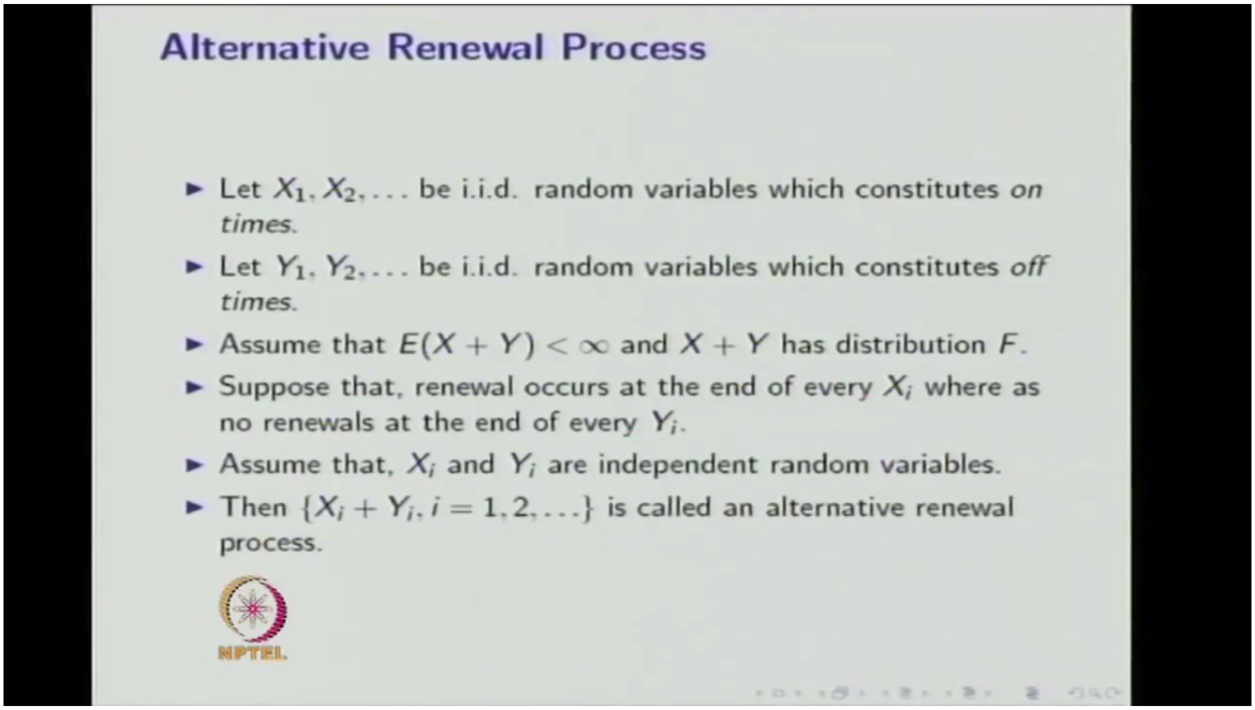
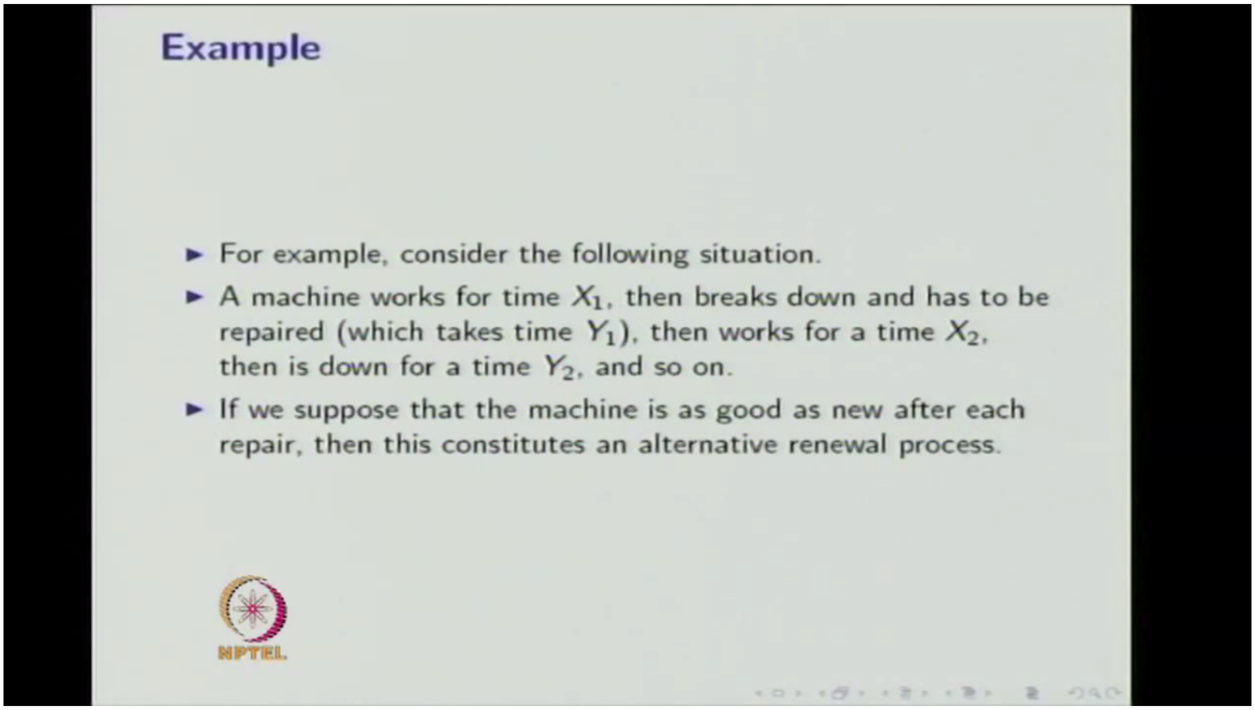
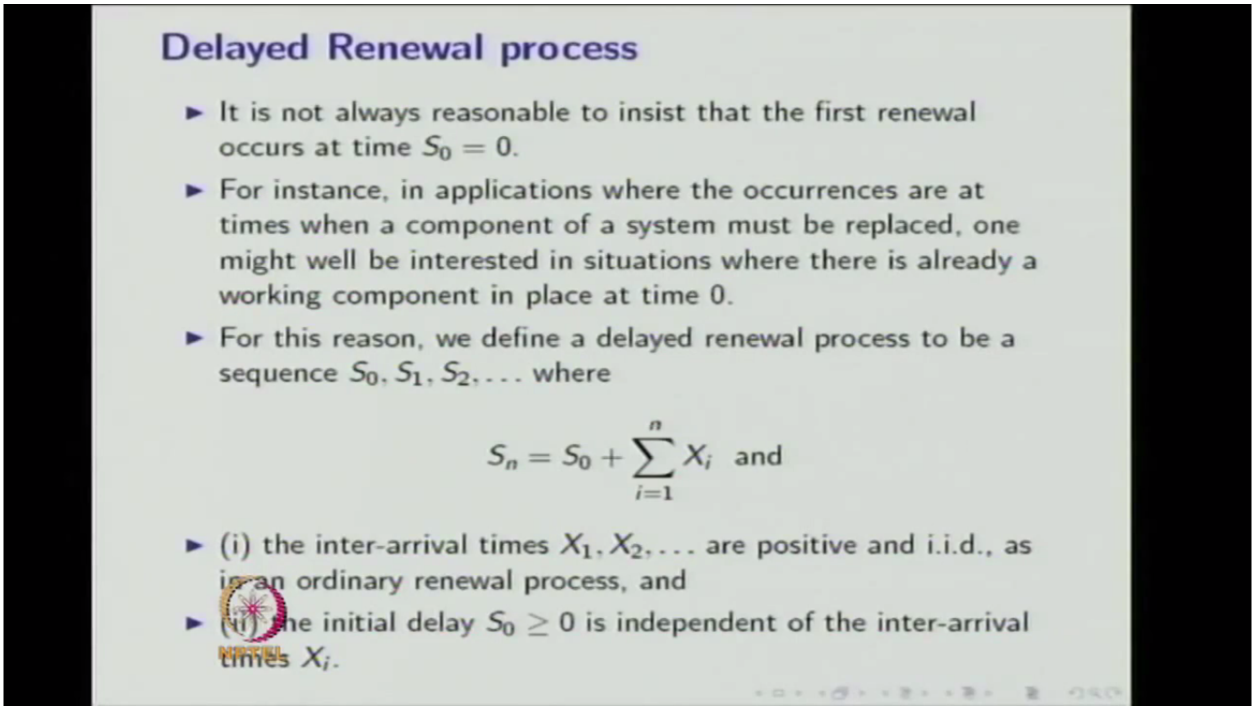


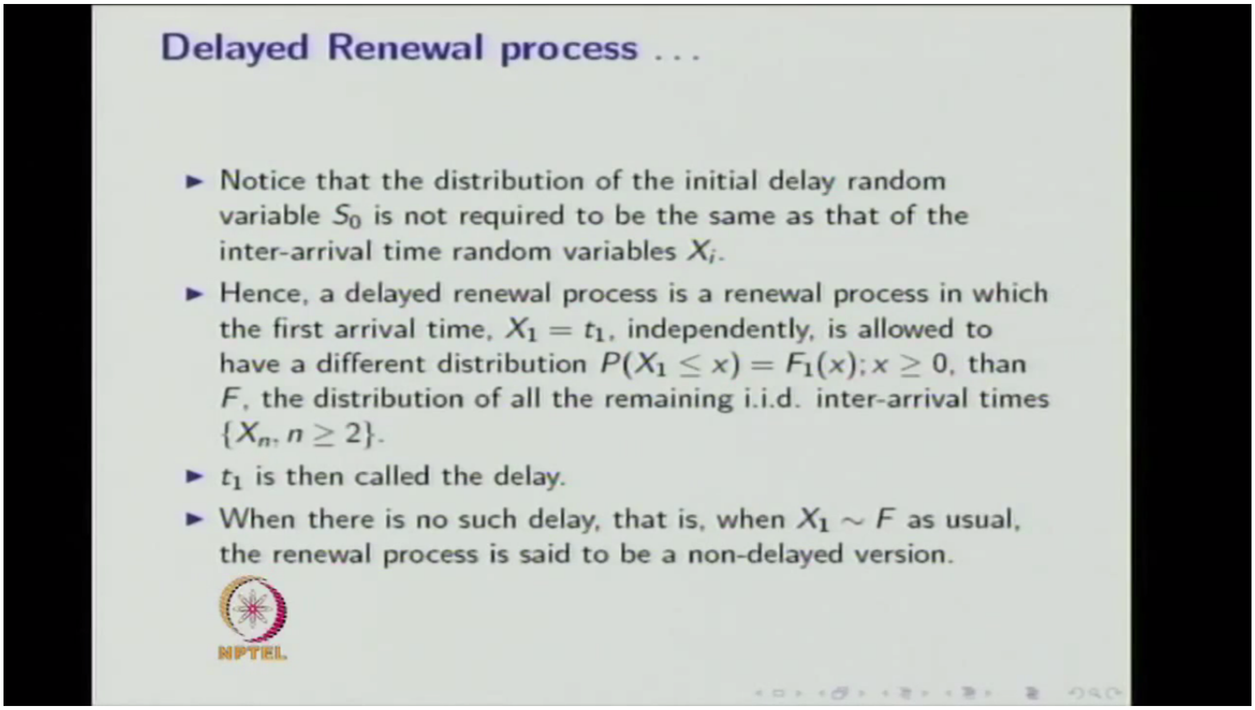
Now we are moving into the one type of renewal process that is called alternating renewal process. Let Xi's are iid random variable which constitute on times and Yi's are iid random variable which constitutes off times. Assume that mean is the existent it is finite and the X plus Y has the distribution F. Suppose that the renewal occurs at the end of every Xi's whereas no arrivals at the end of every Yi's. Assume that Xi's and Yi's are independent random variables also. Then the Xi plus Yi are called the alternative renewal process.



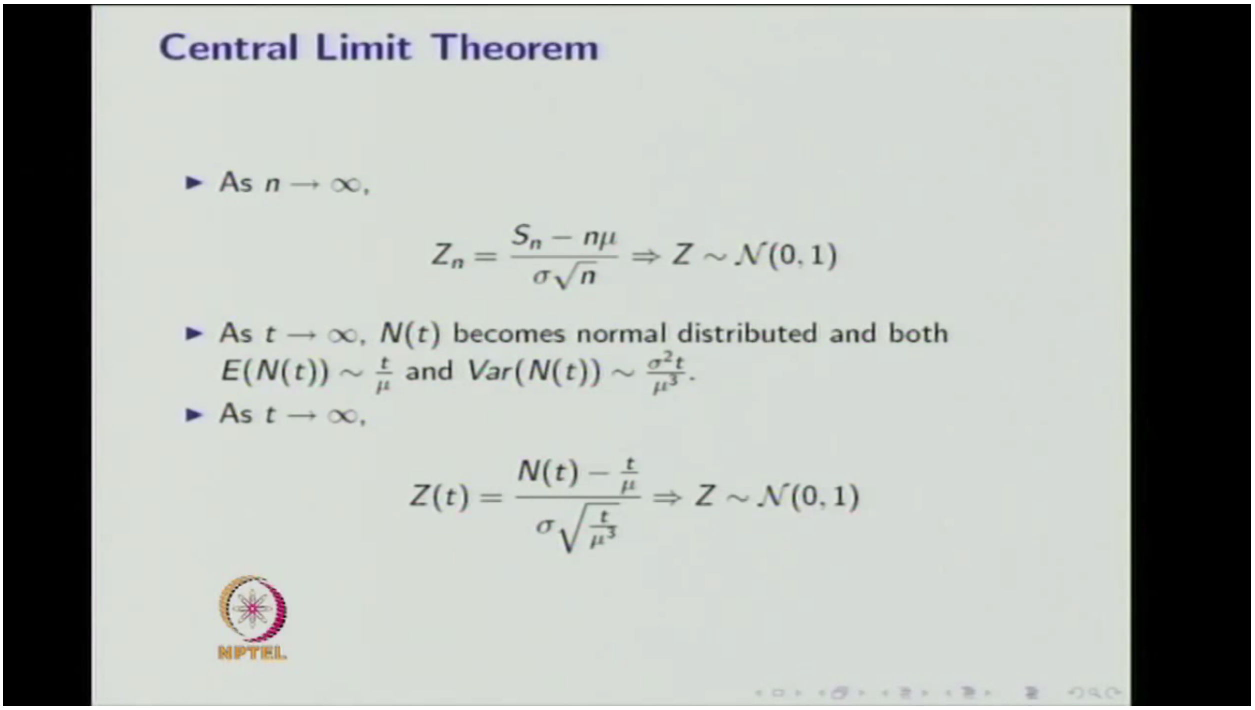
See the example for this situation. A machine works for the time X1 and then breaks down and has to be repaired which takes a time Y1. Then works for the time X2 then it is down for a time Y2. That's a second repair time and so on. So Xi's are nothing but the machine works and the Yi's are nothing but the repair time. If you suppose that the machine is good as new after each repair then this constitute alternative renewal process. So this is the example of alternative renewal process with the proper assumptions.



Now we are moving into the next renewal processes that is delayed renewal process. It is not always reasonable to insist that the first renewal occurs at time S naught that is equal to 0 in the origin itself, at the time 0 itself. For instant in applications where the occurrence are not or at times when a component of the system must be replaced. One might well be interested in situations where there is already a working component in place at time 0. For this reason we defined a delayed renewal process to be a sequence S naught, S1, S2 and so on where Sn is nothing but S naught plus the summation of first n Xi's and the inter arrival times Xi's are positive and iid random variables as in the ordinary renewal process. And the initial delay S naught which is great or equal to 0 that is independent of inter arrival times Xi's.

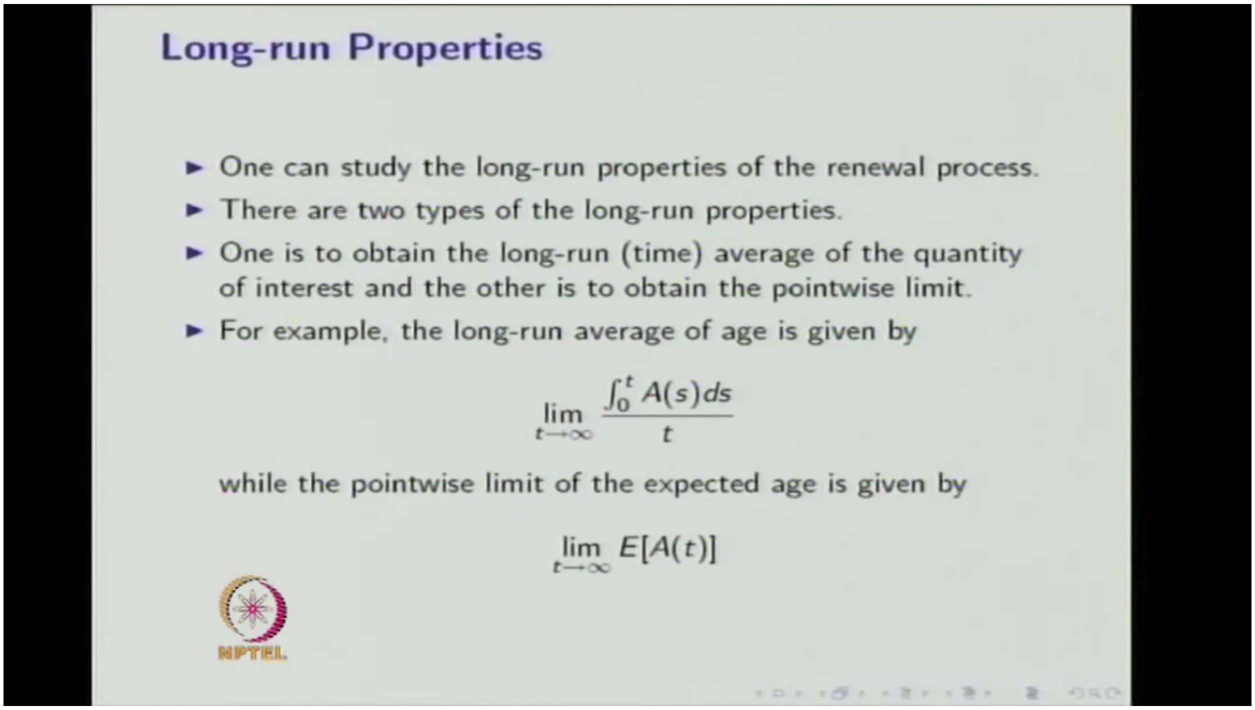


Notice that the distribution of initial delay random variable X naught is not recurred to be the same as that of the inter arrival time random variables Xi's. Hence a delayed renewal process is a renewal process in which the first arrival time X1 is independently and is allowed to have a different distribution that is F1. The distribution of all remaining iid random variables that distribution is F. So the F1 is different from F. The X1 is equal to t1 that is nothing but that delay and there is no such delay then X1 is also distributed in the same way so the distribution of X1 is F as usual. Then the renewal process is said to be a non delayed version.

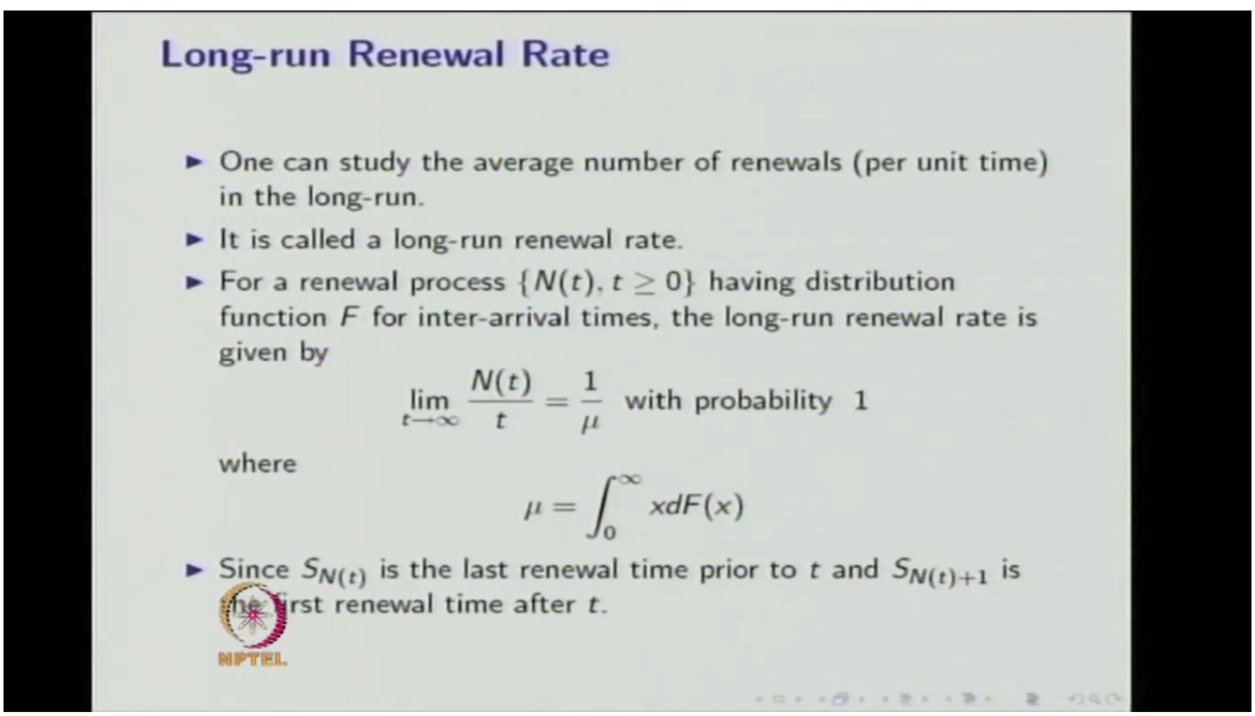


Now we are discussing the central limit theorem on the renewal process. As n tends to infinity the random variable S suffix n that is nothing but the nth time renewal minus n times mu divided by Sigma times square root of n will be normal distribution with the mean 0 and variance 1. So this convergence takes place in a distribution. So this is a weak distribution, weak convergence.

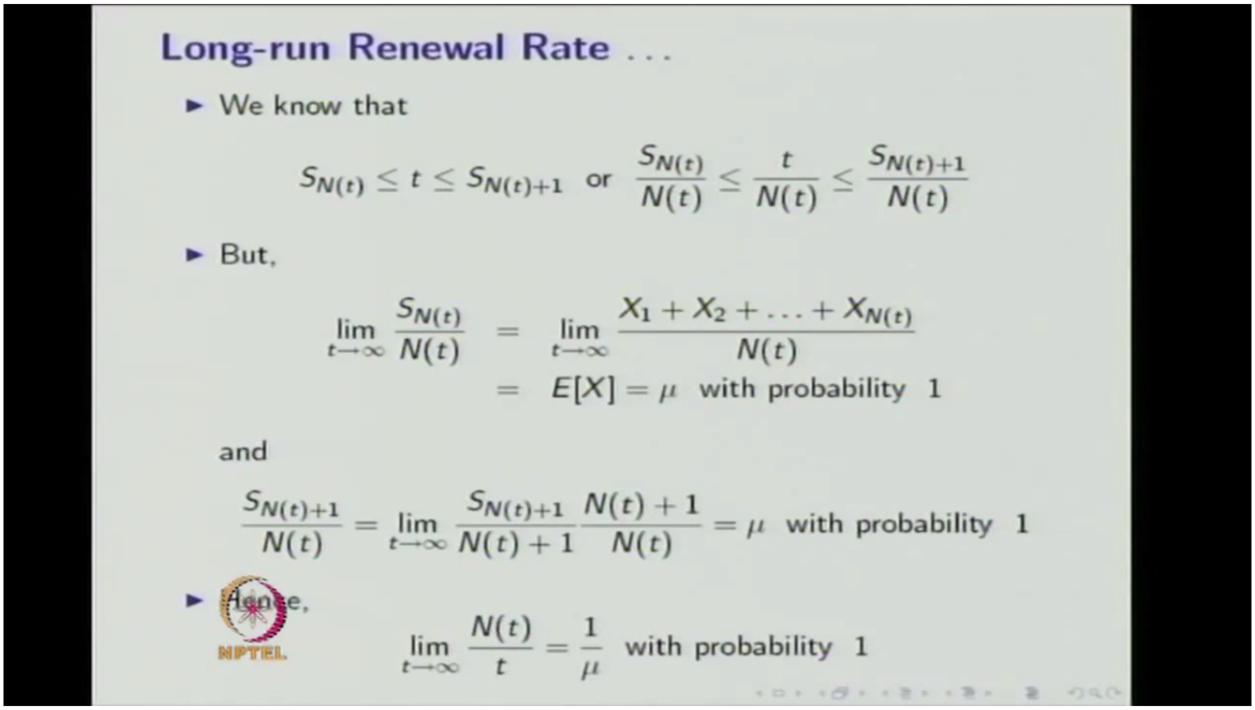
So as n tends to infinity the random variable Sn and the mean of Sn is n times mu and the variance of Sn is Sigma square n and the random variable minus the mean divided by the standard deviation is normal distributed with the mean 0 and variance 1 as n tends to infinity as n tends to infinity the n of t the counting process it's a renewal process becomes a normal distribution in the mean t divided by mu and the variance is Sigma square t divided by mu cube where mu is the mean of inter arrival time as a t tends to infinity the random variable n of t minus t by mu divided by Sigma times square root of t by mu cube will be normal distribution with the mean 0 and the variance 1. So this is also a convergence in distribution.



Now we are going to discuss the long-run properties of renewal process. There are two types of long run properties. One is to obtain the long-run average of the quantity of interest. The other one is to obtain the pointwise limit. For example the long-run average of age that is limit t tends to infinity the integration 0 to t A of s, A of s is the age divided by t. while the pointwise limit of the expected age that is the limit t tends to infinity expectation of A of t.



One can study the average number of renewals per unit time in the long run. It is called the long run renewal rate. For a renewal process having a distribution F for the inter arrival times the long run renewal rate is nothing but limit t tends to infinity n of t divided by t. That will be 1 divided by mu with the probability 1 where mu is nothing but the mean of inter arrival time.



Since S suffix n of t plus last renewal time prior to t and S suffix n of t plus 1 is the first renewal time of t we know the relation of S of n of t with the S of n of t plus 1 and that lies, the t lies between those two renewal times.

You can divide by t you can divide by n of t. Therefore S of n of t divided by n of t less than or equal to t divided by n of t that is less than or equal to S of n plus t plus 1 divided by n of t. Now we can evaluate the first one the S of n of t divided by n of t limit t tends to infinity. That is nothing but the numerator is nothing but the summation of Xi's t n of t denominator is n of t and that is nothing but expectation of X in a long run – in the long run the summation of X1, X2 till X of n t divided by n of T that is nothing but the expectation of X that is same as the mu with the probability 1. And similarly one can evaluate the last value that is S of n of t plus 1 divided by n of t that is also will be mu with the probability 1. Since t lies between S of n of t and S of n of t plus 1 and the throughout divided by n of t in all three therefore as limit t tends to infinity n of t divided by t will be 1 divided by mu with the probability 1.