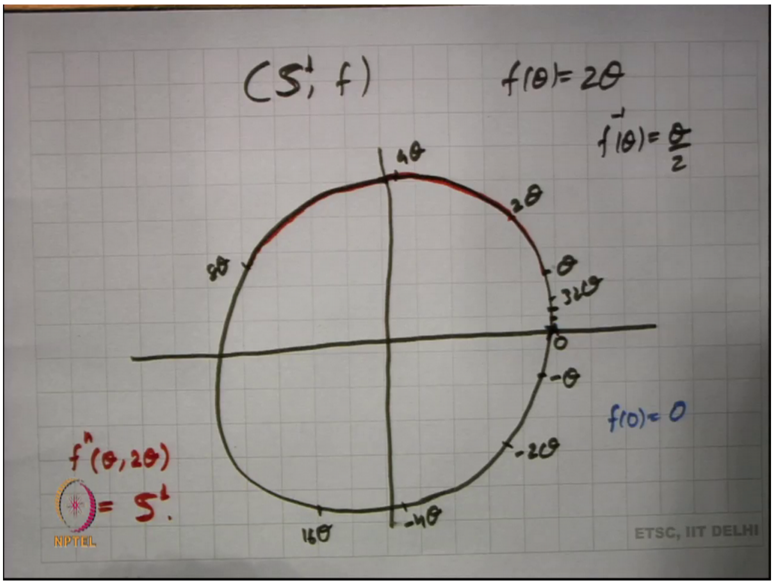

So, you have S 1 and let me look into this map f over here, where I am defining my f of theta equal to 2 theta; what happens in this particular case? So, again let us try to look let is try to draw the figure here. So, we try to draw the figure here, what we find here is your theta is being map to 2 theta right then where is 2 theta being map 2. So, 2 theta will be map to 4 theta, 4 theta will be map to say 8 theta, 8 theta is map to 16 theta and so on. So, we get again back 32 theta, which would come back over here.

So, now look into this fact, now this is a case of a homomorphism here right. So, I can also think of looking back into what it is. So, where is theta map 2? So, it is map to here. So, if I look into f inverse my f inverse of theta it is basically theta by 2 and where does this map 2. So, this maps to somewhere theta gets map to theta by 2, theta by 2 gets map to again theta by 4 and so right.

Now, what happens for if I want to look into the dynamics of this particular system? So, the dynamics of this system is really very simple because, I know that f of 0 is map to 0, where is theta being map right I am always increasing the argument right. For any argument theta the argument is increased by twice its range, what happens to the minus case, what happens to f inverse? f inverse where is my minus theta being map 2. So, if I look into my minus theta here right, my minus theta is map to minus 2 theta minus 2 theta is being map to minus 4 theta and so on, but here I have 0 which is being map to itself.

So, what happens in this particular case what happens in the case of 0, what happens to the rest of the points between 2 theta and theta, if I look into this arc between theta and 2 theta? It is map to a bigger arc right which goes from 2 theta to 4 theta, now this is again being map to a bigger arc between 4 theta and 8 theta. So, somewhere along you get n point in such that, if I look into what happens to fn right of this arc I am talking of this arc between theta and 2 theta right. So, this arc would have covered the whole circle right.

So, fn of this would basically have covered the whole circle it would be whole of S 1, we have a fix point here 0. What happens typically when I am looking into the negative part right looking into them part over here right. So, what is f inverse doing is that between theta and 2 theta the arc the arc length reduces to half right. It meets map to theta by 2, then theta by 2 is map to theta by 2 theta by 4 and so on. So, the points here right the arc length here is decreasing in the negative case the arc length is decreasing, but ultimately what happens to a typical point theta over here, theta is always map to 2 theta 4 theta so and so for; Whether it is close to 0 or it is far away from 0 its always mapped in this particular region and since at some particular point you find that, this particular arc length is converging is basically expanding to whole of S1 right, some point it will take the value theta 0, right.

If we takes the value 0 everywhere right, for every typical theta it takes the value 0 at each and every point we know that 0 remains fixed then everything will be remain fixed right. But the expansion is. So, great at some particular point in between it there definitely will take the value 0, then it will remain stuck at 0 and the rest of the points will again travel back to expand will expand, the rest of the points or maybe the rest of the arc length right will expand again to form the whole of S1. So, x since at each and every point we are doubling the length here right. So, this also can be said called a doubling map right. So, this doubling happens at each and every stage and it covers the whole of S 1.

Analyzing this points analyzing the orbits of theta here become a little bit difficult here because, what happens typically what happens when can we reach 0,we know where for sure very sure that if theta is a rational multiple of is an irrational multiple of 2 pi it will never reach 0, never remains stuck to itself right. So, we know for very sure how or we moving over here right.

So, typically we can think of this fact that the points are moving randomly over here right, the orbit of points here are quite random and we know that they will never come back or basically they are never sticking to 1 point right. You are not finding a periodic point here right theta moving to 2 theta is possible, we can have a theta which goes to 2 theta its possible right, but we are not sure of what the orbits of this particular case r. Again this is a particular case which we shall cover up once again when we look into some more theory, but right now let us move to something else about the system.

So, we want to now look into a typical case, where again we will come back to this example to look for this particular case. So, let us look into some elementary definitions once again.

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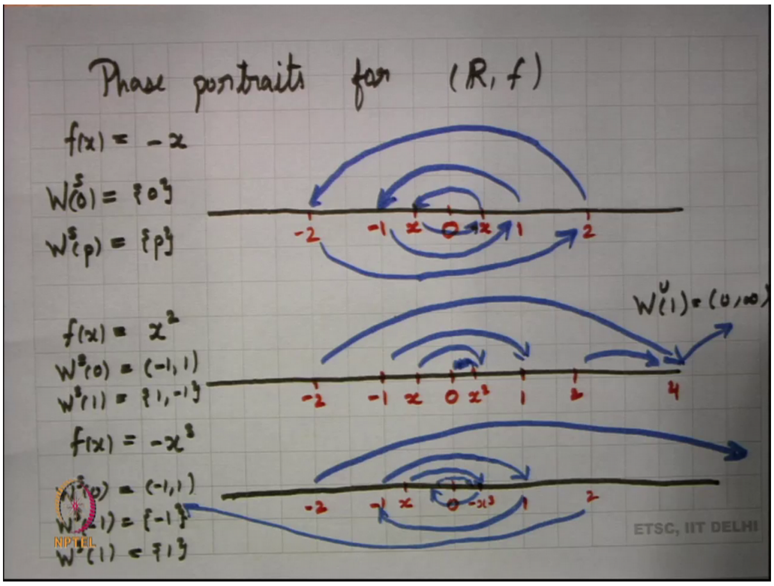

Let p be a periodic point of period n, a point x is called forward asymptotic, if I take f kn of x right it converges to p or I can say that limit as k tends to infinity fn of x is same as p, then we say that a point x is forward asymptotic to p.

Now, we look into the stable set of p. So, I call it as W s of p right which is basically the set of all points right, which are forward asymptotic to p. So, this is basically called the stable set of p. Now let us come back to some examples that we have done earlier, let us look into this case again.

We find here that this 0 is a fix point, what are all the points? So, this is periodic point of period 1, what are all the points which are forward asymptotic to 0 or what is the stable set of 0. So, what here is what is the stable set of 0 in this particular case, there is no point right because which converges to 0; what is the stable set of 0 here right, the stable set of 0 happens to be equal to single be 0 right.

What happens to the stable set of any other now any other point over here happens to be a periodic point of period 2 right. So, I can say that any other point here is p right and what is the stable set of p it happens to be p itself right. So, we find that this basically is p itself; let us look into this particular case fx equal to x square again here my 0 happens to be a fix point, what is the stable set of 0? We find that all points between minus 1 and 1 right, they are basically converging their orbits are converging towards 0, 0 is a fix point that orbits are converging towards 0. So, the stable set of 0 happens to be minus1 1.

Now, we have 1 also is a fix point right, what is the stable set of 1; the stable set of 1 I find 2 points here right 1 and minus 1, the rest of the points do not come under this table set of any other thing, let us look into this particular case fx equal to minus x cube right. What happens to the stable set of 0 again, we know that everything else comes to 0 right. So, this is minus1 1 and again what happens to the stable set of 1, let me now look into the stable set of minus 1 right; just same as minus 1 goes to maps to 1 right and where is 1 map 2.

Student: (Refer time: 47:08)

Its map to minus 1 right, so what happens in this case what is the stable set of minus 1. So, the stable set of minus 1 we find to be.

Student: Minus 1.

Minus 1 right I am looking into a minus 1 as a periodic point. So, if the question is why should it be equal to minus 1 right, we are looking into minus 1 as a periodic point of period 2. So, for example, where does this x to the power 6 converge to, I am now looking into if square of some point converging right. So, what we find is that minus 1 is the only point which comes back to minus 1, the limit tends to minus 1; for no other point the limit tends to minus 1. So, the stable set of minus 1 is minus 1.

What is the stable set of 1 in this particular case? So, the stable set of 1 here will be just 1 because, under fx square right that is my f square turns out to be x to the power6 right, no other point under f square is converging to 1 right, 1 is the only point which converges to 1. So, the stable set of 1 happens to be equal to 1.

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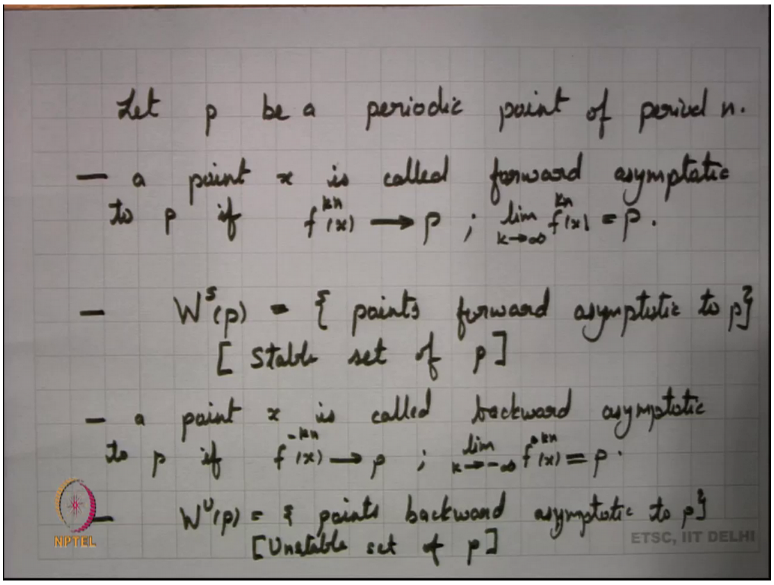

So, I want to look into 1 more definition here since we talked about forward asymptotic, we can also talked about backward asymptotic. So, a point x is called backward asymptotic to p, if f minus kn x tends to p or I can say that take limit as k tends to minus infinity f of minus kn sorry of f of kn x converges to is equal to p.

So, this is basically called the backward orbit and this basically 1 can think of this when f happens to be a homomorphism right and we think of this to be the unstable set of p. So, the unstable set of p is basically the set of all points backward asymptotic, we can think of again let me go back to just 1 example over here, let us look into f square equal to x square right. What I know that 1 also happens to be a fix point here, what is the backward what is the unstable set of 1 in this particular case. So, we find that under the negative iteration right all points from 1 to infinity right and all this points from 0 to infinity all these points are converging to 1 under backward thing right.

I am now looking into backward part right. So, instead of x square it becomes root x right, you find that everything is converging here; what happens to this case? I cannot think of this case right now because, I am looking into x square right I cannot define the negative part, but let us look into what happens over here right, what we find here is that all points right 0 infinity everything is backward asymptotic to 1. Backward asymptotic points can also be said that these are repelling away from the fix point right. So, basically you find very close to them, but that that is moving away from fix point. So, maybe we end up here right and we will come back to this again in the next class.