Mathematics in India: From Vedic Period To Modern Times
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Lecture-7
Aryabhatiya of Aryabhata-Part 1

So far we looked at the mathematical concepts that were available in some of the most ancient text. So Vedas and sulvasutras. The next 3-4 talks we will be focusing our attention on a single text called Aryabhatiya of Aryabhatta.

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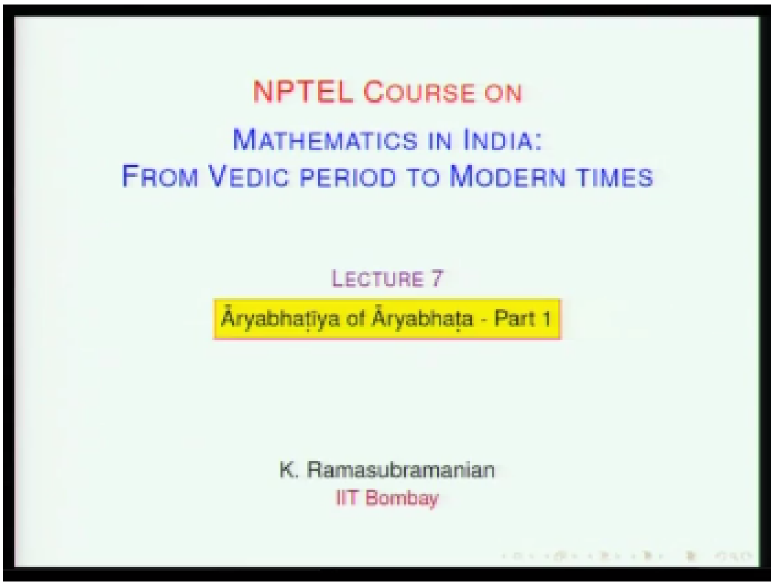

Aryabhatta who was in the later part of the 5th century and his text Aryabhatiya is one the most seminal text on Indian astronomy and mathematics.

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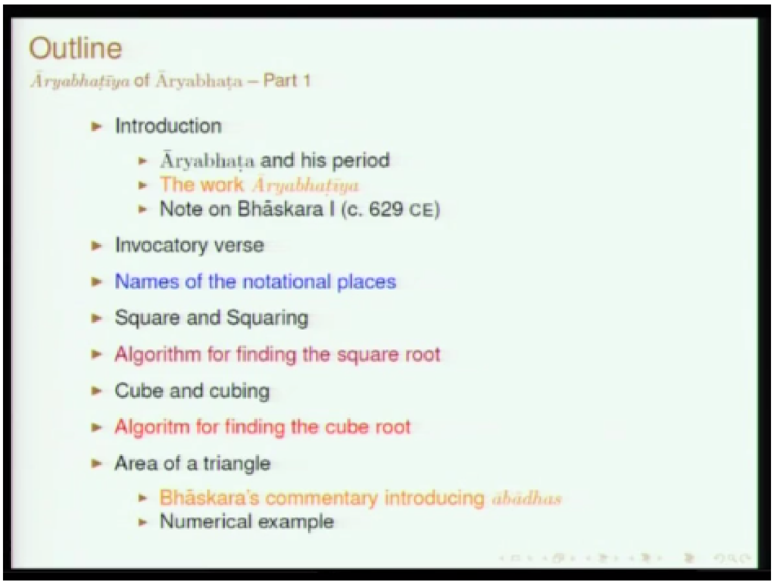

So we will start with the period of Aryabhatta and then we will have a brief description on the work Aryabhatiya. Aryabhatiya as I was mentioning so while discussing this decimal place value system as we compose in a very first time and therefore without it will be extremely difficult for us to understand the contents. So the earliest commentary that is available for this is one by Bhaskara.

So Bhaskara lived in the early part of 7 century and it is a very profound commentary on Aryabhatiya. We will see a brief description of where Bhaskara lived and what was his period and then we will move on to this text Percy. So Aryabhatta first introduces the notational places then he start discovers describing about the fundamental operations, square, squaring, finding out the square root, cube root and so on.

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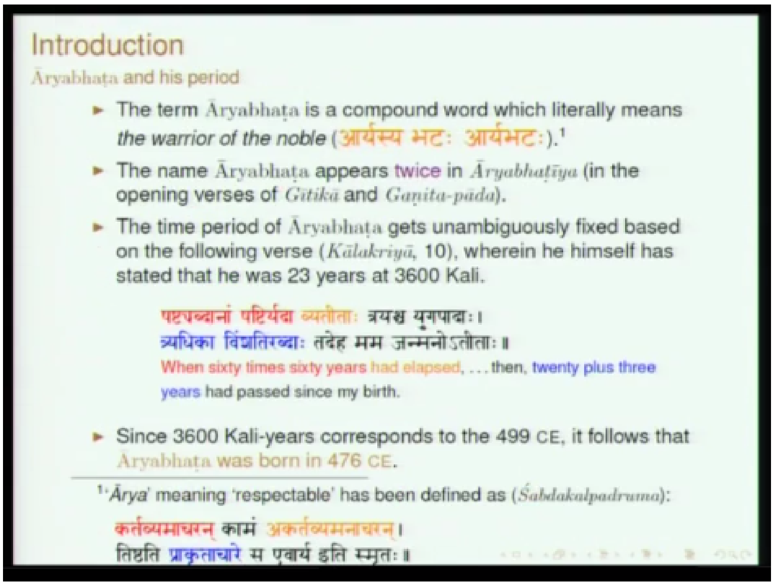

Then he moves on to various other topics. The text Aryabhattiya as such is composed in four parts, we will discuss all that in great detail. But before I move on to that I just wanted to say about the very name Aryabhatta. So (FL) Aryabhatta so this is how one can derive the word Aryabhatta. So bhata normally means a soldier or guardian and so on. So the term arya refers to a noble person.

In fact very interesting definition has been presented in one of the text called shabdakalpadruma. So that have noted down (FL) this is a very beautiful definition. So (FL) the one who performs what is to be performed by thing. So this is not sufficient and you also process in the negative form. So this what is called (FL) have to be done but I will also engage myself in a serious activities that is not acceptable.

And therefore (FL) that is acceptable in the society at that particular point of time, so the one who (FL) so the Aryabhatta cab be derived in 2 ways, so the one who protects that and one who save guardian and so on and so forth any way. So (FL) term can be derived and the very name of the text is based on the name of the author, so sometimes it just happens the text could be completely different (FL).

So here it is called (FL) so that which belongs to Aryabhatta or that which has been composed by Aryabhatta, so that is why it is called (FL). The name Aryabhata appears twice in the text, so which is not quite common in many of the text in the Indian literature, but here write in the beginning of Deepika Pada, which is the first section so he mention his name and again once more in the next section called Ganitha Pada.

The time of Aryabhatta is also pretty clear to us that is no ambiguity because there is a verse in III section (FL) goes like this, (FL) I was 23-years-old at a certain point of time, so that is mention in the first part of the sloga, so (FL) so when 3600 years of the (FL) I was 23-year-old, so when we go back and then find out so what was the beginning of (FL), so it just happens that these 499 AD.

So people are opinion that this verse has been composed in (FL) and therefore Aryabhatta was 23-year-old and he compose this work. So this actually clearly tells that Aryabhatta was born in 4676 AD, coming to Aryabhatiya as I was mentioning, so this is the text which is very clearly datable and it was composed in 499 and as far as we know this is the earliest datable text that is available in full form for us.

So they have been in earlier words in fact Aryabhatta himself towards the end the work he says (FL) he says I entered into the ocean of knowledge with the intellect as board and I lifted up gym. So which means I mean he is trying to point out that there has been earlier literature so from which he has carried out some important things and presented in this work. So this Aryabhatiya is available for us in complete form.

And the earlier works which were available for Aryabhatta have been shot of compiled by varahamihira in his text called panchasiddhantika, will come little later, more or less contemporary. So the systematically discuss the procedures for planetary computation and it has been composed in a very cryptic style and therefore some people prefer (FL) though it is in the form of verses it is in the form of Aryametre.

So the text is many times refer to as sutra itself and there are 4 padas as I said 4 sections Deepikapada 13 verses, Ganitapada 33 verses, Kalakriyapada 5 verses, Golapada 50 verses, in all we find 121 versus composed in arya meter which consists of the text Aryabhatiya and in fact yesterday you might have that Srinivas was mentioning that, so is also called as (FL) 108, so this 108 says it just drop out the 13 verses.

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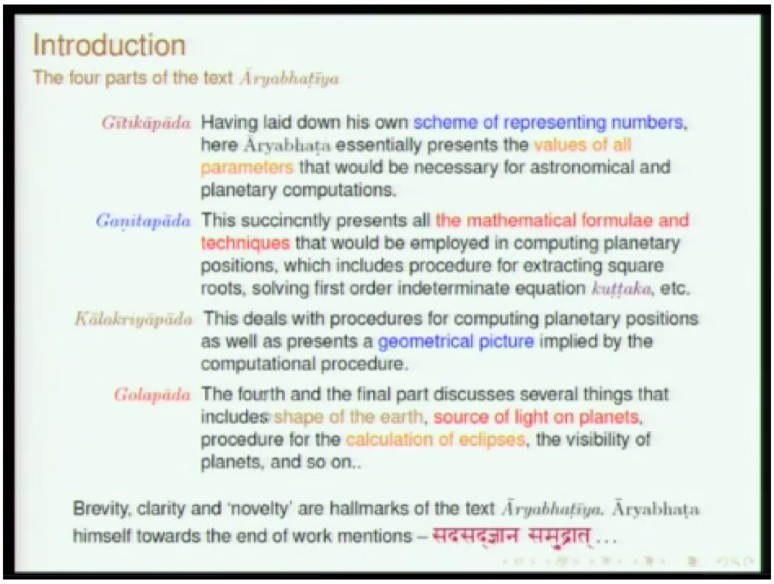

So it just happens to be 108, so we will see it little more in greater detail later, why this 108. So to give you an idea of what are the contents of this 4 chapter. So Gotikapada, so basically he lLays out all the parameters that are essential for doing astronomical computations in the to be described in the later part of the work. Then Ganitapada as a name itself implies discuss all the necessary mathematics that required for doing this planetary competition.

Starts with square, square root, cube, cube root and then it moves on to discuss the solution for indeterminate equations of first order and which is called (FL) which will be discuss the great length by professor Sriram and also myself with you later. In Kalakriyapada we primary find the geometrical picture and implies by the computational procedure, it has been discussed in great detail.

And in Golapada, the variety of topics which are discuss, so it is in fact almost occupy 50% of 50 verses in Golapada, so it discuss various details regarding the shape of the earth. So the source of light and planets calculation of eclipse visibility of planets and so forth. So these are the various topics which are discussed in Golapada. Before I proceed into the text Aryabhatiya I will say a couple of words on Bhaskara commentary.

Because I will be more or less dealing both of them together, so I am not going to take a Bhaskara separately. So we will see a brief note on Bhaskar. Bhaskar as actually return three major works one is Aryabhatta Bhartiya as I was mentioning, the other 2 are sort of independent works but Bhaskara describes them as Aryabhatta (FL) means so it a primarily exposition on what has been described in Aryabhatta.

That is how we describe, those who are called (FL) that these 3 words put together, so a lot of light on the kind of mathematical knowledge as well as the astronomical theories which were present around that period. So Bhaskara's time as we estimated to the 629 AD and his work actually displays a great amount of scholarship and it is really in intellectual peace to be his bhasya. So I had an occasion to go through in great detail in connection with this course.

In fact in one place he says (FL) so he refers to a certain number and you can take this as an exercise now, yesterday we discussed (FL) 0 and then agni (FL) Rama, so 3 Rama and therefore 3 and (FL) and so on you can see this. So based on the last four digits (FL) 1000 times (FL) itself is 4320000 years. So based on some analysis for it has been shown by (FL) that this corresponds to 629 AD.

Bhaskara was just about 140 years after Aryabhatta very close to the period of composition of Aryabhatiya. Then there are references in this work as well as the work of brahmagupta that Aryabhatta had many (FL) so one of the most famous one was (FL) refers to him in various places and also Brahmaputra refers to them ok. This was an introduction.

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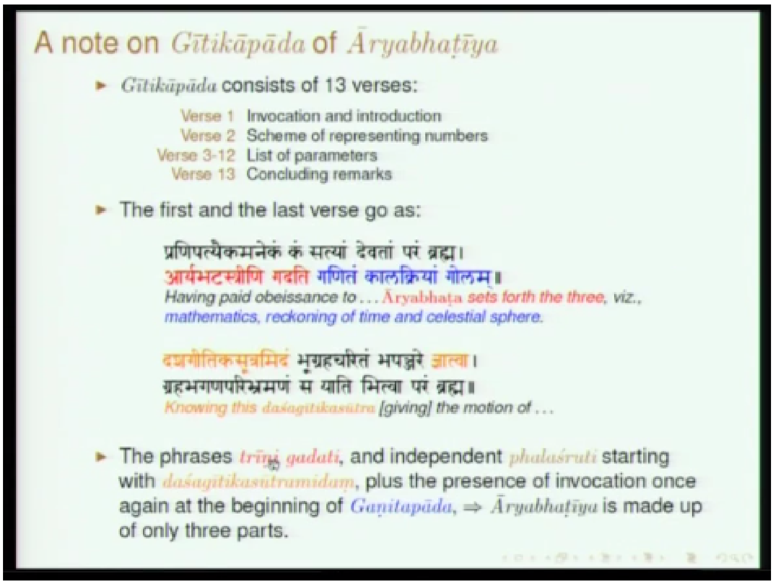

Now we move on to the text Aryabhatiya itself, so the first section Gitikapada consists of 13 versus as I was mentioning, so verse 1 is indicatory work and it goes like (FL) so we find a very clear statement where Aryabhatta himself says Aryabhatta (FL) states, so what does he state. So (FL) so this Gitikapada so which essentially present numbers certain parameters, so is considered to be out of the text in some set.

So the details of person so the numbers because which are required and the kind of find aa sin table etc. are all essential for doing computation, but they need not be integrated with the text perse and you want to understand astronomy. So it is in this sense I mean that it has been segregated out and the last verse goes like this (FL) this is called the (FL) so leaving out the first verse and the last verse and the kind of colours with all that.

She says so once a person knows this (FL) so it is a kind of (FL) but what is the point that I want to convey here was this. So this (FL) very clearly tell that the basic text of Aryabhatiya is leaving out Gitikapada and just consists of 108 verses and there is also a palace with the end in fact one find another invitation at the beginning of Ganitapada. So see invocation basically marks the beginning of the text.

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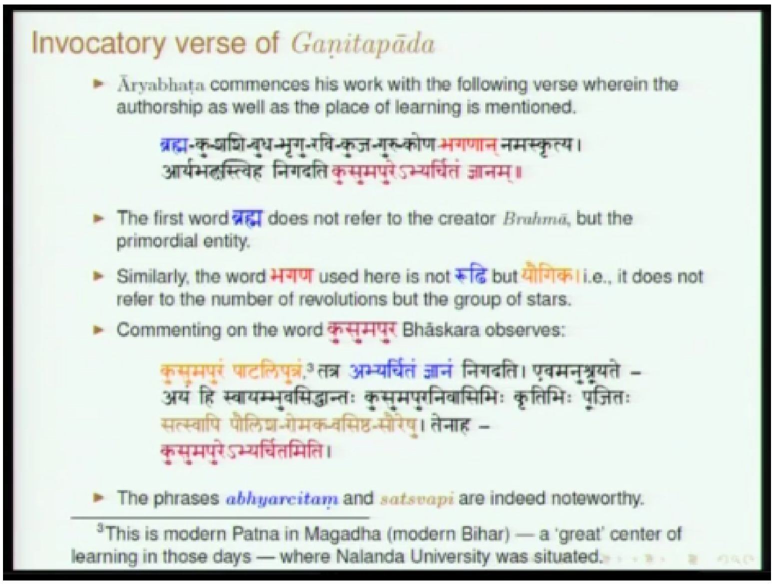

So we find 2 invocation, so one is completely separated out and then again in Ganitapada I will find invocation, so we have basically 3 part of Aryabhatiya, Ganita and Gola, what we are going to discuss here is only Ganitapada ok. The Ganitapada commences with this verse (FL) basically offering my veneration, my veneration to what (FL) you can easily guess, so these are the names for the planets.

So (FL) leave it out, who starts from (FL) refers to moon (FL) then (FL) group of stars constellations, so (FL) to all the celestial bodies and then I start this work Aryabhatta (FL) means states. States what (FL), so Aryabhatta does not playing that I am going to say completely everything out of my head. So it is not that (FL) so all that he says is (FL) so the knowledge which was highly rewired in place called (FL).

And going to narrate here, (FL) So commenting up on these word (FL) in Bihar and earlier it was called (FL) place of great learning where even Nalanda University was existing at a point of time. Aryabhatta says that I actually narrate the knowledge which was highly review in this place and so further Bhaskara says (FL) is being hurt, so what is hurt (FL) in fact here we find reference to all the five (FL) which has been compiled in ah (FL).

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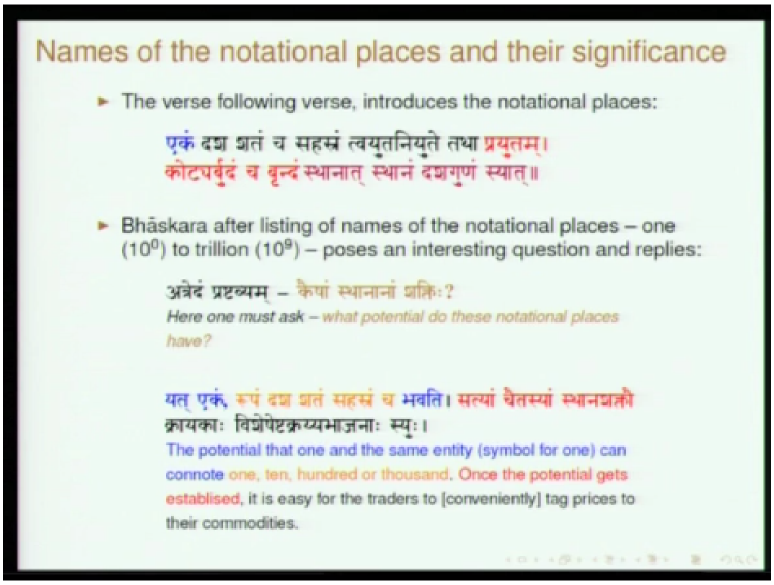

So he says (FL) means Bramha, so (FL) is Brahma Siddhanta, so we having other (FL) so by the people and basing on (FL) Aryabhatta has presented his work. After the invocatory verse the next work basically presents the notational places. So he says (FL) so the names of the various places. In fact this is extremely essential for even deciphering the number which has been given by Aryabhatta.

So he basically tells the means so (FL) 10 to the power 9, the names of the powers of 10 has been listed. There is an interesting discussion made by Bhaskara at this point he says (FL) what is so special about this Shakti means a certain potential. So when we declare something may be something it has been device by you but it has the potential to convey some meaning to you very important meaning.

So he asked the question (FL) so what is the potential that have been associating with this and what purposes, he says (FL). So finally what he is going to convey is so (FL) price something so you can just place it in that place and thereby he conveys something. So that has been something just been created by you it service a very useful purpose in the day today transaction. So that is kind of discussion that represents here.

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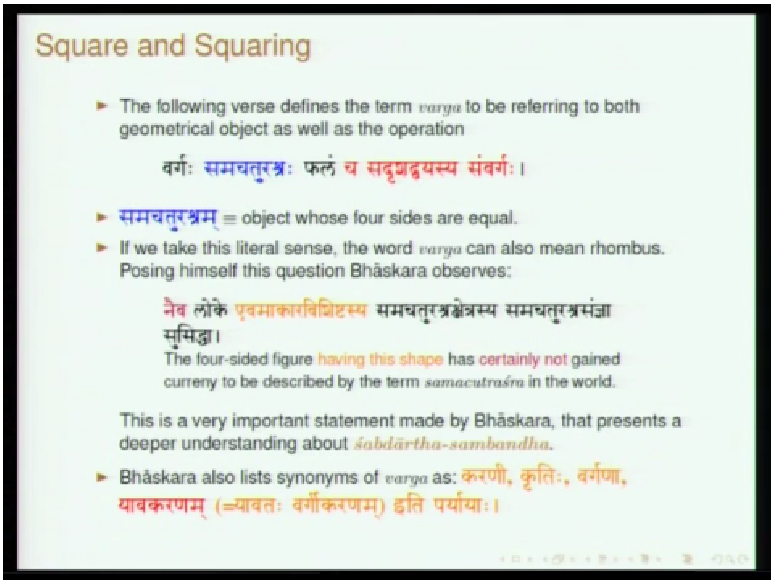

Then he moves on to describe the fundamental operations. So we start with varga and he defines what does the term varga mean, he says (FL) so one is a certain geometrical representation so we have square and we have squaring scaring actually refers to the operation and what is represented by this operation both are created here in this thing, (FL) 4 sided figure, (FL); refers to square (FL) then he says (FL).

So mathematically what does it represent (FL) 2 equal quantities, so the product of 2 equal quantities is what is represented by there and both are referred to as varga ok this, then he ask in the commentary Bhaskara processor interesting question. When you say (FL) so we can also think of a rhombus, so rhombus is also an object which has 4 sides equal. Why does the word varga convert rhombus.

So can it also convert rhombus or not, so this is the question. He says (FL) see in fact this is the very important statement in the sense that in order to understand the meaning of a particular word. So we have to go to the society to whatever sense it has been used by people, he send the world and therefore he says (FL) this particular shape rhombus has been never refer to be people by this word and therefore it does not denote Rhombus.

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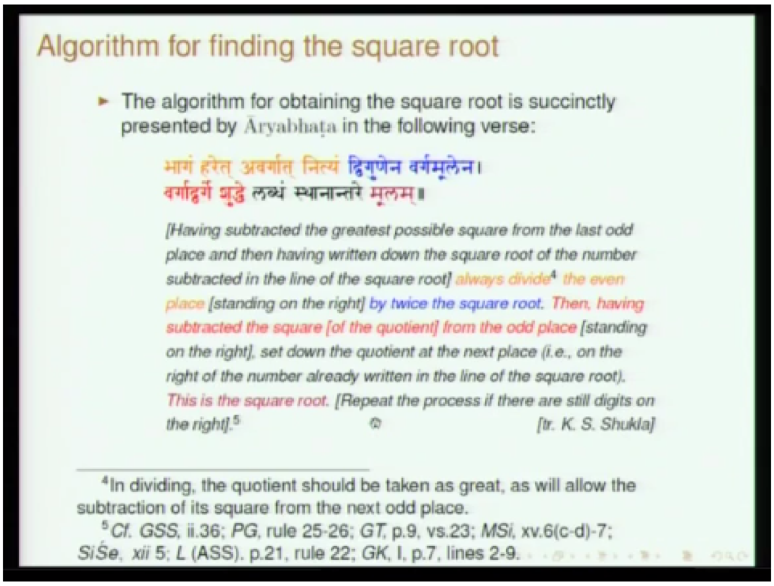

It has the potential only to convert square. Then Bhaskara also (FL) so all these are synonyms for square. Then I move on to describe this algorithm which has been presented by Aryabhatta to extract square root ok, varga refers to square, vargamula refers to square root. So the verse goes like this (FL) in fact yesterday while I discuss to the Aryabhatta systems of representing numbers I said (FL) right.

So (FL) can refer to the square and when we think of the powers of 10, so 10 to the power 0, 1 is a square number, then 10 to the power of 2 is a square number. So similarly in describing the operation which has to be done to extract the square root he clearly states, when we have a certain number which has been given to you, you have to first of all break that into 2 units. So one is the varga part and the other is the (FL).

So in understanding this verse, this has to be kept in mind, so (FL) here refers to that which is 10 to the power of 1, 10 to the power of 3 and so on. So this is called (FL). So the (FL) he says so the first work has to be understood the Vanga which is the square of some number, (FL) subtracted. So the process will be very clear, but I just try to make certain terminology clear before we show a certain example.

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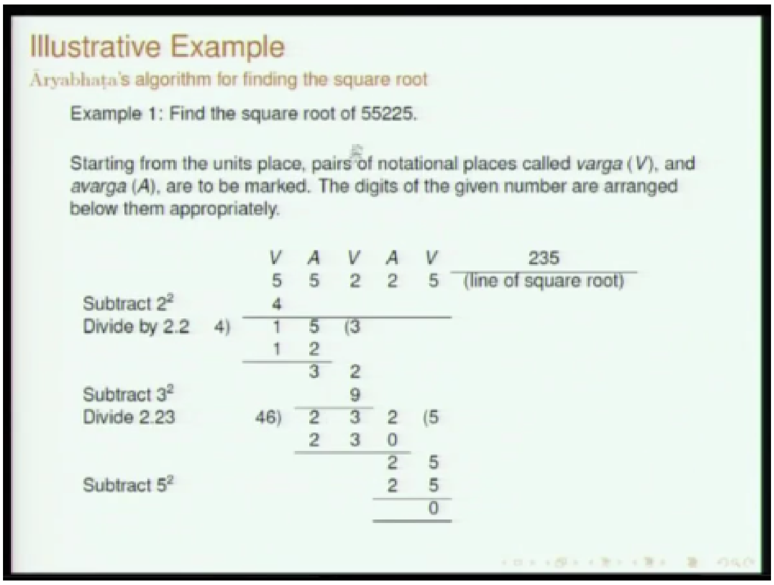

So let us see this example now. So here we have a number 55225 and this has to be written down like this putting varga, avarga, varga, avarga, varga, avarga. So this is what he means by in the sloga varga, avarga and so on. And the procedure to be adopted can be stated in 2 steps then he will move on to the example. So in Bhaskara commentary what we find is (FL) so he says (FL) divide ok.

And he says (FL) so whenever you encounter a certain (FL) thing, so all that you need to do you have to do a certain division and whenever we find this varga you move want to replace all that you need to do is (FL) means you have to remove a certain square. So we will see the algorithm and the basis of the algorithm 2. So in fact Bhaskara very says (FL) a certain number 54321, so this (FL) even place. So this is odd.

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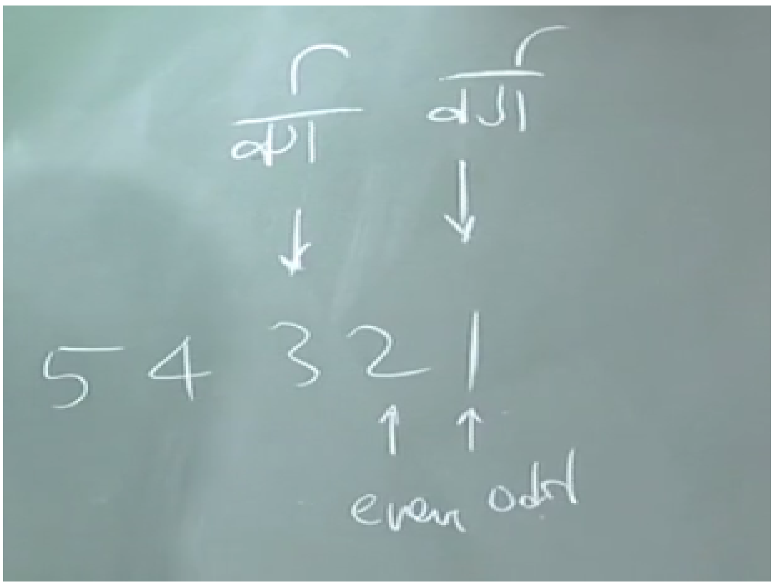

So all the hot places so they are referred to as varga, so this is varga. So all even spaces are referred to us avarga. So this is basically from the fact where the 10 the power of 0, 10 to the power of 2, 10 to the power of 4 and so on. So basically this has to be divided into two, so one is (FL), why is it done. So basically a single digit, so if you take the square of it also can be only two digit therefore, so depending upon the number of digits.

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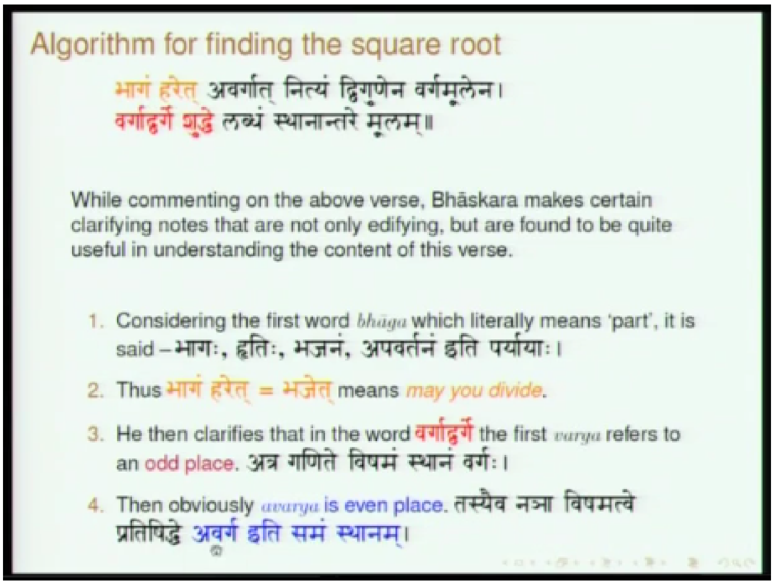

And number which has been presented you first make a guess of how many digits in the square root have. Yes so this is this classification separate into two, so (FL). So the algorithm essentially has 3 steps. So what is states is starting from the least significant digit, group the digits of the number into two. So the first thing that needs to be done is what I was saying. So some the least you just group them into two.

Then from the remaining part the most significant digit, so which constitutes the (FL) so this is pretty evident, so this is (FL) so then this basically is (FL), so whatever be the most significant digit it can be one or it can be 2. So that is just taken in the beginning and so which constitutes the (FL) subtract the maximum square that is possible. So having done that (FL) so long with the reminder, so you bring down the next digit of the (FL).

So this is (FL) then once you remove the maximum square that is possible for remove, for instance when you have 5 to the maximum square that can be removed is 2 square. So one will be remaining and 4 will be remaining in the next step you have to start your operation was 14. So along with the reminder bring down next digit from the (FL) this has to be divided by twice the (FL).

In fact this has been stated (FL) determine here at the first place, this has to be done at every states (FL). So this (FL) so at this stage if you deal with the first two digits or 1 digit so you have to jot down the (FL) whatever you get. So suppose you have 5 by two. So in the next stage so when you do certain operation you will add one more digit to this and that should be considered as (FL) at that stage.

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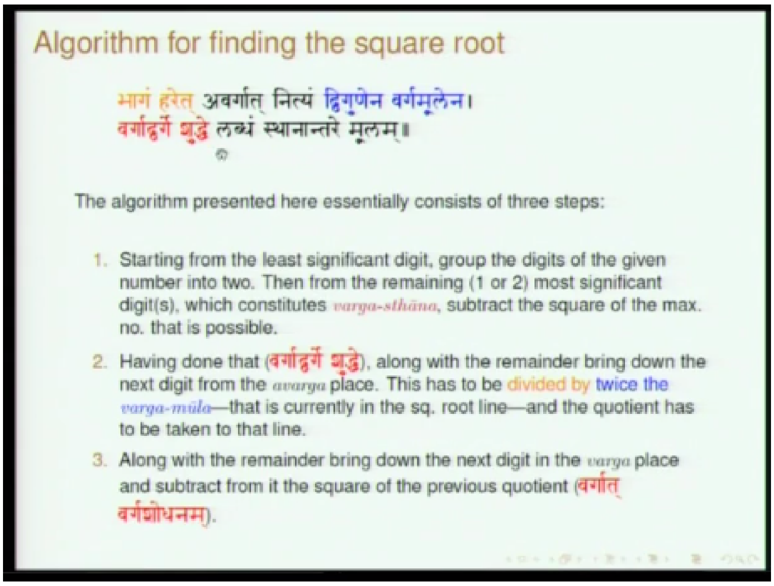

So things will become clear when you look at the example (FL) basic operations 1 is divided by twice that and then remove the square. These are the 2 operation with have to be repeated that for the entire square root of the number. So now we go to the example.

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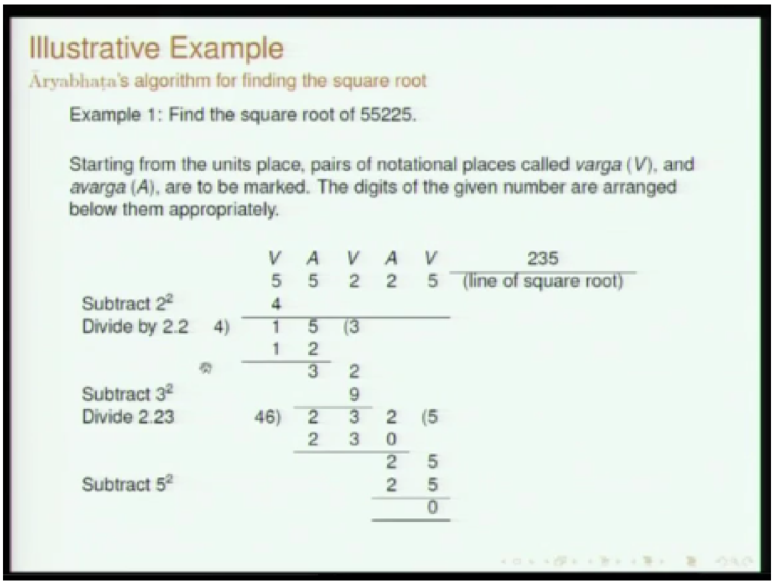

So let us take this number 55225, so we have group in this (FL) and then we have this (FL), so the first things that is to be done is remove the maximum square root that is possible, so 2 square can be removed, so you write 4 and then take the number 2 there. So this is the (FL) so you just keep it somewhere and so 1 is a reminder here, then bring down the next number from the (FL) plays, what you have is 15.

So the operation the stated is (FL) so you have to divide by twice the (FL) so two times 2 at this stage what you have is only 2. So 2 times 2 is 4. So you have to divided by that, so what to get is 3, put 12 and the remainder is 3. Now the next digit has to be brought down 32 is a number that is available. So the (FL) operation is over. So the next operation that is to be done is (FL) taken the (FL) number down.

So (FL) you have to remove the square, the square of what square of the question that you obtained in the previous place, therefore you remove 9, so (FL) so what you have is 23 here and 23 were bringing down (FL) so the (FL) place is got down, so (FL) at this stage the (FL) that you have is 23, (FL) is 2 times 23, so you have to divide by 46 ok. (FL) so what you get is 5, (FL) and then reminder is 2 here.

So you then bring down (FL) at this stage so what you got was 5, so (FL) so you have to remove the square of these and 25 and the remainder is 0. So this is basically the operation of extracting square root at the same way Aryabhatta. So let us take one more example.

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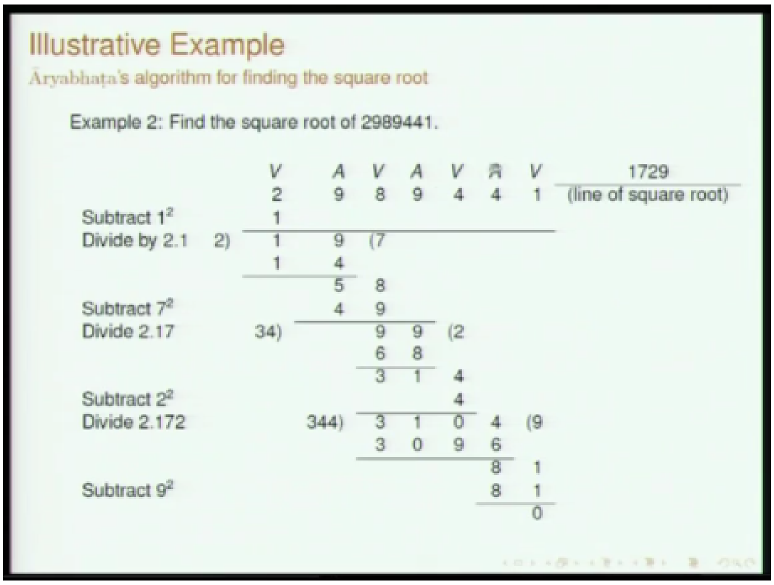

So we have to give into two, so 41 then we have 94, we have 98 and what remains is 2. So what can be extracted out is only one square to remove that you write it 1 and then the next step you bring down the (FL) number, so 19 and twice the (FL) at this stage, so you have to take 2*1/that. So here you can actually have the greater number pulled out, but then keeping in mind that the next step.

You do not get a negative number okay, so if you do that then you have to revert that and then you will be able to do that. So here so what you get is 58 and so what has to be removed here is 7 square (FL) to remove 49, then again you do the same operation twice the (FL) 2 times 17. So you can see that (FL). So these are the 2 operations which have to be repeated.

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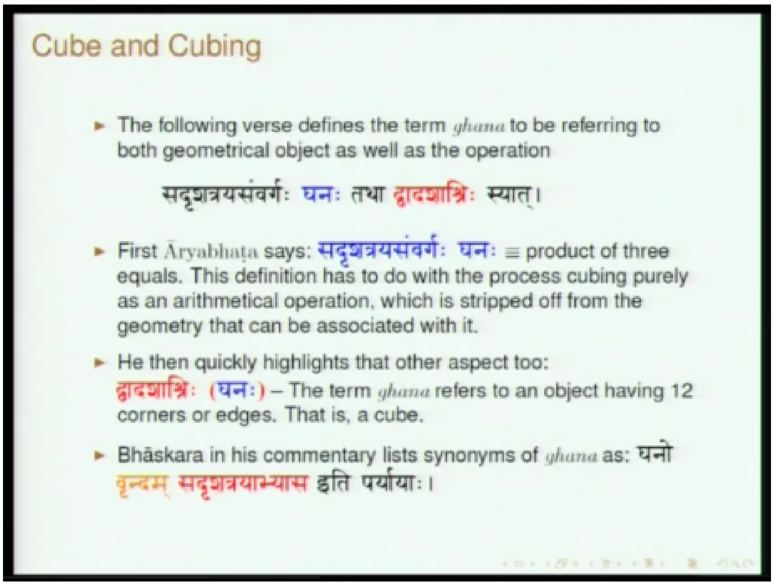

After describing the extraction of square root he moves on to the square, cube, cubing and how to find cube root, so this is the next operation. So (FL) understand clearly lay down the procedure for extracting cube root ok very clearly lay down procedure, so defining what is cube, he says (FL) product. So you have quantity multiply by 3 times, so that gives what is called (FL) similarly. So geometrically what it represents, so a cube what says (FL).

(FL) is an object which has 12 not size, so it is the (FL) kind of line (FL) so both have been nicely stated and (FL), so this is what has been stated by Bhaskara in fact they sometimes (FL) basically a product (FL) is a product of 3 quantities which are one in the same. So this is what is cube, and what is interesting note is, so the Aryabhatta definition of this cube scripting out of the geometrical thing, as quantity you protect the product price and you get this.

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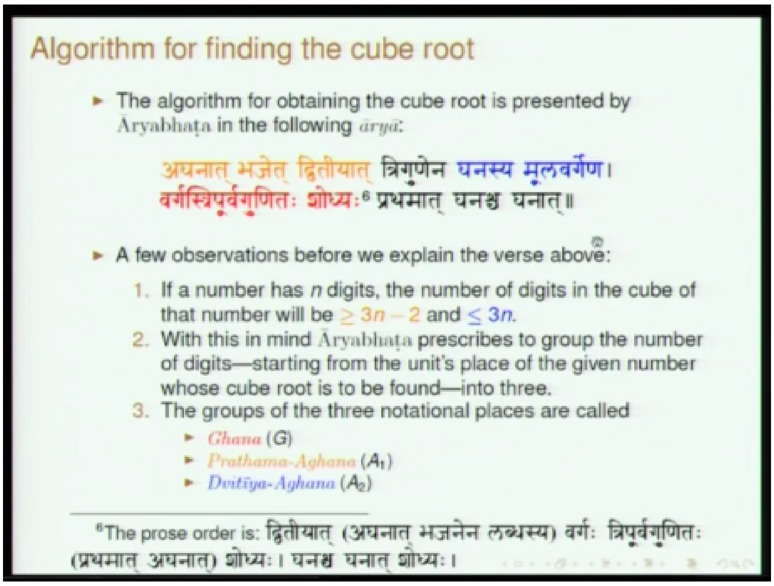

So this is a sort of abstraction, so beyond what is this is a geometrical shape operation has been again beautifully and concisely describe in one word, so he says (FL). So this is the procedure for extracting cube root of a number, see in case of square root so I said you have to break the number given number into 2, 2 digits, so you can usually guess that in case you are extracting cube root you have to break them as units of 3.

So these units of 3 have been assign a certain nomenclature, so there he call it as (FL) so here he uses 3 terms and they are (FL) the name that he uses. (FL) so this has to understand (FL) so from this second (FL) you have to divide ok. So what is to be done (FL) there he said twice the (FL) here thrice ok, so thrice of the (FL) means the (FL) that you get at that stage ok. Then (FL) is be subtracted, so what is to be subtracted.

(FL) so 3 is number 3 and (FL) is previous number so (FL) means 3 times the previous number (FL) you have to subtract, so this has to be at the (FL) and when you come to the gana place (FL) so there from the varga place you have to remove the varga, ok. So here at the ghana place you have to remove the Ghana the cube of something ok. So all that will be clear with example, but the terminology has to be very clear when we read (FL) see this understanding the algorithm will become quite evident.

The moment we understand that the cube of a given number can of course of (FL) ok, if you have three digit number 6 digits, so here it can be out most 3M, and it can be it should be definitely greater than and less than or equal to 3m. So with this is mine so Aryabhatta asked us to divide into groups of 3 and then carry out the operation.

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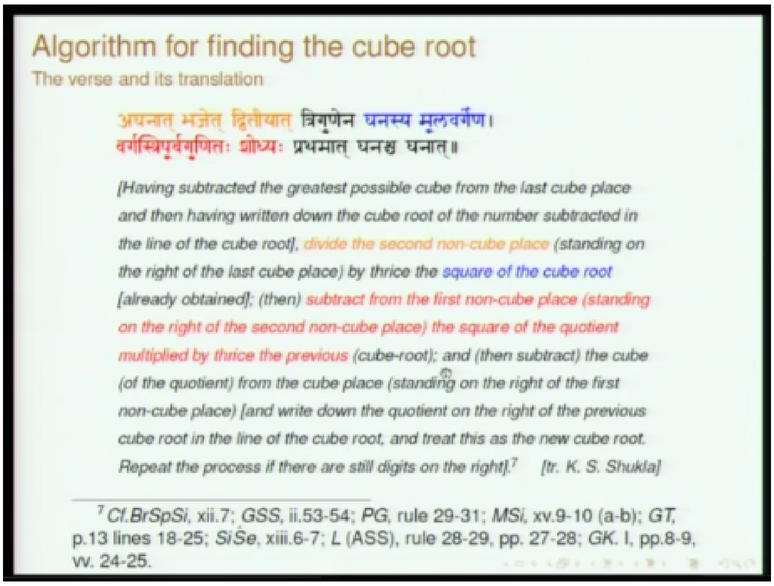

So I leave this is basically translation the work and the essential steps involved I just show this and then I go back to this previously slide, here we have this number 1771561, so the grouping I think I have to put a coma here. So here Ghana, Ghana1, Ghana1, Ghana Ghana1, Ghana2 and whatever remains here should be considered as Ghana ok, so group of 3 start from the least significant and whatever remains with it is one two or three.

That will be considered as Ghana in the most significant place, so in this example we have a Ghana with one number and the first thing that has to be say all that he says he is this possible to be repeated, the process of his stage is with (FL) operation with (FL) have to do operation, (FL) other operation and this operation has to be related the number gets over, so this is the process. We get a square also he said, so with the reference to (FL) do an operation.

There is (FL) place you are doing, this operation has been repeated and you will get the square root, so as an algorithm so what are the steps involved, starting from the units place, having grouped the digits of the number into three, from the remaining 1, 2, or 3 the most significant digits, so the digit is called (FL) there he called (FL) period is called (FL) subtracting maximum soon as possible this has to be done.

So this digit actually forms the most significant digit of the cube root, so then along with the reminder bring down the next digit from the (FL) so once you do this operation this (FL) this number has to be brought down (FL) so this has to be divided by thrice the square of the (FL) obtained so far, in fact if you look at the verse (FL) ok.

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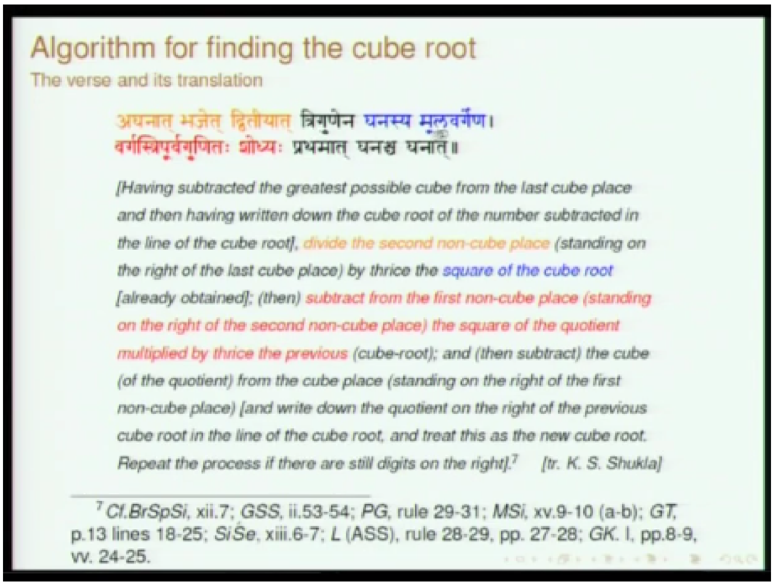

So (FL) multiply by 3, (FL) that for it has to be understood ok, so the (FL) whatever you have written divide thrice the square of the (FL) obtained so far. So the portion forms the next digit of the cube root ok, so this is operation. So whatever portion that we get here that forms the next place to the cube. So along in the remainder again the next digit (FL) has to be brought down, and at this stage so all that he says is (FL).

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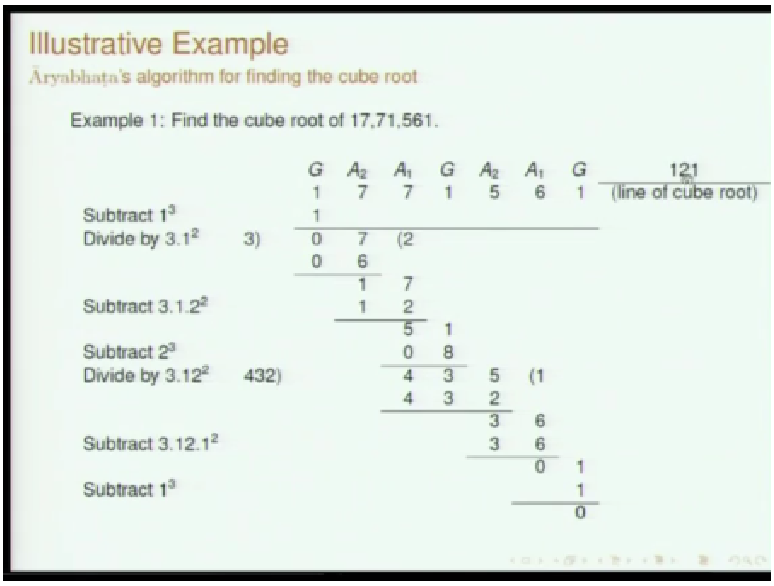

(FL) is the square of this, so whatever we obtain 3 (FL) so 3 is 3 and (FL) is the previous number ok, so in the cube root that you do whatever be the (FL) number the previous number, so (FL) second digit of the cube root, and previously have got one number. So all of he says is 3 times the previous number and the square of this ok (FL) that should be the thing which has to be subtracted (FL).

So this is the prescription for (FL), so we (FL) so this the operation that has to be repeated (FL) so this is all the prescription and the process has to be repeated ok.

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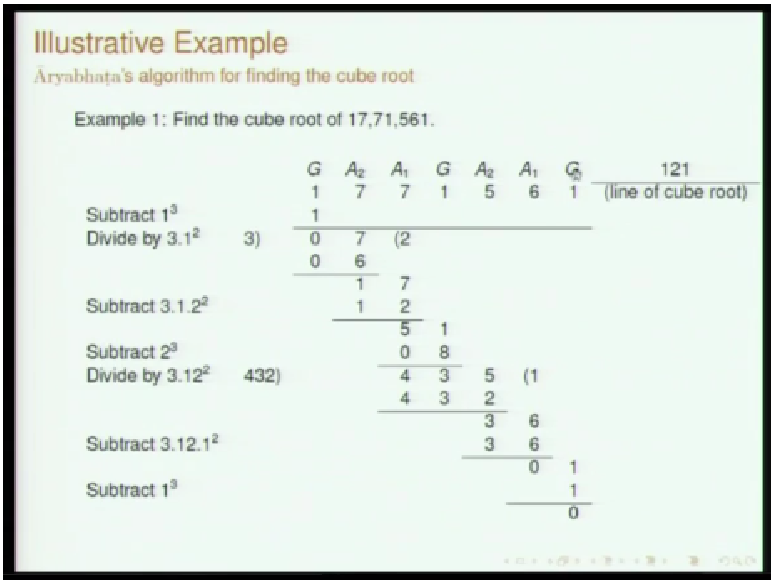

Now let us look at this example, so we have this Ghana place 1, so the maximum cube that can be removed is one cube, remove that we have 0, so you places 1 here and then the next digit the second (FL) that has to be brought down. So here the operation (FL) the square of that (FL) so this has to be divided. When you divide this (FL) 2 whatever you get has to be taken as the next digit in this place.

And the remainder is 1 here to bring down this and this si (FL) and the operation is (FL) so 3 times and (FL) number 1. So at this stage 5 as remainder then you bring down the Ghana place, at the moment you come to (FL) place you have to subtract the Ghana, Ghana of the previous digit, so 2 cube has to be removed from here and you remove so you get 43 here then again repeat the same operation with (FL) ok 3 times 12 square ok.

So you get 1 and then again (FL) 1 square of the last to determine 3 (FL) is 12 here now ok, at this stage when you got 2 the (FL) was 1, so when you move to 1 the (FL) is 12 and the (FL) so it will be 2.

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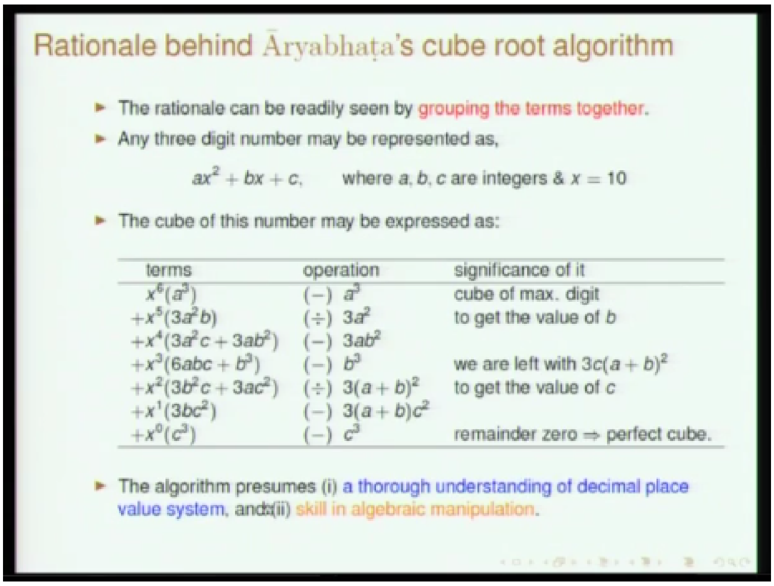

Now let us see what is the rationale behind this procedure which has been given by Aryabhatta, this is straight forward for us to see, so consider a 3 digit number. So this three digit number can be represented as axsquare+bx+c ok x represents 10, fine the cube of this number axsquare+bx+c will have these terms, so we can group them out, it is clear, so that I have done is cube of this and then I have thought of group to them ok as powers of x.

So the maximum thing is going to be so a cube and the this to be multiplied by x to the power of 6, here this is the largest term and coefficient is going to be a cube, when we consider the next x to the power of 5, so the coefficient will be 3 a square b, so then you do x power 4 this will be the coefficient x cube will be the coefficient and this is how the cube of this number can be written down.

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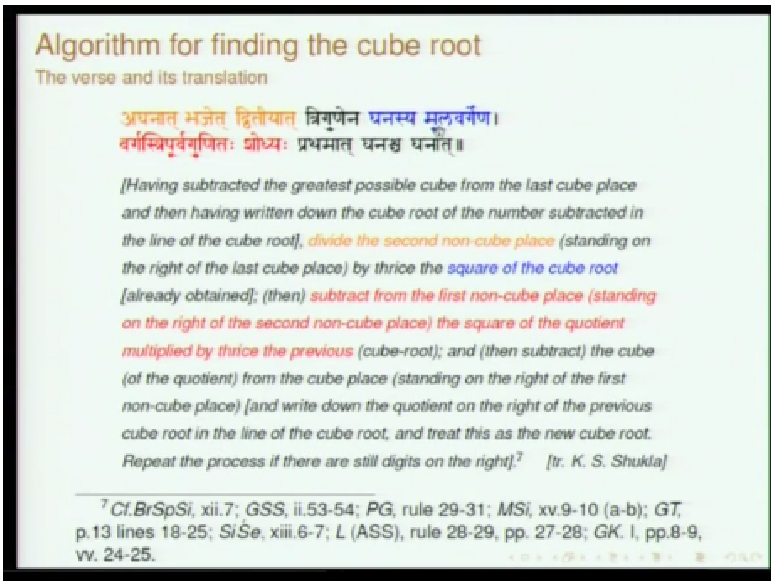

When you look at the operation which has been given by Aryabhatta, so all that he said was see when you have the maximum digit you have to remove the maximum cube that can be remove (FL), so you subtract this, so this is what is operation cube has the maximum digit it has to be done. Then in the next stage all that he said was (FL) you have to divide, see we know the cube of this number.

And what you want to find out is basically ABC, so what the first state to determine a, then you in order to determine b all that you have to do is you have to divide by 3a square. So there is a operation which he says (FL) so you have to do that. So you will basically get b. at this stage so what needs to be done if you have to remove 3 ab square in order to get this ok. So this is basically the principle behind and this process has to be completely repeated.

So at this stage see you have to get c, so then you have to do this 3 times ab square, so this is the basic algebra which explains the process of extraction of cube root as described by Aryabhatta. You can easily see here 3 times a square is a first thing, then (FL) see (FL) is one operation, the other operation is (FL) subtracted, so here (FL) divide, the third operation is (FL) again you have to understand the (FL).

So (FL) 1 division and 2 subtraction, so that you what we can see here, so we have this subtraction in the Ghana place, subtraction in this place, (FL) so this is repeated, and once it is done you have be able to get the cube root of the given number, ok this has been repeated. So at this stage since you are applying a and b 2 things have been obtained, so (FL) a+b and (FL) is c, do (FL).

So this is the operation which has been described, from a different view point if you look at the algorithm (FL) through understanding of the decimal place value system the other way it seems to be impossible for anybody to describe an operation in so systematically by which you will be able to extract the square root and it also sort of indicated as to how they have been able to do this out of algebraic manipulation.

So if Aryabhatta has to given the algorithm, so he should have analyse so very clearly so the process which goes in humming and the way of extracting 3 cube root is the reverse process of it which is what has been presented by Aryabhatta in this beautiful world (FL) so more about Aryabhatta in next lecture, thank you.