Mathematics in India: From Vedic Period to Modern Times
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Lecture – 39
Mathematics in Modern India 1

So, today we will this first lecture on mathematics in Modern India. There will be two lectures covering the modern period roughly from 1750 to the present time. As you can see it is a long period and a lot of historical records and lot of events have happened, so we will have to focus on important things in this period.

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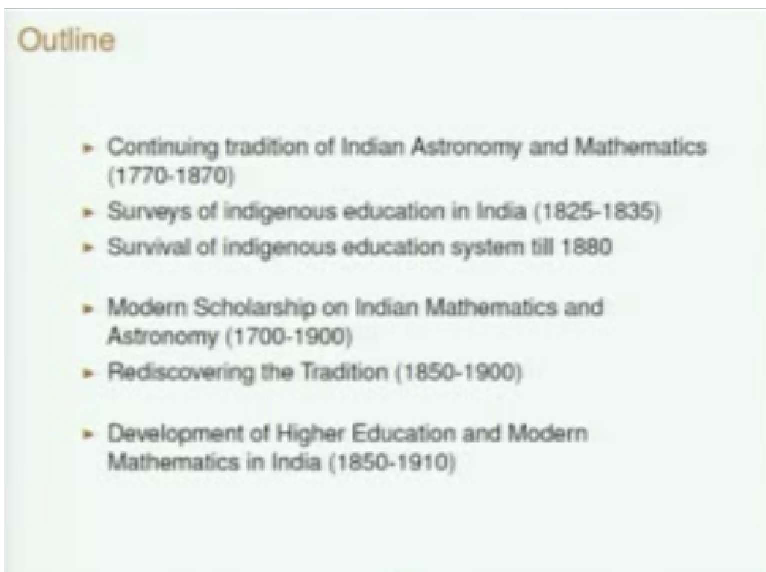

So, I will first say something about the way that the indigenous tradition in astronomy and mathematics continued late into 19th-century. We tell you something about the nature of indigenous education system in India in early part of 19th-century before the British started the English education system here and how that system survived of course very limpingly till the later part of 19th century.

Then, I will say something about the Modern European scholarship on the Indian tradition of astronomy and mathematics during the two centuries, 18th and 19th centuries and then how Indians themselves participated in this rediscovery of tradition in the latter part of 19th century. Then, of course this new system of education that was started in the middle of 19th century, how did that develop, but the main part of today's lecture, we will look into the life and work of Srinivasa Ramanujan and its impact today.

So, till about 50 years ago most modern scholars most textbooks had Indian mathematics died with Bhaskaracharya II around 1150. Later people only wrote a few commentaries or few works here and there. As we have seen in detail in this course very interesting was then in the post Bhaskara period. We have discussed Narayana Pandita. We have discussed Kerala scholar Madhava, Parameshvara and Nilakantha.

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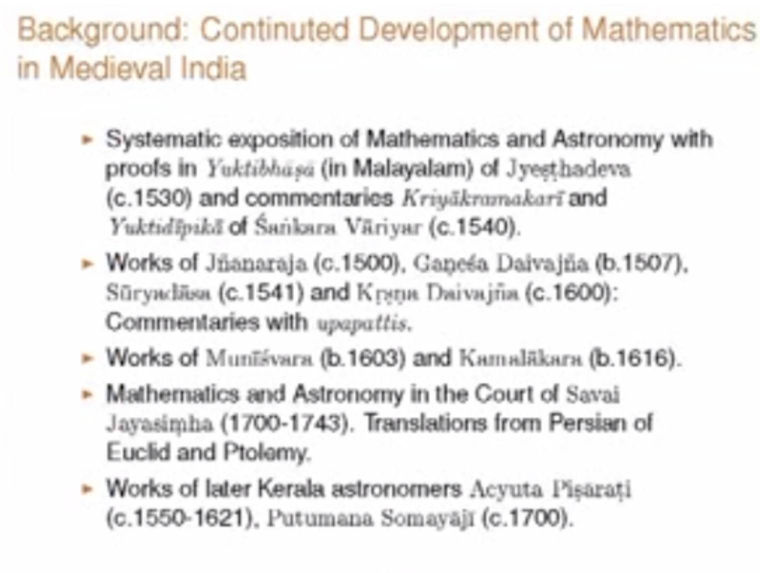

We have given you some (()) (02:11) of the work of Krishna Deva gian, Ganesh Deva gian and given the trigonometrical work of Kamalakara. We have of course not discussed the astronomical efforts of Sawai Jai Singh of Jaipur.

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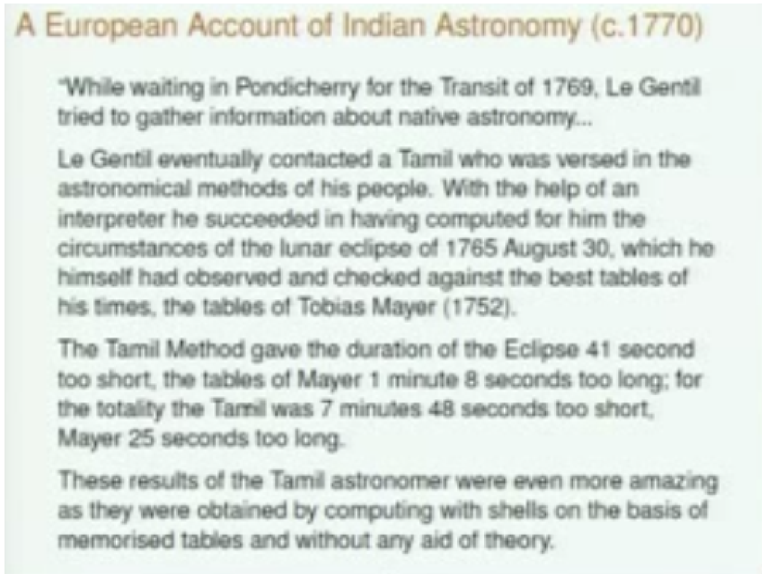

Now, in about 18th century, there were many European observers who came to interact with the traditional Indian astronomers and mathematicians and here is one such Le Gentil was deputed to observe the transit of Venus in Southeast Asia and he went to Pondicherry which was a French enclave and he got an eclipse in 1765 computed by a native astronomer as they call and compared it with the best tables that they had and what Gentil found was that the Indian calculation was 40 seconds too for the duration of the eclipse that Tobias Mayer table gave 1 minute 8 seconds too long, this in 1765.

For the totality the Tamil method was 7 minutes 48 seconds too short, Mayer table was 25 seconds too long and Le Gentil said that this was even more amazing because this Tamil astronomer was computing with cowrie shells, with no tables on hand, on basis of memorised tables.

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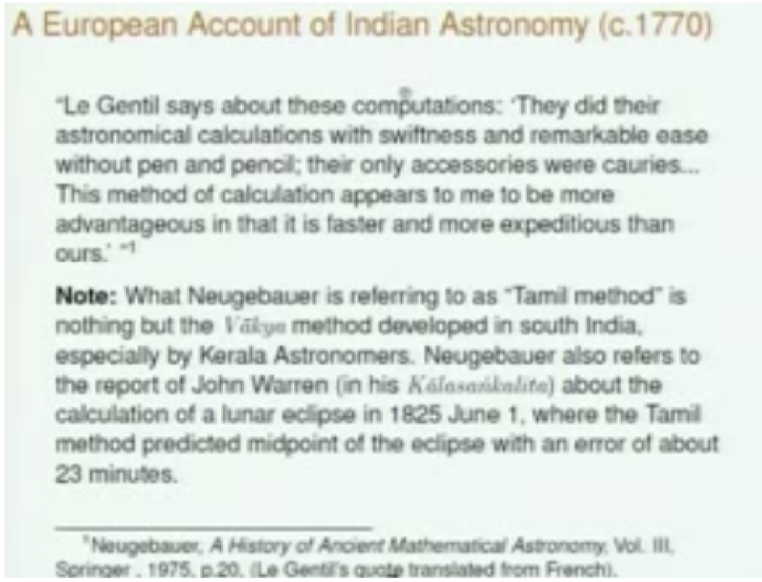

They did their astronomical calculations with swiftness and remarkable ease without pen and pencil. Their only accessories were cowries, that is the shells, cowrie as they call in Kannada. This method of calculation appears to be more advantageous in that it is faster and more expeditious than ours. Now Neugebauer who is calling this Tamil method and all that is actually referring to what is called Vakya method of calculation which was prevalent in most of South India, especially developed by the Kerala astronomers.

Neugebauer (()) (04:11) drawn the above quotation also talks of John Warren's book where he is talking of again in Pondicherry an eclipse being observed in 1825 and the error is about 23 minutes of the calculation.

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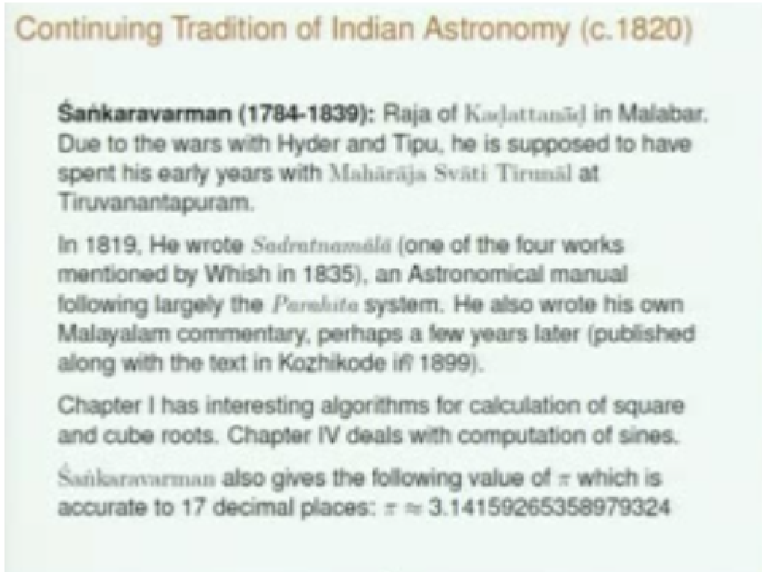

What I was saying is that the traditional methods of calculation were fairly efficient and continued late into 18th and early 19th century and the results were reasonably comparable with whatever was available from the best available tables of contemporary astronomy.

Two important astronomers Sankara Varman in early part of 19th century. He wrote Sadratnamala. It is a (()) (04:47) text in astronomy based on the Parahita system. He also wrote his own Malayalam commentary. There is some interesting discussion of square roots and cube roots. Chapter IV deals with sines and Sankara Varman also gives this value by pi to 17 places that I mentioned.

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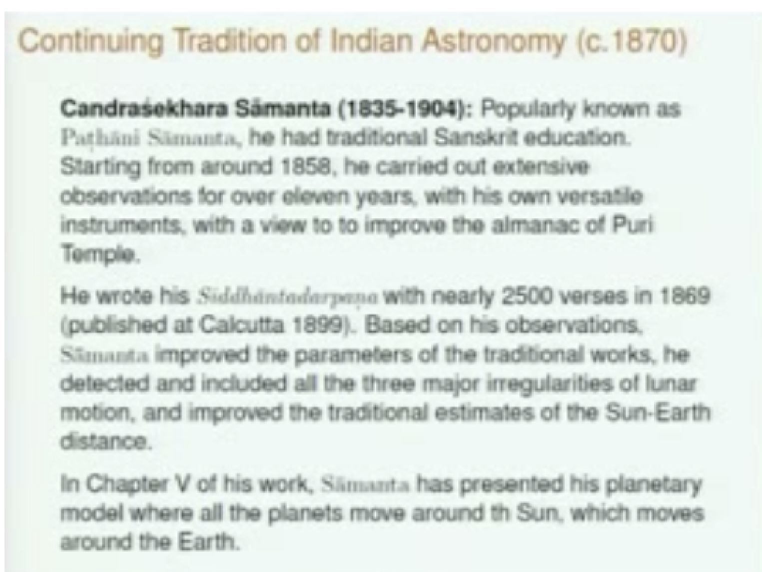

Another important astronomer who continued the indigenous tradition was Chandrashekara Samanta in Orissa called Pathani. He wanted to improve the panchangam of the Puri Temple and he wrote this massive work Siddhanta Darpana. He was all traditionally trained and was not in modern education system in 1869. Based on his observation, he improved the parameters of traditional works.

He detected and included all the three major inequalities of lunar motion and he also improved the traditional estimates of earth-sun distance and in chapter V, Samanta presented a planetary model in five planets go round the sun which was around the earth something like Nilakantha's planetary model which is called an iconic model based upon the name of Tarik Ibrahim.

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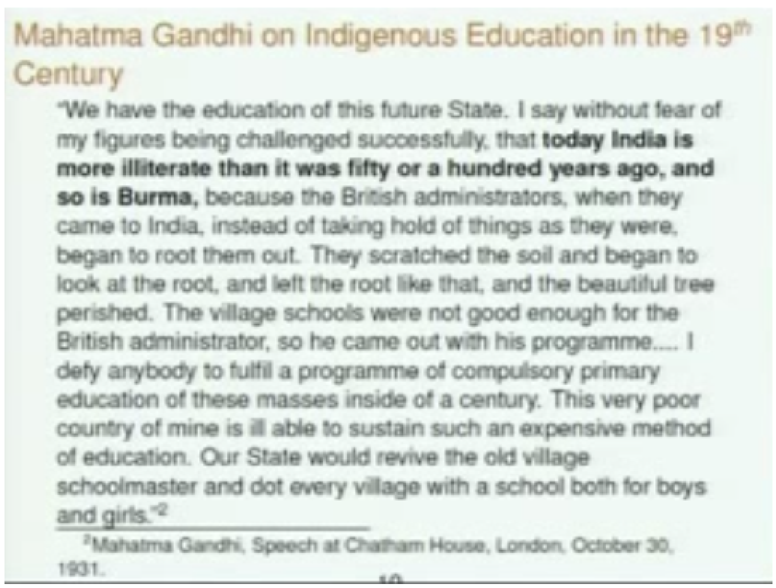

So, there were sort of important astronomers and mathematicians in 19th century but as Mahatma Gandhi declared in 1931 that the great de-education of India which occurred in 19th century. Mahatma Gandhi in a famous speech when he goes for the round table conference in London, he is the making a statement that it was in 1931 India is more illiterate than it was 50 or 100 years ago and so is Burma and that is because the British administrators came and disrupted the indigenous system of education which he calls as a the beautiful tree died because they dug up the roots and left the roots exposed.

What is this indigenous education system in the earlier part of 19th century. So, most observers who visited the administrators who stayed in India reported that almost every village here had a school.

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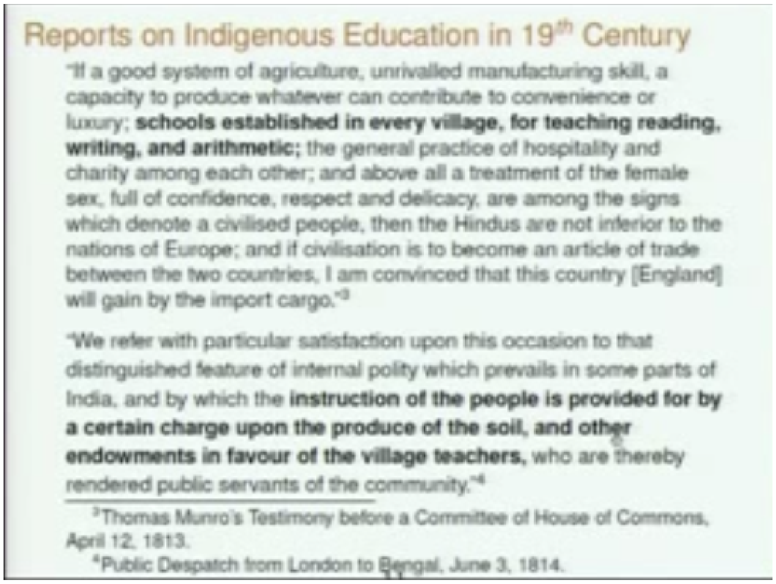

So, here is Thomas Munro declaring 1813 that schools established in every village for teaching, reading, writing and arithmetic or here is a dispatch from the British government in London to Bengal where they are noting that the instruction of the people here is provided for by a certain charge upon the produce of the soil and other endowments in favour of the village teachers who are thereby rendered public servants of the community, something which can hardly be said today that no government servants receive their pay either from Delhi or from Fort St. George and they are not servants of any community.

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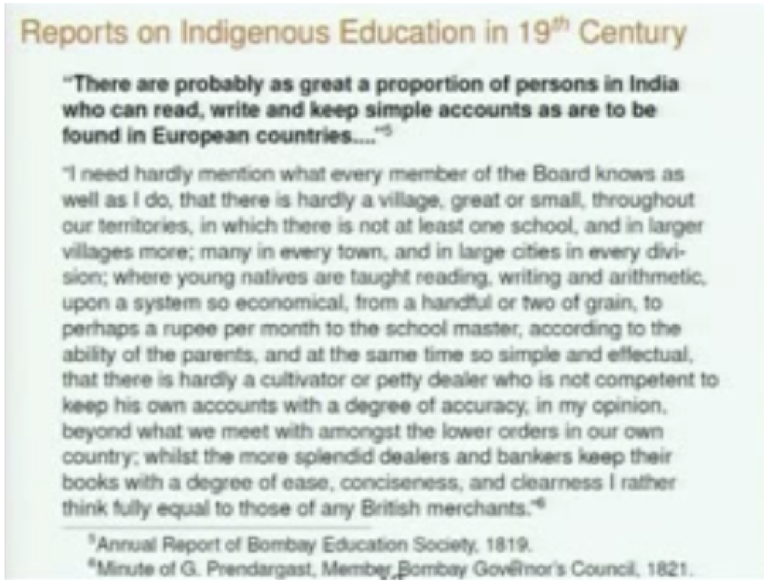

In 1819 Bombay, it was said that there probably as greater proportion of persons in India who can read, write and keep simple accounts as are to be found in European country. So, this is the kind of accounts that administrators and observers were talking about an education system about which the Britishers had nothing to do. They had not started their education system and they were not providing any money for what was there.

And this was a sort of maintained at the societal level, perhaps at a much more sort of (()) (08:07) state than it would have been when the state in the actively promoted and supported in earlier periods.

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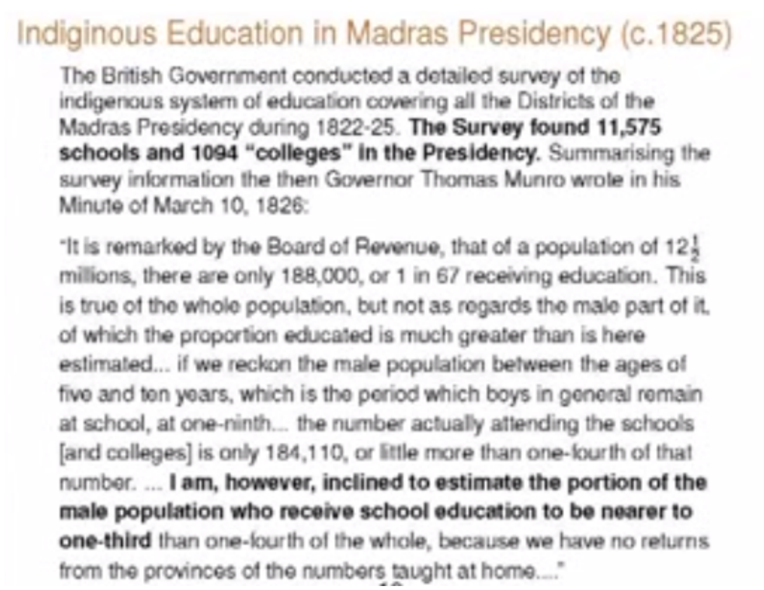

So, to look at this system, various surveys were conducted in the end of first part of 19th century. The survey Madras Presidency went through all the districts in detail and it found in 1825 that there were 11,575 schools and 1094 colleges in Madras Presidency which includes coastal Andhra, whole of Tamil Nadu, northern Kerala and the Ganjam district of Orissa and the Bellary district of Karnataka and Thomas Munro concluded that something like one-third of the boys of school going age are being educated under this system.

The data showed something one-fourth, Thomas Munro said a large number are being educated privately at home also.

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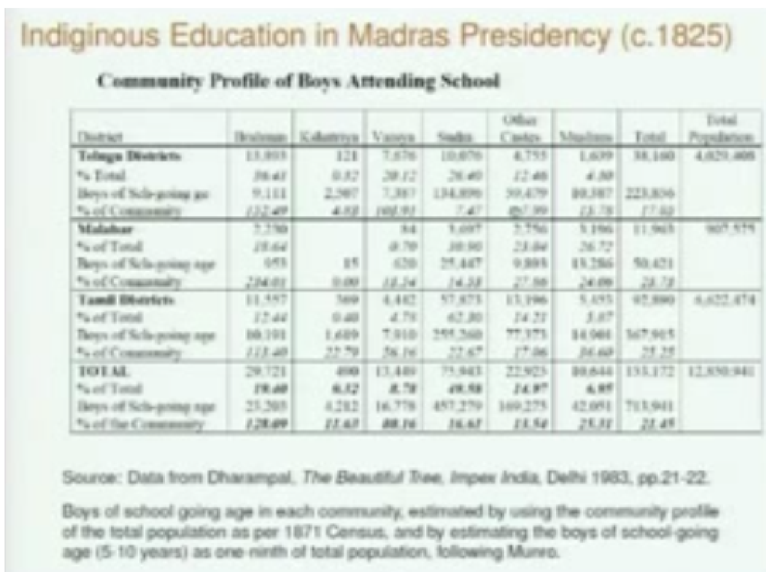

But what was significant about this survey was that about 50% of students belonged to what is called the Shudra community, 15% of the students belonged to what are called the other caste whom we refer to as the scheduled castes and scheduled tribes today. So, the majority of the students 65% came from the non-dvijas category of Indian society.

If you look at percentage of children of school going age of each community being educated, you find that nearly 17% of the boys of school going age of Shudra were receiving education in this public education system in these schools. In Tamil-speaking area, in Tamil Nadu, it was something like 23% and 17% in Tamil Nadu of children of school going age in the other communities, scheduled caste and scheduled tribes were receiving education in it. No wonder

Mahatma Gandhi called it the beautiful tree.

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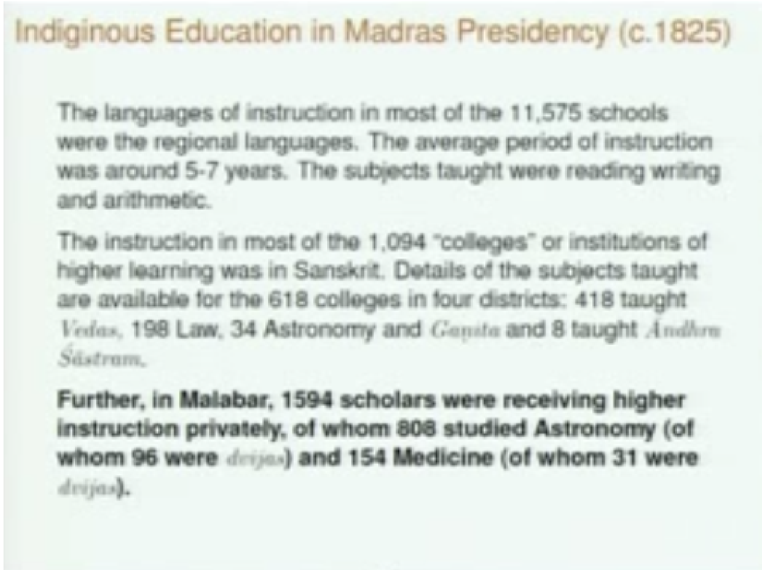

The languages of the instruction were regional languages. The colleges that they were talking 1094 were of course the higher learning was in Sanskrit and various subjects were taught. In Kerala it was interesting that there 800 students privately studying astronomy and about 154 studying medicine and a majority of them were non-dvijas.

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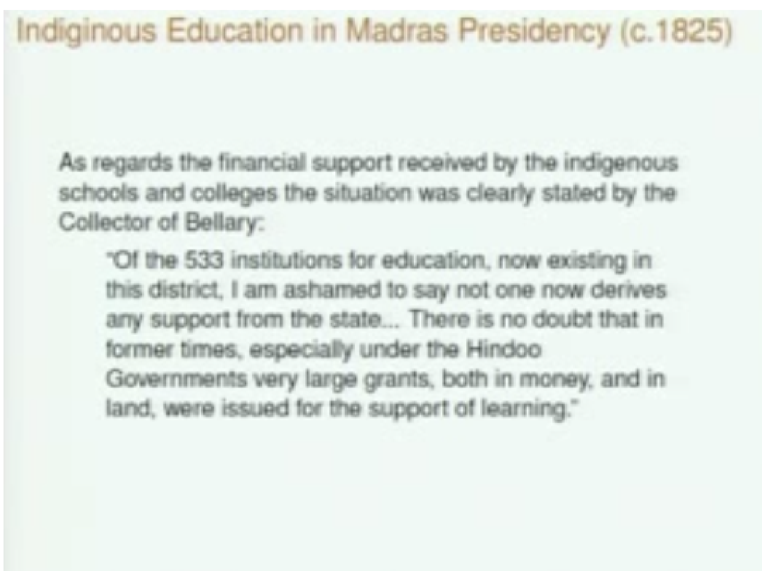

Here is a statement from the Collector of Bellary which says that we are not supporting this system of education even a bit. Of course, it was being supported during the earlier indigenous governments.

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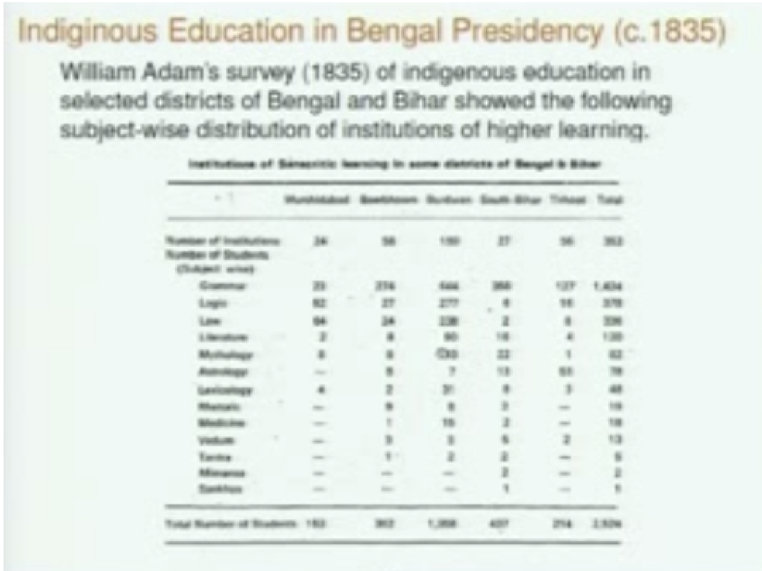

A similar survey was done in Bengal by Williams Adam and he found a large number of schools and colleges but the interesting thing is the kind of subjects that were taught in these colleges. Vyakarana was taught in 10424 institutions of learning, Niyaya in 378, Dharmashastra in 336, Kavya in 120, Purana in 82, Jyotisha in 48 and then Kosha, Alankara, Vaidya, Vedatantra, Mimamsa, all shastras studied but the proportion of students learning Vyakarana, Nyaya, and Dharmashastra were very large.

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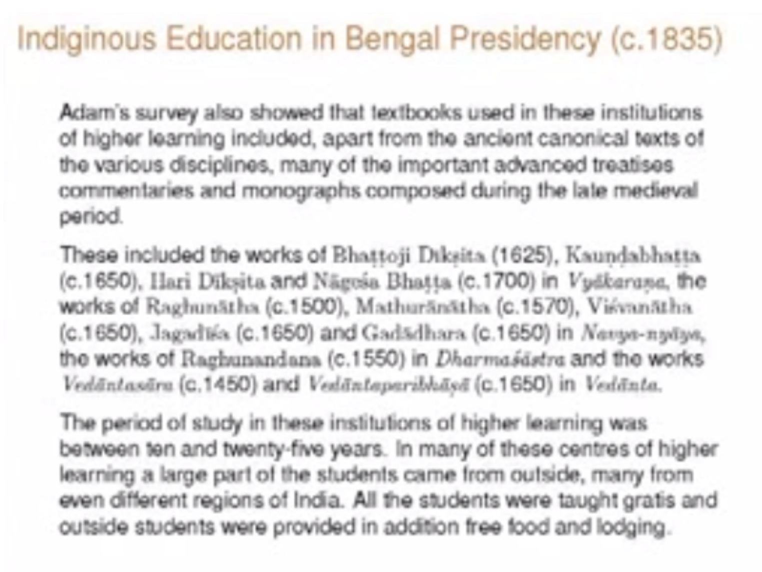

Another thing Adam found in his survey was that the books that these were people were studying in these colleges, the instruction meant to 10 to 25 years where the most advanced text that are written by famous (()) (11:27) 17th century or 18th century by Nagesa Bhatta, Gadadhara, Jagadisa, Kaundabatta, so all these were the works that were being studied in these colleges.

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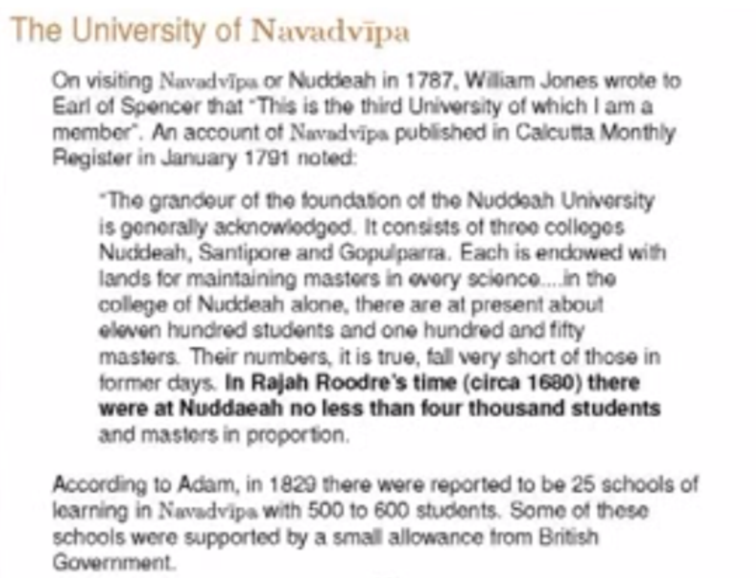

Especially, in Navadvipa everybody said that there is a huge university. Navadvipa is a home of Navya Nyaya, it was place where Chaitanya also was there. It was said in 1700 there were at Navadvipa about 4000 students and around 1000 teachers and in 1790s about 1100 students and 150 teachers and in 1829 there were 25 colleges with 500 to 600 students. This was status of Navadvipa.

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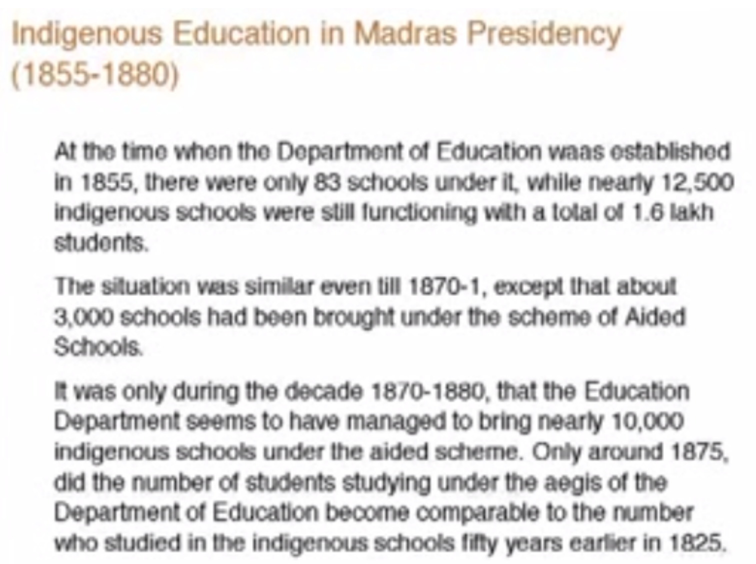

So, this was the state of indigenous education system when these surveys were made but soon enough the British policy was to discourage all these education, at least not provide them any support from the state system and whatever support that the state system was to give was for the new system of education that they wanted to start, mainly based upon English language and English literature.

So, in this new system, in 1855 there were only 83 schools in Madras Presidency; by 1870, about 3000 schools had been brought under the aid scheme. It was only in the decade of 1870 to 1880 about 10,000 of the indigenous schools were brought under the aided scheme. So, the indigenous schools those than 11,000-12,000 had continued almost till 1880 without any state support. So, only around 1880 the number of students studying under the government department of education started becoming comparable to the students in the indigenous schools 50 years earlier.

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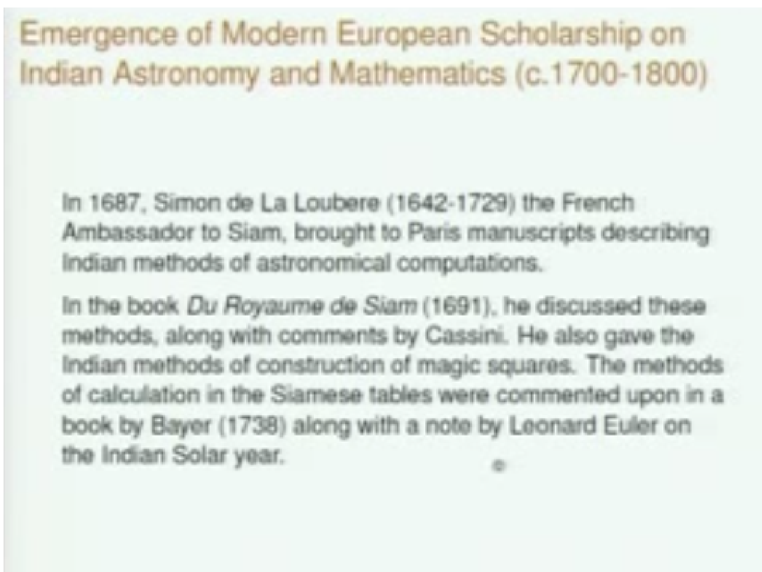

So, we will cause this. Let us say something about, there many slides on various ways in which modern scholars and our own people started knowing about Indian astronomy and mathematics. There are lot of names and lots of name subtext. I have put them down all for completeness sake so that you can really it and get an idea of the kind of work that went on in. So, I will not be going through these lights in detail in the lecture.

Because it will be laborious to merely recounting the names but most students do need to know the great scholars who worked on our indigenous tradition of mathematics and astronomy. So, earliest goes back to Loubere who wrote from Siam about Indian magic squares and Indian methods of planetary tables Le Gentil have already mentioned.

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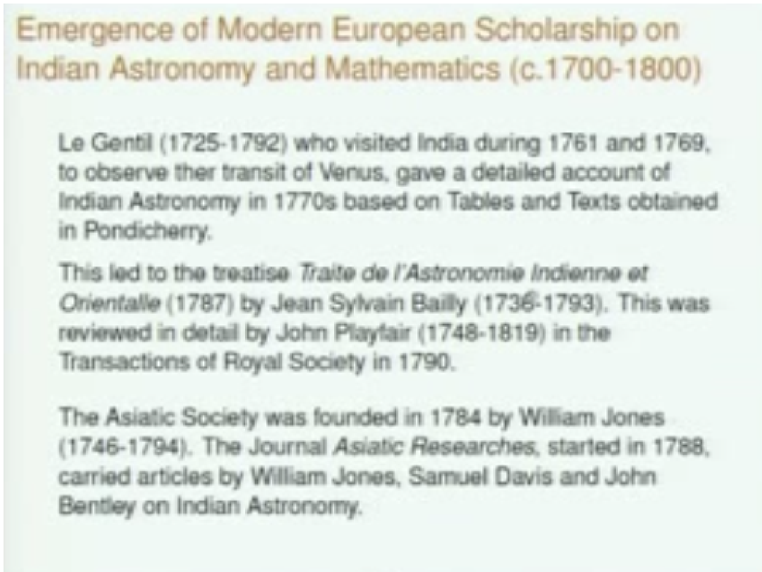

Based upon the French report and the manuscripts, a treatise on Indian astronomy was written by Bailly in French and it was reviewed by John Playfair important (()) (14:27) in Edinburgh in 1790s, but of course in India by then this Asiatic Society was founded and the Journal Asiatic Researchers started reporting from modern scholars reports on nature of Indian science, namely William Jones, Davis, John Bentley, etc were amongst the important people.

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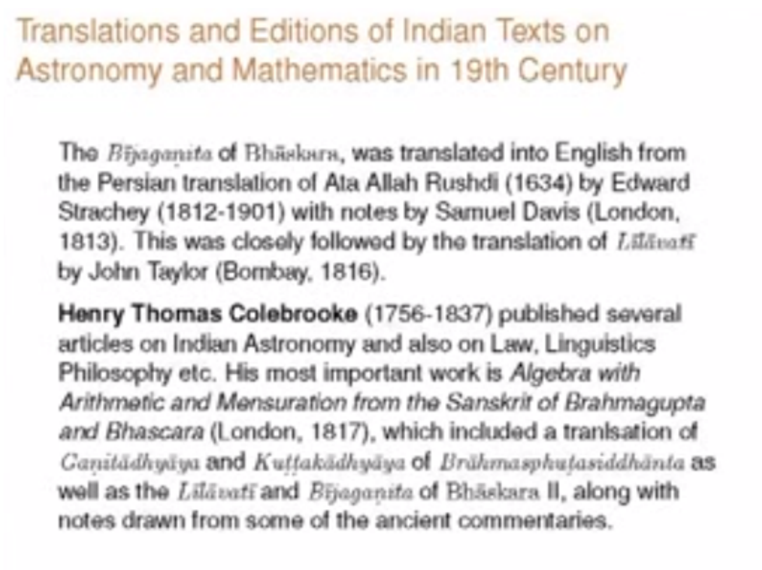

Some books started getting translated in early part of 19th century. Bijaganita was translated from the Persian version by Edward Strachey, but soon enough Henry Thomas Colebrooke translated the Bijaganita and Lilavati of Bhaskara and also the mathematics chapters of Brahmasphutasiddhanta. Colebrooke was also important scholars of law, linguistics, philosophy, etc.

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John Warren discussed the planetary methods of computation based on the Vykaya system in 1825. This book contains an interesting discussion between B. Heyne, Charles Whish, and John Warren on the knowledge of various South Indian pundits, the knowledge of the infinite series and mathematics like that amongst them. Of course, one group believes that this is all borrowed from the west and they are just lying or bluffing as they must have look at some modern books, modern calculus books.

Other group believes no, no, no there are found in Indian text, etc. That kind of debate was going on in 1825.

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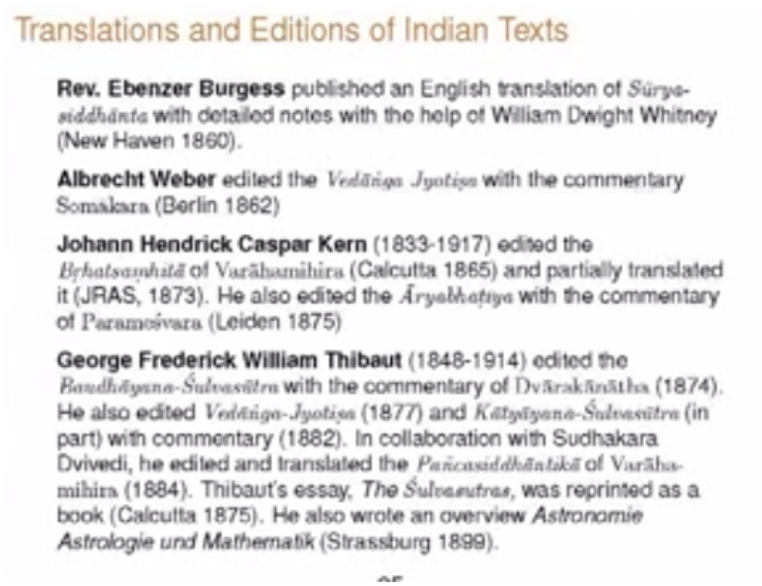

Wilkinson Hall, Burgess, Weber, Kern, Thibaut , there were all major scholars of editing and are translating Indian works in 19th-century.

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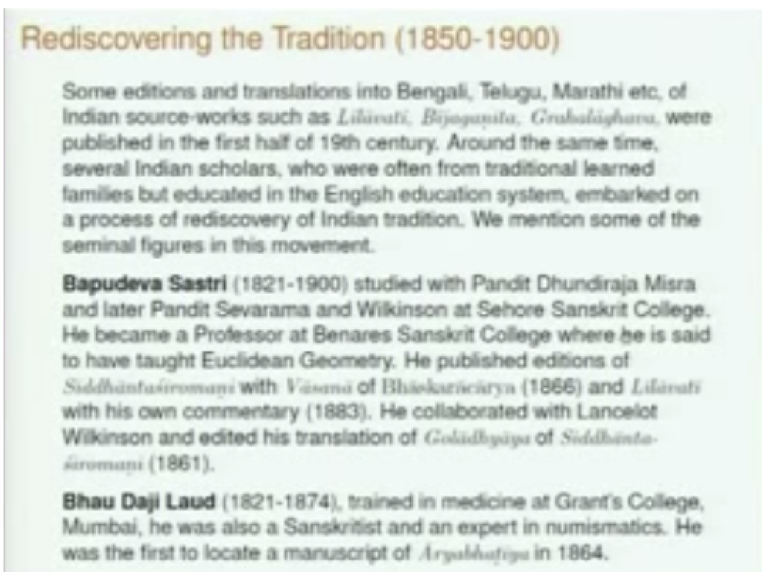

By, then a trend started that books getting published with the translations. The source work like Lilavati, Bijaganita, (()) (16:15) etc. Indian publishers started publishing these works with translation sometimes in Bengali, Telugu, or Marathi, etc. So, enough a set of scholars emerged who are already often from the traditional learned families but were educated in the English education system who started looking into our text.

Bapudeva Sastri is one of the oldest of them. He published the Siddhanta Siromani, then Lilavati.

Bhau Laud discovered the manuscript of the Aryabhatta**.**

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Shankar Balakrishna Dikshit wrote a detail history of Indian astronomy in Marathi which has been translated into English later. Sudhakara Dvivedi was a very major scholar who brought to print about 40-50 major texts of Indian mathematics. In fact, much of our study today owes to the fact that these books were published between 1860 and 1880 and 1900.

I mean the list is large, Lilavati, Karanakutuhala, Yantraraja, Siddhanta-tattvaviveka, Sisyadhivrddhida, Bijaganita, Brhatsamhita, you can see that he was a major scholar who edited and published sometimes with his own commentary are various books.

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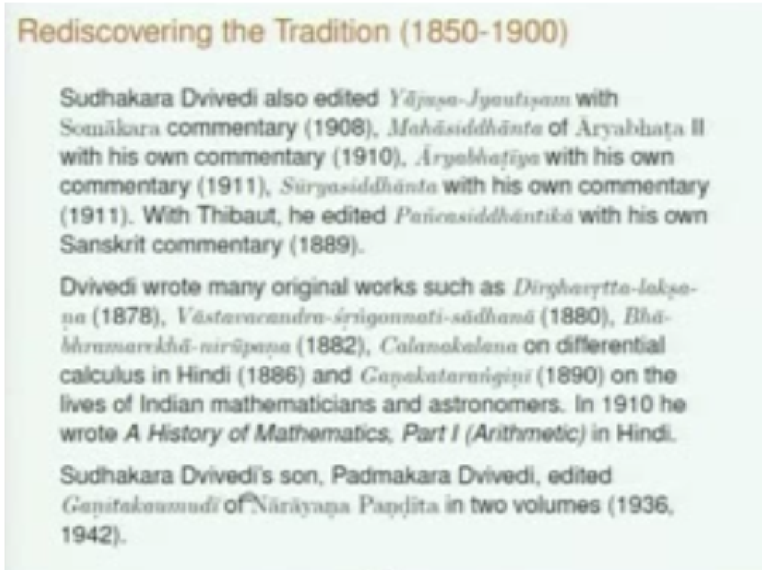

Finally, his son in 1930s published the Ganita Kaumudi of Narayana Pandita also. Sudhakara Dvivedi wrote a couple of books in Hindi also on history of Indian mathematics. So, this is a brief look at what happened between 1700 and 1900 about the study of our older tradition of mathematics by modern scholars.

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Now, let us go to development of modern education in India. So, these Universities of Calcutta, Bombay and Madras were set up in 1857. But most scholars who have studied this understand that these were established mostly as examining bodies with lots of affiliated colleges on the model of the then London University and not only model of the renowned Oxford or Cambridge Universities which were devoted to large amount of teaching and research activities.

So, these were mostly degree awarding and examining bodies, that was what the universities were. So, looking at it Mahendralal Sircar in 1876 started the Indian Association of Cultivation of science with the hope that that will foster research and advanced study amongst the Indians. But even then nothing much perhaps happened in the first 20-25 years of this institution. Most of the time, the main effort of the institution got directed towards developing of science teaching in the college level.

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Development of Higher Education in India (1850-1900)

	1855	1882	1901-2
Universities	-	4	5
Number of Students	-	NA	NA
Arts Colleges	15	38	145
Number of Students	3,246	4,252	17,651
Professional Coleges	13	96	46
Number of Students	912	3,670	5,358
Secondary Schools	169	1,363	5,493
Number of Students	18,335	44,605	6,22,768
Primary Schools	1,202	13,882	97,854
Number of Students	40,041	6,81,835	32,04,336
Special Schools	7	83	1,084
Number of Students	197	2,814	36,380
Total Recognised Institutions	1,406	15,462	1,04,627
Number of Students	62,731	7,37,176	38,86,493

So, in 1855 there were about 15 colleges with 3246 students. By 1900, there were 150 colleges with 17,000 students and 45 professional colleges. You can see that in a table like this. Later on, at leisure you can examine. This gives the growth of education actually primary schools, special schools, etc.

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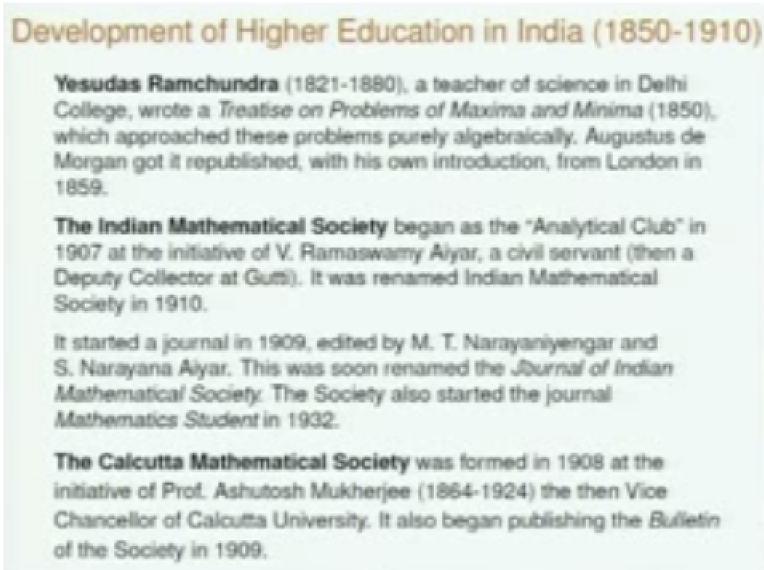

Some scholars in India started working on modern subjects. One person was Yesudas Ramchundra, a teacher of science in Delhi, who wrote a treatise on problems of maxima and minima. He tried to approach these problems algebraically. De Morgan, the British famous logician and mathematician got impressed with it. He got it re-published in London. By the time 20th century, it was only by then that some serious research activity and study of the higher sciences started in India.

The Indian Mathematical Society began as analytical club in 1907 at the initiative of V. Ramaswamy Aiyar. It started a journal in 1909, the Journal of Indian Mathematical Society. After the 25th year of the society, they started a popular journal for students. Some material was coming in Journal of Indian Mathematical Society itself which was separated and the mathematic student was started. Soon enough in good competition, the Calcutta Mathematical Society was started in 1908 by Professor Ashutosh Mukherjee's initiative. It also began the bulletining of the Calcutta Society.

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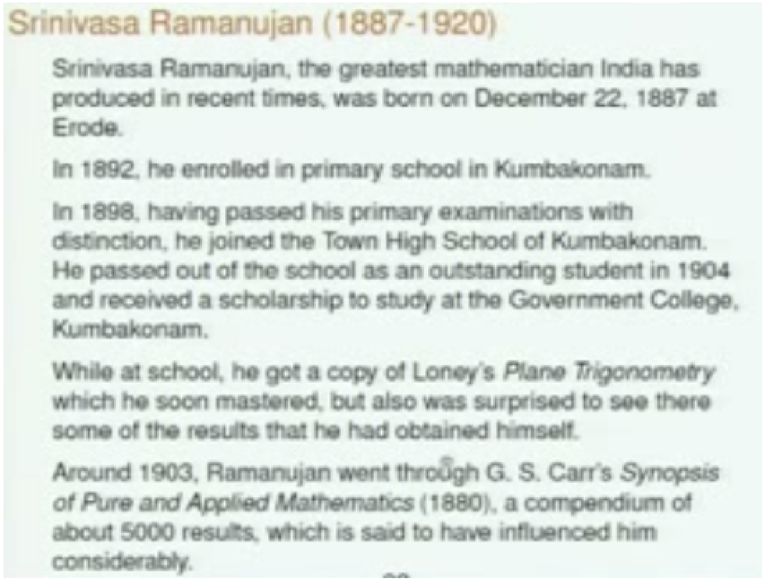

We will discuss the development of modern mathematics in India from about 1905 in the next lecture. I would like to go straight away to discuss the work of Srinivasa Ramanujan who if we examine carefully, you will see that in some very fundamental sense he is in continuity with the older tradition of mathematics and science in India. So, this talk will be devoted to the work of Ramanujan.

Ramanujan was born in December 1887. He enrolled in school in 1892 in Kumbakonam and he did fairly well in school. He topped, he received a scholarship to study in the local college and while at school he looks right up Loney's Trignometry and he found that he had himself discovered many of these results. Towards late period in school, he looked up G. S. Carr's Synopsis of Pure and Applied Mathematics which was a sort of textbook for the Cambridge Wrangler examination listing about 5000 in result which is said to have influenced him considerably.

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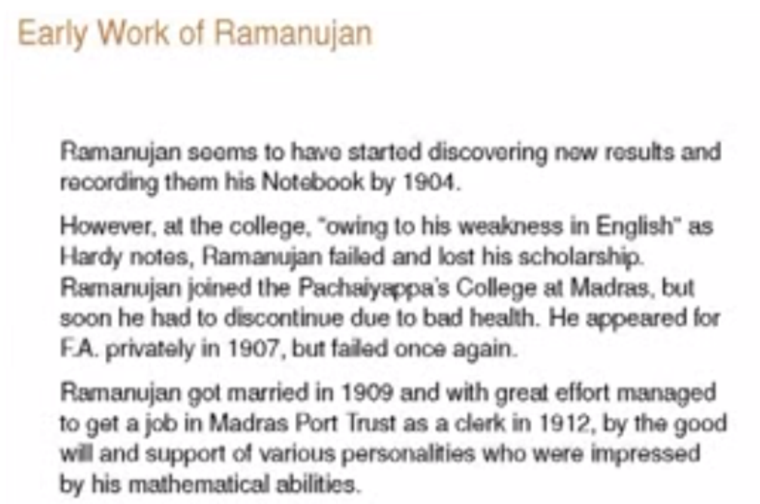

He seems to have started discovering new results and recording them in his notebook towards the later period when he was in school itself. Now, once he went to college owing to his weakness in English as Hardy notes, he totally failed, lost his scholarship. He came and Pachaiyappa's College in Chennai but that also he could not complete. He appeared for F.A. privately, failed. He got married in 1909.

He continued his mathematical work and he went and met various influential people showing them the work and some of them did help him and by the goodwill of his supporters, he at least obtained a job in Madras Port Trust as a clerk in 1912.

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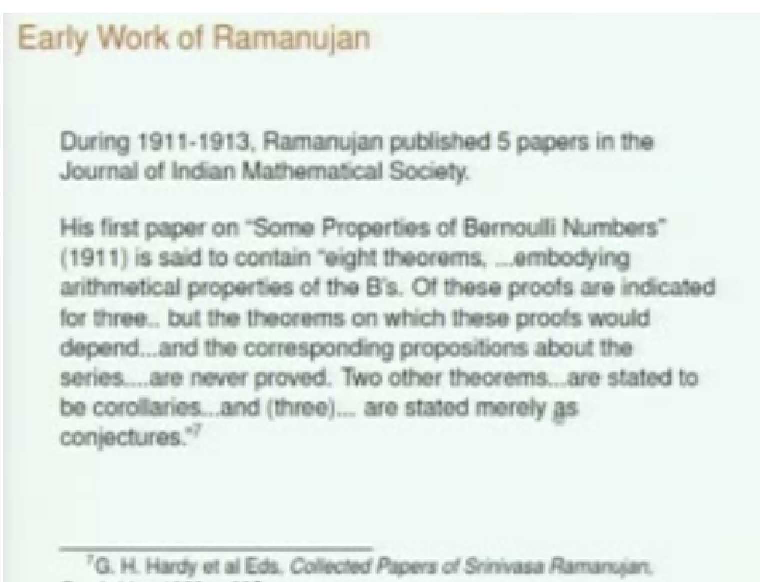

He published about five papers during 1911 to 1913. The first paper was on Properties of Bernoulli Numbers which is supposed to contain many interesting results.

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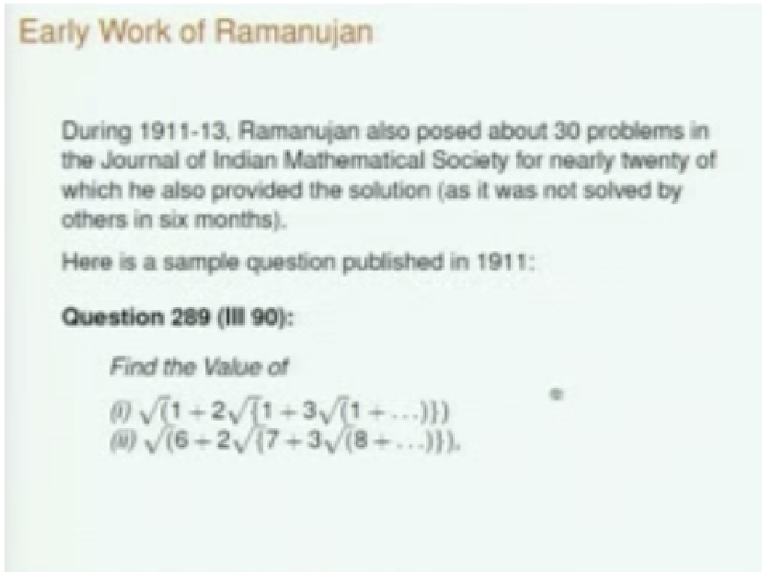

He also sent about 30 problems to the Journal of Indian Mathematical Society. I am just giving one to show you the flavour such kind of problems you have already seen in Narayana Pandita. So, one question he posed in 1911 was find the value of square root of 1+ square root of 1+3 times square root of 1+ etc. So, the square root runs over the entire.

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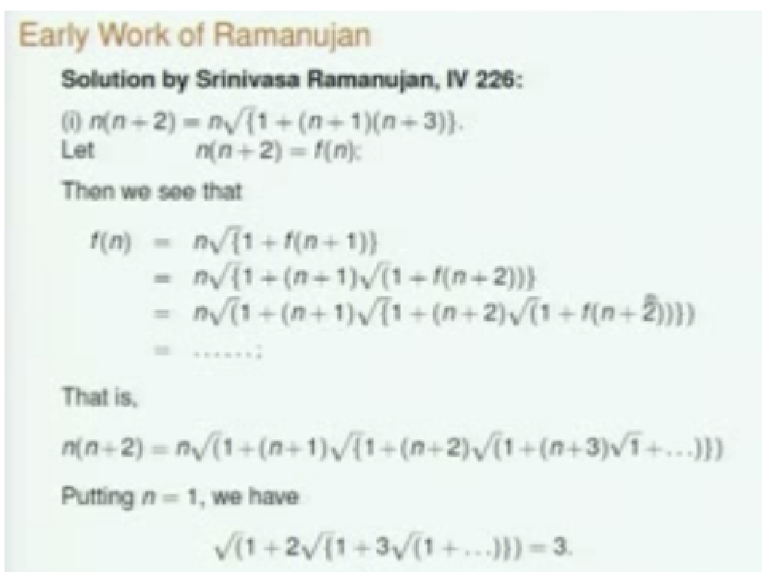

So, people were supposed to answer. If no answer can in six months, Ramanujan would give the solution and the person who pose the problem and so the way Ramanujan solved it was, he first saw the simple the identity n to n+1 is n times this much and define n to n+1 as fn and immediately in terms of the fn, he was able to get by equation a relation and so the particular quantity here could be directly shown to be n to n+2, put n=1 and you get the beautiful result. Square root of 1+twice square root of 1+thrice square root of etc=3.

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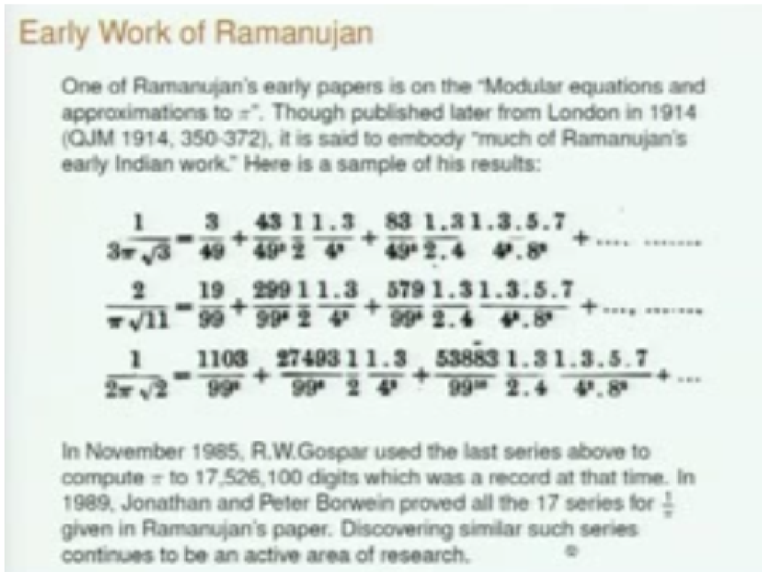

Similarly, the other problem also. There is a paper called Modular equations and approximations to pi which was published at Ramanujan (()) (23:46) in quarterly journal, but this is also said to be the kind of work that he was engaged in while in India and in 1985 this last series was used by Gospar to compute pi to 17 million digits which was a record at that time and Borwein proved all the 17 series that Ramanujan had given in this paper for 1/pi in 1989. Discovering similar series continues to be an active area of mathematical research, okay.

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Continuing Ramanujan story, in 1912 he sent a sample of his research to one professor M.J.M. Hill in the University College London through the Professor of Madras College of Engineering which is the current Hindi college. Professor Hill wrote back that Ramanujan has fallen into the pitfalls of divergent series and he should consult Bromwich's book on infinite series. Ramanujan again wrote to Baker and Hobson in Cambridge and it seems he did not get any response.

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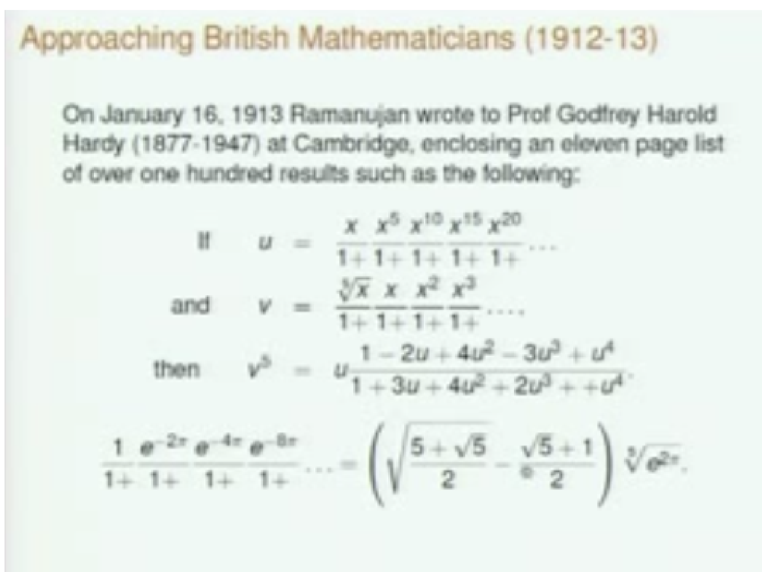

So, in January 1913, Ramanujan wrote to Godfrey Harold Hardy at Cambridge. He enclosed an 11-page supplement which had more than 100 results. So, I am just displaying two of the results of Ramanujan. This is a continued fraction. This is another continued fraction and he has a certain relation between them and this is a continued fraction and this is another analytical expression. These were two of the kind of hundred formulae that the Ramanujan sent to Hardy in January 1930.

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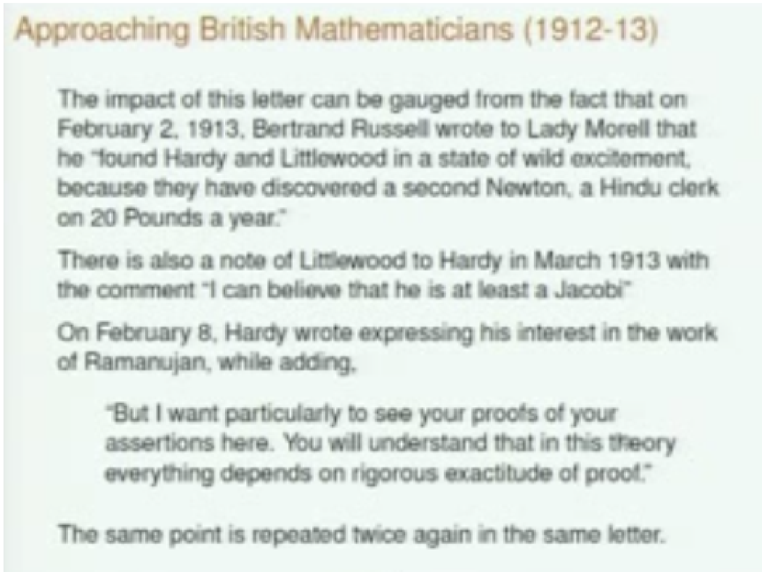

The impact of this letter can be gauged from the fact that on February 2, Lord Bertrand Russell wrote to Lady Morell that he found Hardy and Littlewood in a state of wild excitement because they have discovered a second Newton a Hindu clerk on 20 pounds a year. So, in fact, even there is a comment of Littlewood to Hardy in March that looking at his work I can believe that he is at least a Jacobi. Jacobi is a great analyst of 19th century.

So, Hardy wrote back to Ramanujan expressing interest in his work but he added but I want particularly to see the proofs of your assertions here. You will understand that in this theory everything depends on rigorous exactitude of proof and this is a point that Hardy repeated about two three times in the same letter.

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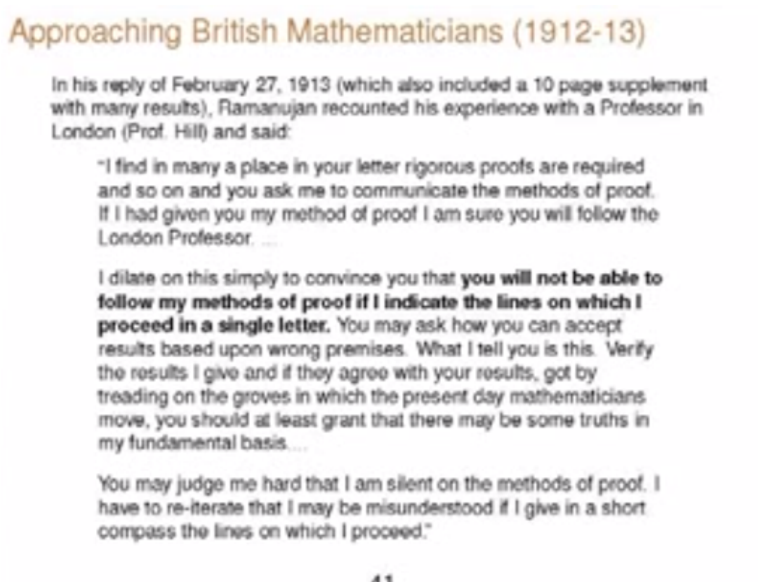

So, Ramanujan sends a reply back with another 10-page supplement with various results. The main point he is trying to tell him is that you will not be able to follow my methods or proof if I indicate the lines on which I proceed in a single letter that of course I follow. There is a method, there are various sorts of ways in which I am arriving at them, but it cannot be put down in a letter.

Then, he just threw a challenge. Verify the results that I give and if agree with your results, got by treading on the groves in which the present day mathematicians move, you should at least grant that there may be some truth in my fundamental basis. So, it is one of the few letters by Ramanujan. He is asserting something about himself that is he has a method and he has an approach.

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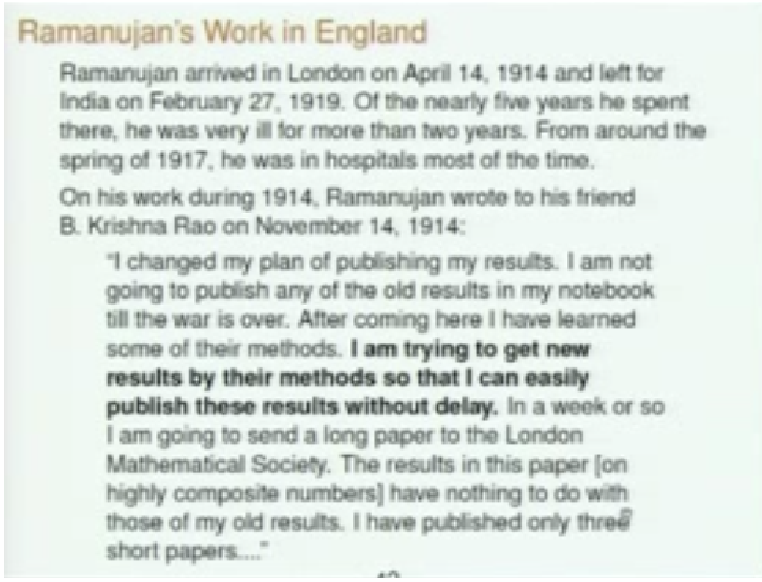

So, Ramanujan arrived in London in April 1914. He left for India back in February 1919. Of these five years, he was very ill for more than two years, almost three years I think. From around the spring of 1917 most of the time he spent in hospitals. On his work during the end of the year that he reached after six-seven months, he wrote to his friend B. Krishna Rao saying that I have changed my plan of publishing my results.

I am not going to publish any of old results in my notebook till the war is over. The war started in 1914 and goes on till 1918. I am trying to get new results by their methods, so that I can easily publish these results without delay.

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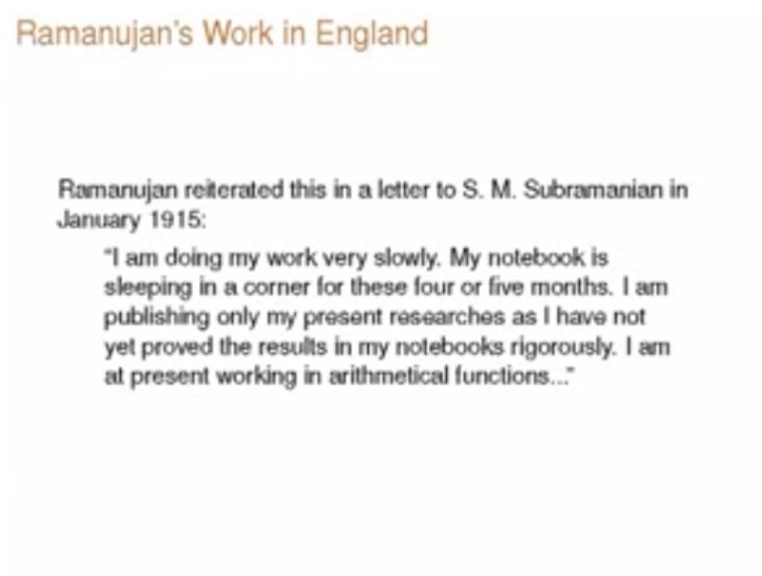

Same thing Ramanujan writes to another friend Subramanian in a letter in 1915, that my notebook is lying in a corner, I am not going to take it out and publish any of the results. So, whatever work he did in England was with the new methods and the new way with the interaction with the British mathematicians and of course with this great acumen and intuition.

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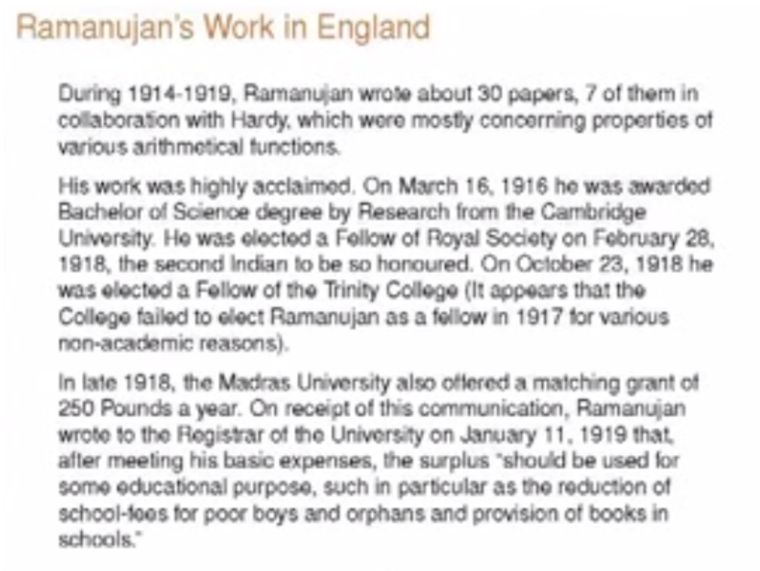

So, during 1914 to 1919, Ramanujan wrote about 30 papers, seven of them in collaboration with Hardy which were mostly concerning properties of arithmetical functions. We will get a chance to have a glimpse of few of them. His work was highly acclaimed. In March, he was awarded BSC by Research from Cambridge University for his paper on highly composite number. He was elected in FRS in 1918, the second Indian to be so honoured. The earlier person was a Parsi gentleman in mid 19th century.

In October 1918, he was elected as fellow of Trinity College. It seems that they failed to elect him a fellow the previous year and that was some heartburns for Ramanujan. So, in late 1918, Madras University offered him a matching grant of 250 pounds. The Cambridge Fellowship gave him a 250-pound annual fellowship being a fellow of Trinity College, so Madras University also gave that.

So, Ramanujan writes a very moving letter to the registrar saying that after meeting his basic expense, the surplus should be used for some educational purpose, such in particular as the reduction of school fees for poor boys and orphans and provision of books in schools.

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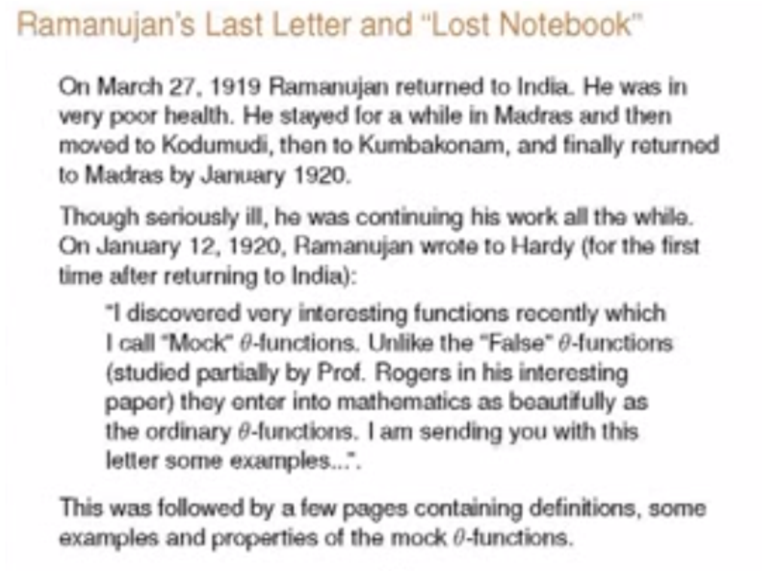

So, in March 1919, Ramanujan returns to India. He is in very poor health, stays a bit in Madras, then goes to Kodumudi, then goes to Kumbakonam and finally returns to Madras by January. At that time, he wrote to Hardy for the first time after returning to India. He returned in March 1919. I discovered very interesting functions recently which I call mock theta functions. Unlike the false theta functions studied by Professor Rogers.

They enter into mathematics as beautifully as the ordinary theta function. I am sending you with this letter some examples. Again, few pages of results enclosed.

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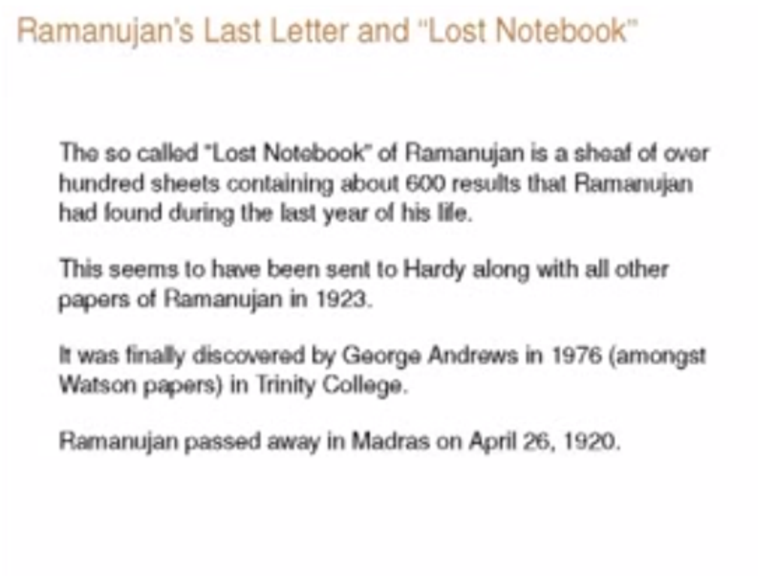

There is something called the Lost notebook of Ramanujan. This is a collection of more than 100 pages many of them written on both sides having about 600 results. This consists of the work that he was going after he returned from England to India. These papers were sent to Hardy 1923. It was finally discovered by George Andrews in 1976 in amongst the Watson papers in Trinity College. So, Ramanujan passed away in April 1920.

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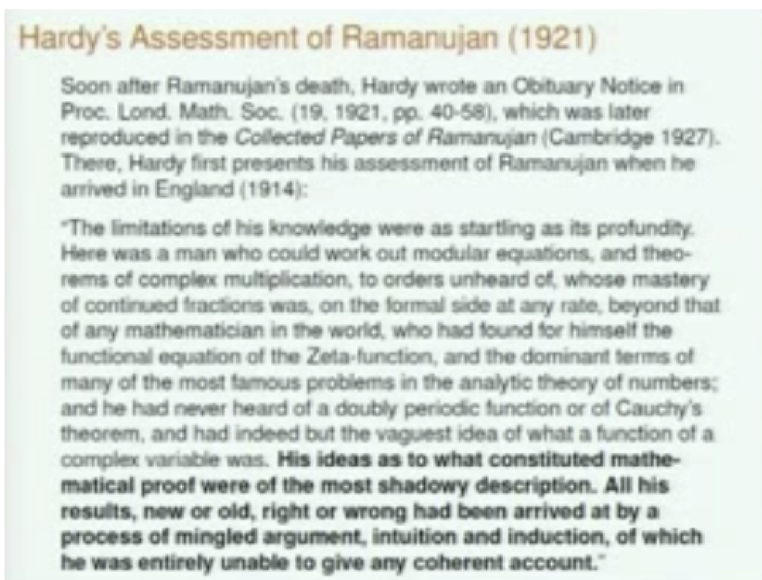

Now, soon after Ramanujan's death, Hardy wrote an Obituary and there he tried to give an estimate of Ramanujan what he thought of him when he came to England and what was made out of him later and how he understands his work and this is a very interesting document. It has to be read very carefully. I am just giving a few quotations. So, Hardy is saying that Ramanujan's ideas as to what constituted mathematical proof were of the most shadowy description.

All his results, new or older, right or wrong had been arrived at by a process of mingled argument, intuition and induction, all these things written in the first obituary that he is writing on this man, of which he was entirely unable to give any coherent account.

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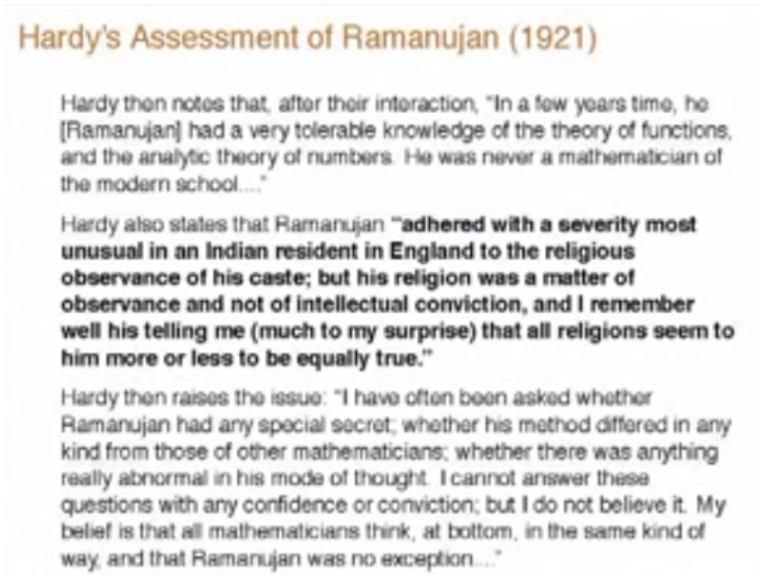

There is another opt quoted statement by Hardy which seems to make much of the statement that Ramanujan told him at some context that all religions seem to him more or less equally true. Hardy to took to mean that he really did not believe in any religion and he says, Ramanujan adhered with a severity most unusual in an Indian resident in England to the religious observance of his caste but his religion was a matter observance and not of intellectual conviction.

And the evidence for this is that I remember well is telling me much to my surprise that all religions seem to him more or less to be equally true. So, this is kind maturity that Hardy has in trying to understand this man.

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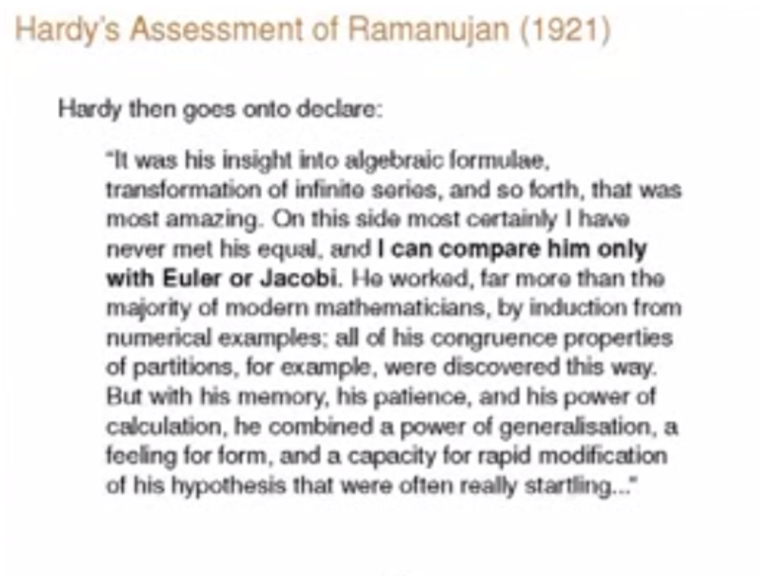

Then, Hardy says of course that Ramanujan was extraordinary with his insight in algebraic formulae, transformation of infinite series and so far so forth. On this side most certainly I have never met his equal and I can compare him only with Euler or Jacobi. But immediately he has to say he worked far more than the majority of modern mathematicians by induction from numerical examples but it was his memory, his patience and his power, not his mathematical acumen that gave him all with this results.

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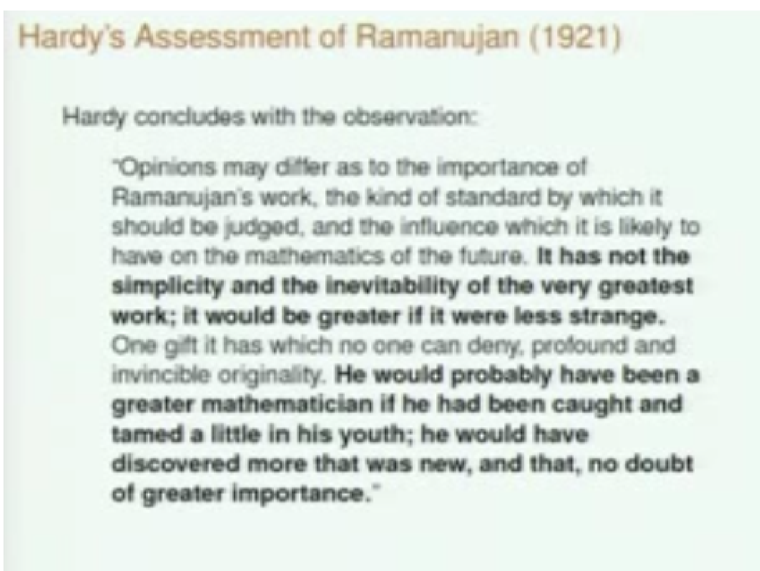

He concludes the whole thing by saying Ramanujan did wonderful work but that cannot be called very great work or amongst the greatest work. It has not the simplicity and inevitability of the very greatest work. It would be greater if it were less strange. He would have probably been a greater mathematician if he had been caught and tamed a little in his youth. He would have discovered more that was new and that no doubt of greater importance.

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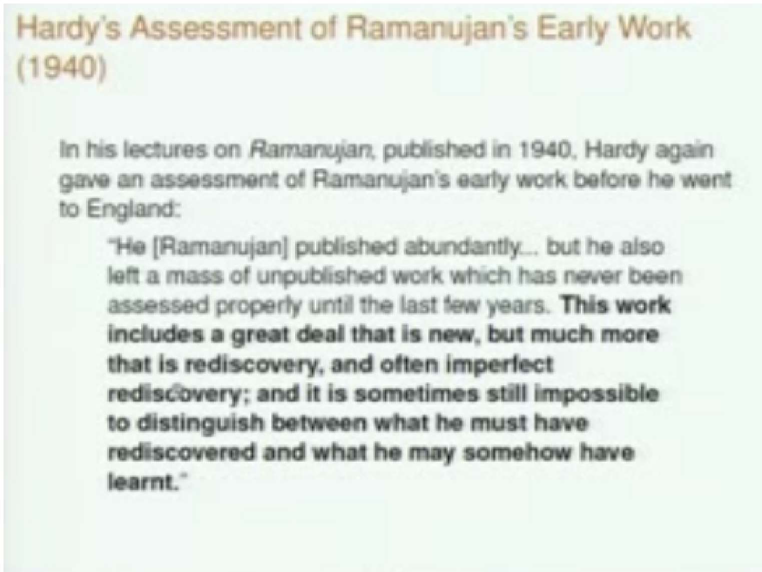

So, this was an assessment of Ramanujan's were given immediately after his death and this was reprinted in the collected works of Ramanujan that Hardy, Seshu Aiyar and Wilson published in 1927. Again, 15 years later, Hardy had an occasion to write an estimate of Ramanujan. This was a series of lectures he delivered in Harvard and this came out as a book in 1940. Here Hardy is somewhat even more free in trying to explain how Ramanujan, according to him, was when he reached England.

Ramanujan left a mass of unpublished work, which he is referring to his notebooks, which has never been assessed properly until the last few years. This work includes a great deal that is new but much more that is rediscovery and often imperfect rediscovery and it is sometimes still impossible to distinguish between what he must have rediscovered and what he may somehow have learnt is even sort of, accusing no plagiarism that have recorded some results without the knowledge that so.

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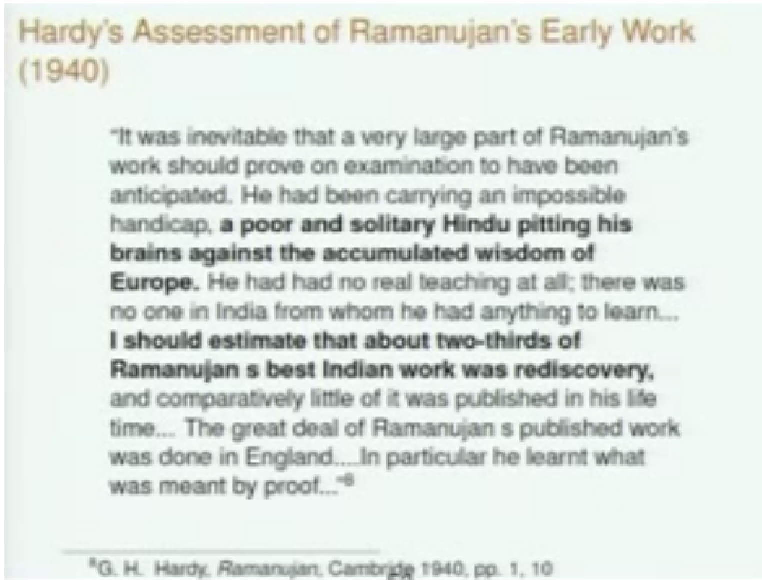

Then he says he had been carrying on an impossible handicap, a poor and solitary Hindu pitting his brains against the accumulated wisdom of Europe. Hardy was no ordinary racist or imperialist. He was after all a great anti-war champion. He was a liberal sort of intellectual, but the times were such that even he could not resist coming up with this kind of a wonderful praise, poor and solitary Hindu pitting his brains against accumulated wisdom of Europe. I should estimate that about two-thirds of Ramanujan's best Indian work was rediscovered.

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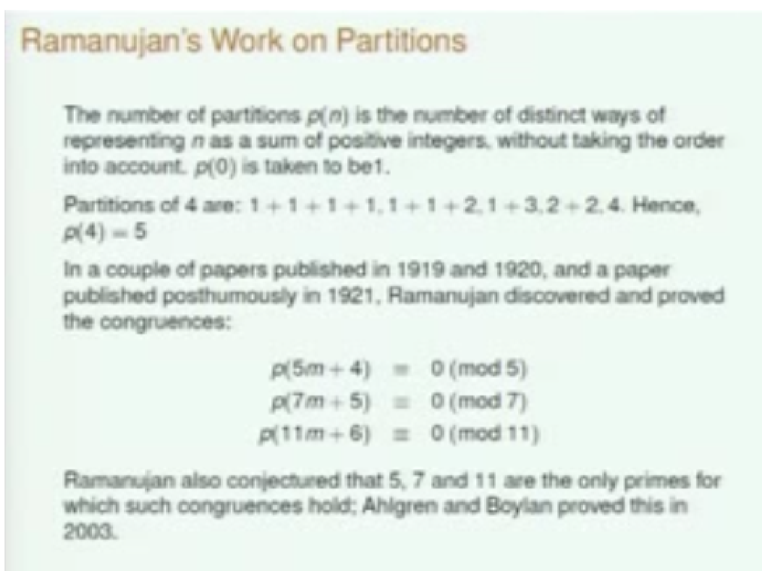

Anyway, so now let us go quickly to discuss something of Ramanujan's work. We will come back to the notebook a little bit later, the works that he did prior to going to England which is contained in the notebooks, we will come to it towards the end of it. So, one of the important work of Ramanujan is on partitions. So, the number of partitions p of n is the number of distinct ways in which n can be written as sum of positive integer without taking the order into account.

So, it is not like the mass number rows in the (()) (35:33) where the order of the elements also is important, 2+1+1 will be treated differently than 1+2+1, GLL and LGL will be taken as two different partitions. So, these are repetitions not taken into account. So, the number of partitions of 4 are 1+1+1+1, 1+1+2, 1+3, 2+2 and 4. So, there are five partitions of 4. If you try to write partitions of 10, you will immediately find that itself is a huge number. In a couple of papers published in 1919 and 1920.

And a paper published posthumously in 1921, Ramanujan discovered and proved these three very important congruences.

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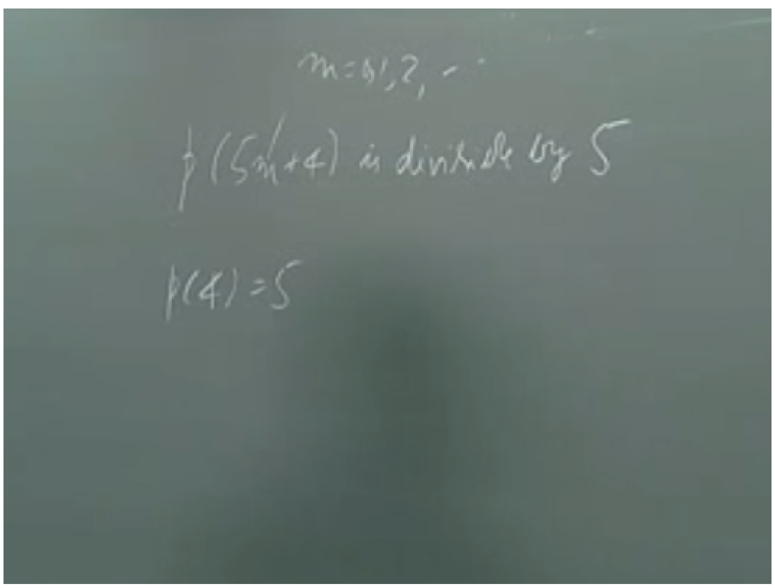

What is meant by congruence, that is just that is divisible by 5, that is what it means. So, in particular p of 4=5, then I put m=0. Use of p of 4+5 that is divisible by 5, but Ramanujan is saying for any m the number of partitions of the p of 5m+4 is divisible by 5, that is what this equation means. Similarly, the number of partitions of a number integer of the type 7m+5 is divisible by 7.

The number of partitions of number of type 11m+6 is divisible by 11, these were three congruence property. He conjectured 5, 7 and 11 are the only primes for which such congruences hold that was proved about 70-80 years later.

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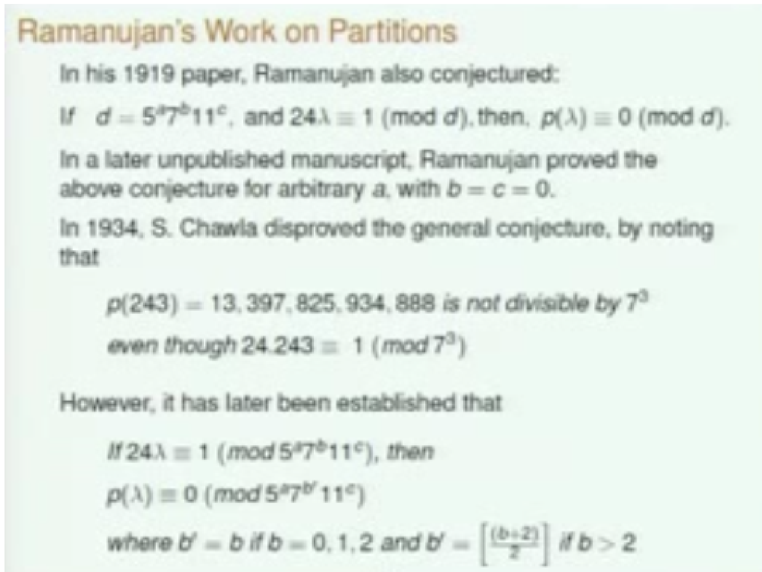

In his 1919 paper, Ramanujan also wrote down conjecture, this is somewhat complex. If d is an integer of 5 to the power a 7 to the power b 11 to the power c and if lambda is a number such that 24 times lambda when divided d gives a reminder of 1, that is what this equation means then the partitions of lambda are divisible by d. This was the conjecture. In an unpublished manuscript, Ramanujan tried to prove it for arbitrary a with b=c=0. So, Chawla in 1934 disproved this conjecture.

He showed that p of 243 (()) (37:57) number, the partitions of 243 are of the order of 10 to the power 14, huge number. He showed that the 7 power relation is not fully satisfied and later on it has been shown that Ramanujan was essentially correct. In this congruence equation, you have to just change the b to a b prime and b prime is either same as b if b=0, 1, 2 and b prime is of this form.

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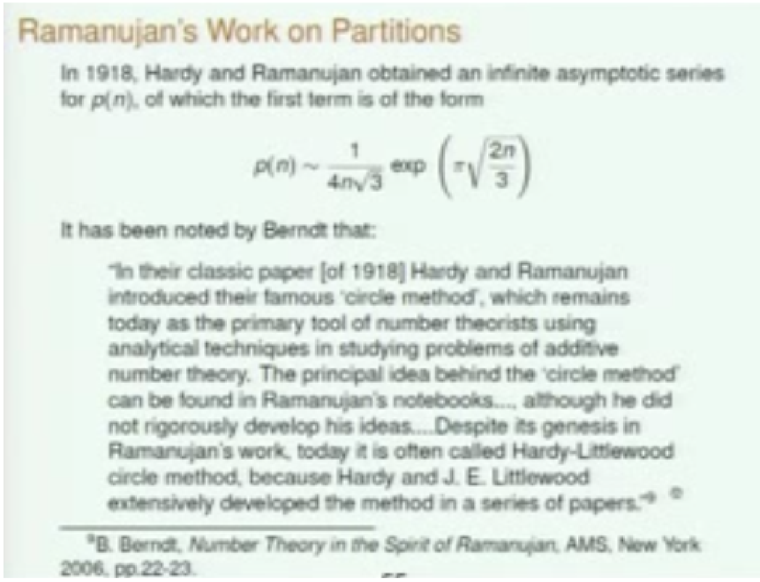

In 1918, Ramanujan and Hardy wrote a monumental paper which was (()) (38:31). It gave an infinite asymptotic series for p of n, that is this is the dominant term for large N for p of n. So, p of n goes by e to the power –root n/1/n. For this they use what is called the circle method and later on Bruce Brendt who looked up Ramanujan's notebooks found the elements of circle method in Ramanujan's notebooks.

And he says that it is unfortunate that it is called Hardy Littlewood circle method because Hardy and Littlewood wrote many more papers on this afterwards.

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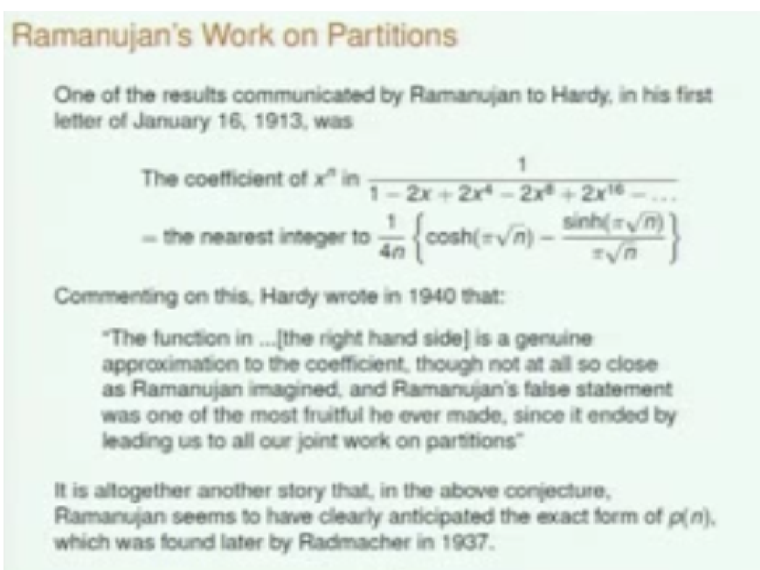

But more interesting was the letter than Ramanujan wrote to Hardy, the first letter he wrote had a proposition like this, again a very interesting kind of proposition. The coefficient of x to the power n in this complicated quantity 1/1-2x+2x 4 etc. is the integer nearest to 1/4n cos sign hyperbolic pi root n and sin hyperbolic pi root n. This is at the basis of a Ramanujan and Hardy's work on that Hardy acknowledges it in 1940.

But in another matter in this Ramanujan had a much better result, than what was proved in Hardy-Ramanujan paper. He had the exact form of the partition p of n which was later on proved by Radmacher. I will come back to this question a bit later.

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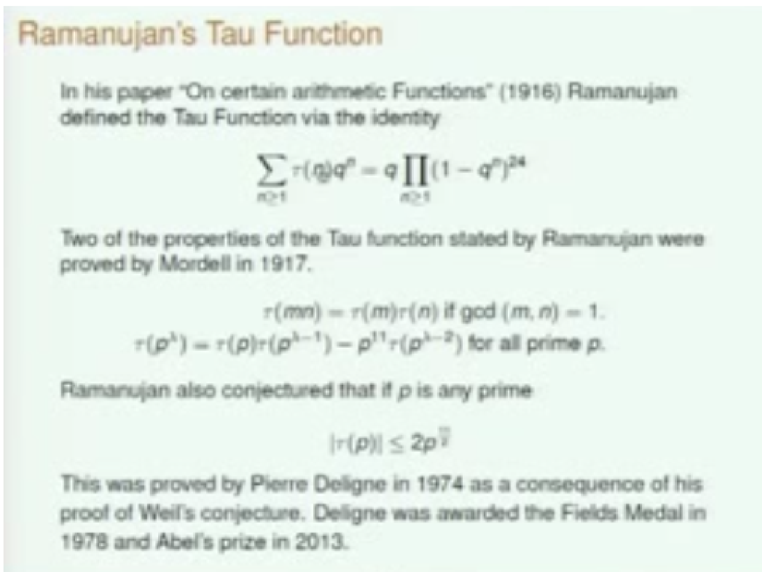

Now, another interesting thing of Ramanujan which is often talked about is something called the tau function. This is the definition of the tau function. It is a function of integers n. So, left hand side is power series like this, right hand side is an infinite product like this. Ramanujan mentioned two properties of the tau functions, they were proved by model, the next year he wrote define the tau function in 1916.

Ramanujan conjectured that if p is any prime tau p is bounded by p to the power 11/2. This was a very famous conjecture. It was proved by Deligne in 1974 as a consequence of another very famous conjecture known as Weil's conjecture for which Deligne was awarded the Fields medal and this year Deligne has been given the Abel's prize, the current sort of equivalent of Nobel prize in mathematics.

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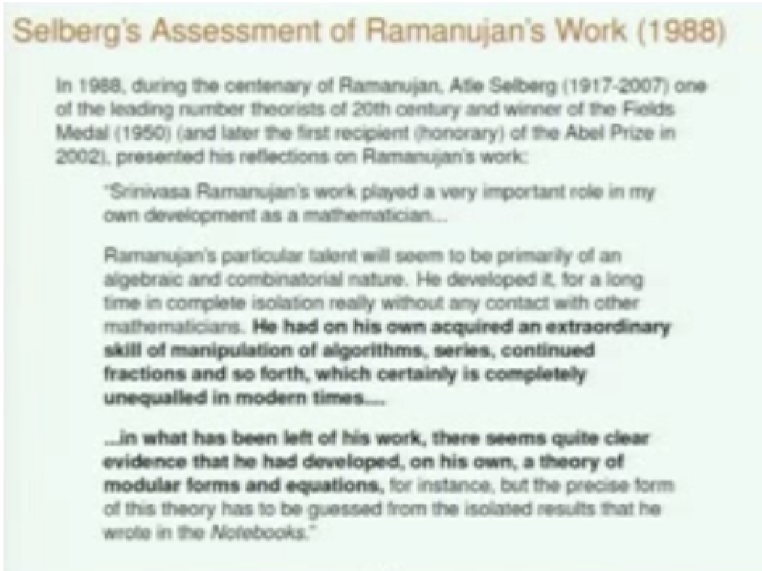

These are some of the kind of works that Ramanujan did, the tau function, partition function, etc. in his papers published from England. During the time of Ramanujan centenary, an assessment started getting made of the kind of the impact his work had, what was the nature of his work and Selberg is one of the leading number theorists of 20th century, a Fields medallist in 1950. He was the first honorary recipient of Abel prize in 2002.

He explained how his work very highly influenced by Ramanujan's. How somebody his father or someone gave him the collected papers of Ramanujan and that sort of set him on a big trail of work and according to Selberg, Ramanujan on his own had acquired an extraordinary skill of manipulation of algorithm, continued fractions and so forth which certainly is completely unequalled in modern times.

Then he says, in what has been left of his work, there seems quite clear evidence that he had developed on his own a theory of modular forms and equation. It is a very important area in mathematics. So, Ramanujan was essentially a modular forms man.

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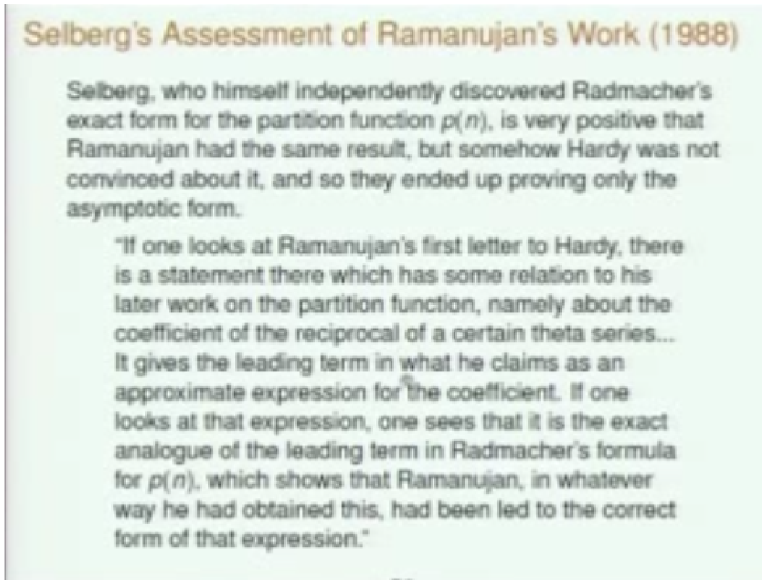

Then, Selberg comes to this question that this partition function p of n, he says if one looks at Ramanujan's first letter to Hardy, there is a statement there which has some relation to his latter work on partition function. It gives a leading term in what he claims as an approximate expression to the coefficient. If one looks at this expression, one sees that it is the exact analogue of the leading term in the Radmacher's formula for p of n. Radmacher's was the exact result which was proved in 1930s.

Selberg had independently proved the same result. Hardy and Ramanujan had given the asymptotic form. Ramanujan, in whatever we had obtained this, had been let to the correct form of that expression.

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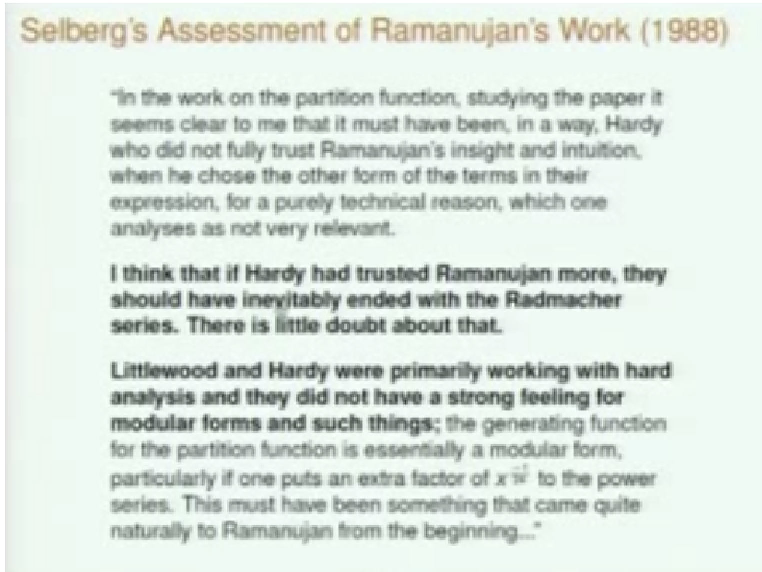

Then, he says that it was Hardy who seems to have moved away of the conjecture Ramanujan made and proved a simpler result that he could prove. So, for technical reasons he proved something much less. I think that if Hardy had trusted Ramanujan more, they should have inevitably ended with the Radmacher series. There is little doubt about it. Unfortunately, I mean Selberg does not say so.

Littlewood and Hardy were primarily working with hard analysis and they did not have a strong feeling for modular forms and such things.

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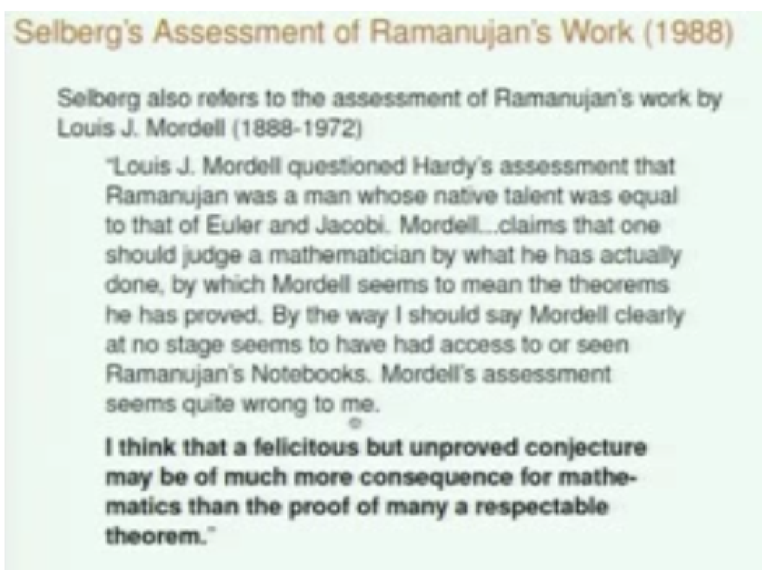

Similarly, Louis Mordell, an important British mathematician took the issue with Hardy comparing Ramanujan with Euler and Jacobi. So, Selberg tries to say that Mordell really has not seen Ramanujan's notebooks and he says Mordell seems to go by the fact that a mathematician should be judged by the number of theorems that he has proved.

So, Selberg says I think that a felicitous but unproved conjecture may be of much more consequence for mathematics than the proof of many a respectable theorem. I view the (()) (43:57) to have said in the 19th century or early 20th century.

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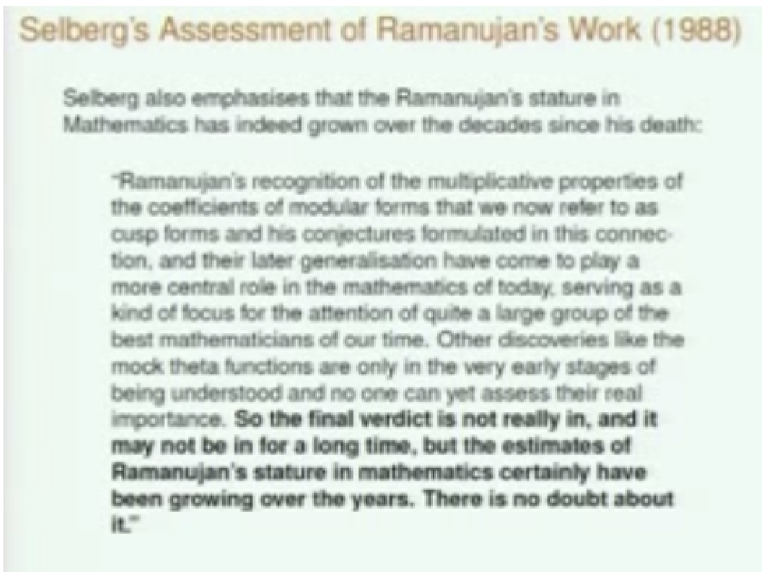

Selberg is saying that we are still studying this mock theta functions and things like that. So, really an assessment of Ramanujan's work cannot really be given. So, the final verdict is not really in and it may not be in for a long time, but the estimates of Ramanujan's stature in mathematics certainly have been growing over the years, there is no doubt about it.

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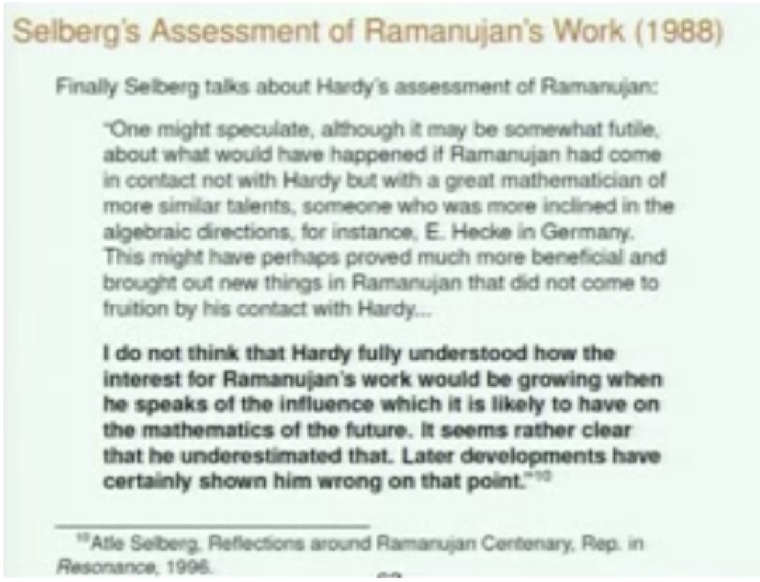

Then, referring to the fact that Hardy said that Ramanujan's work could have been more greater if it were not that strange and if it were more mainstream, etc. He says I do not think that Hardy fully understood how the interest for Ramanujan's work would be growing when he speaks of the influence which it is likely to have on the mathematics of the future. It seems rather clear that he underestimated that. Later developments have certainly shown him wrong on that point.

Selberg is sort of a person well known for understatement but definitive understatement. He will not make a hyper (()) (44:58) but he makes his point very clearly. He even had said that it would have been better if Ramanujan had gone to someone more sympathetic who had a more inclination in algebraic directions; for instance, E. Hecke in Germany. E. Hecke is well-known for work on modular forms.

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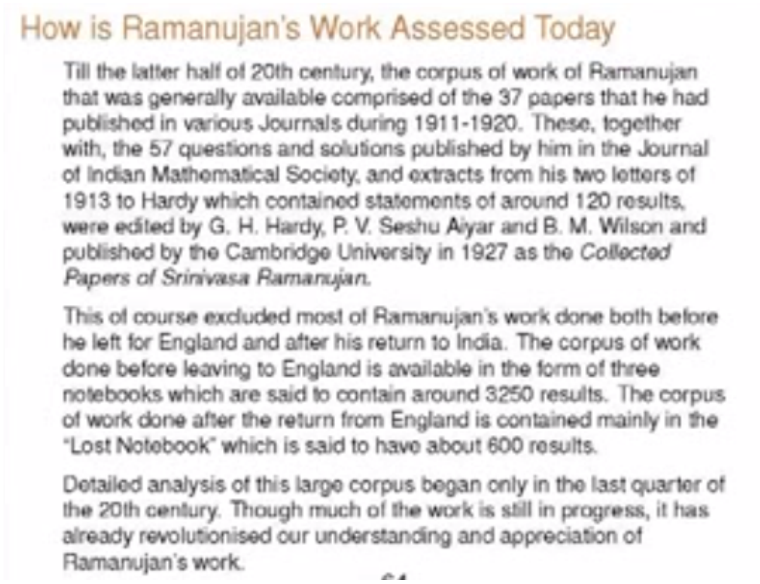

So, by 1988, the assessment of Ramanujan's work looks very, very different from the way it was assessed by Hardy in 1921. Now, how is it assessed today, so next five-six minutes here. The issue is the collected works of Ramanujan which was published in 1927 had only 37 papers and then the 57 questions that he wrote in mathematical society and it included the two letters that he wrote to Hardy in 1913 which contained about 120 results, most of them taken from his notebooks without proofs.

This excluded almost the entire corpus of the work of Ramanujan that was done either before he went to England with the notebooks or the Lost Notebook in which he wrote down the result after he came back from England. The notebooks that were there had about 3300 results, the notebooks on which he wrote down his results prior to going to England and this Lost Notebook had about 600 results.

This entire corpus in fact has been seriously analysed only in the last 25 years and so we are in a position to better estimate Ramanujan's work.

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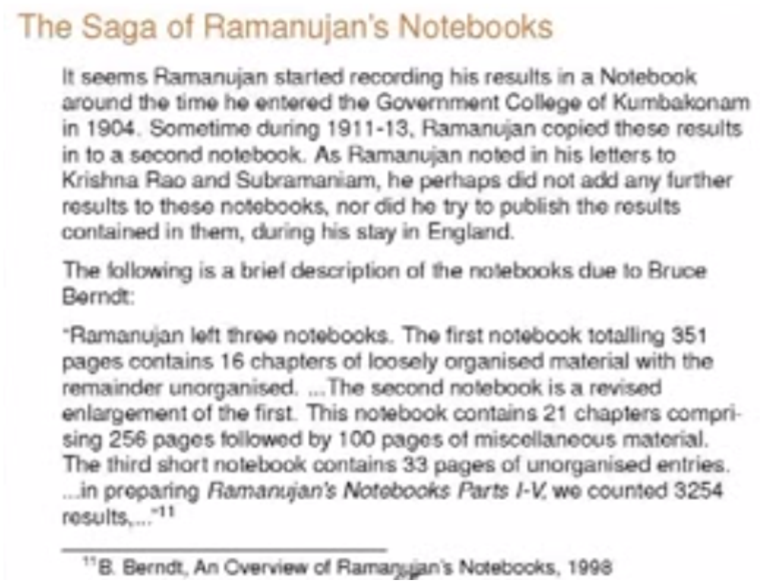

So, as I said then itself Ramanujan started recording this in around 1904. Sometime in 1911 or 1913, he even made a copy of it. So, there are 351 pages, 16 chapters, the second notebooks have 21 chapters 256 pages, third notebook has 33 pages, in all about 3254 results.

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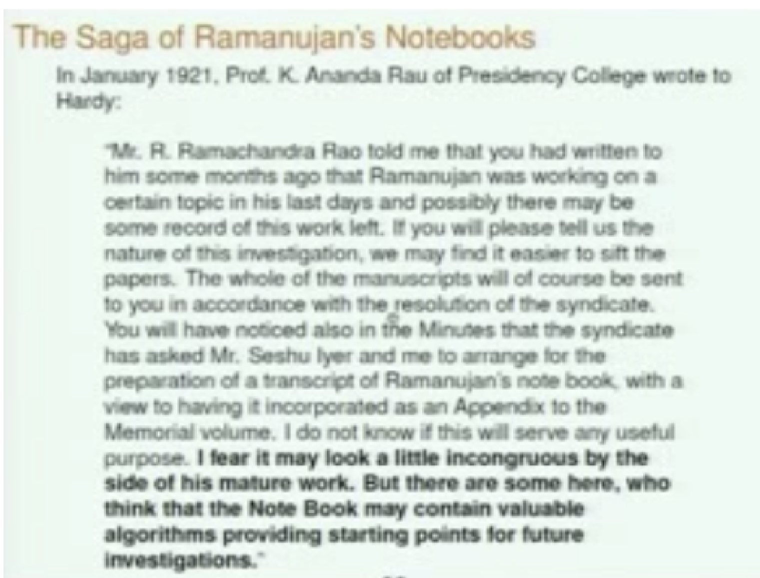

Now, in 1921, Professor Ananda Rau of Presidency College wrote to Hardy, Hardy had enquired about Ramanujan had mentioned about this mock theta functions in the letter in 1921 to Hardy. So, Hardy was enquiring whether there is material on it. So, he says we will look it up and then he says now Madras University has passed a resolution, we are sending you all his notebooks to be published.

Ananda Rau of course adds his own view that I fear it may look a little incongruous by the side of his mature work, mature work meaning the work that he did in collaboration with the British mathematicians in England but still many people here in Madras seem to think that the notebooks may contain some valuable insights.

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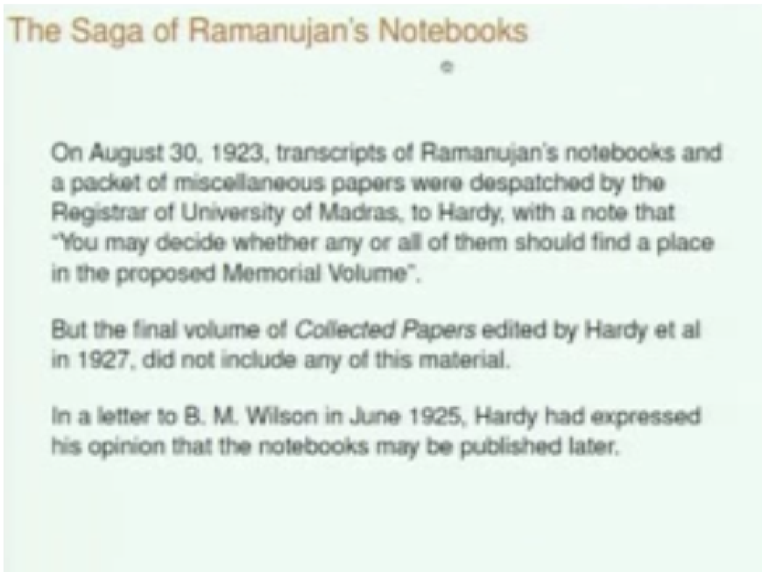

So, all these notebooks were sent in 1923 to England. I think some of them were returned to India to Ranganathan by Hardy. In 1927, the collected papers were edited but the material of this notebook was not included in the collected papers. Now, when the collected papers got published, it had these two letters of Ramanujan which had this 120 results. So, many people started looking at them and started proving them and wondering what else is contained in the notebook. So, there was a clamour for the publication of this notebook.

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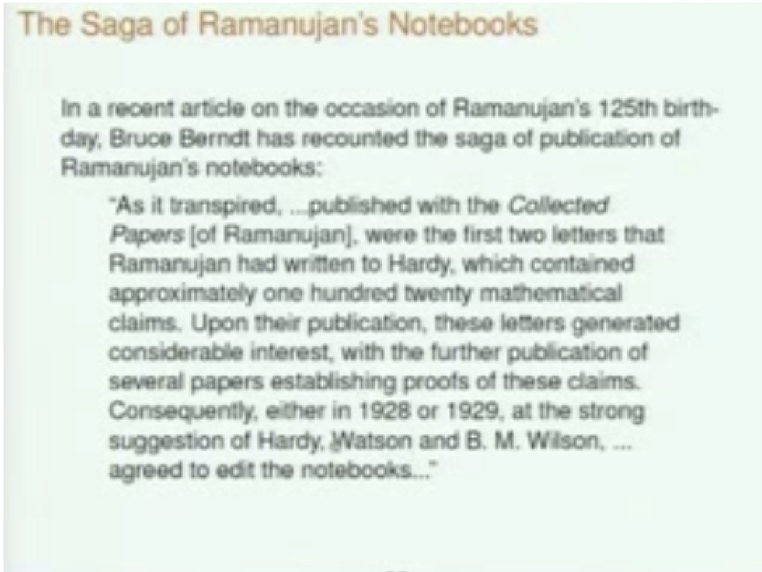

So, around 1930, Hardy asked Watson and Wilson to try and edit it.

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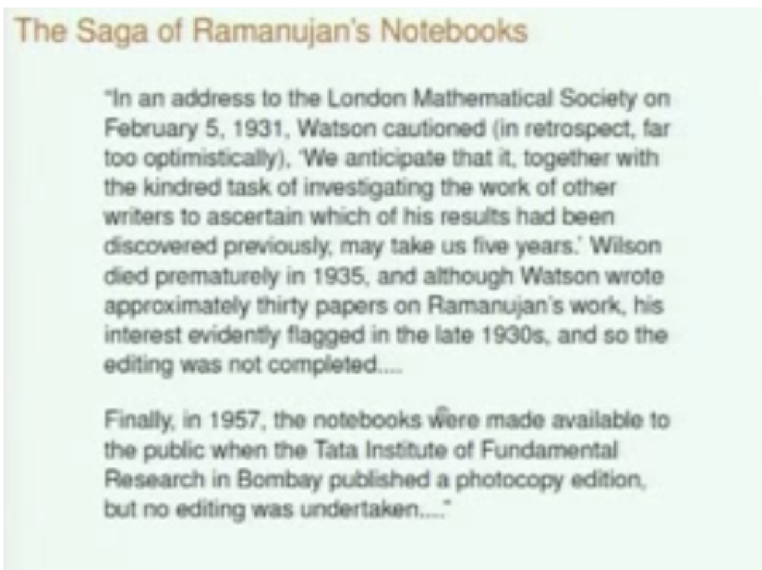

Somehow, this did not go through. Watson wrote about 30 papers from Ramanujan's work but somehow they could not complete this. So, in 1957, that Tata Institute of Fundamental Research in Bombay published a facsimile edition, a photocopy of the book but no editing was undertaken. A second edition of it has been published last year during 125th year of Ramanujan.

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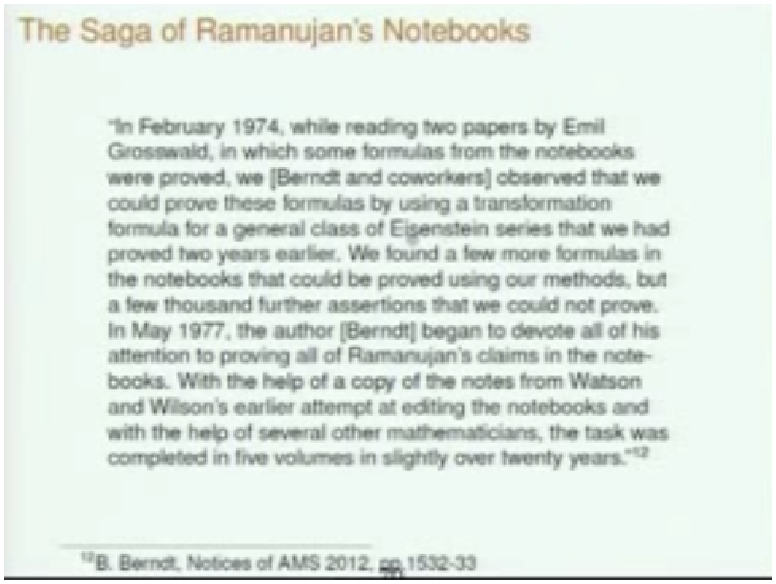

So, Bruce Brendt says that it was only in 1974 he started looking at some results in the notebook and he found that there were many more that he did not know and he started proving one or two of them. Then, he realised that this is going to be a huge work. So, he spent the next 20 years from May 1977. He used all the material that Watson and Wilson had left. Huge amount of notes had been made on Ramanujan's notebook, especially by Wilson and wanted to edit that and with the aide of all that, is about five volumes these notebooks with a commentary were published.

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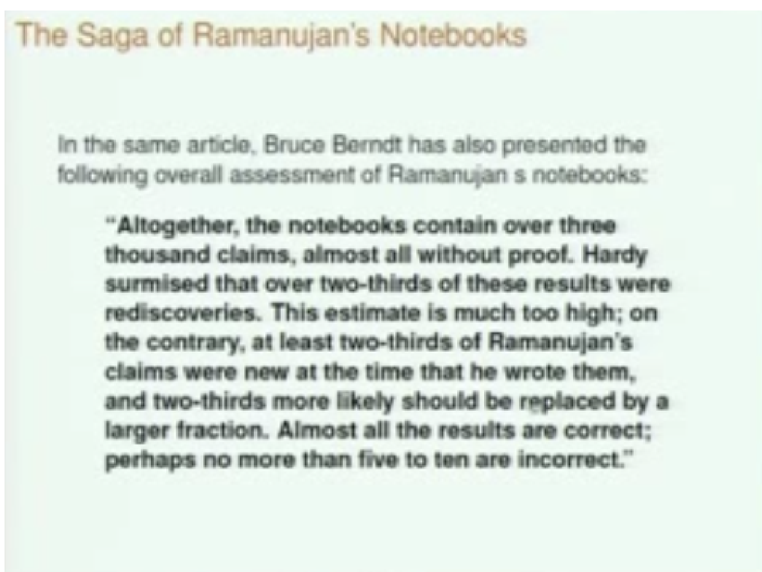

So, now comes what Bruce Brendt understands the notebooks in which Ramanujan wrote down his results prior to going to England. Altogether, the notebooks contain over 3000 claims, almost all without proof. Hardy surmised that over two-thirds of these results were rediscoveries, I mentioned that. This estimate is much too high.

On the contrary, at least two-thirds of the Ramanujan's claims were new at the time that he wrote down, that is between 1904 to 1913 and two-thirds more likely should be replaced by a larger fraction, almost all the results are correct perhaps no more than 5 to 10 are incorrect, in a mass of 3250 results.

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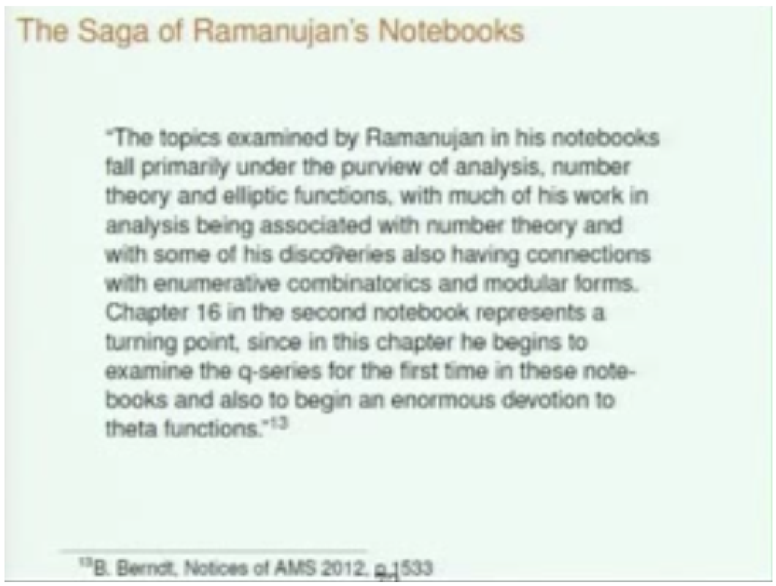

Then, he describes what are the topics that contains. This is an article written in the context of the 125th year of Ramanujan.

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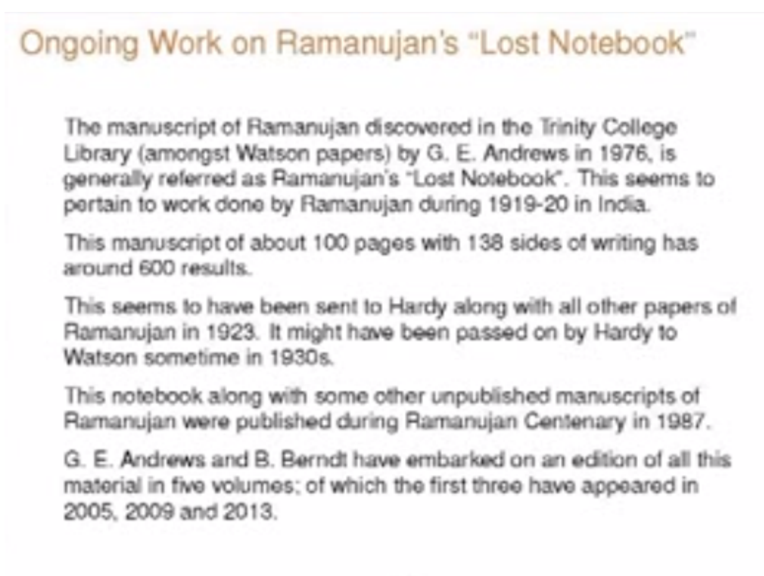

Now, there is this Lost Notebook of Ramanujan which was discovered in 1976 by George Andrews. It has about 600 results and facsimile of this notebook was published during Ramanujan centenary by Narosa publishers. Now, Andrews and Brendt have embarked on the edition of this material in five volumes, three volumes have appeared now.

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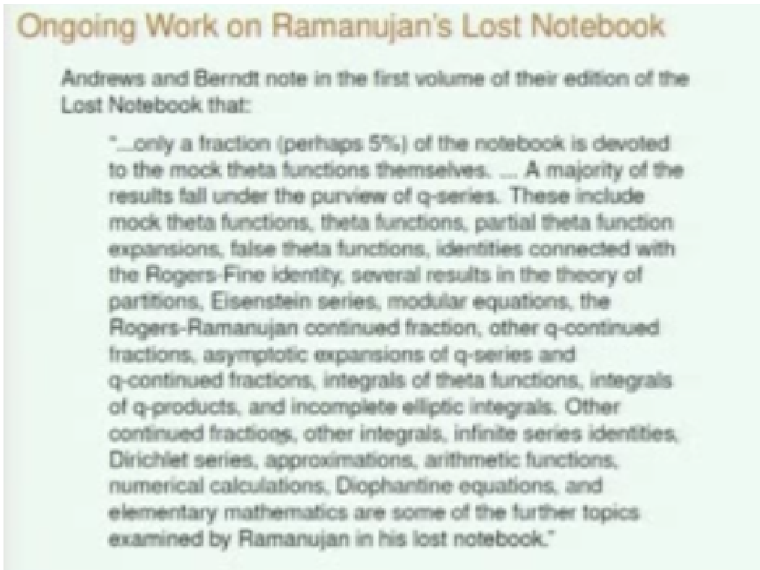

Again, the list to the kind of works, mock theta function is only a small part of this Lost Notebook, they are only 5%. There are many more things it contains, so this work is still going on.

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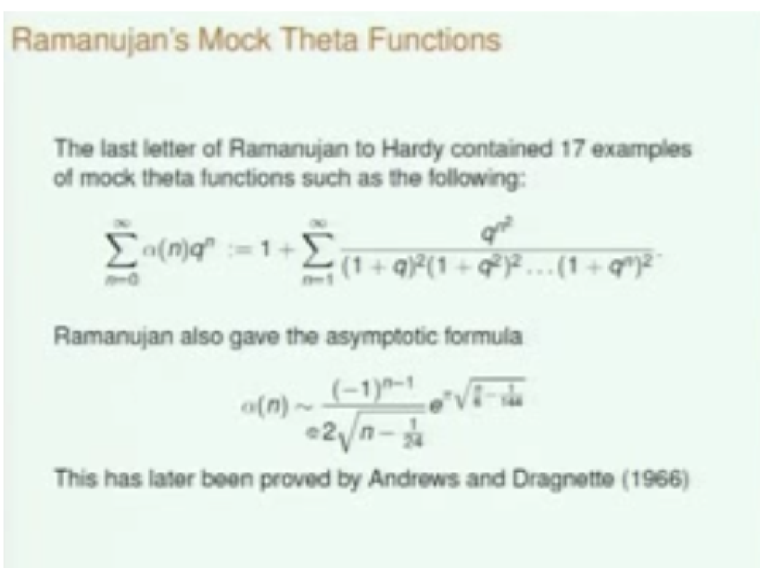

Now, on mock theta function itself insomuch is talked about, what is a mock theta function. So, this last letter to Hardy had 17 examples of functions like this. Each of them is a mock theta function and Ramanujan gave this formula for this function which was proved in 1966 by Andrews.

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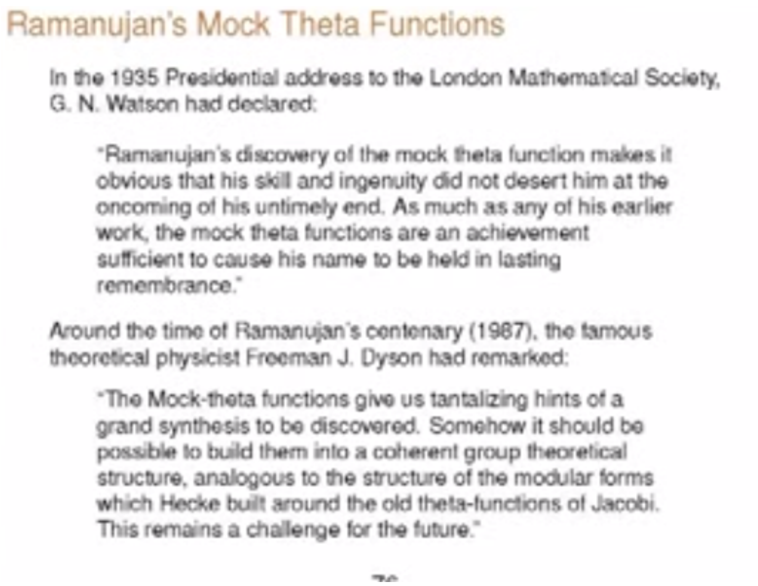

In 1935, G. N. Watson had said that Ramanujan's discovery of mock theta functions makes it obvious that his skill and ingenuity did not desert him at the oncoming of his untimely end. As much as any of his earlier work, the mock theta function are an achievement sufficient to cause his name to be held in lasting remembrance. During the centenary, Freeman J. Dyson, a very famous theoretical physicist said that some new theory is expected out of this mock theta function.

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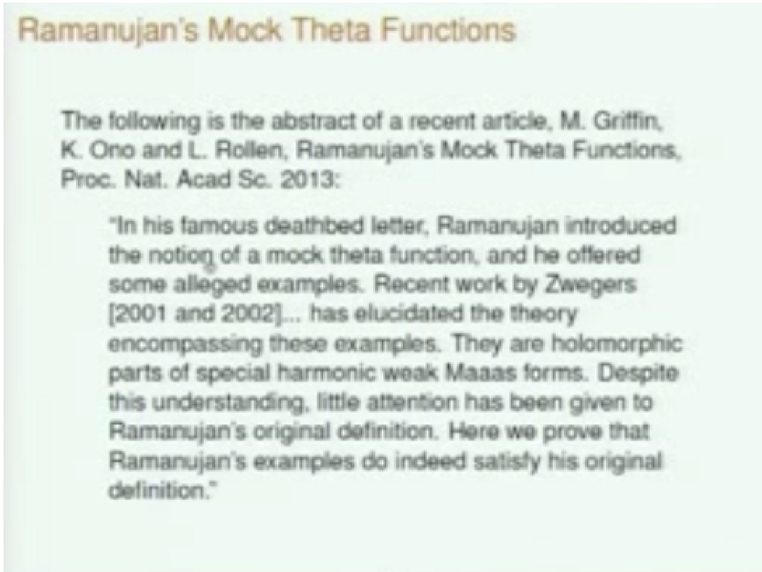

Recently last month I think, there is a paper by a famous Japanese scholar, K. Ono on Ramanujan's mock theta functions in his famous deathbed letter, this is the abstract of that paper, Ramanujan introduced the notion of a mock theta function and he offered some examples. Recent work by Zwegers in 2001 and 2002 has elucidated the theory encompassing these. They are holomorphic parts of special harmonic week Maass forms, this is not misspelling.

These are special mathematical terminology. Despite this understanding, little attention has been given to Ramanujan's original definite of mock theta function. Here, that is in this paper of 2013 we are proving that Ramanujan's examples do indeed satisfy his original definition.

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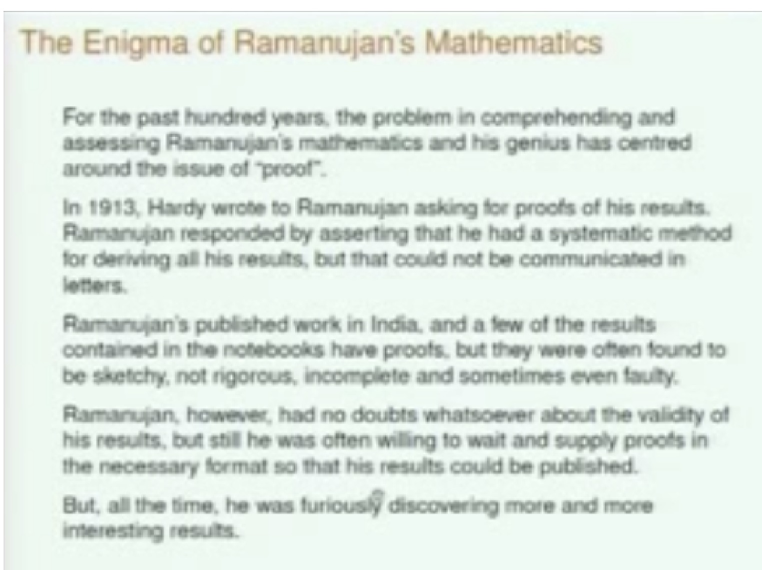

So, there is a sort of scenario we are now in regarding Ramanujan's work. Now, what has been the cause of the tremendous confusion in trying to understand and almost made Ramanujan in enigma is the issue that everybody is asking Ramanujan has this result but where are the proofs. In fact, this started in 1913 itself with Hardy telling Ramanujan your results are fine but unless you send me the proofs what is the use.

The published work in India and his notebooks have some proofs here and there but most of them are sketchy, not rigorous, incomplete and sometimes even said to be faulty. Now, Ramanujan himself had no doubts whatsoever about the validity of his results and as he wrote to his friends that he was even willing to wait and supply proofs in the necessary format so that they can be published. But all the time, he was more furiously engaged in discovering newer and newer results.

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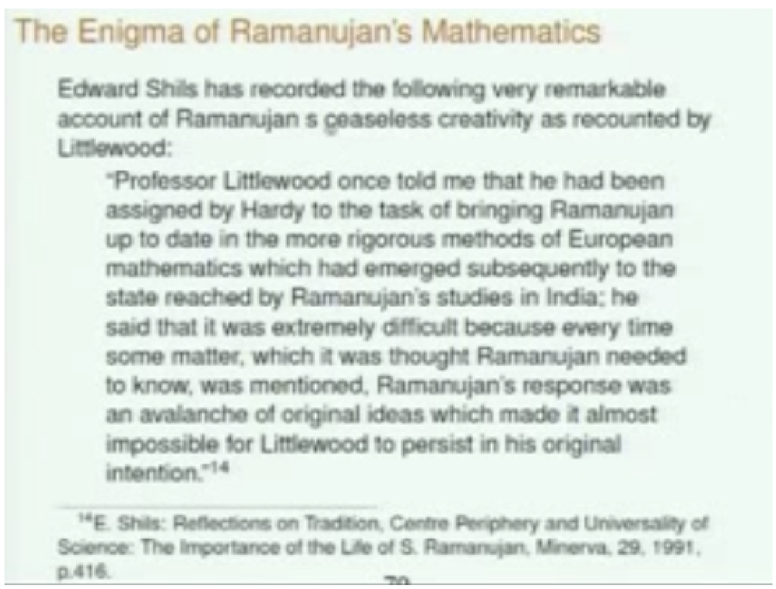

There is one interesting story documented in a paper by Shils of a conversation that he had with Littlewood. Littlewood was a colleague of Hardy in Cambridge at that time. Professor Littlewood once told me that is Edward Shils, that he had been assigned by Hardy to the task of bringing Ramanujan up to date in the more rigourous methods of European mathematics which had emerged subsequently to the state reached by Ramanujan's studies in India.

He said that it was extremely difficult because every time some matter which it was thought

Ramanujan needed to know was mentioned, Ramanujan's response was an avalanche of original ideas which made it almost impossible for Littlewood to persist in his original intention.

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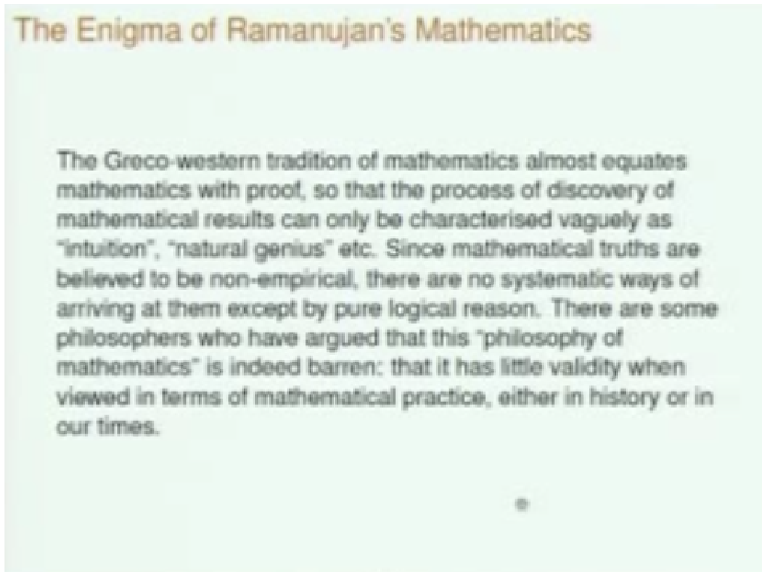

So, much of this cause of difficulty and confusion is with the fact that the Greco-western tradition of mathematics almost equates all mathematics with proof. So, the process of discovery of mathematical results can only understood very vaguely. It can be described as intuition, natural genius. If you seen the writings of Hardy, you will find that here is a man who does not know any proof but he is filled with intuition, he is filled with natural genius.

So, since mathematical tools are believed to be non-empirical, there is no systematic way of arriving them, except by pure logical reason. So, this is the sort of dominate understanding of mathematics which does not throw any light on mathematical discovery; and for the same reason, it cannot explain most of the history of mathematics or not even the contemporary mathematical practice as to how one comes up with the result or how somebody else correct some result and things like that.

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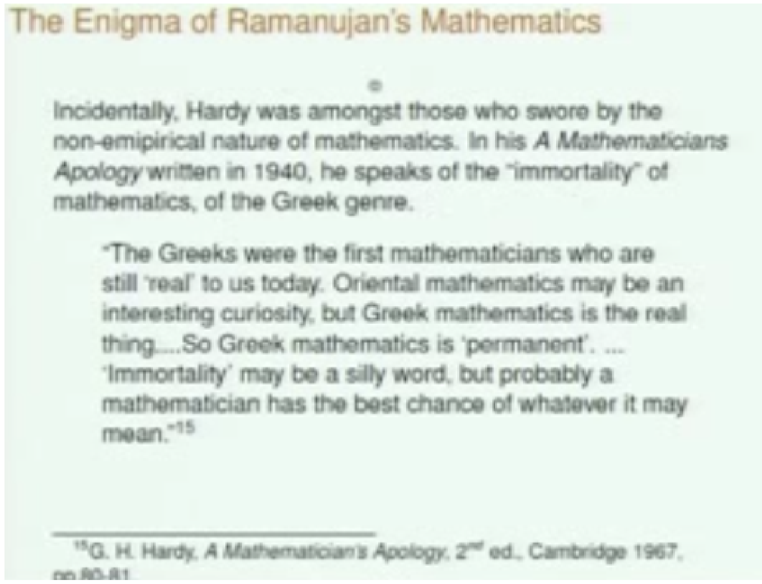

Incidentally, Hardy was one of those people who really swore by the non-empirical nature mathematics. This quotation was given yesterday. It is the second half of the quotation which is more interesting. The Greeks were the first mathematicians who are still real to us today. Oriental mathematics may be an interesting curiosity but Greek mathematics is the reality. This is his view of the Oriental mathematics, that is alright.

So, Greek mathematics is permanent. Hardy, you understand, is a great agnostic, great skeptic. So, Greek mathematics is permanent. Immortality may be a silly word but probably a mathematician has the best chance of whatever it may mean. So, if mathematical tools that are going to stay forever and forever and forever.

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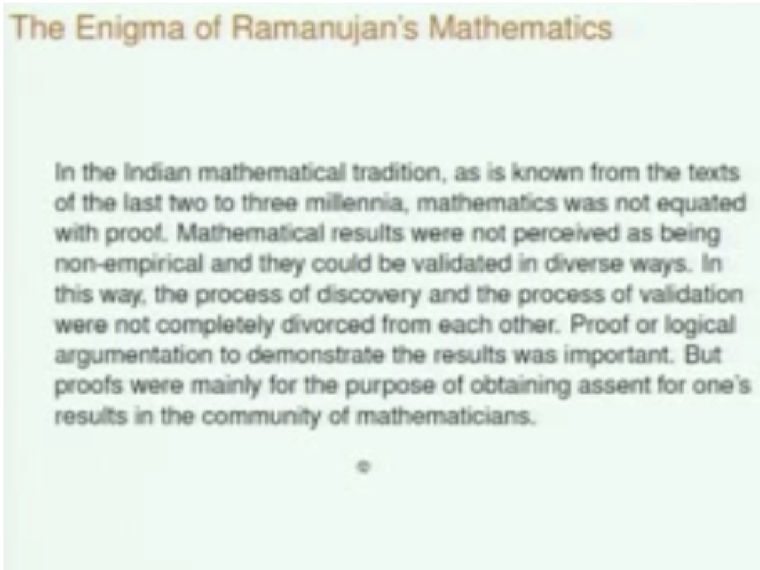

On the contrary in the Indian tradition, at least as known from the text of the last 2000-3000 years, mathematics was not equated with proof. Mathematical results were not perceived as being non-empirical and they could be validated in diverse ways. In this way, the process of discovery and the process of validation were not completely diverged from each other.

Of course, we have to understand it much more, if we have to make a claim like that. Proof of logical argumentation to demonstrate the results was important but they were mainly for obtaining the community's assent for once result.

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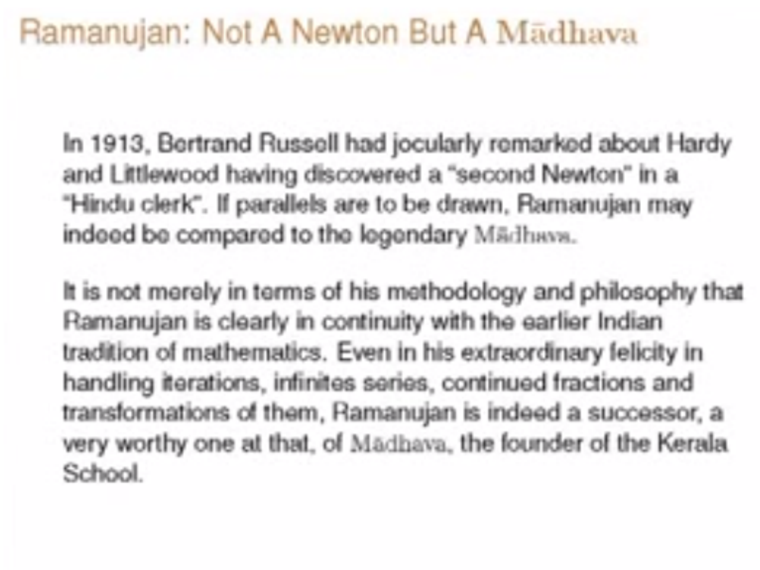

Therefore, we have to conclude something. We can go back to this wonderful joke of Bertrand

Russell that Hardy and Littlewood had discovered Newton in a Hindu class but the kind of outline given of Ramanujan's work and the kind of outline of the earlier work of Kerala School perhaps does convince you that it is not merely in terms of his methodology or philosophy that Ramanujan is clearly in continuity with the Indian tradition of mathematics. Even in his extraordinary felicity in handling iterations, algorithms, infinite series, continued fractions, transformations of them, etc., he is a successor of Madhava, the founder of Kerala School.

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So, this is the collected papers. These are the lectures by Hardy. Then, these are the notebooks of Ramanujan edited by Bruce Brendt in five volumes from 1985 to 1998. These are his letters edited by Brendt.

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References

8. R. Kanigal, *The Man Who Knew Infinity: The Life of the Genius Ramanujan*, Scribner, New York 1995.
9. B. C. Berndt and R. A. Rankin, *Ramanujan: Essays and Surveys*, AMS, Providence 2001.
10. G. E. Andrews and B. C. Berndt, *Ramanujan's Lost Notebook*, Volumes 1-3, Springer, New York 2005-2013.
11. Krishnaswami Alladi (ed), Srinivasa Ramanujan Going Strong at 125, Notices of the AMS, 59, December 2012, pp.1522-37 and ibid. 60, January 1913, pp.10-22 [Articles by K. Alladi, G. E. Andrews, B. C. Berndt, J. M. Borwein, K. Ono, K. Soundararajan, R. C. Vaughan and S. O. Warnaar]

The well-known biography of Robert Kanigal which is I think being reprinted last year. Brendt and Rankin have a set of collective essays. This Lost Notebook is still coming three volumes of it have appeared so far. This was a recent collection of articles during the time of Ramanujan's 125th year. Thank you.