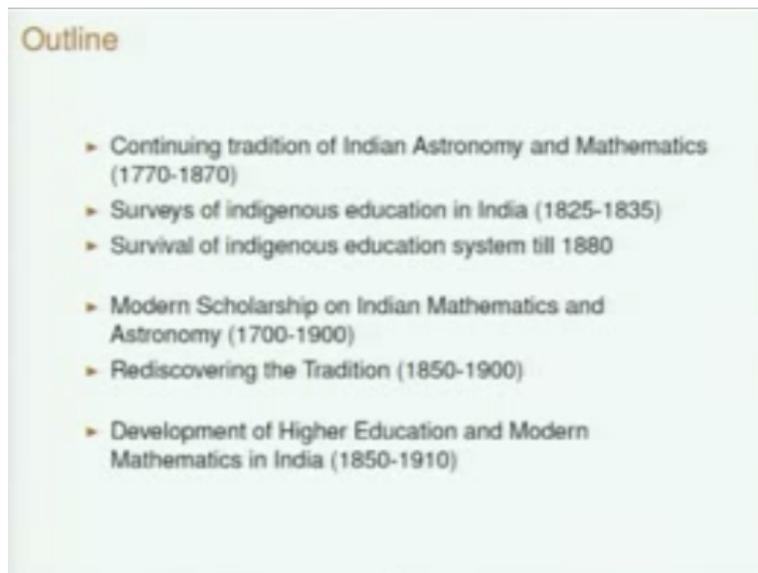


Mathematics in India: From Vedic Period to Modern Times
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Lecture – 39
Mathematics in Modern India 1

So, today we will this first lecture on mathematics in Modern India. There will be two lectures covering the modern period roughly from 1750 to the present time. As you can see it is a long period and a lot of historical records and lot of events have happened, so we will have to focus on important things in this period.

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So, I will first say something about the way that the indigenous tradition in astronomy and mathematics continued late into 19th-century. We tell you something about the nature of indigenous education system in India in early part of 19th-century before the British started the English education system here and how that system survived of course very limpingly till the later part of 19th century.

Then, I will say something about the Modern European scholarship on the Indian tradition of astronomy and mathematics during the two centuries, 18th and 19th centuries and then how Indians themselves participated in this rediscovery of tradition in the latter part of 19th century. Then, of course this new system of education that was started in the middle of 19th century, how

did that develop, but the main part of today's lecture, we will look into the life and work of Srinivasa Ramanujan and its impact today.

So, till about 50 years ago most modern scholars most textbooks had Indian mathematics died with Bhaskaracharya II around 1150. Later people only wrote a few commentaries or few works here and there. As we have seen in detail in this course very interesting was then in the post Bhaskara period. We have discussed Narayana Pandita. We have discussed Kerala scholar Madhava, Parameshvara and Nilakantha.

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Background: Continued Development of Mathematics in Medieval India

- ▶ Systematic exposition of Mathematics and Astronomy with proofs in *Yuktibhāṣā* (in Malayalam) of Jyeṣṭhadeva (c.1530) and commentaries *Kriyākramakarī* and *Yuktidīpikā* of Śaṅkara Vāriyar (c.1540).
- ▶ Works of Jīnanaraja (c.1500), Gaṇeśa Daivajña (b.1507), Sūryasūtra (c.1541) and Kṛṣṇa Daivajña (c.1600): Commentaries with *upapattis*.
- ▶ Works of Munīśvara (b.1603) and Kamalākara (b.1616).
- ▶ Mathematics and Astronomy in the Court of Sawai Jayasīnha (1700-1743). Translations from Persian of Euclid and Ptolemy.
- ▶ Works of later Kerala astronomers Acyuta Piṣaraṭi (c.1550-1621), Putumana Somayāji (c.1700).

We have given you some (()) (02:11) of the work of Krishna Deva gian, Ganesh Deva gian and given the trigonometrical work of Kamalakara. We have of course not discussed the astronomical efforts of Sawai Jai Singh of Jaipur.

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A European Account of Indian Astronomy (c.1770)

"While waiting in Pondicherry for the Transit of 1769, Le Gentil tried to gather information about native astronomy...

Le Gentil eventually contacted a Tamil who was versed in the astronomical methods of his people. With the help of an interpreter he succeeded in having computed for him the circumstances of the lunar eclipse of 1765 August 30, which he himself had observed and checked against the best tables of his times, the tables of Tobias Mayer (1752).

The Tamil Method gave the duration of the Eclipse 41 second too short, the tables of Mayer 1 minute 8 seconds too long; for the totality the Tamil was 7 minutes 48 seconds too short, Mayer 25 seconds too long.

These results of the Tamil astronomer were even more amazing as they were obtained by computing with shells on the basis of memorised tables and without any aid of theory.

Now, in about 18th century, there were many European observers who came to interact with the traditional Indian astronomers and mathematicians and here is one such Le Gentil was deputed to observe the transit of Venus in Southeast Asia and he went to Pondicherry which was a French enclave and he got an eclipse in 1765 computed by a native astronomer as they call and compared it with the best tables that they had and what Gentil found was that the Indian calculation was 40 seconds too for the duration of the eclipse that Tobias Mayer table gave 1 minute 8 seconds too long, this in 1765.

For the totality the Tamil method was 7 minutes 48 seconds too short, Mayer table was 25 seconds too long and Le Gentil said that this was even more amazing because this Tamil astronomer was computing with cowrie shells, with no tables on hand, on basis of memorised tables.

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A European Account of Indian Astronomy (c.1770)

'Le Gentil says about these computations: 'They did their astronomical calculations with swiftness and remarkable ease without pen and pencil; their only accessories were cauries... This method of calculation appears to me to be more advantageous in that it is faster and more expeditious than ours.'¹

Note: What Neugebauer is referring to as "Tamil method" is nothing but the *Vākya* method developed in south India, especially by Kerala Astronomers. Neugebauer also refers to the report of John Warren (in his *Kālasākalita*) about the calculation of a lunar eclipse in 1825 June 1, where the Tamil method predicted midpoint of the eclipse with an error of about 23 minutes.

¹Neugebauer, *A History of Ancient Mathematical Astronomy*, Vol. III, Springer, 1975, p.20. (Le Gentil's quote translated from French).

They did their astronomical calculations with swiftness and remarkable ease without pen and pencil. Their only accessories were cowries, that is the shells, cowrie as they call in Kannada. This method of calculation appears to be more advantageous in that it is faster and more expeditious than ours. Now Neugebauer who is calling this Tamil method and all that is actually referring to what is called Vakya method of calculation which was prevalent in most of South India, especially developed by the Kerala astronomers.

Neugebauer (()) (04:11) drawn the above quotation also talks of John Warren's book where he is talking of again in Pondicherry an eclipse being observed in 1825 and the error is about 23 minutes of the calculation.

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Continuing Tradition of Indian Astronomy (c.1820)

Śaṅkaravarman (1784-1839): Raja of Kaṣṭṭanū in Malabar. Due to the wars with Hyder and Tipu, he is supposed to have spent his early years with Mahārāja Svāti Tirumāḷ at Tiruvananthapuram.

In 1819, He wrote *Sadratnamālā* (one of the four works mentioned by Whish in 1835), an Astronomical manual following largely the *Parahita* system. He also wrote his own Malayalam commentary, perhaps a few years later (published along with the text in Kozhikode in 1899).

Chapter I has interesting algorithms for calculation of square and cube roots. Chapter IV deals with computation of sines.

Śaṅkaravarman also gives the following value of π which is accurate to 17 decimal places: $\pi \approx 3.14159265358979324$

What I was saying is that the traditional methods of calculation were fairly efficient and continued late into 18th and early 19th century and the results were reasonably comparable with whatever was available from the best available tables of contemporary astronomy.

Two important astronomers Sankara Varman in early part of 19th century. He wrote Sadratnamala. It is a (()) (04:47) text in astronomy based on the Parahita system. He also wrote his own Malayalam commentary. There is some interesting discussion of square roots and cube roots. Chapter IV deals with sines and Sankara Varman also gives this value by pi to 17 places that I mentioned.

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Continuing Tradition of Indian Astronomy (c.1870)

Candraśekhara Śāmanṭa (1835-1904): Popularly known as Pūthāni Śāmanṭa, he had traditional Sanskrit education. Starting from around 1858, he carried out extensive observations for over eleven years, with his own versatile instruments, with a view to to improve the almanac of Puri Temple.

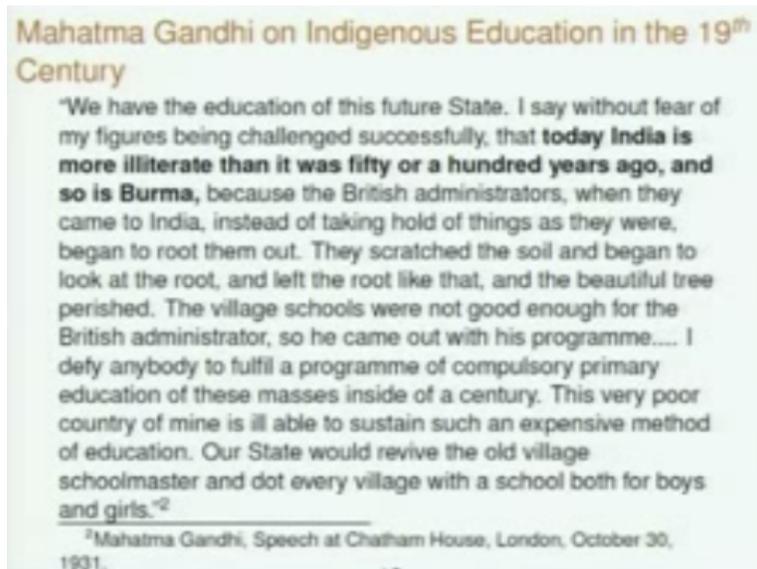
He wrote his *Siddhāntadarpaṇa* with nearly 2500 verses in 1869 (published at Calcutta 1899). Based on his observations, Śāmanṭa improved the parameters of the traditional works, he detected and included all the three major irregularities of lunar motion, and improved the traditional estimates of the Sun-Earth distance.

In Chapter V of his work, Śāmanṭa has presented his planetary model where all the planets move around the Sun, which moves around the Earth.

Another important astronomer who continued the indigenous tradition was Chandrashekara Samanta in Orissa called Pathani. He wanted to improve the panchangam of the Puri Temple and he wrote this massive work Siddhanta Darpana. He was all traditionally trained and was not in modern education system in 1869. Based on his observation, he improved the parameters of traditional works.

He detected and included all the three major inequalities of lunar motion and he also improved the traditional estimates of earth-sun distance and in chapter V, Samanta presented a planetary model in five planets go round the sun which was around the earth something like Nilakantha's planetary model which is called an iconic model based upon the name of Tarik Ibrahim.

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So, there were sort of important astronomers and mathematicians in 19th century but as Mahatma Gandhi declared in 1931 that the great de-education of India which occurred in 19th century. Mahatma Gandhi in a famous speech when he goes for the round table conference in London, he is the making a statement that it was in 1931 India is more illiterate than it was 50 or 100 years ago and so is Burma and that is because the British administrators came and disrupted the indigenous system of education which he calls as a the beautiful tree died because they dug up the roots and left the roots exposed.

What is this indigenous education system in the earlier part of 19th century. So, most observers

who visited the administrators who stayed in India reported that almost every village here had a school.

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Reports on Indigenous Education in 19th Century

"If a good system of agriculture, unrivalled manufacturing skill, a capacity to produce whatever can contribute to convenience or luxury; **schools established in every village, for teaching reading, writing, and arithmetic**; the general practice of hospitality and charity among each other; and above all a treatment of the female sex, full of confidence, respect and delicacy, are among the signs which denote a civilised people, then the Hindus are not inferior to the nations of Europe; and if civilisation is to become an article of trade between the two countries, I am convinced that this country [England] will gain by the import cargo."³

"We refer with particular satisfaction upon this occasion to that distinguished feature of internal polity which prevails in some parts of India, and by which the **instruction of the people is provided for by a certain charge upon the produce of the soil, and other endowments in favour of the village teachers**, who are thereby rendered public servants of the community."⁴

³Thomas Munro's Testimony before a Committee of House of Commons, April 12, 1813.
⁴Public Despatch from London to Bengal, June 3, 1814.

So, here is Thomas Munro declaring 1813 that schools established in every village for teaching, reading, writing and arithmetic or here is a dispatch from the British government in London to Bengal where they are noting that the instruction of the people here is provided for by a certain charge upon the produce of the soil and other endowments in favour of the village teachers who are thereby rendered public servants of the community, something which can hardly be said today that no government servants receive their pay either from Delhi or from Fort St. George and they are not servants of any community.

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Reports on Indigenous Education in 19th Century

"There are probably as great a proportion of persons in India who can read, write and keep simple accounts as are to be found in European countries...."⁵

"I need hardly mention what every member of the Board knows as well as I do, that there is hardly a village, great or small, throughout our territories, in which there is not at least one school, and in larger villages more; many in every town, and in large cities in every division; where young natives are taught reading, writing and arithmetic, upon a system so economical, from a handful or two of grain, to perhaps a rupee per month to the school master, according to the ability of the parents, and at the same time so simple and effectual, that there is hardly a cultivator or petty dealer who is not competent to keep his own accounts with a degree of accuracy, in my opinion, beyond what we meet with amongst the lower orders in our own country; whilst the more splendid dealers and bankers keep their books with a degree of ease, conciseness, and clearness I rather think fully equal to those of any British merchants."⁶

⁵Annual Report of Bombay Education Society, 1819.

⁶Minute of G. Prendargast, Member, Bombay Governor's Council, 1821.

In 1819 Bombay, it was said that there probably as greater proportion of persons in India who can read, write and keep simple accounts as are to be found in European country. So, this is the kind of accounts that administrators and observers were talking about an education system about which the Britishers had nothing to do. They had not started their education system and they were not providing any money for what was there.

And this was a sort of maintained at the societal level, perhaps at a much more sort of ((08:07)) state than it would have been when the state in the actively promoted and supported in earlier periods.

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Indigenous Education in Madras Presidency (c.1825)

The British Government conducted a detailed survey of the indigenous system of education covering all the Districts of the Madras Presidency during 1822-25. **The Survey found 11,575 schools and 1094 "colleges" in the Presidency.** Summarising the survey information the then Governor Thomas Munro wrote in his Minute of March 10, 1826:

"It is remarked by the Board of Revenue, that of a population of 12½ millions, there are only 188,000, or 1 in 67 receiving education. This is true of the whole population, but not as regards the male part of it, of which the proportion educated is much greater than is here estimated... if we reckon the male population between the ages of five and ten years, which is the period which boys in general remain at school, at one-ninth... the number actually attending the schools [and colleges] is only 184,110, or little more than one-fourth of that number. ... **I am, however, inclined to estimate the portion of the male population who receive school education to be nearer to one-third** than one-fourth of the whole, because we have no returns from the provinces of the numbers taught at home...."

So, to look at this system, various surveys were conducted in the end of first part of 19th century. The survey Madras Presidency went through all the districts in detail and it found in 1825 that there were 11,575 schools and 1094 colleges in Madras Presidency which includes coastal Andhra, whole of Tamil Nadu, northern Kerala and the Ganjam district of Orissa and the Bellary district of Karnataka and Thomas Munro concluded that something like one-third of the boys of school going age are being educated under this system.

The data showed something one-fourth, Thomas Munro said a large number are being educated privately at home also.

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Indigenous Education in Madras Presidency (c.1825)

Community Profile of Boys Attending School

District	Brahmins	Kshatriyas	Vaisyas	Shudras	Other Castes	Muslims	Total	Total Population
Telugu Districts	11,893	121	7,876	10,876	4,753	1,639	38,168	4,879,408
% of Total	30.47	0.32	20.67	28.49	12.46	4.30		
Boys of School-going age	9,111	2,307	7,387	134,896	39,479	40,387	223,836	
% of Community	222.49	4.82	169.91	7.47	467.99	23.78	77.62	
Malabar	7,750		84	9,697	7,750	3,186	11,963	907,874
% of Total	23.64		0.79	28.90	23.84	26.72		
Boys of School-going age	953	15	620	25,447	9,893	13,284	59,421	
% of Community	236.01	0.00	22.24	26.22	27.88	24.08	22.72	
Tamil Districts	11,597	509	4,442	57,873	13,196	3,453	97,669	6,622,474
% of Total	27.44	0.40	4.79	62.30	24.72	3.67		
Boys of School-going age	16,191	1,639	7,910	299,260	77,373	14,944	367,946	
% of Community	113.40	22.79	26.16	22.67	27.86	24.69	23.22	
TOTAL	29,721	490	13,440	75,943	22,925	10,644	153,172	12,850,940
% of Total	19.40	0.32	8.79	49.58	24.87	4.89		
Boys of School-going age	23,203	4,282	16,778	457,279	149,275	42,691	713,941	
% of the Community	228.69	23.67	89.24	26.67	23.54	25.32	22.45	

Source: Data from Dharampal, *The Beautiful Tree*, Imper India, Delhi 1963, pp.21-22.

Boys of school going age in each community, estimated by using the community profile of the total population as per 1871 Census, and by estimating the boys of school-going age (5-10 years) as one-ninth of total population, following Munro.

But what was significant about this survey was that about 50% of students belonged to what is called the Shudra community, 15% of the students belonged to what are called the other caste whom we refer to as the scheduled castes and scheduled tribes today. So, the majority of the students 65% came from the non-dvijyas category of Indian society.

If you look at percentage of children of school going age of each community being educated, you find that nearly 17% of the boys of school going age of Shudra were receiving education in this public education system in these schools. In Tamil-speaking area, in Tamil Nadu, it was something like 23% and 17% in Tamil Nadu of children of school going age in the other communities, scheduled caste and scheduled tribes were receiving education in it. No wonder

Mahatma Gandhi called it the beautiful tree.

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Indigenous Education in Madras Presidency (c.1825)

The languages of instruction in most of the 11,575 schools were the regional languages. The average period of instruction was around 5-7 years. The subjects taught were reading writing and arithmetic.

The instruction in most of the 1,094 "colleges" or institutions of higher learning was in Sanskrit. Details of the subjects taught are available for the 618 colleges in four districts: 418 taught *Vedas*, 198 Law, 34 Astronomy and *Ganita* and 8 taught *Andhra Śāstram*.

Further, in Malabar, 1594 scholars were receiving higher instruction privately, of whom 808 studied Astronomy (of whom 96 were *dvijas*) and 154 Medicine (of whom 31 were *dvijas*).

The languages of the instruction were regional languages. The colleges that they were talking 1094 were of course the higher learning was in Sanskrit and various subjects were taught. In Kerala it was interesting that there 800 students privately studying astronomy and about 154 studying medicine and a majority of them were non-dvijas.

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Indigenous Education in Madras Presidency (c.1825)

As regards the financial support received by the indigenous schools and colleges the situation was clearly stated by the Collector of Bellary:

"Of the 533 institutions for education, now existing in this district, I am ashamed to say not one now derives any support from the state... There is no doubt that in former times, especially under the Hindoo Governments very large grants, both in money, and in land, were issued for the support of learning."

Here is a statement from the Collector of Bellary which says that we are not supporting this system of education even a bit. Of course, it was being supported during the earlier indigenous governments.

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Indigenous Education in Bengal Presidency (c.1835)

William Adam's survey (1835) of indigenous education in selected districts of Bengal and Bihar showed the following subject-wise distribution of institutions of higher learning.

Institutions of Sanskrit learning in some districts of Bengal & Bihar

	Murshidabad	Bankura	Burdwan	South Bihar	Tripura	Total
Number of Institutions	24	55	150	27	55	311
Number of Students (Students were)						
Grammar	25	274	644	360	127	1,424
Logic	82	27	277	6	16	378
Law	84	24	128	2	6	336
Literature	2	6	80	16	4	120
Maths	5	6	120	22	1	154
Astronomy	—	6	7	13	53	79
Language	4	2	31	8	3	48
Medicine	—	6	6	2	—	16
Metaphysics	—	1	16	2	—	19
Vedant	—	3	5	6	2	16
Yoga	—	1	2	2	—	6
Mimamsa	—	—	—	2	—	2
Santhia	—	—	—	1	—	1
Total Number of Students	152	362	1,266	427	214	2,821

A similar survey was done in Bengal by Williams Adam and he found a large number of schools and colleges but the interesting thing is the kind of subjects that were taught in these colleges. Vyakarana was taught in 10424 institutions of learning, Niyaya in 378, Dharmashastra in 336, Kavya in 120, Purana in 82, Jyotisha in 48 and then Kosha, Alankara, Vaidya, Vedatantra, Mimamsa, all shastras studied but the proportion of students learning Vyakarana, Nyaya, and Dharmashastra were very large.

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Indigenous Education in Bengal Presidency (c.1835)

Adam's survey also showed that textbooks used in these institutions of higher learning included, apart from the ancient canonical texts of the various disciplines, many of the important advanced treatises commentaries and monographs composed during the late medieval period.

These included the works of Bhaṭṭoji Dikṣita (1625), Kaundabhaṭṭa (c.1650), Hari Dikṣita and Nagoṣa Bhaṭṭa (c.1700) in *Vyākaraṇa*, the works of Raghunātha (c.1500), Mathurānātha (c.1570), Viśvanātha (c.1650), Jagadīśa (c.1650) and Gadādhara (c.1650) in *Naiya-nyāya*, the works of Raghunandana (c.1550) in *Dharmasāstra* and the works *Vedāntasūtra* (c.1450) and *Vedāntaparibhāṣā* (c.1650) in *Vedānta*.

The period of study in these institutions of higher learning was between ten and twenty-five years. In many of these centres of higher learning a large part of the students came from outside, many from even different regions of India. All the students were taught gratis and outside students were provided in addition free food and lodging.

Another thing Adam found in his survey was that the books that these were people were studying in these colleges, the instruction meant to 10 to 25 years where the most advanced text that are

written by famous (()) (11:27) 17th century or 18th century by Nagesa Bhatta, Gadadhara, Jagadisa, Kaundabatta, so all these were the works that were being studied in these colleges.

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The University of Navadvīpa

On visiting Navadvīpa or Nuddeah in 1787, William Jones wrote to Earl of Spencer that "This is the third University of which I am a member". An account of Navadvīpa published in Calcutta Monthly Register in January 1791 noted:

"The grandeur of the foundation of the Nuddeah University is generally acknowledged. It consists of three colleges Nuddeah, Santipore and Gopulparra. Each is endowed with lands for maintaining masters in every science....in the college of Nuddeah alone, there are at present about eleven hundred students and one hundred and fifty masters. Their numbers, it is true, fall very short of those in former days. **In Rajah Roodre's time (circa 1680) there were at Nuddaeah no less than four thousand students and masters in proportion.**

According to Adam, in 1829 there were reported to be 25 schools of learning in Navadvīpa with 500 to 600 students. Some of these schools were supported by a small allowance from British Government.

Especially, in Navadvīpa everybody said that there is a huge university. Navadvīpa is a home of Navya Nyaya, it was place where Chaitanya also was there. It was said in 1700 there were at Navadvīpa about 4000 students and around 1000 teachers and in 1790s about 1100 students and 150 teachers and in 1829 there were 25 colleges with 500 to 600 students. This was status of Navadvīpa.

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Indigenous Education in Madras Presidency (1855-1880)

At the time when the Department of Education was established in 1855, there were only 83 schools under it, while nearly 12,500 indigenous schools were still functioning with a total of 1.6 lakh students.

The situation was similar even till 1870-1, except that about 3,000 schools had been brought under the scheme of Aided Schools.

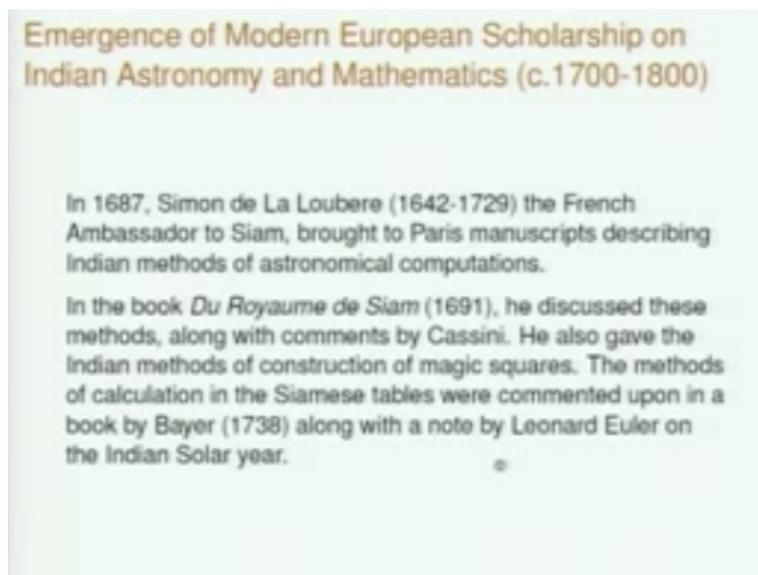
It was only during the decade 1870-1880, that the Education Department seems to have managed to bring nearly 10,000 indigenous schools under the aided scheme. Only around 1875, did the number of students studying under the aegis of the Department of Education become comparable to the number who studied in the indigenous schools fifty years earlier in 1825.

So, this was the state of indigenous education system when these surveys were made but soon

enough the British policy was to discourage all these education, at least not provide them any support from the state system and whatever support that the state system was to give was for the new system of education that they wanted to start, mainly based upon English language and English literature.

So, in this new system, in 1855 there were only 83 schools in Madras Presidency; by 1870, about 3000 schools had been brought under the aid scheme. It was only in the decade of 1870 to 1880 about 10,000 of the indigenous schools were brought under the aided scheme. So, the indigenous schools those than 11,000-12,000 had continued almost till 1880 without any state support. So, only around 1880 the number of students studying under the government department of education started becoming comparable to the students in the indigenous schools 50 years earlier.

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So, we will cause this. Let us say something about, there many slides on various ways in which modern scholars and our own people started knowing about Indian astronomy and mathematics. There are lot of names and lots of name subtext. I have put them down all for completeness sake so that you can really it and get an idea of the kind of work that went on in. So, I will not be going through these lights in detail in the lecture.

Because it will be laborious to merely recounting the names but most students do need to know the great scholars who worked on our indigenous tradition of mathematics and astronomy. So,

earliest goes back to Loubere who wrote from Siam about Indian magic squares and Indian methods of planetary tables Le Gentil have already mentioned.

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Emergence of Modern European Scholarship on Indian Astronomy and Mathematics (c.1700-1800)

Le Gentil (1725-1792) who visited India during 1761 and 1769, to observe the transit of Venus, gave a detailed account of Indian Astronomy in 1770s based on Tables and Texts obtained in Pondicherry.

This led to the treatise *Traite de l'Astronomie Indienne et Orientale* (1787) by Jean Sylvain Bailly (1736-1793). This was reviewed in detail by John Playfair (1748-1819) in the Transactions of Royal Society in 1790.

The Asiatic Society was founded in 1784 by William Jones (1746-1794). The Journal *Asiatic Researches*, started in 1788, carried articles by William Jones, Samuel Davis and John Bentley on Indian Astronomy.

Based upon the French report and the manuscripts, a treatise on Indian astronomy was written by Bailly in French and it was reviewed by John Playfair in Edinburgh in 1790s, but of course in India by then this Asiatic Society was founded and the Journal Asiatic Researches started reporting from modern scholars reports on nature of Indian science, namely William Jones, Davis, John Bentley, etc were amongst the important people.

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Translations and Editions of Indian Texts on Astronomy and Mathematics in 19th Century

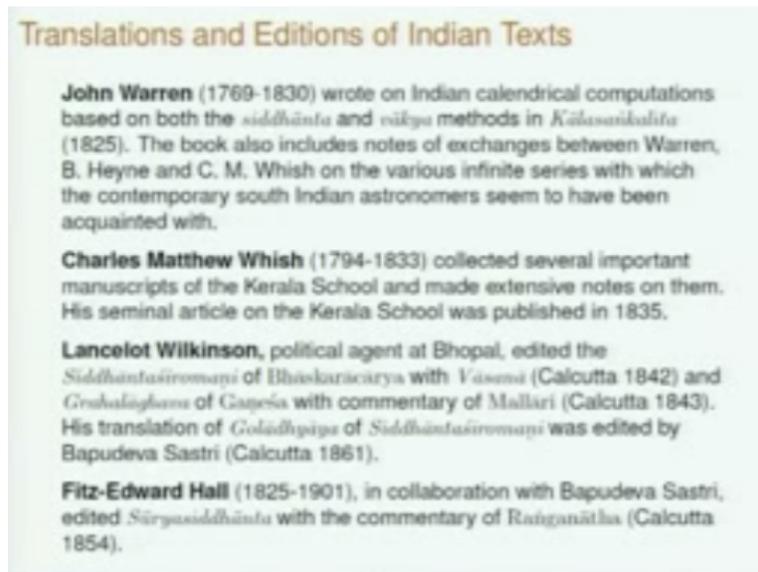
The *Bijaganita* of Bhāskara, was translated into English from the Persian translation of Ata Allah Rushdi (1634) by Edward Strachey (1812-1901) with notes by Samuel Davis (London, 1813). This was closely followed by the translation of *Lilāvati* by John Taylor (Bombay, 1816).

Henry Thomas Colebrooke (1756-1837) published several articles on Indian Astronomy and also on Law, Linguistics Philosophy etc. His most important work is *Algebra with Arithmetic and Mensuration from the Sanskrit of Brahmagupta and Bhāskara* (London, 1817), which included a translation of *Caṇḍīādhyāya* and *Kuṭṭakādhyāya* of *Brāhmasphuṭasiddhānta* as well as the *Lilāvati* and *Bijaganita* of Bhāskara II, along with notes drawn from some of the ancient commentaries.

Some books started getting translated in early part of 19th century. Bijaganita was translated

from the Persian version by Edward Strachey, but soon enough Henry Thomas Colebrooke translated the Bijaganita and Lilavati of Bhaskara and also the mathematics chapters of Brahmasphutasiddhanta. Colebrooke was also important scholars of law, linguistics, philosophy, etc.

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John Warren discussed the planetary methods of computation based on the Vykaya system in 1825. This book contains an interesting discussion between B. Heyne, Charles Whish, and John Warren on the knowledge of various South Indian pundits, the knowledge of the infinite series and mathematics like that amongst them. Of course, one group believes that this is all borrowed from the west and they are just lying or bluffing as they must have look at some modern books, modern calculus books.

Other group believes no, no, no there are found in Indian text, etc. That kind of debate was going on in 1825.

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Translations and Editions of Indian Texts

Rev. Ebenzer Burgess published an English translation of *Sūrya-siddhānta* with detailed notes with the help of William Dwight Whitney (New Haven 1860).

Albrecht Weber edited the *Vedānga Jyotiṣa* with the commentary Somakara (Berlin 1862)

Johann Hendrick Caspar Kern (1833-1917) edited the *Bṛhatsaṃhitā* of Varāhamihira (Calcutta 1865) and partially translated it (JRAS, 1873). He also edited the *Āryabhaṭṭiya* with the commentary of Paramośvara (Leiden 1875)

George Frederick William Thibaut (1848-1914) edited the *Baudhāyana-Śulvasūtra* with the commentary of Dvārakānātha (1874). He also edited *Vedānga-Jyotiṣa* (1877) and *Kātyāyana-Śulvasūtra* (in part) with commentary (1882). In collaboration with Sudhakar Dvivedi, he edited and translated the *Pañcasiddhāntikā* of Varāhamihira (1884). Thibaut's essay, *The Śulvasūtras*, was reprinted as a book (Calcutta 1875). He also wrote an overview *Astronomie Astrologie und Mathematik* (Strassburg 1899).

Wilkinson Hall, Burgess, Weber, Kern, Thibaut, there were all major scholars of editing and are translating Indian works in 19th-century.

(Refer Slide Time: 16:05)

Rediscovering the Tradition (1850-1900)

Some editions and translations into Bengali, Telugu, Marathi etc. of Indian source-works such as *Lilāvati*, *Bijaganita*, *Grubalighana*, were published in the first half of 19th century. Around the same time, several Indian scholars, who were often from traditional learned families but educated in the English education system, embarked on a process of rediscovery of Indian tradition. We mention some of the seminal figures in this movement.

Bapudeva Sastri (1821-1900) studied with Pandit Dhundiraja Misra and later Pandit Sevarama and Wilkinson at Sehore Sanskrit College. He became a Professor at Benares Sanskrit College where he is said to have taught Euclidean Geometry. He published editions of *Siddhāntasīromani* with *Vāsanā* of Bhaṭṭakavīrya (1866) and *Lilāvati* with his own commentary (1883). He collaborated with Lancelot Wilkinson and edited his translation of *Golādhyāya* of *Siddhāntasīromani* (1861).

Bhau Daji Laud (1821-1874), trained in medicine at Grant's College, Mumbai, he was also a Sanskritist and an expert in numismatics. He was the first to locate a manuscript of *Āryabhaṭṭiya* in 1864.

By, then a trend started that books getting published with the translations. The source work like *Lilavati*, *Bijaganita*, (()) (16:15) etc. Indian publishers started publishing these works with translation sometimes in Bengali, Telugu, or Marathi, etc. So, enough a set of scholars emerged who are already often from the traditional learned families but were educated in the English education system who started looking into our text.

Bapudeva Sastri is one of the oldest of them. He published the *Siddhanta Siromani*, then *Lilavati*.

Bhau Laud discovered the manuscript of the Aryabhata.

(Refer Slide Time: 16:51)

Rediscovering the Tradition (1850-1900)

Shankar Balakrishna Dikshit (1853-1898), a mathematics teacher and Principal of Teacher's Training College, Pune, wrote a voluminous history of Indian Astronomy, *Bharatiya Jyotisa Sastracha Pracinna ani Arvaca Itihasa* in Marathi (Pune, 1896). Along with Robert Sewell, he also wrote on the *Indian Calendar* (London, 1896)

Sudhakara Dvivedi (1855-1910) studied with Pandits Devakrishna and Bapudeva Sastri at Benares Sanskrit College and became a Professor there. He edited a very large number of ancient texts which became the main source for all later studies.

Some of the important texts edited by Dvivedi are: *Lilavati* (1878), *Karanakutuhala* of Brahmagosvara II (1881), *Yantraraja* with Malayendu's commentary (1882), *Siddhanta-tattvaviveka* with Sanyasani of Kamalakra (1885), *Sisyadhivrdhida* of Lalla (1886), *Bijaganita* with commentary (1888), *Brhatsamhita* with Utpala's commentary (1895-7), *Trisatika* of Sriharsha (1899), *Karapaprakasa* of Brahmasdeva (1899), *Brhatsamhitasiddhanta* with his own Sanskrit commentary (1902) and *Grabalaghava* with commentaries of Mallari and Visvanatha (1904).

Shankar Balakrishna Dikshit wrote a detail history of Indian astronomy in Marathi which has been translated into English later. Sudhakara Dvivedi was a very major scholar who brought to print about 40-50 major texts of Indian mathematics. In fact, much of our study today owes to the fact that these books were published between 1860 and 1880 and 1900.

I mean the list is large, Lilavati, Karanakutuhala, Yantraraja, Siddhanta-tattvaviveka, Sisyadhivrdhida, Bijaganita, Brhatsamhita, you can see that he was a major scholar who edited and published sometimes with his own commentary are various books.

(Refer Slide Time: 17:33)

Rediscovering the Tradition (1850-1900)

Sudhakara Dvivedi also edited *Yājñasa-Jyotiṣam* with Somākara commentary (1908), *Mahāsiddhānta* of Āryabhaṭa II with his own commentary (1910), *Āryabhaṭīya* with his own commentary (1911), *Sūryasiddhānta* with his own commentary (1911). With Thibaut, he edited *Pañcisiddhāntikā* with his own Sanskrit commentary (1889).

Dvivedi wrote many original works such as *Dirghavytta-lakṣaṇa* (1878), *Vāstavaśāstra-śrīgṇanīti-sādhana* (1880), *Bhāṣyamarekhā-nirūpaṇa* (1882), *Calamakalena* on differential calculus in Hindi (1886) and *Gapakatarangini* (1890) on the lives of Indian mathematicians and astronomers. In 1910 he wrote *A History of Mathematics, Part I (Arithmetic)* in Hindi.

Sudhakara Dvivedi's son, Padmakara Dvivedi, edited *Gapitakaumudī* of Nārāyaṇa Paṇḍita in two volumes (1936, 1942).

Finally, his son in 1930s published the *Ganita Kaumudi* of Narayana Pandita also. Sudhakara Dvivedi wrote a couple of books in Hindi also on history of Indian mathematics. So, this is a brief look at what happened between 1700 and 1900 about the study of our older tradition of mathematics by modern scholars.

(Refer Slide Time: 17:58)

Development of Higher Education in India (1850-1900)

The Universities of Calcutta, Bombay and Madras were set up in 1857.

It has often been remarked that these (and the later Universities in India) were established as examining bodies with affiliated colleges on the model of the then London University and not on the model of the renowned Oxford and Cambridge Universities with extensive research and teaching activities.

The Indian Association of Cultivation of Science was established by Mahendra Lal Sircar (1833-1904) in 1876 with the object of enabling Indians "to cultivate science in all its departments with a view to its advancement by original research". However, during the first thirty years, the main efforts of the institution were directed towards the development of science teaching at the collegiate level.

In 1855, there were 15 Arts Colleges with 3246 students, and 13 Professional Colleges with 912 students. By 1901, there were 5 Universities, 145 Arts Colleges with 17,651 students and 46 Professional Colleges with 5,358 students in the whole of India (including Burma).

Now, let us go to development of modern education in India. So, these Universities of Calcutta, Bombay and Madras were set up in 1857. But most scholars who have studied this understand that these were established mostly as examining bodies with lots of affiliated colleges on the model of the then London University and not only model of the renowned Oxford or Cambridge Universities which were devoted to large amount of teaching and research activities.

So, these were mostly degree awarding and examining bodies, that was what the universities were. So, looking at it Mahendralal Sircar in 1876 started the Indian Association of Cultivation of science with the hope that that will foster research and advanced study amongst the Indians. But even then nothing much perhaps happened in the first 20-25 years of this institution. Most of the time, the main effort of the institution got directed towards developing of science teaching in the college level.

(Refer Slide Time: 19:11)

Development of Higher Education in India (1850-1900)

	1855	1882	1901-2
Universities	-	4	5
Number of Students	-	NA	NA
Arts Colleges	15	38	145
Number of Students	3,246	4,252	17,651
Professional Colleges	13	96	46
Number of Students	912	3,670	5,358
Secondary Schools	169	1,363	5,493
Number of Students	18,335	44,605	6,22,768
Primary Schools	1,202	13,882	97,854
Number of Students	40,041	6,81,835	32,04,336
Special Schools	7	83	1,084
Number of Students	197	2,814	36,380
Total Recognised Institutions	1,406	15,462	1,04,627
Number of Students	62,731	7,37,176	38,86,493

So, in 1855 there were about 15 colleges with 3246 students. By 1900, there were 150 colleges with 17,000 students and 45 professional colleges. You can see that in a table like this. Later on, at leisure you can examine. This gives the growth of education actually primary schools, special schools, etc.

(Refer Slide Time: 19:20)

Development of Higher Education in India (1850-1910)

Yesudas Ramchundra (1821-1880), a teacher of science in Delhi College, wrote a *Treatise on Problems of Maxima and Minima* (1850), which approached these problems purely algebraically. Augustus de Morgan got it republished, with his own introduction, from London in 1859.

The Indian Mathematical Society began as the "Analytical Club" in 1907 at the initiative of V. Ramaswamy Aiyar, a civil servant (then a Deputy Collector at Gutt). It was renamed Indian Mathematical Society in 1910.

It started a journal in 1909, edited by M. T. Narayaniyengar and S. Narayana Aiyar. This was soon renamed the *Journal of Indian Mathematical Society*. The Society also started the journal *Mathematics Student* in 1932.

The Calcutta Mathematical Society was formed in 1908 at the initiative of Prof. Ashutosh Mukherjee (1864-1924) the then Vice Chancellor of Calcutta University. It also began publishing the *Bulletin of the Society* in 1909.

Some scholars in India started working on modern subjects. One person was Yesudas Ramchundra, a teacher of science in Delhi, who wrote a treatise on problems of maxima and minima. He tried to approach these problems algebraically. De Morgan, the British famous logician and mathematician got impressed with it. He got it re-published in London. By the time 20th century, it was only by then that some serious research activity and study of the higher sciences started in India.

The Indian Mathematical Society began as analytical club in 1907 at the initiative of V. Ramaswamy Aiyar. It started a journal in 1909, the Journal of Indian Mathematical Society. After the 25th year of the society, they started a popular journal for students. Some material was coming in Journal of Indian Mathematical Society itself which was separated and the mathematic student was started. Soon enough in good competition, the Calcutta Mathematical Society was started in 1908 by Professor Ashutosh Mukherjee's initiative. It also began the bulletining of the Calcutta Society.

(Refer Slide Time: 20:32)

Srinivasa Ramanujan (1887-1920)

Srinivasa Ramanujan, the greatest mathematician India has produced in recent times, was born on December 22, 1887 at Erode.

In 1892, he enrolled in primary school in Kumbakonam.

In 1898, having passed his primary examinations with distinction, he joined the Town High School of Kumbakonam. He passed out of the school as an outstanding student in 1904 and received a scholarship to study at the Government College, Kumbakonam.

While at school, he got a copy of Loney's *Plane Trigonometry* which he soon mastered, but also was surprised to see there some of the results that he had obtained himself.

Around 1903, Ramanujan went through G. S. Carr's *Synopsis of Pure and Applied Mathematics* (1880), a compendium of about 5000 results, which is said to have influenced him considerably.

We will discuss the development of modern mathematics in India from about 1905 in the next lecture. I would like to go straight away to discuss the work of Srinivasa Ramanujan who if we examine carefully, you will see that in some very fundamental sense he is in continuity with the older tradition of mathematics and science in India. So, this talk will be devoted to the work of Ramanujan.

Ramanujan was born in December 1887. He enrolled in school in 1892 in Kumbakonam and he did fairly well in school. He topped, he received a scholarship to study in the local college and while at school he looks right up Loney's Trigonometry and he found that he had himself discovered many of these results. Towards late period in school, he looked up G. S. Carr's Synopsis of Pure and Applied Mathematics which was a sort of textbook for the Cambridge Wrangler examination listing about 5000 in result which is said to have influenced him considerably.

(Refer Slide Time: 21:40)

Early Work of Ramanujan

Ramanujan seems to have started discovering new results and recording them in his Notebook by 1904.

However, at the college, "owing to his weakness in English" as Hardy notes, Ramanujan failed and lost his scholarship.

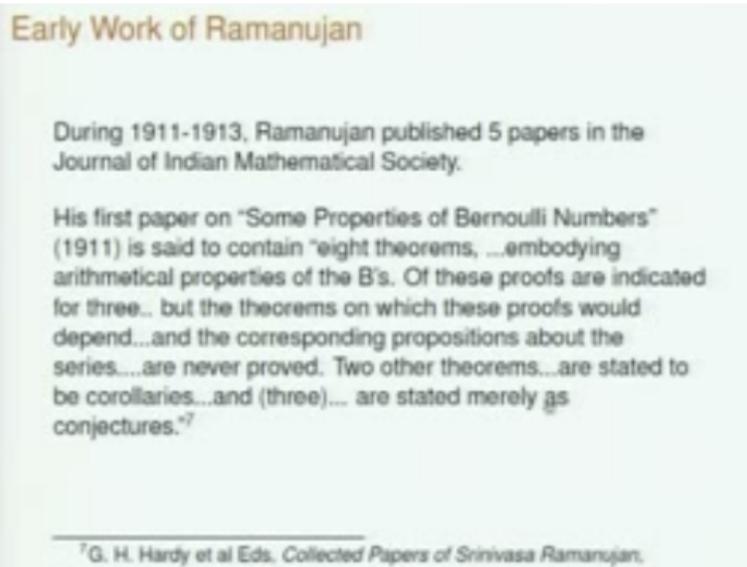
Ramanujan joined the Pachaiyappa's College at Madras, but soon he had to discontinue due to bad health. He appeared for F.A. privately in 1907, but failed once again.

Ramanujan got married in 1909 and with great effort managed to get a job in Madras Port Trust as a clerk in 1912, by the good will and support of various personalities who were impressed by his mathematical abilities.

He seems to have started discovering new results and recording them in his notebook towards the later period when he was in school itself. Now, once he went to college owing to his weakness in English as Hardy notes, he totally failed, lost his scholarship. He came and Pachaiyappa's College in Chennai but that also he could not complete. He appeared for F.A. privately, failed. He got married in 1909.

He continued his mathematical work and he went and met various influential people showing them the work and some of them did help him and by the goodwill of his supporters, he at least obtained a job in Madras Port Trust as a clerk in 1912.

(Refer Slide Time: 22:25)



Early Work of Ramanujan

During 1911-1913, Ramanujan published 5 papers in the Journal of Indian Mathematical Society.

His first paper on "Some Properties of Bernoulli Numbers" (1911) is said to contain "eight theorems, ...embodying arithmetical properties of the B's. Of these proofs are indicated for three.. but the theorems on which these proofs would depend...and the corresponding propositions about the series...are never proved. Two other theorems...are stated to be corollaries...and (three)... are stated merely as conjectures."⁷

⁷G. H. Hardy et al Eds. *Collected Papers of Srinivasa Ramanujan*, Cambridge 1927, p. 225.

He published about five papers during 1911 to 1913. The first paper was on Properties of Bernoulli Numbers which is supposed to contain many interesting results.

(Refer Slide Time: 22:35)

Early Work of Ramanujan

During 1911-13, Ramanujan also posed about 30 problems in the Journal of Indian Mathematical Society for nearly twenty of which he also provided the solution (as it was not solved by others in six months).

Here is a sample question published in 1911:

Question 289 (III 90):

Find the Value of

(i) $\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + \dots}}}$

(ii) $\sqrt{6 + 2\sqrt{7 + 3\sqrt{8 + \dots}}}$.

He also sent about 30 problems to the Journal of Indian Mathematical Society. I am just giving one to show you the flavour such kind of problems you have already seen in Narayana Pandita. So, one question he posed in 1911 was find the value of square root of 1+ square root of 1+3 times square root of 1+ etc. So, the square root runs over the entire.

(Refer Slide Time: 22:59)

Early Work of Ramanujan

Solution by Srinivasa Ramanujan, IV 226:

(i) $n(n+2) = n\sqrt{1 + (n+1)(n+3)}$.

Let $n(n+2) = f(n)$;

Then we see that

$$\begin{aligned} f(n) &= n\sqrt{1 + f(n+1)} \\ &= n\sqrt{1 + (n+1)\sqrt{1 + f(n+2)}} \\ &= n\sqrt{1 + (n+1)\sqrt{1 + (n+2)\sqrt{1 + f(n+2)}}} \\ &= \dots \end{aligned}$$

That is,

$$n(n+2) = n\sqrt{1 + (n+1)\sqrt{1 + (n+2)\sqrt{1 + (n+3)\sqrt{1 + \dots}}}}$$

Putting $n = 1$, we have

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + \dots}}} = 3.$$

So, people were supposed to answer. If no answer can in six months, Ramanujan would give the solution and the person who pose the problem and so the way Ramanujan solved it was, he first

saw the simple the identity n to $n+1$ is n times this much and define n to $n+1$ as f_n and immediately in terms of the f_n , he was able to get by equation a relation and so the particular quantity here could be directly shown to be n to $n+2$, put $n=1$ and you get the beautiful result. Square root of 1+twice square root of 1+thrice square root of etc=3.

(Refer Slide Time: 23:38)

Early Work of Ramanujan

One of Ramanujan's early papers is on the "Modular equations and approximations to π ". Though published later from London in 1914 (QJM 1914, 350-372), it is said to embody "much of Ramanujan's early Indian work." Here is a sample of his results:

$$\frac{1}{3\pi\sqrt{3}} = \frac{3}{49} + \frac{43 \cdot 1 \cdot 1 \cdot 3}{49^2 \cdot 4^2} + \frac{83 \cdot 1 \cdot 3 \cdot 1 \cdot 3 \cdot 5 \cdot 7}{49^3 \cdot 2 \cdot 4 \cdot 4^2 \cdot 8^2} + \dots$$

$$\frac{2}{\pi\sqrt{11}} = \frac{19}{99} + \frac{299 \cdot 1 \cdot 1 \cdot 3}{99^2 \cdot 2 \cdot 4^2} + \frac{579 \cdot 1 \cdot 3 \cdot 1 \cdot 3 \cdot 5 \cdot 7}{99^3 \cdot 2 \cdot 4 \cdot 4^2 \cdot 8^2} + \dots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1103}{99^2} + \frac{27493 \cdot 1 \cdot 1 \cdot 3}{99^3 \cdot 2 \cdot 4^2} + \frac{53883 \cdot 1 \cdot 3 \cdot 1 \cdot 3 \cdot 5 \cdot 7}{99^4 \cdot 2 \cdot 4 \cdot 4^2 \cdot 8^2} + \dots$$

In November 1985, R.W.Gospar used the last series above to compute π to 17,526,100 digits which was a record at that time. In 1989, Jonathan and Peter Borwein proved all the 17 series for $\frac{1}{\pi}$ given in Ramanujan's paper. Discovering similar such series continues to be an active area of research.

Similarly, the other problem also. There is a paper called Modular equations and approximations to pi which was published at Ramanujan (()) (23:46) in quarterly journal, but this is also said to be the kind of work that he was engaged in while in India and in 1985 this last series was used by Gospar to compute pi to 17 million digits which was a record at that time and Borwein proved all the 17 series that Ramanujan had given in this paper for $1/\pi$ in 1989. Discovering similar series continues to be an active area of mathematical research, okay.

(Refer Slide Time: 24:23)

Approaching British Mathematicians (1912-13)

In 1912, Ramanujan sent a sample of his results to Prof. M. J. M. Hill of University College, London, through Prof. C. L. T. Griffith of the Madras College of Engineering.

Prof. Hill wrote back that Ramanujan had "fallen into the pitfalls of ...divergent series" and advised that he consult Bromwich's book on infinite series.

Ramanujan is said to have also contacted Profs H. F. Baker and E. W. Hobson at Cambridge and received no response.

Continuing Ramanujan story, in 1912 he sent a sample of his research to one professor M.J.M. Hill in the University College London through the Professor of Madras College of Engineering which is the current Hindi college. Professor Hill wrote back that Ramanujan has fallen into the pitfalls of divergent series and he should consult Bromwich's book on infinite series. Ramanujan again wrote to Baker and Hobson in Cambridge and it seems he did not get any response.

(Refer Slide Time: 24:54)

Approaching British Mathematicians (1912-13)

On January 16, 1913 Ramanujan wrote to Prof Godfrey Harold Hardy (1877-1947) at Cambridge, enclosing an eleven page list of over one hundred results such as the following:

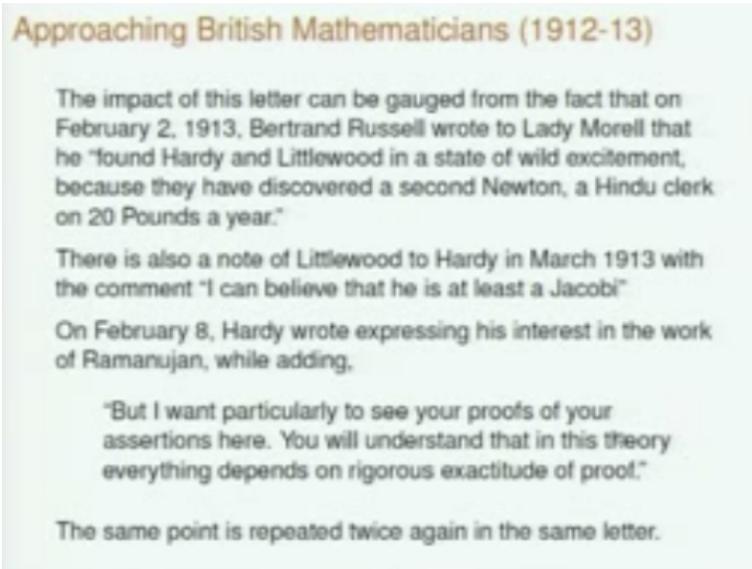
$$\begin{aligned} \text{If } u &= \frac{x}{1+} \frac{x^5}{1+} \frac{x^{10}}{1+} \frac{x^{15}}{1+} \frac{x^{20}}{1+} \dots \\ \text{and } v &= \frac{\sqrt[5]{x}}{1+} \frac{x}{1+} \frac{x^2}{1+} \frac{x^3}{1+} \dots \\ \text{then } v^5 &= u \frac{1 - 2u + 4u^2 - 3u^3 + u^4}{1 + 3u + 4u^2 + 2u^3 + u^4}. \end{aligned}$$

$$\frac{1}{1+} \frac{e^{-2x}}{1+} \frac{e^{-4x}}{1+} \frac{e^{-8x}}{1+} \dots = \left(\sqrt{\frac{5 + \sqrt{5}}{2}} - \frac{\sqrt{5} + 1}{2} \right) \sqrt[5]{e^{2x}}.$$

So, in January 1913, Ramanujan wrote to Godfrey Harold Hardy at Cambridge. He enclosed an 11-page supplement which had more than 100 results. So, I am just displaying two of the results of Ramanujan. This is a continued fraction. This is another continued fraction and he has a certain relation between them and this is a continued fraction and this is another analytical

expression. These were two of the kind of hundred formulae that the Ramanujan sent to Hardy in January 1930.

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Approaching British Mathematicians (1912-13)

The impact of this letter can be gauged from the fact that on February 2, 1913, Bertrand Russell wrote to Lady Morell that he "found Hardy and Littlewood in a state of wild excitement, because they have discovered a second Newton, a Hindu clerk on 20 Pounds a year."

There is also a note of Littlewood to Hardy in March 1913 with the comment "I can believe that he is at least a Jacobi"

On February 8, Hardy wrote expressing his interest in the work of Ramanujan, while adding,

"But I want particularly to see your proofs of your assertions here. You will understand that in this theory everything depends on rigorous exactitude of proof."

The same point is repeated twice again in the same letter.

The impact of this letter can be gauged from the fact that on February 2, Lord Bertrand Russell wrote to Lady Morell that he found Hardy and Littlewood in a state of wild excitement because they have discovered a second Newton a Hindu clerk on 20 pounds a year. So, in fact, even there is a comment of Littlewood to Hardy in March that looking at his work I can believe that he is at least a Jacobi. Jacobi is a great analyst of 19th century.

So, Hardy wrote back to Ramanujan expressing interest in his work but he added but I want particularly to see the proofs of your assertions here. You will understand that in this theory everything depends on rigorous exactitude of proof and this is a point that Hardy repeated about two three times in the same letter.

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Approaching British Mathematicians (1912-13)

In his reply of February 27, 1913 (which also included a 10 page supplement with many results), Ramanujan recounted his experience with a Professor in London (Prof. Hill) and said:

"I find in many a place in your letter rigorous proofs are required and so on and you ask me to communicate the methods of proof. If I had given you my method of proof I am sure you will follow the London Professor.

I dilate on this simply to convince you that **you will not be able to follow my methods of proof if I indicate the lines on which I proceed in a single letter.** You may ask how you can accept results based upon wrong premises. What I tell you is this. Verify the results I give and if they agree with your results, got by treading on the groves in which the present day mathematicians move, you should at least grant that there may be some truths in my fundamental basis....

You may judge me hard that I am silent on the methods of proof. I have to re-iterate that I may be misunderstood if I give in a short compass the lines on which I proceed."

..

So, Ramanujan sends a reply back with another 10-page supplement with various results. The main point he is trying to tell him is that you will not be able to follow my methods or proof if I indicate the lines on which I proceed in a single letter that of course I follow. There is a method, there are various sorts of ways in which I am arriving at them, but it cannot be put down in a letter.

Then, he just threw a challenge. Verify the results that I give and if agree with your results, got by treading on the groves in which the present day mathematicians move, you should at least grant that there may be some truth in my fundamental basis. So, it is one of the few letters by Ramanujan. He is asserting something about himself that is he has a method and he has an approach.

(Refer Slide Time: 27:19)

Ramanujan's Work in England

Ramanujan arrived in London on April 14, 1914 and left for India on February 27, 1919. Of the nearly five years he spent there, he was very ill for more than two years. From around the spring of 1917, he was in hospitals most of the time.

On his work during 1914, Ramanujan wrote to his friend B. Krishna Rao on November 14, 1914:

"I changed my plan of publishing my results. I am not going to publish any of the old results in my notebook till the war is over. After coming here I have learned some of their methods. **I am trying to get new results by their methods so that I can easily publish these results without delay.** In a week or so I am going to send a long paper to the London Mathematical Society. The results in this paper [on highly composite numbers] have nothing to do with those of my old results. I have published only three short papers...."

So, Ramanujan arrived in London in April 1914. He left for India back in February 1919. Of these five years, he was very ill for more than two years, almost three years I think. From around the spring of 1917 most of the time he spent in hospitals. On his work during the end of the year that he reached after six-seven months, he wrote to his friend B. Krishna Rao saying that I have changed my plan of publishing my results.

I am not going to publish any of old results in my notebook till the war is over. The war started in 1914 and goes on till 1918. I am trying to get new results by their methods, so that I can easily publish these results without delay.

(Refer Slide Time: 28:05)

Ramanujan's Work in England

Ramanujan reiterated this in a letter to S. M. Subramanian in January 1915:

"I am doing my work very slowly. My notebook is sleeping in a corner for these four or five months. I am publishing only my present researches as I have not yet proved the results in my notebooks rigorously. I am at present working in arithmetical functions..."

Same thing Ramanujan writes to another friend Subramanian in a letter in 1915, that my notebook is lying in a corner, I am not going to take it out and publish any of the results. So, whatever work he did in England was with the new methods and the new way with the interaction with the British mathematicians and of course with this great acumen and intuition.

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Ramanujan's Work in England

During 1914-1919, Ramanujan wrote about 30 papers, 7 of them in collaboration with Hardy, which were mostly concerning properties of various arithmetical functions.

His work was highly acclaimed. On March 16, 1916 he was awarded Bachelor of Science degree by Research from the Cambridge University. He was elected a Fellow of Royal Society on February 28, 1918, the second Indian to be so honoured. On October 23, 1918 he was elected a Fellow of the Trinity College (It appears that the College failed to elect Ramanujan as a fellow in 1917 for various non-academic reasons).

In late 1918, the Madras University also offered a matching grant of 250 Pounds a year. On receipt of this communication, Ramanujan wrote to the Registrar of the University on January 11, 1919 that, after meeting his basic expenses, the surplus "should be used for some educational purpose, such in particular as the reduction of school-fee for poor boys and orphans and provision of books in schools."

So, during 1914 to 1919, Ramanujan wrote about 30 papers, seven of them in collaboration with Hardy which were mostly concerning properties of arithmetical functions. We will get a chance to have a glimpse of few of them. His work was highly acclaimed. In March, he was awarded BSC by Research from Cambridge University for his paper on highly composite number. He was elected in FRS in 1918, the second Indian to be so honoured. The earlier person was a Parsi gentleman in mid 19th century.

In October 1918, he was elected as fellow of Trinity College. It seems that they failed to elect him a fellow the previous year and that was some heartburns for Ramanujan. So, in late 1918, Madras University offered him a matching grant of 250 pounds. The Cambridge Fellowship gave him a 250-pound annual fellowship being a fellow of Trinity College, so Madras University also gave that.

So, Ramanujan writes a very moving letter to the registrar saying that after meeting his basic expense, the surplus should be used for some educational purpose, such in particular as the

reduction of school fees for poor boys and orphans and provision of books in schools.

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Ramanujan's Last Letter and "Lost Notebook"

On March 27, 1919 Ramanujan returned to India. He was in very poor health. He stayed for a while in Madras and then moved to Kodumudi, then to Kumbakonam, and finally returned to Madras by January 1920.

Though seriously ill, he was continuing his work all the while. On January 12, 1920, Ramanujan wrote to Hardy (for the first time after returning to India):

"I discovered very interesting functions recently which I call "Mock" θ -functions. Unlike the "False" θ -functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary θ -functions. I am sending you with this letter some examples...".

This was followed by a few pages containing definitions, some examples and properties of the mock θ -functions.

So, in March 1919, Ramanujan returns to India. He is in very poor health, stays a bit in Madras, then goes to Kodumudi, then goes to Kumbakonam and finally returns to Madras by January. At that time, he wrote to Hardy for the first time after returning to India. He returned in March 1919. I discovered very interesting functions recently which I call mock theta functions. Unlike the false theta functions studied by Professor Rogers.

They enter into mathematics as beautifully as the ordinary theta function. I am sending you with this letter some examples. Again, few pages of results enclosed.

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Ramanujan's Last Letter and "Lost Notebook"

The so called "Lost Notebook" of Ramanujan is a sheaf of over hundred sheets containing about 600 results that Ramanujan had found during the last year of his life.

This seems to have been sent to Hardy along with all other papers of Ramanujan in 1923.

It was finally discovered by George Andrews in 1976 (amongst Watson papers) in Trinity College.

Ramanujan passed away in Madras on April 26, 1920.

There is something called the Lost notebook of Ramanujan. This is a collection of more than 100 pages many of them written on both sides having about 600 results. This consists of the work that he was going after he returned from England to India. These papers were sent to Hardy 1923. It was finally discovered by George Andrews in 1976 in amongst the Watson papers in Trinity College. So, Ramanujan passed away in April 1920.

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Hardy's Assessment of Ramanujan (1921)

Soon after Ramanujan's death, Hardy wrote an Obituary Notice in Proc. Lond. Math. Soc. (19, 1921, pp. 40-58), which was later reproduced in the *Collected Papers of Ramanujan* (Cambridge 1927). There, Hardy first presents his assessment of Ramanujan when he arrived in England (1914):

"The limitations of his knowledge were as startling as its profundity. Here was a man who could work out modular equations, and theorems of complex multiplication, to orders unheard of, whose mastery of continued fractions was, on the formal side at any rate, beyond that of any mathematician in the world, who had found for himself the functional equation of the Zeta-function, and the dominant terms of many of the most famous problems in the analytic theory of numbers; and he had never heard of a doubly periodic function or of Cauchy's theorem, and had indeed but the vaguest idea of what a function of a complex variable was. **His ideas as to what constituted mathematical proof were of the most shadowy description. All his results, new or old, right or wrong had been arrived at by a process of mingled argument, intuition and induction, of which he was entirely unable to give any coherent account.**"

Now, soon after Ramanujan's death, Hardy wrote an Obituary and there he tried to give an estimate of Ramanujan what he thought of him when he came to England and what was made out of him later and how he understands his work and this is a very interesting document. It has to be read very carefully. I am just giving a few quotations. So, Hardy is saying that Ramanujan's

ideas as to what constituted mathematical proof were of the most shadowy description.

All his results, new or older, right or wrong had been arrived at by a process of mingled argument, intuition and induction, all these things written in the first obituary that he is writing on this man, of which he was entirely unable to give any coherent account.

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Hardy's Assessment of Ramanujan (1921)

Hardy then notes that, after their interaction, "In a few years time, he [Ramanujan] had a very tolerable knowledge of the theory of functions, and the analytic theory of numbers. He was never a mathematician of the modern school...."

Hardy also states that Ramanujan "**adhered with a severity most unusual in an Indian resident in England to the religious observance of his caste; but his religion was a matter of observance and not of intellectual conviction, and I remember well his telling me (much to my surprise) that all religions seem to him more or less to be equally true.**"

Hardy then raises the issue: "I have often been asked whether Ramanujan had any special secret; whether his method differed in any kind from those of other mathematicians; whether there was anything really abnormal in his mode of thought. I cannot answer these questions with any confidence or conviction; but I do not believe it. My belief is that all mathematicians think, at bottom, in the same kind of way, and that Ramanujan was no exception...."

There is another opt quoted statement by Hardy which seems to make much of the statement that Ramanujan told him at some context that all religions seem to him more or less equally true. Hardy to took to mean that he really did not believe in any religion and he says, Ramanujan adhered with a severity most unusual in an Indian resident in England to the religious observance of his caste but his religion was a matter observance and not of intellectual conviction.

And the evidence for this is that I remember well is telling me much to my surprise that all religions seem to him more or less to be equally true. So, this is kind maturity that Hardy has in trying to understand this man.

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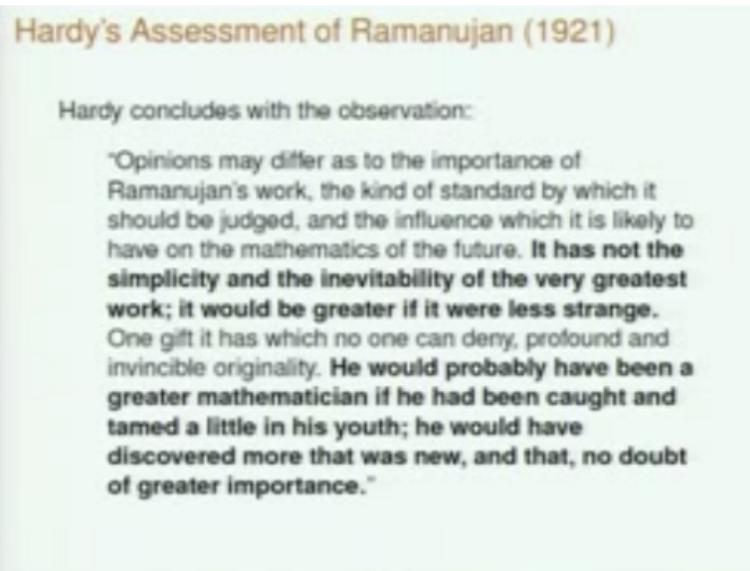
Hardy's Assessment of Ramanujan (1921)

Hardy then goes onto declare:

"It was his insight into algebraic formulae, transformation of infinite series, and so forth, that was most amazing. On this side most certainly I have never met his equal, and **I can compare him only with Euler or Jacobi**. He worked, far more than the majority of modern mathematicians, by induction from numerical examples; all of his congruence properties of partitions, for example, were discovered this way. But with his memory, his patience, and his power of calculation, he combined a power of generalisation, a feeling for form, and a capacity for rapid modification of his hypothesis that were often really startling..."

Then, Hardy says of course that Ramanujan was extraordinary with his insight in algebraic formulae, transformation of infinite series and so far so forth. On this side most certainly I have never met his equal and I can compare him only with Euler or Jacobi. But immediately he has to say he worked far more than the majority of modern mathematicians by induction from numerical examples but it was his memory, his patience and his power, not his mathematical acumen that gave him all with this results.

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Hardy's Assessment of Ramanujan (1921)

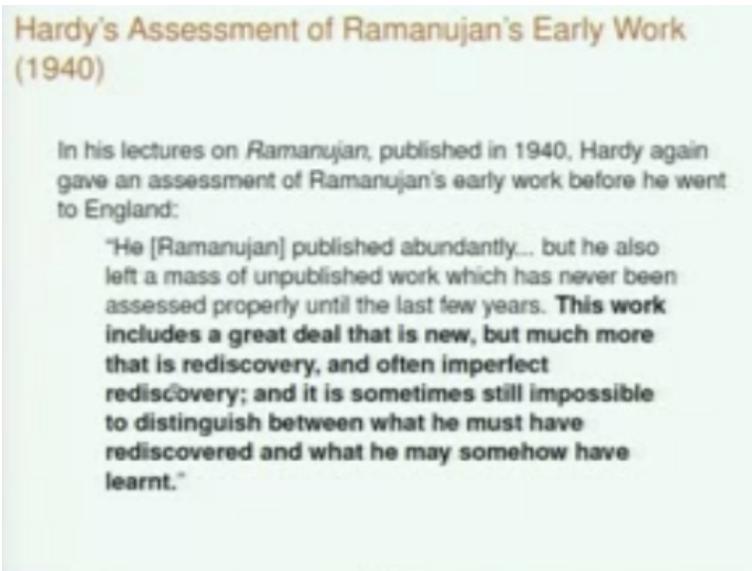
Hardy concludes with the observation:

"Opinions may differ as to the importance of Ramanujan's work, the kind of standard by which it should be judged, and the influence which it is likely to have on the mathematics of the future. **It has not the simplicity and the inevitability of the very greatest work; it would be greater if it were less strange.** One gift it has which no one can deny, profound and invincible originality. **He would probably have been a greater mathematician if he had been caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt of greater importance.**"

He concludes the whole thing by saying Ramanujan did wonderful work but that cannot be called very great work or amongst the greatest work. It has not the simplicity and inevitability of the very greatest work. It would be greater if it were less strange. He would have probably been a

greater mathematician if he had been caught and tamed a little in his youth. He would have discovered more that was new and that no doubt of greater importance.

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So, this was an assessment of Ramanujan's work given immediately after his death and this was reprinted in the collected works of Ramanujan that Hardy, Seshu Aiyar and Wilson published in 1927. Again, 15 years later, Hardy had an occasion to write an estimate of Ramanujan. This was a series of lectures he delivered in Harvard and this came out as a book in 1940. Here Hardy is somewhat even more free in trying to explain how Ramanujan, according to him, was when he reached England.

Ramanujan left a mass of unpublished work, which he is referring to his notebooks, which has never been assessed properly until the last few years. This work includes a great deal that is new but much more that is rediscovery and often imperfect rediscovery and it is sometimes still impossible to distinguish between what he must have rediscovered and what he may somehow have learnt is even sort of, accusing no plagiarism that have recorded some results without the knowledge that so.

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Hardy's Assessment of Ramanujan's Early Work (1940)

"It was inevitable that a very large part of Ramanujan's work should prove on examination to have been anticipated. He had been carrying an impossible handicap, **a poor and solitary Hindu pitting his brains against the accumulated wisdom of Europe**. He had had no real teaching at all; there was no one in India from whom he had anything to learn... **I should estimate that about two-thirds of Ramanujan's best Indian work was rediscovery**, and comparatively little of it was published in his life time... The great deal of Ramanujan's published work was done in England...In particular he learnt what was meant by proof..."⁶

⁶G. H. Hardy, *Ramanujan*, Cambridge 1940, pp. 1, 10

Then he says he had been carrying on an impossible handicap, a poor and solitary Hindu pitting his brains against the accumulated wisdom of Europe. Hardy was no ordinary racist or imperialist. He was after all a great anti-war champion. He was a liberal sort of intellectual, but the times were such that even he could not resist coming up with this kind of a wonderful praise, poor and solitary Hindu pitting his brains against accumulated wisdom of Europe. I should estimate that about two-thirds of Ramanujan's best Indian work was rediscovered.

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Ramanujan's Work on Partitions

The number of partitions $p(n)$ is the number of distinct ways of representing n as a sum of positive integers, without taking the order into account. $p(0)$ is taken to be 1.

Partitions of 4 are: $1 + 1 + 1 + 1, 1 + 1 + 2, 1 + 3, 2 + 2, 4$. Hence, $p(4) = 5$

In a couple of papers published in 1919 and 1920, and a paper published posthumously in 1921, Ramanujan discovered and proved the congruences:

$$\begin{aligned}p(5m + 4) &= 0 \pmod{5} \\p(7m + 5) &= 0 \pmod{7} \\p(11m + 6) &= 0 \pmod{11}\end{aligned}$$

Ramanujan also conjectured that 5, 7 and 11 are the only primes for which such congruences hold; Ahlgren and Boylan proved this in 2003.

Anyway, so now let us go quickly to discuss something of Ramanujan's work. We will come back to the notebook a little bit later, the works that he did prior to going to England which is contained in the notebooks, we will come to it towards the end of it. So, one of the important

work of Ramanujan is on partitions. So, the number of partitions p of n is the number of distinct ways in which n can be written as sum of positive integer without taking the order into account.

So, it is not like the mass number rows in the (∞) (35:33) where the order of the elements also is important, $2+1+1$ will be treated differently than $1+2+1$, GLL and LGL will be taken as two different partitions. So, these are repetitions not taken into account. So, the number of partitions of 4 are $1+1+1+1$, $1+1+2$, $1+3$, $2+2$ and 4. So, there are five partitions of 4. If you try to write partitions of 10, you will immediately find that itself is a huge number. In a couple of papers published in 1919 and 1920.

And a paper published posthumously in 1921, Ramanujan discovered and proved these three very important congruences.

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$m=0, 2, \dots$
 $p(5m+4)$ is divisible by 5
 $p(4)=5$

What is meant by congruence, that is just that is divisible by 5, that is what it means. So, in particular p of $4=5$, then I put $m=0$. Use of p of $4+5$ that is divisible by 5, but Ramanujan is saying for any m the number of partitions of the p of $5m+4$ is divisible by 5, that is what this equation means. Similarly, the number of partitions of a number integer of the type $7m+5$ is divisible by 7.

The number of partitions of number of type $11m+6$ is divisible by 11, these were three

congruence property. He conjectured 5, 7 and 11 are the only primes for which such congruences hold that was proved about 70-80 years later.

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Ramanujan's Work on Partitions

In his 1919 paper, Ramanujan also conjectured:
 If $d = 5^a 7^b 11^c$, and $24\lambda \equiv 1 \pmod{d}$, then, $p(\lambda) \equiv 0 \pmod{d}$.

In a later unpublished manuscript, Ramanujan proved the above conjecture for arbitrary a , with $b = c = 0$.

In 1934, S. Chowla disproved the general conjecture, by noting that

$$p(243) = 13,397,825,934,888 \text{ is not divisible by } 7^3$$

even though $24 \cdot 243 \equiv 1 \pmod{7^3}$

However, it has later been established that

$$\text{If } 24\lambda \equiv 1 \pmod{5^a 7^{b'} 11^c}, \text{ then}$$

$$p(\lambda) \equiv 0 \pmod{5^a 7^{b'} 11^c}$$

where $b' = b$ if $b = 0, 1, 2$ and $b' = \lfloor \frac{b+2}{2} \rfloor$ if $b > 2$

In his 1919 paper, Ramanujan also wrote down conjecture, this is somewhat complex. If d is an integer of 5 to the power a 7 to the power b 11 to the power c and if λ is a number such that 24 times λ when divided d gives a remainder of 1, that is what this equation means then the partitions of λ are divisible by d . This was the conjecture. In an unpublished manuscript, Ramanujan tried to prove it for arbitrary a with $b=c=0$. So, Chowla in 1934 disproved this conjecture.

He showed that p of 243 (()) (37:57) number, the partitions of 243 are of the order of 10^{14} , huge number. He showed that the 7 power relation is not fully satisfied and later on it has been shown that Ramanujan was essentially correct. In this congruence equation, you have to just change the b to a b prime and b prime is either same as b if $b=0, 1, 2$ and b prime is of this form.

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Ramanujan's Work on Partitions

In 1918, Hardy and Ramanujan obtained an infinite asymptotic series for $p(n)$, of which the first term is of the form

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

It has been noted by Berndt that:

"In their classic paper [of 1918] Hardy and Ramanujan introduced their famous 'circle method', which remains today as the primary tool of number theorists using analytical techniques in studying problems of additive number theory. The principal idea behind the 'circle method' can be found in Ramanujan's notebooks..., although he did not rigorously develop his ideas....Despite its genesis in Ramanujan's work, today it is often called Hardy-Littlewood circle method, because Hardy and J. E. Littlewood extensively developed the method in a series of papers."⁹ ©

⁹B. Berndt, *Number Theory in the Spirit of Ramanujan*, AMS, New York 2006, pp.22-23.

In 1918, Ramanujan and Hardy wrote a monumental paper which was (()) (38:31). It gave an infinite asymptotic series for p of n , that is this is the dominant term for large N for p of n . So, p of n goes by e to the power $-\text{root } n/1/n$. For this they use what is called the circle method and later on Bruce Brendt who looked up Ramanujan's notebooks found the elements of circle method in Ramanujan's notebooks.

And he says that it is unfortunate that it is called Hardy Littlewood circle method because Hardy and Littlewood wrote many more papers on this afterwards.

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Ramanujan's Work on Partitions

One of the results communicated by Ramanujan to Hardy, in his first letter of January 16, 1913, was

The coefficient of x^n in $\frac{1}{1 - 2x + 2x^4 - 2x^8 + 2x^{16} - \dots}$
 is the nearest integer to $\frac{1}{4n} \left\{ \cosh(\pi\sqrt{n}) - \frac{\sinh(\pi\sqrt{n})}{\pi\sqrt{n}} \right\}$

Commenting on this, Hardy wrote in 1940 that:

"The function in ...[the right hand side] is a genuine approximation to the coefficient, though not at all so close as Ramanujan imagined, and Ramanujan's false statement was one of the most fruitful he ever made, since it ended by leading us to all our joint work on partitions"

It is altogether another story that, in the above conjecture, Ramanujan seems to have clearly anticipated the exact form of $p(n)$, which was found later by Radmacher in 1937.

But more interesting was the letter than Ramanujan wrote to Hardy, the first letter he wrote had a

proposition like this, again a very interesting kind of proposition. The coefficient of x to the power n in this complicated quantity $1/(1-2x+2x^4)$ etc. is the integer nearest to $1/(4n) \cos(\pi/\sqrt{n})$ and $\sin(\pi/\sqrt{n})$. This is at the basis of a Ramanujan and Hardy's work on that Hardy acknowledges it in 1940.

But in another matter in this Ramanujan had a much better result, than what was proved in Hardy-Ramanujan paper. He had the exact form of the partition p of n which was later on proved by Radmacher. I will come back to this question a bit later.

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Ramanujan's Tau Function

In his paper "On certain arithmetic Functions" (1916) Ramanujan defined the Tau Function via the identity

$$\sum_{n \geq 1} \tau(n) q^n = q \prod_{n \geq 1} (1 - q^n)^{24}$$

Two of the properties of the Tau function stated by Ramanujan were proved by Mordell in 1917.

$$\tau(mn) = \tau(m)\tau(n) \text{ if } \gcd(m, n) = 1.$$

$$\tau(p^3) = \tau(p)\tau(p^2) - p^{11}\tau(p)$$

Ramanujan also conjectured that if p is any prime

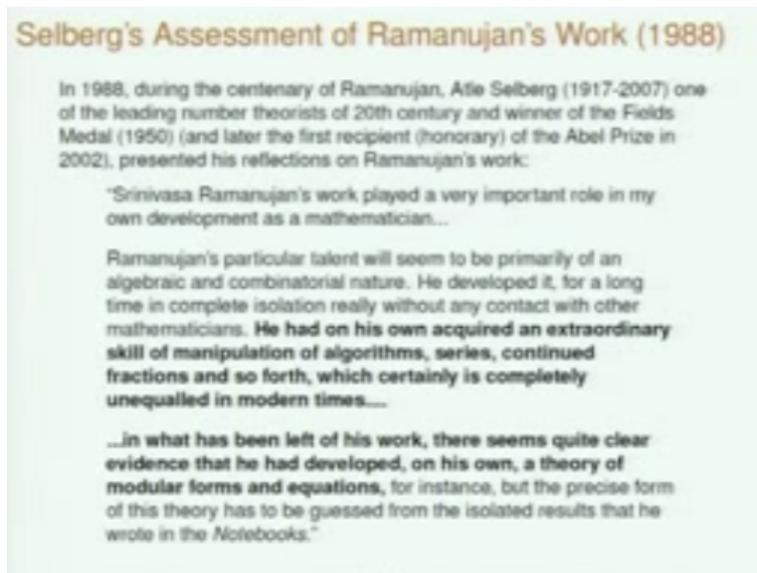
$$|\tau(p)| \leq 2p^{11/2}$$

This was proved by Pierre Deligne in 1974 as a consequence of his proof of Weil's conjecture. Deligne was awarded the Fields Medal in 1978 and Abel's prize in 2013.

Now, another interesting thing of Ramanujan which is often talked about is something called the tau function. This is the definition of the tau function. It is a function of integers n . So, left hand side is power series like this, right hand side is an infinite product like this. Ramanujan mentioned two properties of the tau functions, they were proved by Mordell, the next year he wrote define the tau function in 1916.

Ramanujan conjectured that if p is any prime $\tau(p)$ is bounded by p to the power $11/2$. This was a very famous conjecture. It was proved by Deligne in 1974 as a consequence of another very famous conjecture known as Weil's conjecture for which Deligne was awarded the Fields medal and this year Deligne has been given the Abel's prize, the current sort of equivalent of Nobel prize in mathematics.

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These are some of the kind of works that Ramanujan did, the tau function, partition function, etc. in his papers published from England. During the time of Ramanujan centenary, an assessment started getting made of the kind of the impact his work had, what was the nature of his work and Selberg is one of the leading number theorists of 20th century, a Fields medallist in 1950. He was the first honorary recipient of Abel prize in 2002.

He explained how his work very highly influenced by Ramanujan's. How somebody his father or someone gave him the collected papers of Ramanujan and that sort of set him on a big trail of work and according to Selberg, Ramanujan on his own had acquired an extraordinary skill of manipulation of algorithm, continued fractions and so forth which certainly is completely unequalled in modern times.

Then he says, in what has been left of his work, there seems quite clear evidence that he had developed on his own a theory of modular forms and equation. It is a very important area in mathematics. So, Ramanujan was essentially a modular forms man.

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Selberg's Assessment of Ramanujan's Work (1988)

Selberg, who himself independently discovered Radmacher's exact form for the partition function $p(n)$, is very positive that Ramanujan had the same result, but somehow Hardy was not convinced about it, and so they ended up proving only the asymptotic form.

"If one looks at Ramanujan's first letter to Hardy, there is a statement there which has some relation to his later work on the partition function, namely about the coefficient of the reciprocal of a certain theta series... It gives the leading term in what he claims as an approximate expression for the coefficient. If one looks at that expression, one sees that it is the exact analogue of the leading term in Radmacher's formula for $p(n)$, which shows that Ramanujan, in whatever way he had obtained this, had been led to the correct form of that expression."

Then, Selberg comes to this question that this partition function p of n , he says if one looks at Ramanujan's first letter to Hardy, there is a statement there which has some relation to his latter work on partition function. It gives a leading term in what he claims as an approximate expression to the coefficient. If one looks at this expression, one sees that it is the exact analogue of the leading term in the Radmacher's formula for p of n . Radmacher's was the exact result which was proved in 1930s.

Selberg had independently proved the same result. Hardy and Ramanujan had given the asymptotic form. Ramanujan, in whatever we had obtained this, had been let to the correct form of that expression.

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Selberg's Assessment of Ramanujan's Work (1988)

"In the work on the partition function, studying the paper it seems clear to me that it must have been, in a way, Hardy who did not fully trust Ramanujan's insight and intuition, when he chose the other form of the terms in their expression, for a purely technical reason, which one analyses as not very relevant.

I think that if Hardy had trusted Ramanujan more, they should have inevitably ended with the Radmacher series. There is little doubt about that.

Littlewood and Hardy were primarily working with hard analysis and they did not have a strong feeling for modular forms and such things; the generating function for the partition function is essentially a modular form, particularly if one puts an extra factor of $x^{-1/24}$ to the power series. This must have been something that came quite naturally to Ramanujan from the beginning..."

Then, he says that it was Hardy who seems to have moved away of the conjecture Ramanujan made and proved a simpler result that he could prove. So, for technical reasons he proved something much less. I think that if Hardy had trusted Ramanujan more, they should have inevitably ended with the Radmacher series. There is little doubt about it. Unfortunately, I mean Selberg does not say so.

Littlewood and Hardy were primarily working with hard analysis and they did not have a strong feeling for modular forms and such things.

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Selberg's Assessment of Ramanujan's Work (1988)

Selberg also refers to the assessment of Ramanujan's work by Louis J. Mordell (1888-1972)

"Louis J. Mordell questioned Hardy's assessment that Ramanujan was a man whose native talent was equal to that of Euler and Jacobi. Mordell...claims that one should judge a mathematician by what he has actually done, by which Mordell seems to mean the theorems he has proved. By the way I should say Mordell clearly at no stage seems to have had access to or seen Ramanujan's Notebooks. Mordell's assessment seems quite wrong to me.

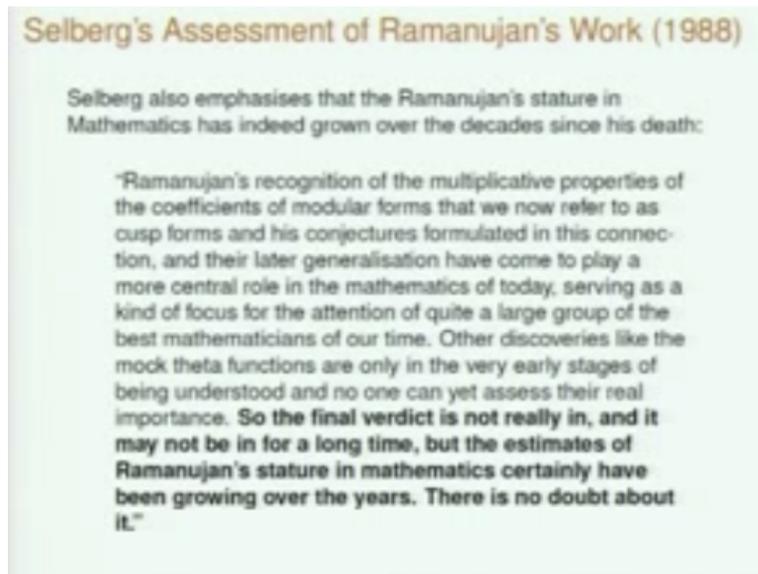
I think that a felicitous but unproved conjecture may be of much more consequence for mathematics than the proof of many a respectable theorem."

Similarly, Louis Mordell, an important British mathematician took the issue with Hardy

comparing Ramanujan with Euler and Jacobi. So, Selberg tries to say that Mordell really has not seen Ramanujan's notebooks and he says Mordell seems to go by the fact that a mathematician should be judged by the number of theorems that he has proved.

So, Selberg says I think that a felicitous but unproved conjecture may be of much more consequence for mathematics than the proof of many a respectable theorem. I view the (()) (43:57) to have said in the 19th century or early 20th century.

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Selberg is saying that we are still studying this mock theta functions and things like that. So, really an assessment of Ramanujan's work cannot really be given. So, the final verdict is not really in and it may not be in for a long time, but the estimates of Ramanujan's stature in mathematics certainly have been growing over the years, there is no doubt about it.

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Selberg's Assessment of Ramanujan's Work (1988)

Finally Selberg talks about Hardy's assessment of Ramanujan:

"One might speculate, although it may be somewhat futile, about what would have happened if Ramanujan had come in contact not with Hardy but with a great mathematician of more similar talents, someone who was more inclined in the algebraic directions, for instance, E. Hecke in Germany. This might have perhaps proved much more beneficial and brought out new things in Ramanujan that did not come to fruition by his contact with Hardy...

I do not think that Hardy fully understood how the interest for Ramanujan's work would be growing when he speaks of the influence which it is likely to have on the mathematics of the future. It seems rather clear that he underestimated that. Later developments have certainly shown him wrong on that point."¹⁰

¹⁰Atle Selberg, Reflections around Ramanujan Centenary, Rep. in Resonance, 1996.

Then, referring to the fact that Hardy said that Ramanujan's work could have been more greater if it were not that strange and if it were more mainstream, etc. He says I do not think that Hardy fully understood how the interest for Ramanujan's work would be growing when he speaks of the influence which it is likely to have on the mathematics of the future. It seems rather clear that he underestimated that. Later developments have certainly shown him wrong on that point.

Selberg is sort of a person well known for understatement but definitive understatement. He will not make a hyper (()) (44:58) but he makes his point very clearly. He even had said that it would have been better if Ramanujan had gone to someone more sympathetic who had a more inclination in algebraic directions; for instance, E. Hecke in Germany. E. Hecke is well-known for work on modular forms.

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How is Ramanujan's Work Assessed Today

Till the latter half of 20th century, the corpus of work of Ramanujan that was generally available comprised of the 37 papers that he had published in various Journals during 1911-1920. These, together with, the 57 questions and solutions published by him in the Journal of Indian Mathematical Society, and extracts from his two letters of 1913 to Hardy which contained statements of around 120 results, were edited by G. H. Hardy, P. V. Seshu Aiyar and B. M. Wilson and published by the Cambridge University in 1927 as the *Collected Papers of Srinivasa Ramanujan*.

This of course excluded most of Ramanujan's work done both before he left for England and after his return to India. The corpus of work done before leaving to England is available in the form of three notebooks which are said to contain around 3250 results. The corpus of work done after the return from England is contained mainly in the "Lost Notebook" which is said to have about 600 results.

Detailed analysis of this large corpus began only in the last quarter of the 20th century. Though much of the work is still in progress, it has already revolutionised our understanding and appreciation of Ramanujan's work.

So, by 1988, the assessment of Ramanujan's work looks very, very different from the way it was assessed by Hardy in 1921. Now, how is it assessed today, so next five-six minutes here. The issue is the collected works of Ramanujan which was published in 1927 had only 37 papers and then the 57 questions that he wrote in mathematical society and it included the two letters that he wrote to Hardy in 1913 which contained about 120 results, most of them taken from his notebooks without proofs.

This excluded almost the entire corpus of the work of Ramanujan that was done either before he went to England with the notebooks or the Lost Notebook in which he wrote down the result after he came back from England. The notebooks that were there had about 3300 results, the notebooks on which he wrote down his results prior to going to England and this Lost Notebook had about 600 results.

This entire corpus in fact has been seriously analysed only in the last 25 years and so we are in a position to better estimate Ramanujan's work.

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The Saga of Ramanujan's Notebooks

It seems Ramanujan started recording his results in a Notebook around the time he entered the Government College of Kumbakonam in 1904. Sometime during 1911-13, Ramanujan copied these results in to a second notebook. As Ramanujan noted in his letters to Krishna Rao and Subramaniam, he perhaps did not add any further results to these notebooks, nor did he try to publish the results contained in them, during his stay in England.

The following is a brief description of the notebooks due to Bruce Berndt:

"Ramanujan left three notebooks. The first notebook totalling 351 pages contains 16 chapters of loosely organised material with the remainder unorganised. ...The second notebook is a revised enlargement of the first. This notebook contains 21 chapters comprising 256 pages followed by 100 pages of miscellaneous material. The third short notebook contains 33 pages of unorganised entries. ...in preparing *Ramanujan's Notebooks Parts I-V*, we counted 3254 results..."¹¹

¹¹B. Berndt, *An Overview of Ramanujan's Notebooks*, 1998

So, as I said then itself Ramanujan started recording this in around 1904. Sometime in 1911 or 1913, he even made a copy of it. So, there are 351 pages, 16 chapters, the second notebooks have 21 chapters 256 pages, third notebook has 33 pages, in all about 3254 results.

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The Saga of Ramanujan's Notebooks

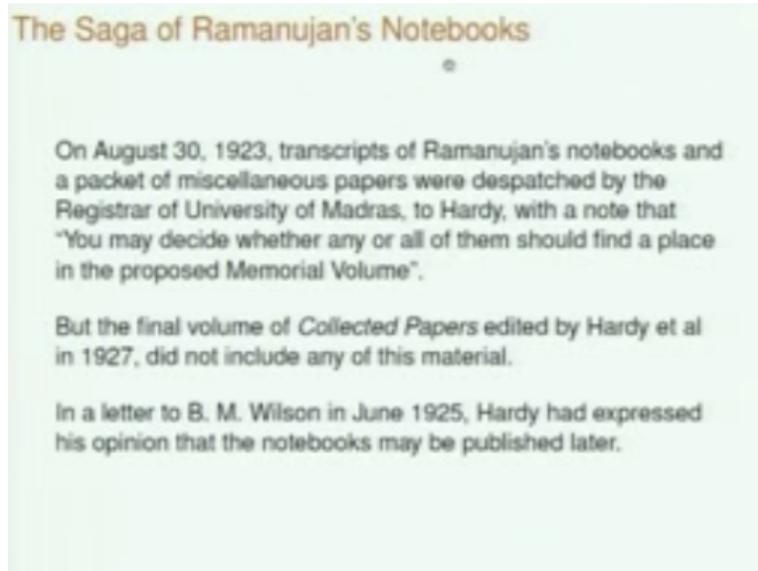
In January 1921, Prof. K. Ananda Rau of Presidency College wrote to Hardy:

"Mr. R. Ramachandra Rao told me that you had written to him some months ago that Ramanujan was working on a certain topic in his last days and possibly there may be some record of this work left. If you will please tell us the nature of this investigation, we may find it easier to sift the papers. The whole of the manuscripts will of course be sent to you in accordance with the resolution of the syndicate. You will have noticed also in the Minutes that the syndicate has asked Mr. Seshu Iyer and me to arrange for the preparation of a transcript of Ramanujan's note book, with a view to having it incorporated as an Appendix to the Memorial volume. I do not know if this will serve any useful purpose. **I fear it may look a little incongruous by the side of his mature work. But there are some here, who think that the Note Book may contain valuable algorithms providing starting points for future investigations.**"

Now, in 1921, Professor Ananda Rau of Presidency College wrote to Hardy, Hardy had enquired about Ramanujan had mentioned about this mock theta functions in the letter in 1921 to Hardy. So, Hardy was enquiring whether there is material on it. So, he says we will look it up and then he says now Madras University has passed a resolution, we are sending you all his notebooks to be published.

Ananda Rau of course adds his own view that I fear it may look a little incongruous by the side of his mature work, mature work meaning the work that he did in collaboration with the British mathematicians in England but still many people here in Madras seem to think that the notebooks may contain some valuable insights.

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So, all these notebooks were sent in 1923 to England. I think some of them were returned to India to Ranganathan by Hardy. In 1927, the collected papers were edited but the material of this notebook was not included in the collected papers. Now, when the collected papers got published, it had these two letters of Ramanujan which had this 120 results. So, many people started looking at them and started proving them and wondering what else is contained in the notebook. So, there was a clamour for the publication of this notebook.

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The Saga of Ramanujan's Notebooks

In a recent article on the occasion of Ramanujan's 125th birthday, Bruce Berndt has recounted the saga of publication of Ramanujan's notebooks:

"As it transpired, ...published with the *Collected Papers* [of Ramanujan], were the first two letters that Ramanujan had written to Hardy, which contained approximately one hundred twenty mathematical claims. Upon their publication, these letters generated considerable interest, with the further publication of several papers establishing proofs of these claims. Consequently, either in 1928 or 1929, at the strong suggestion of Hardy, Watson and B. M. Wilson, ... agreed to edit the notebooks..."

So, around 1930, Hardy asked Watson and Wilson to try and edit it.

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The Saga of Ramanujan's Notebooks

"In an address to the London Mathematical Society on February 5, 1931, Watson cautioned (in retrospect, far too optimistically), 'We anticipate that it, together with the kindred task of investigating the work of other writers to ascertain which of his results had been discovered previously, may take us five years.' Wilson died prematurely in 1935, and although Watson wrote approximately thirty papers on Ramanujan's work, his interest evidently flagged in the late 1930s, and so the editing was not completed...."

Finally, in 1957, the notebooks were made available to the public when the Tata Institute of Fundamental Research in Bombay published a photocopy edition, but no editing was undertaken...."

Somehow, this did not go through. Watson wrote about 30 papers from Ramanujan's work but somehow they could not complete this. So, in 1957, that Tata Institute of Fundamental Research in Bombay published a facsimile edition, a photocopy of the book but no editing was undertaken. A second edition of it has been published last year during 125th year of Ramanujan.

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The Saga of Ramanujan's Notebooks

"In February 1974, while reading two papers by Emil Grosswald, in which some formulas from the notebooks were proved, we [Berndt and coworkers] observed that we could prove these formulas by using a transformation formula for a general class of Eigenstein series that we had proved two years earlier. We found a few more formulas in the notebooks that could be proved using our methods, but a few thousand further assertions that we could not prove. In May 1977, the author [Berndt] began to devote all of his attention to proving all of Ramanujan's claims in the notebooks. With the help of a copy of the notes from Watson and Wilson's earlier attempt at editing the notebooks and with the help of several other mathematicians, the task was completed in five volumes in slightly over twenty years."¹²

¹²B. Berndt, Notices of AMS 2012, pp. 1532-33

So, Bruce Brendt says that it was only in 1974 he started looking at some results in the notebook and he found that there were many more that he did not know and he started proving one or two of them. Then, he realised that this is going to be a huge work. So, he spent the next 20 years from May 1977. He used all the material that Watson and Wilson had left. Huge amount of notes had been made on Ramanujan's notebook, especially by Wilson and wanted to edit that and with the aide of all that, is about five volumes these notebooks with a commentary were published.

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The Saga of Ramanujan's Notebooks

In the same article, Bruce Berndt has also presented the following overall assessment of Ramanujan's notebooks:

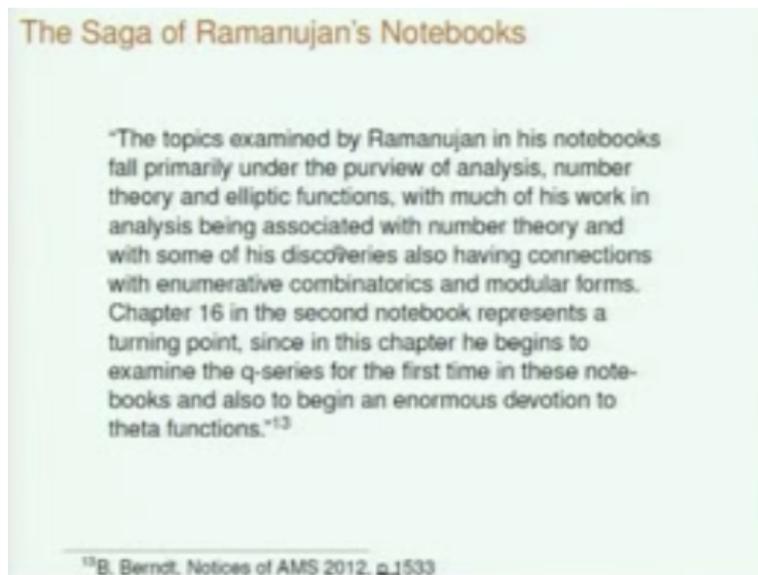
"Altogether, the notebooks contain over three thousand claims, almost all without proof. Hardy surmised that over two-thirds of these results were rediscoveries. This estimate is much too high; on the contrary, at least two-thirds of Ramanujan's claims were new at the time that he wrote them, and two-thirds more likely should be replaced by a larger fraction. Almost all the results are correct; perhaps no more than five to ten are incorrect."

So, now comes what Bruce Brendt understands the notebooks in which Ramanujan wrote down his results prior to going to England. Altogether, the notebooks contain over 3000 claims, almost all without proof. Hardy surmised that over two-thirds of these results were rediscoveries, I

mentioned that. This estimate is much too high.

On the contrary, at least two-thirds of the Ramanujan's claims were new at the time that he wrote down, that is between 1904 to 1913 and two-thirds more likely should be replaced by a larger fraction, almost all the results are correct perhaps no more than 5 to 10 are incorrect, in a mass of 3250 results.

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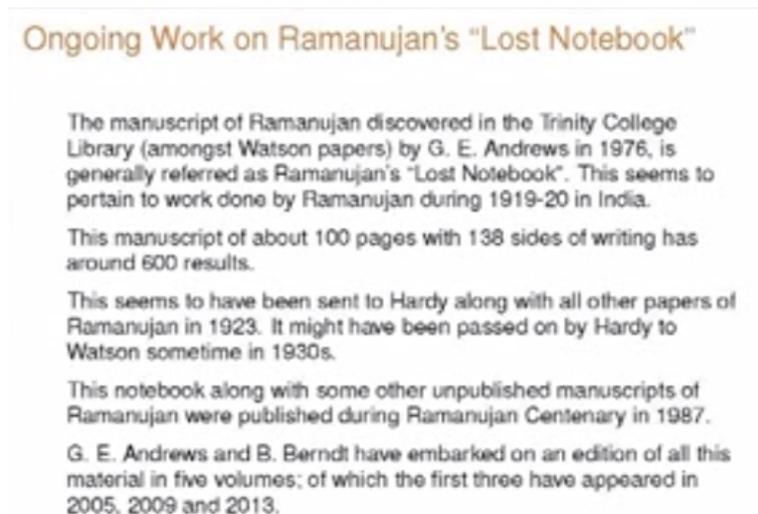
The Saga of Ramanujan's Notebooks

"The topics examined by Ramanujan in his notebooks fall primarily under the purview of analysis, number theory and elliptic functions, with much of his work in analysis being associated with number theory and with some of his discoveries also having connections with enumerative combinatorics and modular forms. Chapter 16 in the second notebook represents a turning point, since in this chapter he begins to examine the q-series for the first time in these notebooks and also to begin an enormous devotion to theta functions."¹³

¹³B. Berndt, Notices of AMS 2012, p.1533

Then, he describes what are the topics that contains. This is an article written in the context of the 125th year of Ramanujan.

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Ongoing Work on Ramanujan's "Lost Notebook"

The manuscript of Ramanujan discovered in the Trinity College Library (amongst Watson papers) by G. E. Andrews in 1976, is generally referred as Ramanujan's "Lost Notebook". This seems to pertain to work done by Ramanujan during 1919-20 in India.

This manuscript of about 100 pages with 138 sides of writing has around 600 results.

This seems to have been sent to Hardy along with all other papers of Ramanujan in 1923. It might have been passed on by Hardy to Watson sometime in 1930s.

This notebook along with some other unpublished manuscripts of Ramanujan were published during Ramanujan Centenary in 1987.

G. E. Andrews and B. Berndt have embarked on an edition of all this material in five volumes; of which the first three have appeared in 2005, 2009 and 2013.

Now, there is this Lost Notebook of Ramanujan which was discovered in 1976 by George Andrews. It has about 600 results and facsimile of this notebook was published during Ramanujan centenary by Narosa publishers. Now, Andrews and Berndt have embarked on the edition of this material in five volumes, three volumes have appeared now.

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Ongoing Work on Ramanujan's Lost Notebook

Andrews and Berndt note in the first volume of their edition of the Lost Notebook that:

"...only a fraction (perhaps 5%) of the notebook is devoted to the mock theta functions themselves. ... A majority of the results fall under the purview of q-series. These include mock theta functions, theta functions, partial theta function expansions, false theta functions, identities connected with the Rogers-Fine identity, several results in the theory of partitions, Eisenstein series, modular equations, the Rogers-Ramanujan continued fraction, other q-continued fractions, asymptotic expansions of q-series and q-continued fractions, integrals of theta functions, integrals of q-products, and incomplete elliptic integrals. Other continued fractions, other integrals, infinite series identities, Dirichlet series, approximations, arithmetic functions, numerical calculations, Diophantine equations, and elementary mathematics are some of the further topics examined by Ramanujan in his lost notebook."

Again, the list to the kind of works, mock theta function is only a small part of this Lost Notebook, they are only 5%. There are many more things it contains, so this work is still going on.

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Ramanujan's Mock Theta Functions

The last letter of Ramanujan to Hardy contained 17 examples of mock theta functions such as the following:

$$\sum_{n=0}^{\infty} \alpha(n)q^n := 1 + \sum_{n=1}^{\infty} \frac{q^{n^2}}{(1+q)^2(1+q^2)^2 \dots (1+q^n)^2}$$

Ramanujan also gave the asymptotic formula

$$\alpha(n) \sim \frac{(-1)^{n-1}}{2\sqrt{n - \frac{1}{24}}} e^{\pi\sqrt{\frac{1}{3}(n - \frac{1}{24})}}$$

This has later been proved by Andrews and Dragnette (1966)

Now, on mock theta function itself insomuch is talked about, what is a mock theta function. So,

this last letter to Hardy had 17 examples of functions like this. Each of them is a mock theta function and Ramanujan gave this formula for this function which was proved in 1966 by Andrews.

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Ramanujan's Mock Theta Functions

In the 1935 Presidential address to the London Mathematical Society, G. N. Watson had declared:

"Ramanujan's discovery of the mock theta function makes it obvious that his skill and ingenuity did not desert him at the oncoming of his untimely end. As much as any of his earlier work, the mock theta functions are an achievement sufficient to cause his name to be held in lasting remembrance."

Around the time of Ramanujan's centenary (1987), the famous theoretical physicist Freeman J. Dyson had remarked:

"The Mock-theta functions give us tantalizing hints of a grand synthesis to be discovered. Somehow it should be possible to build them into a coherent group theoretical structure, analogous to the structure of the modular forms which Hecke built around the old theta-functions of Jacobi. This remains a challenge for the future."

In 1935, G. N. Watson had said that Ramanujan's discovery of mock theta functions makes it obvious that his skill and ingenuity did not desert him at the oncoming of his untimely end. As much as any of his earlier work, the mock theta function are an achievement sufficient to cause his name to be held in lasting remembrance. During the centenary, Freeman J. Dyson, a very famous theoretical physicist said that some new theory is expected out of this mock theta function.

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Ramanujan's Mock Theta Functions

The following is the abstract of a recent article, M. Griffin, K. Ono and L. Rollen, Ramanujan's Mock Theta Functions, Proc. Nat. Acad. Sci. 2013:

"In his famous deathbed letter, Ramanujan introduced the notion of a mock theta function, and he offered some alleged examples. Recent work by Zwegers [2001 and 2002]... has elucidated the theory encompassing these examples. They are holomorphic parts of special harmonic weak Maass forms. Despite this understanding, little attention has been given to Ramanujan's original definition. Here we prove that Ramanujan's examples do indeed satisfy his original definition."

Recently last month I think, there is a paper by a famous Japanese scholar, K. Ono on Ramanujan's mock theta functions in his famous deathbed letter, this is the abstract of that paper, Ramanujan introduced the notion of a mock theta function and he offered some examples. Recent work by Zwegers in 2001 and 2002 has elucidated the theory encompassing these. They are holomorphic parts of special harmonic weak Maass forms, this is not misspelling.

These are special mathematical terminology. Despite this understanding, little attention has been given to Ramanujan's original definite of mock theta function. Here, that is in this paper of 2013 we are proving that Ramanujan's examples do indeed satisfy his original definition.

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The Enigma of Ramanujan's Mathematics

For the past hundred years, the problem in comprehending and assessing Ramanujan's mathematics and his genius has centred around the issue of "proof".

In 1913, Hardy wrote to Ramanujan asking for proofs of his results. Ramanujan responded by asserting that he had a systematic method for deriving all his results, but that could not be communicated in letters.

Ramanujan's published work in India, and a few of the results contained in the notebooks have proofs, but they were often found to be sketchy, not rigorous, incomplete and sometimes even faulty.

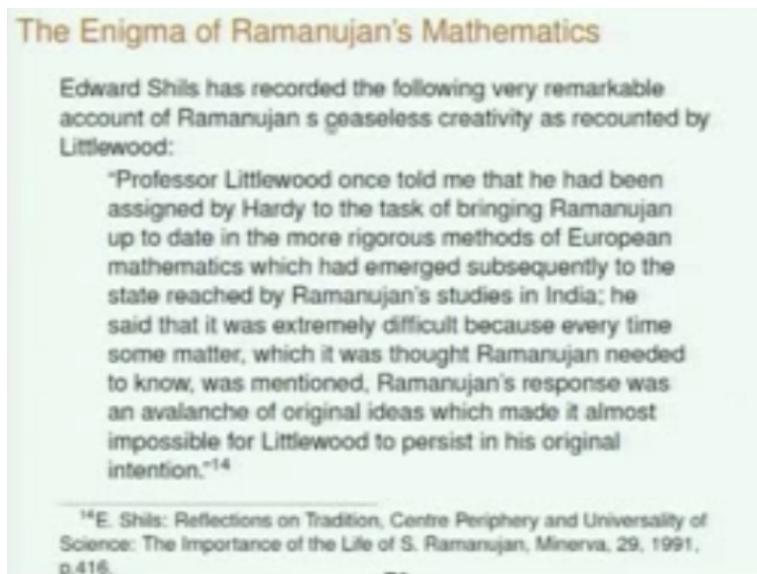
Ramanujan, however, had no doubts whatsoever about the validity of his results, but still he was often willing to wait and supply proofs in the necessary format so that his results could be published.

But, all the time, he was furiously discovering more and more interesting results.

So, there is a sort of scenario we are now in regarding Ramanujan's work. Now, what has been the cause of the tremendous confusion in trying to understand and almost made Ramanujan in enigma is the issue that everybody is asking Ramanujan has this result but where are the proofs. In fact, this started in 1913 itself with Hardy telling Ramanujan your results are fine but unless you send me the proofs what is the use.

The published work in India and his notebooks have some proofs here and there but most of them are sketchy, not rigorous, incomplete and sometimes even said to be faulty. Now, Ramanujan himself had no doubts whatsoever about the validity of his results and as he wrote to his friends that he was even willing to wait and supply proofs in the necessary format so that they can be published. But all the time, he was more furiously engaged in discovering newer and newer results.

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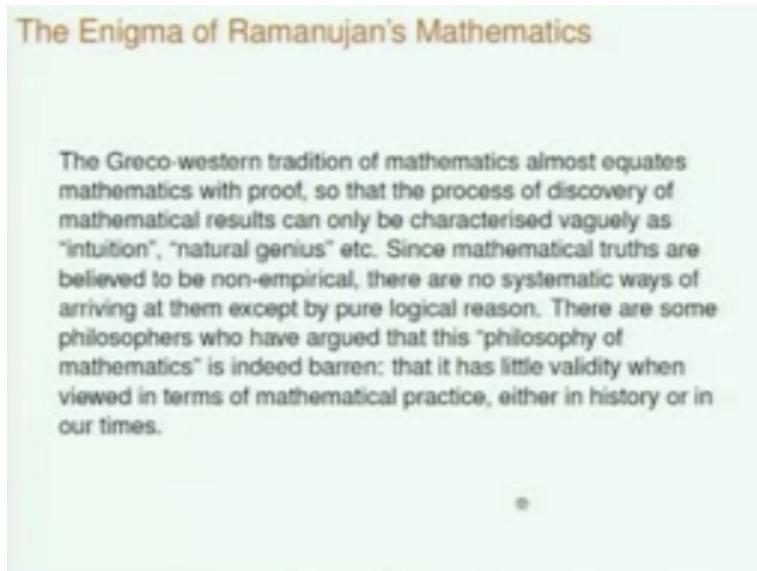


There is one interesting story documented in a paper by Shils of a conversation that he had with Littlewood. Littlewood was a colleague of Hardy in Cambridge at that time. Professor Littlewood once told me that is Edward Shils, that he had been assigned by Hardy to the task of bringing Ramanujan up to date in the more rigorous methods of European mathematics which had emerged subsequently to the state reached by Ramanujan's studies in India.

He said that it was extremely difficult because every time some matter which it was thought

Ramanujan needed to know was mentioned, Ramanujan's response was an avalanche of original ideas which made it almost impossible for Littlewood to persist in his original intention.

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So, much of this cause of difficulty and confusion is with the fact that the Greco-western tradition of mathematics almost equates all mathematics with proof. So, the process of discovery of mathematical results can only understood very vaguely. It can be described as intuition, natural genius. If you seen the writings of Hardy, you will find that here is a man who does not know any proof but he is filled with intuition, he is filled with natural genius.

So, since mathematical tools are believed to be non-empirical, there is no systematic way of arriving them, except by pure logical reason. So, this is the sort of dominate understanding of mathematics which does not throw any light on mathematical discovery; and for the same reason, it cannot explain most of the history of mathematics or not even the contemporary mathematical practice as to how one comes up with the result or how somebody else correct some result and things like that.

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The Enigma of Ramanujan's Mathematics

Incidentally, Hardy was amongst those who swore by the non-empirical nature of mathematics. In his *A Mathematician's Apology* written in 1940, he speaks of the "immortality" of mathematics, of the Greek genre.

"The Greeks were the first mathematicians who are still 'real' to us today. Oriental mathematics may be an interesting curiosity, but Greek mathematics is the real thing....So Greek mathematics is 'permanent'. ... 'Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean."¹⁵

¹⁵G. H. Hardy, *A Mathematician's Apology*, 2nd ed., Cambridge 1967, pp. 80-81.

Incidentally, Hardy was one of those people who really swore by the non-empirical nature mathematics. This quotation was given yesterday. It is the second half of the quotation which is more interesting. The Greeks were the first mathematicians who are still real to us today. Oriental mathematics may be an interesting curiosity but Greek mathematics is the reality. This is his view of the Oriental mathematics, that is alright.

So, Greek mathematics is permanent. Hardy, you understand, is a great agnostic, great skeptic. So, Greek mathematics is permanent. Immortality may be a silly word but probably a mathematician has the best chance of whatever it may mean. So, if mathematical tools that are going to stay forever and forever and forever.

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The Enigma of Ramanujan's Mathematics

In the Indian mathematical tradition, as is known from the texts of the last two to three millennia, mathematics was not equated with proof. Mathematical results were not perceived as being non-empirical and they could be validated in diverse ways. In this way, the process of discovery and the process of validation were not completely divorced from each other. Proof or logical argumentation to demonstrate the results was important. But proofs were mainly for the purpose of obtaining assent for one's results in the community of mathematicians.

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On the contrary in the Indian tradition, at least as known from the text of the last 2000-3000 years, mathematics was not equated with proof. Mathematical results were not perceived as being non-empirical and they could be validated in diverse ways. In this way, the process of discovery and the process of validation were not completely diverged from each other.

Of course, we have to understand it much more, if we have to make a claim like that. Proof of logical argumentation to demonstrate the results was important but they were mainly for obtaining the community's assent for once result.

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Ramanujan: Not A Newton But A Mādhava

In 1913, Bertrand Russell had jocularly remarked about Hardy and Littlewood having discovered a "second Newton" in a "Hindu clerk". If parallels are to be drawn, Ramanujan may indeed be compared to the legendary Mādhava.

It is not merely in terms of his methodology and philosophy that Ramanujan is clearly in continuity with the earlier Indian tradition of mathematics. Even in his extraordinary felicity in handling iterations, infinite series, continued fractions and transformations of them, Ramanujan is indeed a successor, a very worthy one at that, of Mādhava, the founder of the Kerala School.

Therefore, we have to conclude something. We can go back to this wonderful joke of Bertrand

Russell that Hardy and Littlewood had discovered Newton in a Hindu class but the kind of outline given of Ramanujan's work and the kind of outline of the earlier work of Kerala School perhaps does convince you that it is not merely in terms of his methodology or philosophy that Ramanujan is clearly in continuity with the Indian tradition of mathematics. Even in his extraordinary felicity in handling iterations, algorithms, infinite series, continued fractions, transformations of them, etc., he is a successor of Madhava, the founder of Kerala School.

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References

1. S. Ramanujan, *Collected Papers*, Cambridge University Press, Cambridge 1927.
2. G. H. Hardy, *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work*, Cambridge 1940.
3. S. Nurullah and J. P. Naik, *A History of Education in India*, Macmillan, New Delhi 1951.
4. Dharampal, *The Beautiful Tree: Indigenous Indian Education in the Eighteenth Century*, Impex, New Delhi 1983.
5. S. N. Sen, Survey of Studies in European Languages, in S. N. Sen and K. S. Shukla, Eds., *History of Astronomy*, INSA, New Delhi 1985, pp.49-121.
6. B. C. Berndt, *Ramanujan's Notebooks*, Parts I-V, Springer New York 1985-1998.
7. B. C. Berndt and R. A. Rankin, *Ramanujan: Letters and Commentary*, AMS, Providence 1995.

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So, this is the collected papers. These are the lectures by Hardy. Then, these are the notebooks of Ramanujan edited by Bruce Brendt in five volumes from 1985 to 1998. These are his letters edited by Brendt.

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References

8. R. Kanigal, *The Man Who Knew Infinity: The Life of the Genius Ramanujan*, Scribner, New York 1995.
9. B. C. Berndt and R. A. Rankin, *Ramanujan: Essays and Surveys*, AMS, Providence 2001.
10. G. E. Andrews and B. C. Berndt, *Ramanujan's Lost Notebook*, Volumes 1-3, Springer, New York 2005-2013.
11. Krishnaswami Alladi (ed), Srinivasa Ramanujan Going Strong at 125, *Notices of the AMS*, 59, December 2012, pp.1522-37 and *ibid.* 60, January 2013, pp.10-22 [Articles by K. Alladi, G. E. Andrews, B. C. Berndt, J. M. Borwein, K. Ono, K. Soundararajan, R. C. Vaughan and S. O. Warnaar]

The well-known biography of Robert Kanigal which is I think being reprinted last year. Brendt and Rankin have a set of collective essays. This Lost Notebook is still coming three volumes of it have appeared so far. This was a recent collection of articles during the time of Ramanujan's 125th year. Thank you.