Mathematics in India: From Vedic Period to Modern Times
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Lecture-19
Development of Combinatorics 2

So, we will continue the discussion on combinatorics and so in the we saw first that (FL) and his (FL) start at this subject of (FL) and he did this (FL) of the syllabic meters and then he talked about the other (FL) also the (FL) then the same thing was repeated for the moric meters or the (FL) by (FL) Hemacandra and others. Now parallely the same kind of discussion started in music also and in some 11, 12th centuries works there are some talks of this kind of (FL).

And combinatorics in music, but really the text in which will be entire subject appears almost in a completely finished and perfect form is the very famous text (FL).

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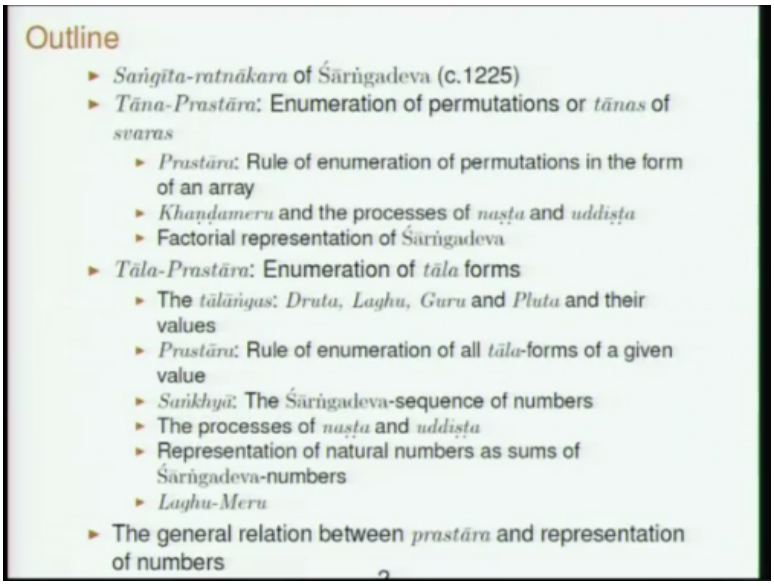

So, (FL) considers 2 different kinds of combinatorial problems one is (FL) of (FL) he also considers (FL) of (FL) are the rhythms. So, you can see both are going to be discuss somewhat extensively in this top. In the end I will make some remarks on the relation between the idea of a (FL) and theory of representation of numbers.

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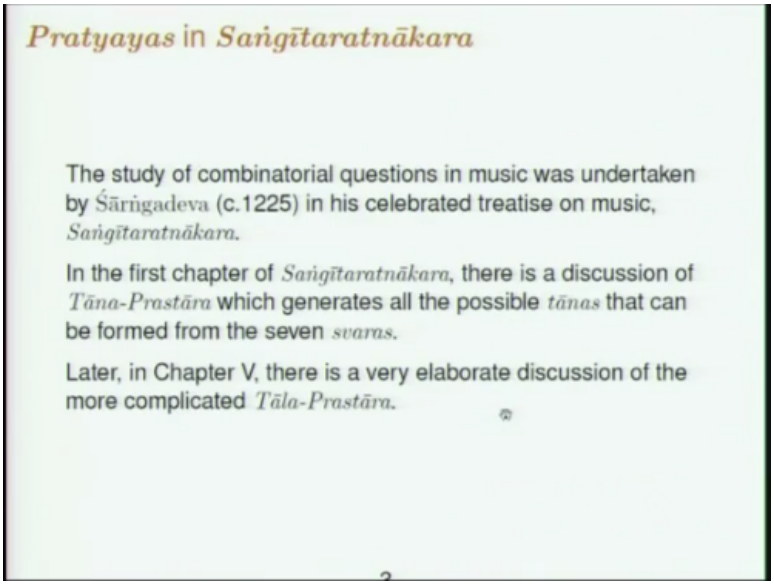

(FL) his (FL) is one of the most famous treatise on music in India, it is said to be prior to the period when classical Indian music divided into the karnatic and the Hindustani form. So, it is a canonical text for both thus streams . He was a (FL) Brahmin by this end they I think came to devagiri 1 or 2 generations earlier to him and he was his father was in the (FL) yadava king (FL) court, (FL) is said to have ruled in devagiri from 12:10 to 12:45. So, (FL) were should be around 12:30, 12:40 (FL) is a treatise like (FL) the first chapter is (FL).

Next there is a (FL). Then there is 1 on compositions then we have a chapter on (FL) and finally there is a chapter on (FL), so it is a complete treatise. The sections on (FL) is very small in the (FL) it is somewhere very in the middle of this (FL) but the section on (FL) is rarely very long in the (FL). There are 2 very famous commentaries one by (FL) and one by (FL) both of around 14th century. I think (FL) was the contemporary of king Devaraya in vijayanagar.

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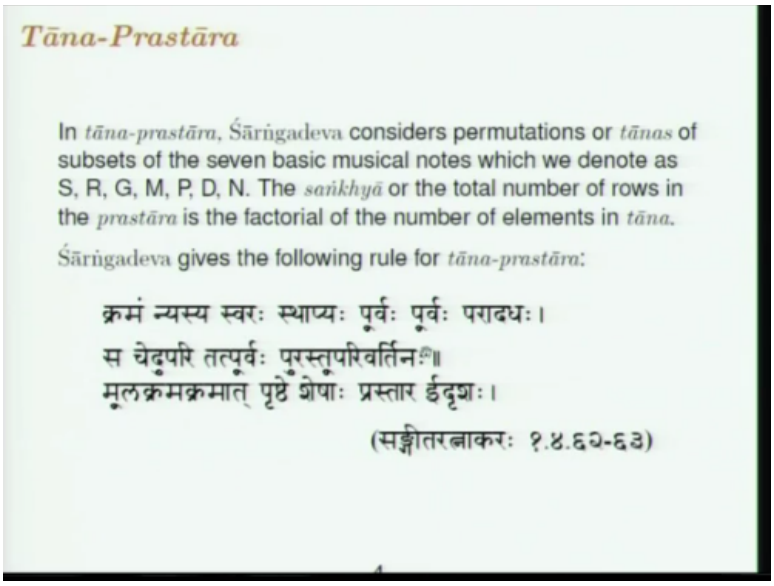

So, we can straight away enter into the discussion of what is (FL), so basically what (FL) is talking is again a mathematical problem that is why it is not much discussed in music circles though this section is a important section in (FL) book.

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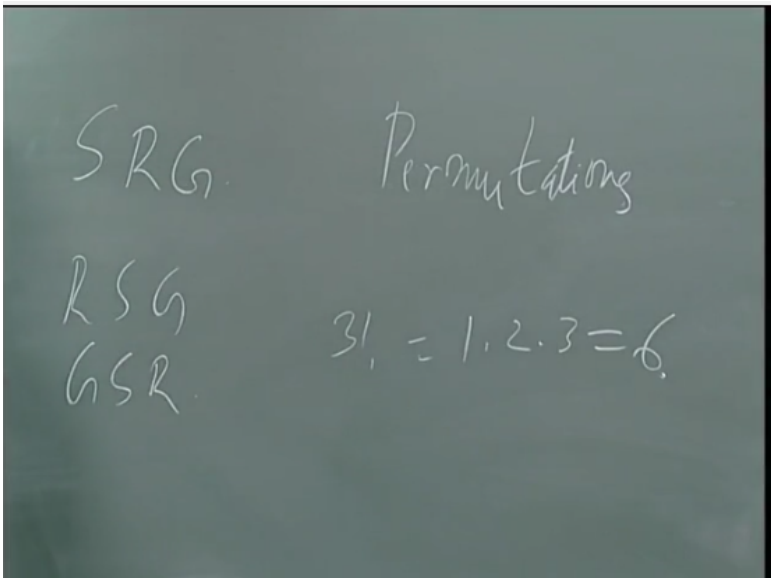

So, you take any 3 (FL) say (FL) now you can render them in various different orders, so in normal mathematical language they will called permutations, so (FL) etc., so, you have 3 (FL) there are a natural order between them and then there is permutation and so the number of permutation here is 3 factorial, so the moment you add the fourth (FL).

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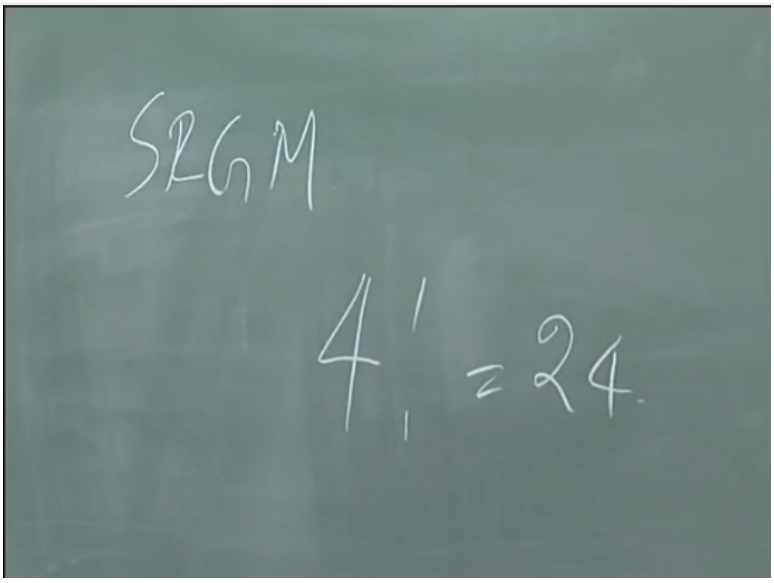

The number of permutations will be 4 factorial it will become 24 and like that up to (FL) you can reach 5047 factorial, now as regards this 6 it does not matter you can always identify them but suppose I ask you to write down all possible permutation of (FL) so are called the (FL), so a phrase like (FL) is called a (FL) and rendering them in a different order than in the natural order is called (FL) and you need a (FL) for all the (FL) that is you need a listing of all this.

So, if you have 5 (FL) you have 120 permutations, so if I ask you to write them down again as I said the problem will be you would come maybe you will write 30, 40 and then you do not know whether the next one that you are writing as already been written down earlier or you has not been written down you have to check. So, there has to be a rule by which you generate all this (FL).

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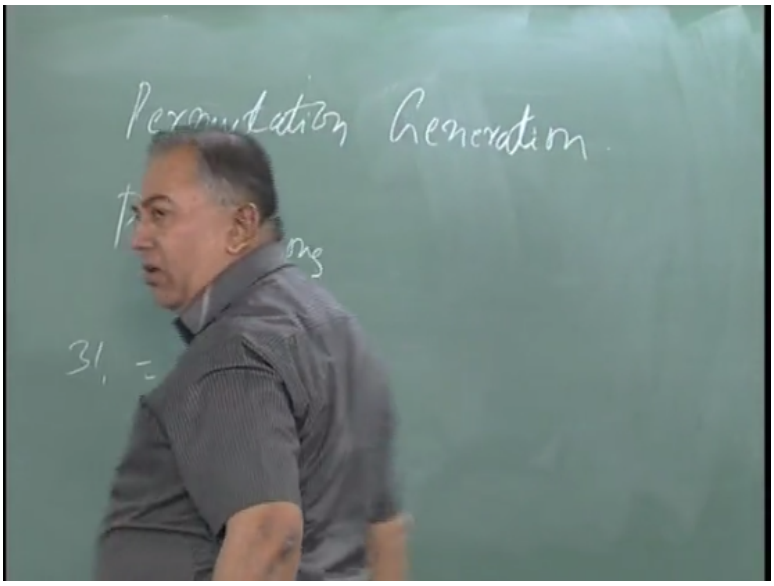

So, in mathematics this called permutation generation today. So, this fourth volume of (FL) alt of computer programming is devoted a section loop 2 sections are devoted to permutation in combination generation, to give algorithms and then study them and in fact the oldest algorithm for permutation generation in the perfect one is due to (FL) and this is the algorithm (FL). So, you start first with the (FL) in natural order (FL).

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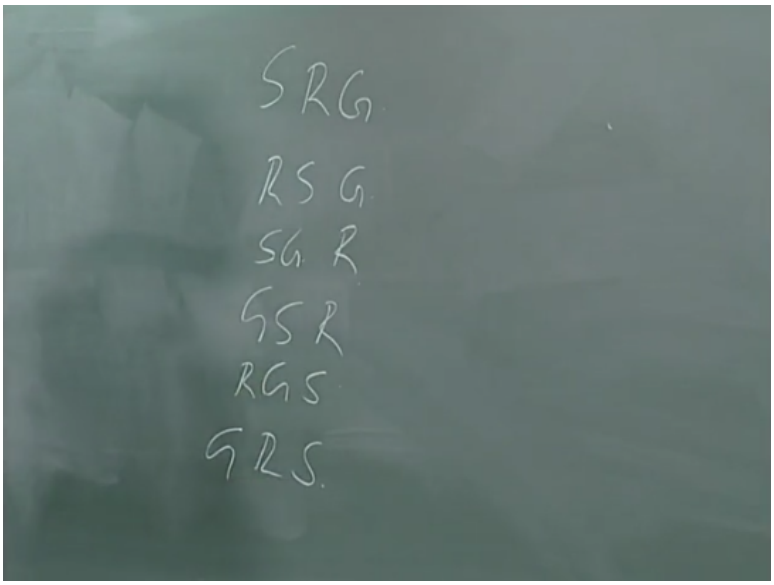

And you should end up with (FL) in reverse order, so that is the last row of the (FL).

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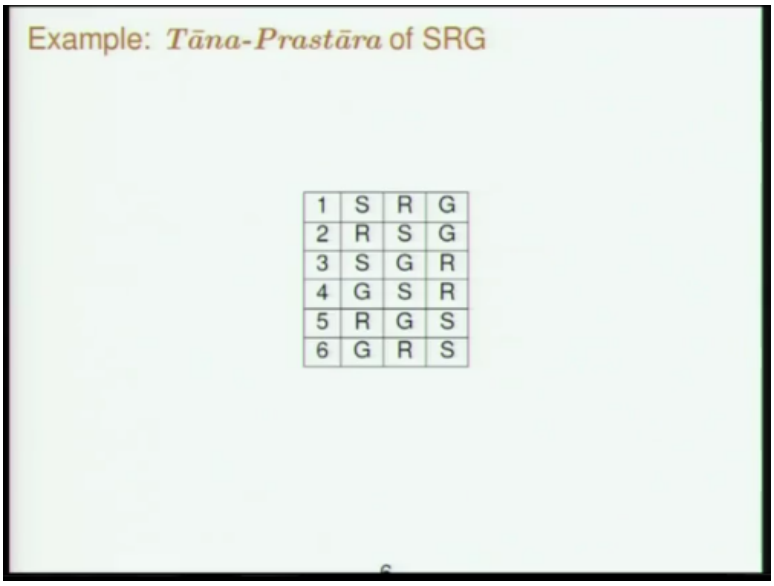

So, the (FL) for this looks like this, so what is the rule that (FL) is giving to go from (FL) to (FL), so what is the rule by which (FL) is saying you go from 1 row of this to the next row. So, stated in verse the rule is as follows, so as flow is write all the (FL) in the natural order.

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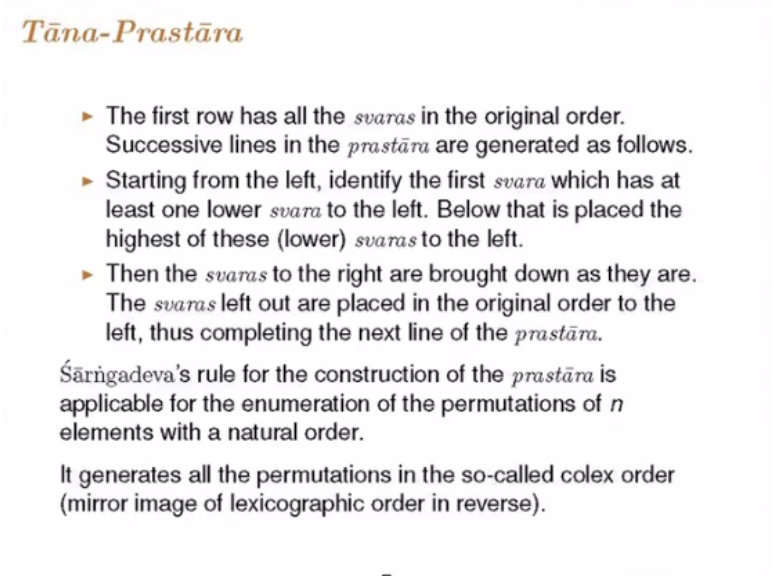

Then starting from the left identify the first (FL) which has at least 1 (FL) lower to the left, below that is placed the highest of this lower (FL) to the left. Then the (FL) in the right are brought down as they are, so this part of (FL) is similar. In every (FL) rule you do something at some point whatever is do the right is brought down as it is, then there is some simple rule to fix whatever is to the left, the (FL) left out of placed in the original order to the left.

So, this rule of (FL) is indeed applicable to any (FL) meaning for any permutation of a n ordered object.

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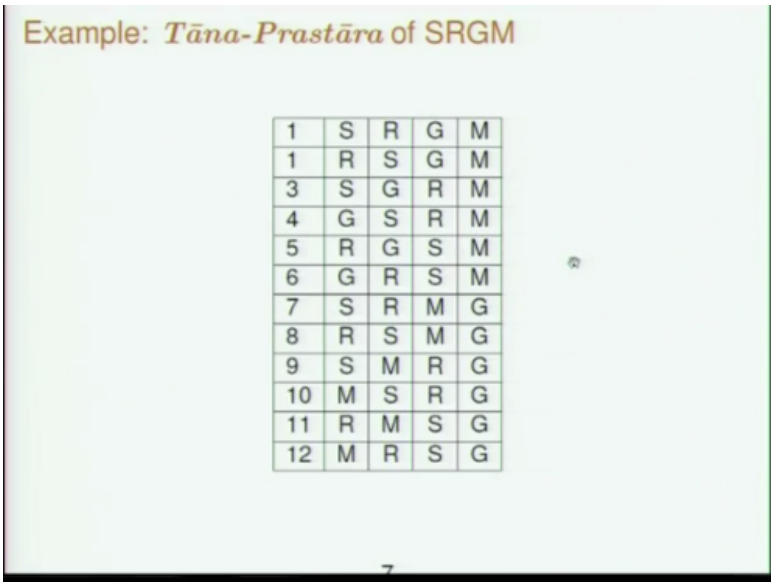

So, let us look at this (FL) 4 objects (FL), so what is (FL) saying from the left you start looking whenever the (FL) appear in an ascending order, so he had a (FL) itself it is appearing in an ascending order before (FL) put down the lowest of the (FL) highest of this (FL) below (FL) here. So, below (FL) you put this (FL) bring down this (FL) and (FL) as it is and remaining (FL) are put in ascending order to the left.

So, second row is (FL) is simple, next (FL) how to go to (FL) from (FL), so (FL) the ascending order is happening it (FL) is descending (FL) is ascending, so below this G I have to put the largest of this highest of this that is (FL). I bring down the 1 to right as it is at in the left whatever is left out (FL) and (FL) already been finish, so (FL) and (FL) have to be put in the natural order, so that is (FL) next row.

From (FL), so (FL) itself is a natural ascending order, so below (FL) put this R bring down the (FL) and (FL) whatever is left out (FL) is put to the left. Next row (FL), so (FL) is in the ascending order below (FL) put the highest below (FL) in the left that is (FL) bring down the (FL) as it is whatever is left out (FL) is here (FL) have to be filled in here. (FL), so (FL) is in

ascending order below the (FL) put the (FL) are brought down from the above what is left out (FL) is placed to the left.

(FL) so (FL) is all in this ending only (FL) is ascending, so before the (FL) put the highest that is below (FL) which lies to the left that is (FL), so before the (FL) put the (FL) there is nothing else on the right to be brought down the remaining (FL) have to puts in the ascending order (FL), so like this you go on.

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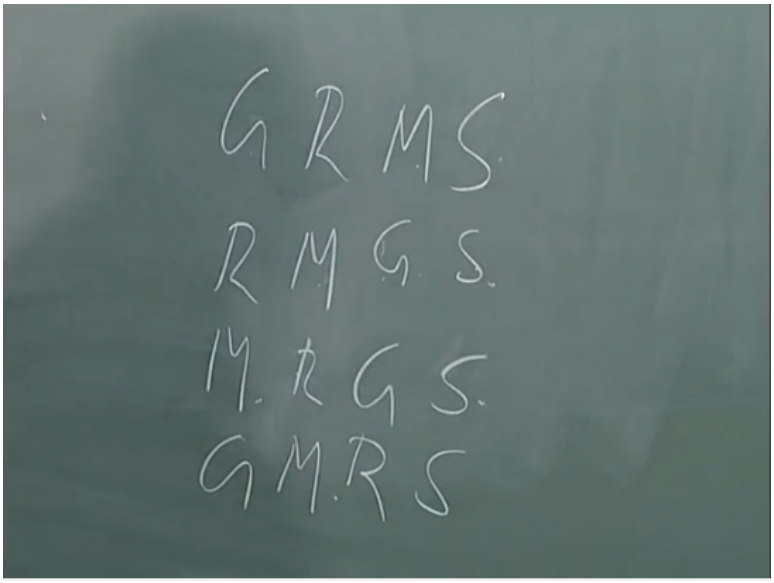

So, you can take any line (FL) suppose what is the next line, ascending order (FL) to (FL) is ascending, so highest before the (FL) is (FL) is to be brought down the (FL) is to be left as it is (FL) and (FL) over (FL) and (FL) is only remaining, the next row is I am sorry (FL) and (FL) are what is left out, so (FL) is the next row (FL) is an ascending order before the (FL) whatever is to the left which is lower than (FL) should be brought R (FL) and (FL) as the R, so this next row is (FL).

Next row (FL) is in ascending order before the (FL) put the (FL) is to be brought down as it is (FL) and (FL) are what is left out, so like that take from any point you can stall follow (FL) ruling keep going down. So, this is the rule, this can be applied to any permutation of n objects which have a natural order among them, so you instead of (FL) you could have done 1,2,3,4.

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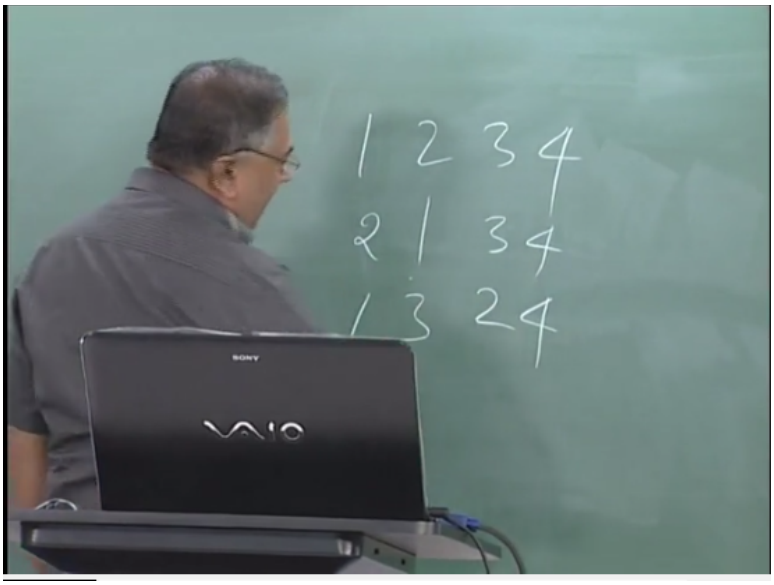

1, 2 put the 1 there 3, 4, 2 is left, 2, 1, 3 1, 3 is ascending put that 2 there bring down 1, 3, so like that you can go on. So, any permutation of any n objects which have a natural order and in fact the same rule even applies when you have reputations (FL) you can do a (FL) of that obviously that is not be n factorial that will be n factorial by n1 factorial, n2 factorial number of identical object, so that is called listing of multi sites.

Those are sites which were objects can repeat and their order, so even that can be done by the same rule of (FL) in fact later on if we have time while discussing (FL) of Narayapanditha he has explicitly consider the (FL) of permutations with reputations we will do, so that is the rule of (FL) that is you order all the (FL) of the (FL) in the form of a (()) (12:55) going from 1 till the n factorial point.

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Example: *Tāna-Prastāra* of SRGM (contd.)

13	S	G	M	R
14	G	S	M	R
15	S	M	G	R
16	M	S	G	R
17	G	M	S	R
18	M	G	S	R
19	R	G	M	S
20	G	R	M	S
21	R	M	G	S
22	M	R	G	S
23	G	M	R	S
24	M	G	R	S

And as you can see in this the first 6 end with a (FL) the next 6 end with a (FL) and the next 6 end with a (FL) and the last 6 end with a (FL) and what is 6, 6 is the factorial below 4 that is 3 factorial and inside each of those blocks you will see they will group under 2 factorial inside that they will group under 1 factorial. So, the factorial numbers naturally appear in the construction of this (FL) and so just as the binary (FL) was natural in the syllabic meter (FL).

The (FL) are the (FL) numbers were the natural units in the (FL) the factorials of integers are the natural units which will discuss which will be used in discussing the (FL) of (FL).

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Tāna-Prastāra

In *tāna-prastāra*, Śārṅgadeva considers permutations or *tānas* of subsets of the seven basic musical notes which we denote as S, R, G, M, P, D, N. The *saṅkhyā* or the total number of rows in the *prastāra* is the factorial of the number of elements in *tāna*.

Śārṅgadeva gives the following rule for *tāna-prastāra*:

क्रमं न्यस्य स्वरः स्थाप्यः पूर्वः पूर्वः परादधः।
स चेदुपरि तत्पूर्वः पुरस्तुपरिवर्तिनः॥
मूलक्रमक्रमात् पृष्ठे शेषाः प्रस्तार ईदृशः।

(सङ्गीतरत्नाकरः २.४.६२-६३)

So, let us just understand the rule of (FL) put the first row in which this (FL) all in the (FL), so from left it there is a 1 going up put 1 below that below that (FL) bring down all the things to the right as they are (FL) the other (FL) which are left out are put in the natural order to the left. So, the complete rule of (FL) is clearly encoded in this beautiful verse. Now obviously I am constructed the (FL) we have to do the standard problems of given a row number what is the permutation that appears what is the (FL) that appears in that row.

Given a (FL) or a permutation in which row of (FL) thus that appear, so that is the next thing, so incidentally some history of this. So, this way of ordering the permutations is what is called the colex ordering in today.

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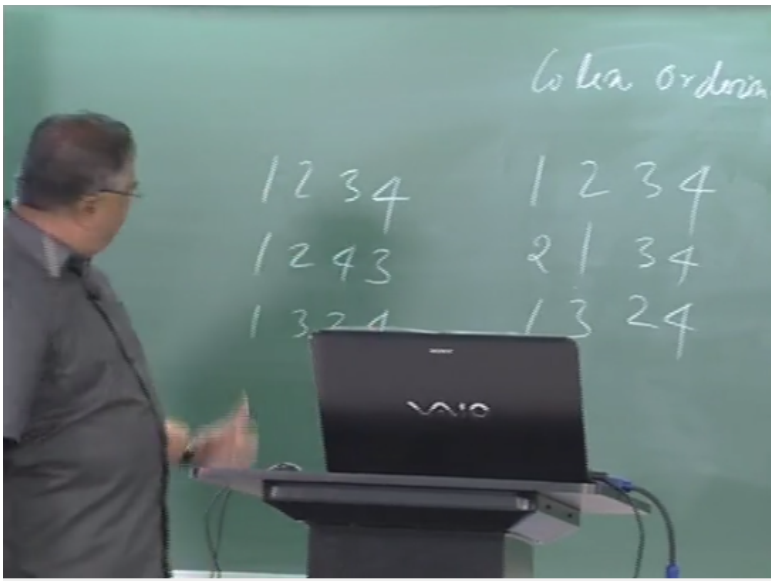

Now there is a simple ordering called the lexico graphical ordering today, so you can start with 1, 2, 3, 4 what could be the next thing in lexico graphic ordering, you 1, 2, 4, 3 right then 1, 3, 2,4 , so like that it goes from the left you keep on choosing the lowest one only in the end you alter the things in the natural order. So, that way of ordering things is called lexico graphic ordering what (FL) has done.

He is a mirror image of the inversion of the lexico graphic ordering that is why it is called the colex ordering in mirror image of colex ordering. So, if you take the last one the mirror image of this is (FL) you take the next one the mirror image of this is (FL) you take the third one (FL), so

as you can see the when you read the (FL) from the bottom the mirror image is the same as the what is called the lexico graphic ordering.

So, (FL) prescription for permutation generation is what is called the mirror image of the co-lexico graphic ordering of permutations. In fact when 16, 17th century lots of books were written on trying to list all permutations of a 8 objects and 9 objects and invariably there would errors found in the books they did not have a systematic algorithm for writing down all the permutations and I think some Frenchmen discovered 1 algorithm around late 18th century that is the sort of beginning of the subject in Europe.

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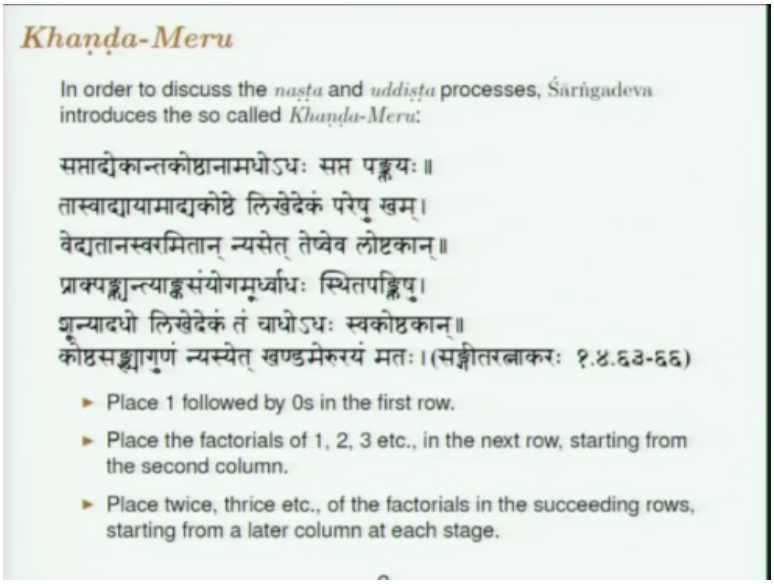

So, now what is the way in which (FL) discusses (FL) and (FL). So, as I said (FL) and (FL) is given the row number find out the (FL) that is (FL) is given the (FL) find out the row and which it appears. And for that (FL) uses something called the (FL) let us not read verse.

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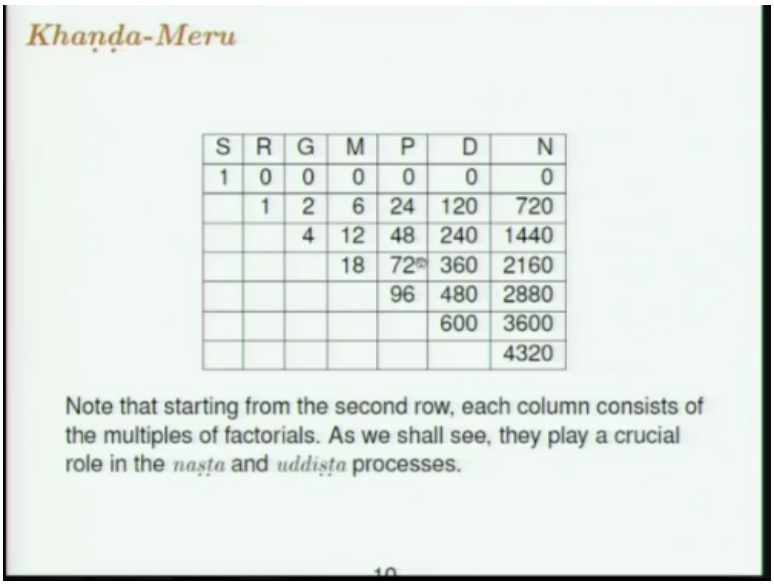

But just look at the meru in (()) (17:34) the first row is all 1 and 0, 0, 0, second row is all the factorials leaving out the first column 1 factorial, 2 factorial, 3 factorial, 4 factorial, 5 factorial, 6 factorial. Third row leave out 2 columns twice the 2 factorial twice the 3 factorial, twice the 4 factorial, next row is 3 times the factorial, 4 times the factorial, 5 times the factorial, so that is the construction of the (FL) of (FL). It is like just you are going to divide this things by factorials.

So, it is like just like a multiplication table of factorials with which you will grade the things and identify the (FL) meaning do the (FL) and (FL). The mathematical operation just that in the case of (FL).

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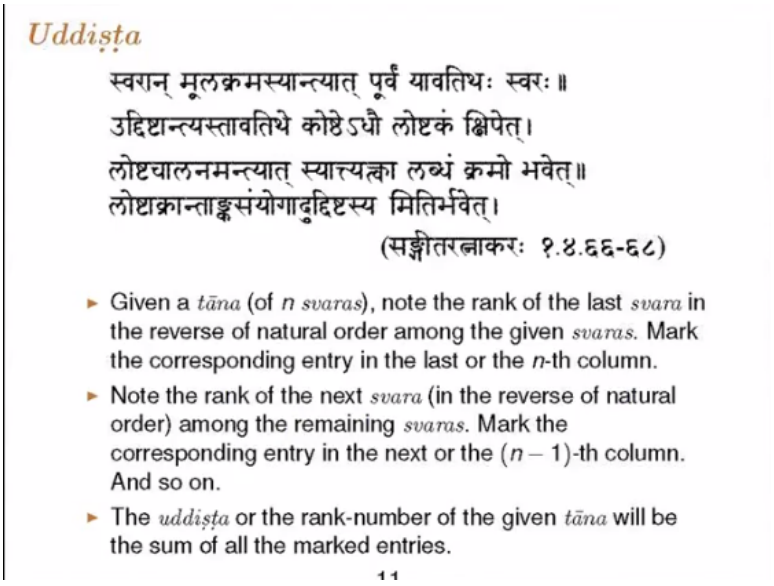

The idea you rend on dividing by 2 that was the you are doing in the (FL) and (FL) here you go dividing by factorials and instead of doing the division you use this (FL). We will again not read the thing.

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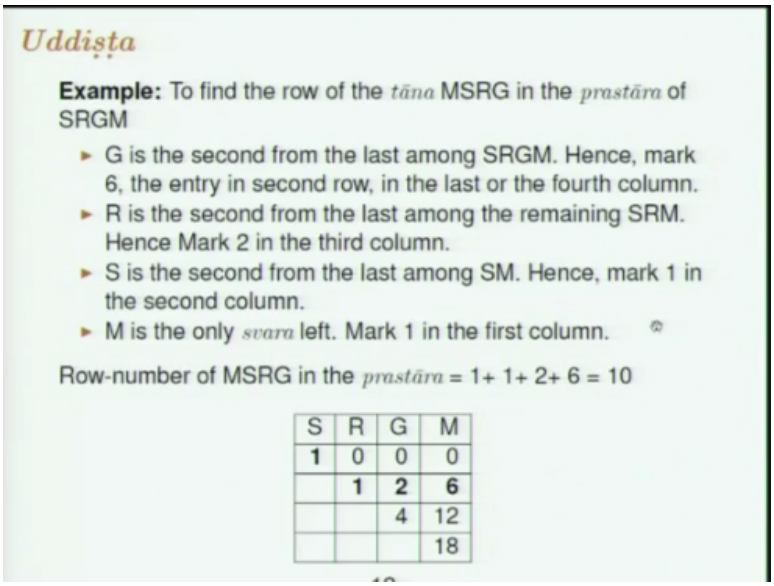

But take an example and understand how the (FL) problem is to be done to find the row of the (FL) in the (FL) of (FL). So, this is a (FL) problem for (FL) you first write down (FL) like this (FL) 1, 0, 0, 0 factorials trice the factorials 3 times the factorial then what you do look up the last (FL) in the end and then identify in the reverse order where it stands among (FL). So, among (FL) is the second in the reverse order.

So, from the top you mark the second row, then (FL) is over (FL) is all that is left. So, you take (FL) now identify amongst the left out (FL) that is (FL) what is the position of (FL) in the reverse order. So, (FL) to (FL) is the second in the reverse order amongst the remaining (FL). So, again mark in the second column in the second row in the next column. Then what is the (FL), so what is (FL) in the reverse order amongst (FL) it is first it is I am sorry (FL) is what we are doing.

So, what is left is (FL) and (FL), so what is (FL) in the among (FL) that is the second, so again the second row whatever is left is in the first row, so you have marked this 4 numbers add all of

them 1+1+2+6 is 10 that is the row of (FL) in the (FL) is in the 10th row of the (FL). So, using (FL) so we can take another example let us keep the (FL).

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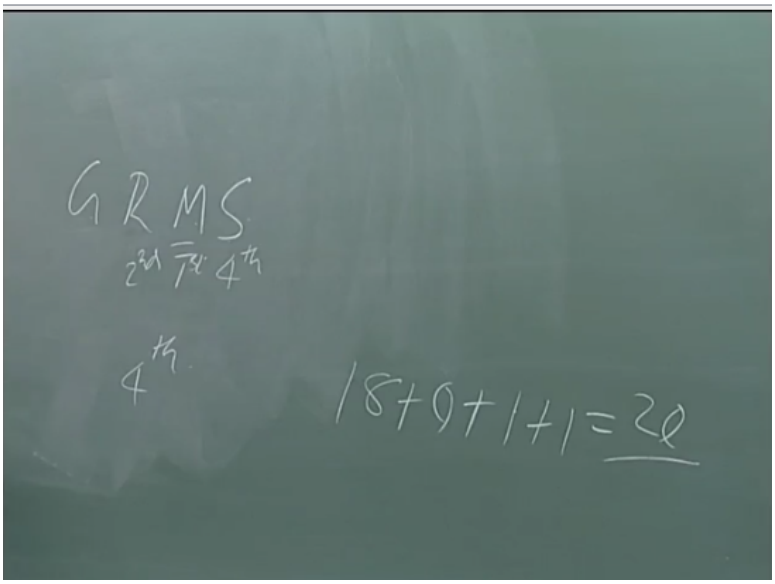

So, let us take some other thing (FL), so what is the row of this in the (FL) of (FL), so (FL) is the fourth among (FL), so you have to identify 1,2,3,4 you have to take the 8 oh it will not come that way, so you have to take this 18, so this will be 18+(FL) amongst (FL) is the first. So, you will just get a 0 in the second entry (FL) is the first in the reverse then you have (FL) amongst (FL) and (FL) is the second from the top therefore you have to take the second row in the next column that is 1.

So, this was the 4th, this was the 1st, this was the 2nd and this you will be whatever is left that is just this one here. So, this what you have, so you should obtain this in the 20th row of the (FL) let us go and see (FL) is in the 20th row of, so this is the method of doing (FL) with the (FL) and (FL) is essentially device for dividing the number by factorials and identifying where it appears. Now (FL) again we will do it with an example.

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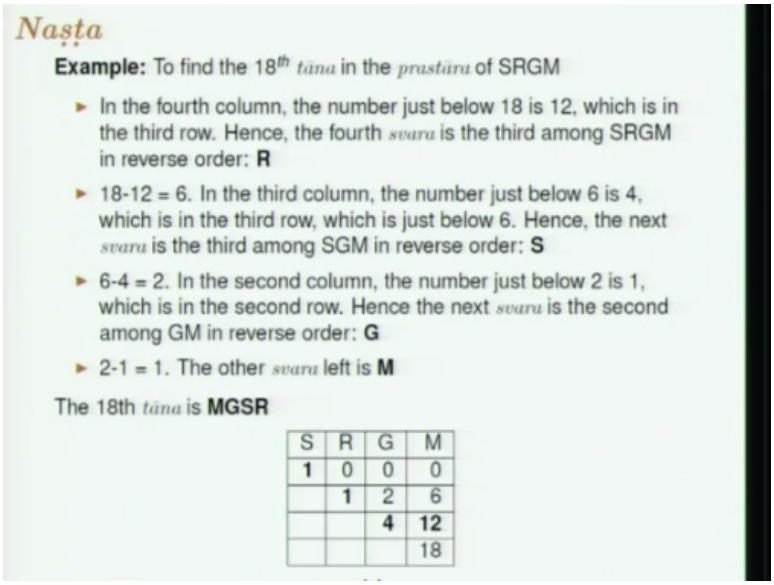

So, find out the 18th (FL) in the (FL) of (FL), so what you do the rule is as follows, in the 14th in the last column identify the number below the number you are looking for, so the number below 18 is 12 then subtract that12 from 18 you are left with 6, then in the next column look for the number below 6 that is 4, 6-4 is 2 take this 2 in the next column look for the number below 2 that is 1, 2-1 is 1 this one.

So, you obtain these 4 numbers, from these 4 numbers how do you identify the (FL) the permutation. So, what do you do same thing same interpretation right. This is the third row, so in (FL) the last entry of the permutation is the third in the reverse order from the end, so that has to be (FL) among (FL) is the third in the reverse order from the end, this is the third row that is coming, so identify the (FL).

Next this is also third, so among (FL) is the third, so next is (FL), so you have identify (FL) then you have identify (FL). Then you have the second row here mark, so amongst what is that we have finished with (FL) and (FL), so only (FL) and (FL) are left, so what is the second in the reverse order amongst (FL) and (FL) that is (FL), and therefore you have the third entry taken (FL) what is left with is.

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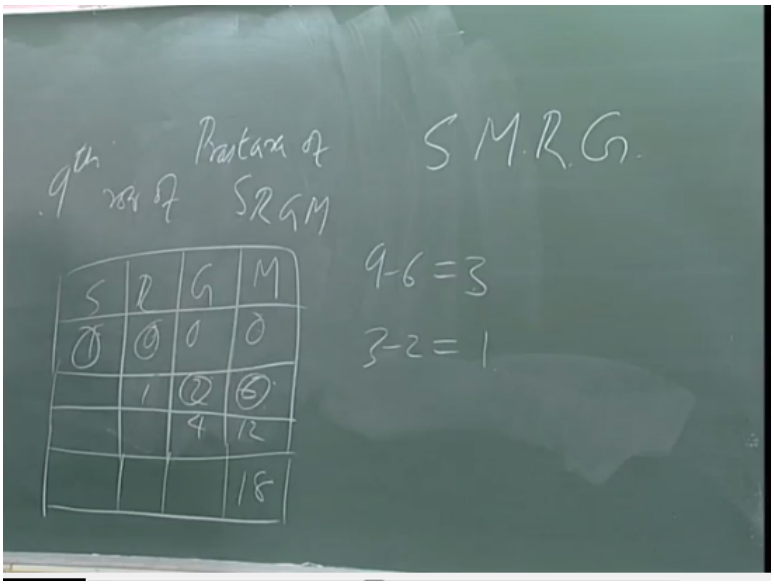

So, we can do one more example, so find out say in the ninth row of (FL), so what you will get in the column you will get 6 right, so last column just one more row. So, you obtain this 6, 9-6 is 3. So, in the next column you are again obtaining this 2 which is just below that 3 this 6 is just below the 9, then 3-2 is 1 and this 0 is just below that 1 and this one has to used. So, the last entry of the permutation is the second the reverse order in (FL) that is (FL).

And what is left (FL) amongst them again the second that is (FL) and what is left (FL) amongst them the first is the reverse order that is (FL), so your permutation should be (FL) let us see okay that this in the that is the ninth row of the (FL). So, in (FL) and (FL) are just done from the (FL). That is modeless it because there is no (FL) why there is no (FL) because every tana has all the elements and only once and therefore there is nothing like how many times any particular elements appears all elements appear once and only once in the permutations.

So, there is no further (FL) but just all this that (FL) and whatever we have done is essentially based upon a very beautiful representation of numbers. So, I will just tell you to in very briefly and we will proceed.

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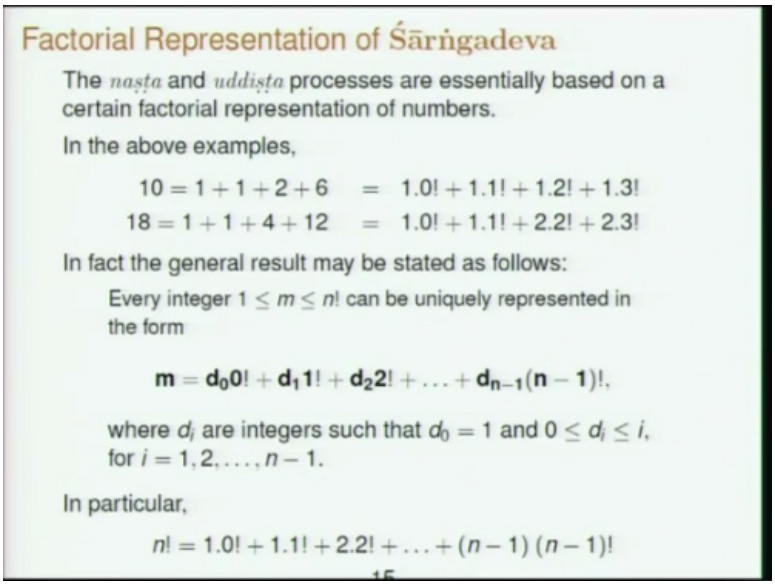

So, in both the (FL) and (FL) examples that I had given the number 10 must broken down into 1+1+2 into 6 the number 18 is broken down into 1+1 into 4 into 12 that is each number is broken down a sum of multiples of factorials and this is a it behind both the (FL) and (FL) process and this can be generalized into a statement which is actually implicit in the construction of the (FL) and in the (FL) and (FL) process. That every integer can be uniquely represented in the form d0 kind 0 factorial.

O factorial is taken to be 1, d1 into 1 factorial, d2 into 2 factorial etc., dn-1 into n-1 factorial where di's are numbers less than or equal to i, di is less than equal to i it can be 0, d0 is always 1, so this kind of a unique representation exist and that gives a 1 to 1 map between the numbers, the rows and the a permutations, so that is at the (FL) and (FL). In particular you have this very beautiful relation n factorial is 1 into 0 factorial+1 into 1 factorial, 2 into 2 factorial etc., okay.

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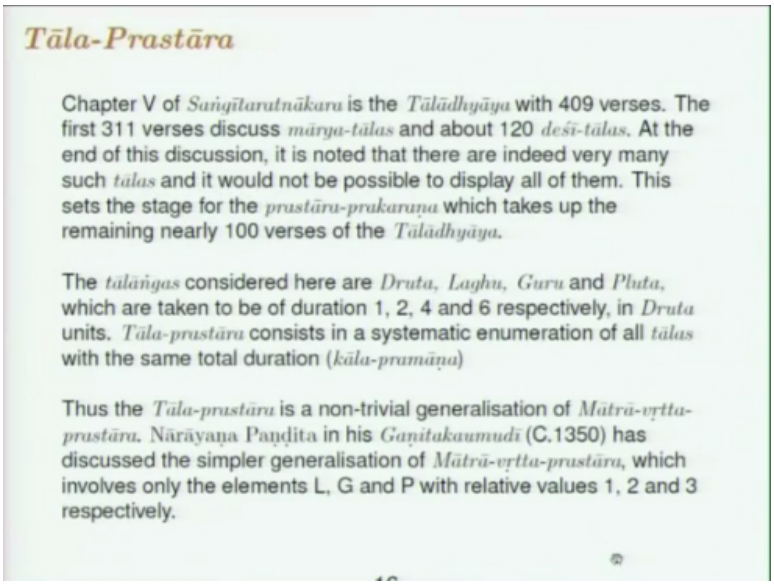

So, we are half done actually we are done with the simpler forum (FL) that (FL) give that all this in about 10, 12 verses something like that. And incidentally what is (FL) for this (FL) is just n factorial right (FL) is indeed something which much more fascinating much more complex and the theory that (FL) has given in fact is more general in even what later people would do, the chapter 5 of (FL) is (FL) it has 409 verses.

The first 311 verses discuss what are known as (FL) and about 120 (FL) at the end all this (FL) says no the number of (FL) are numerable and all that, so they will use some mathematics that you can do with them which will give you an idea of the kind of variety of (FL) that are possible and so his thoughts with the discussion of (FL) and he does various other things. The units of (FL) that (FL) is considering are 4 and that is what is interesting.

And that is somewhat different from the way the (FL) units are in **bo** today. The units is considering the (FL) is considering 4 (FL).

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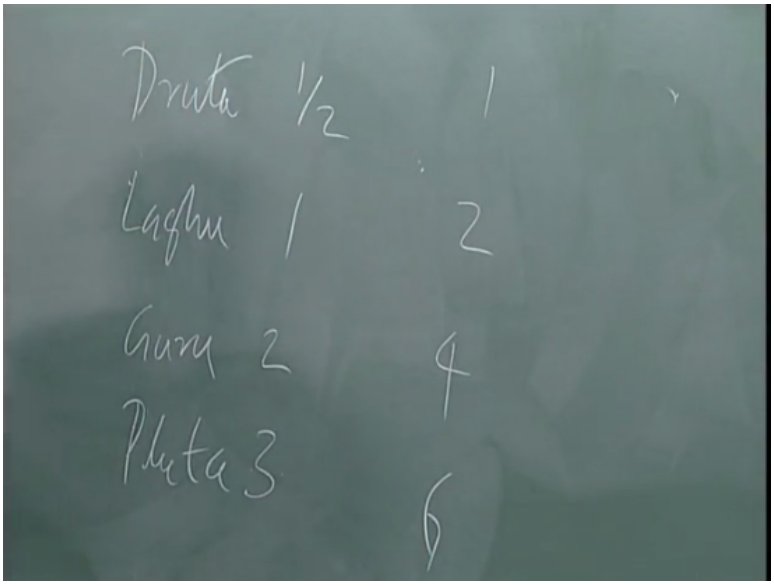

This are the 4 (FL) that (FL) is considering he is giving them actually that temporal measure of half, 1, 2 and 3, so if we put this down in integers instead of this is in the ratio 1:4:6. So, there are 4 (FL) units that (FL) is considering (FL) and (FL).

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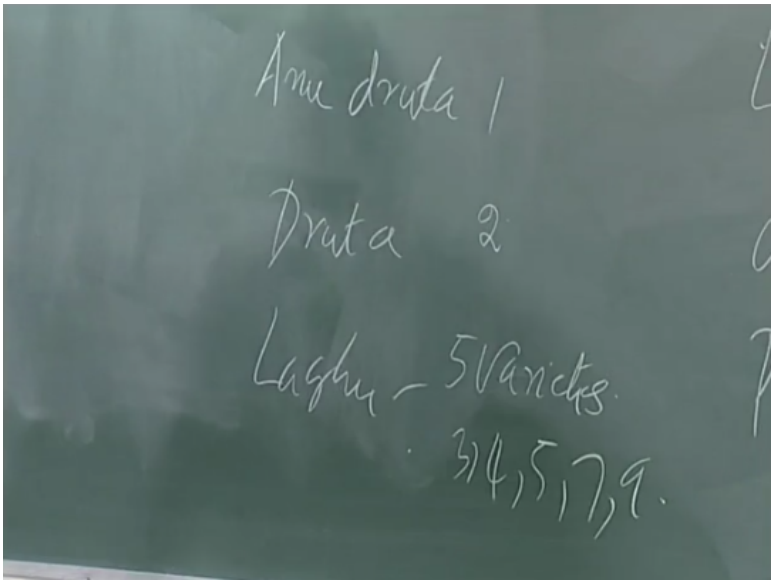

Now in today's music we know there are 3 units in the what are called the (FL) which are involving karnatic music since the time of (FL). There is something called (FL) there is (FL) then there is (FL) and the (FL) are made out of these 3 components and this (FL) can be are 5 varieties right (FL) and (FL) is up to units this can be of units 3, 4, 5, 7 and 9 and this is of unit 1. So, these are the (FL) units currently in bough in the karnatic music as it is practice.

So, when these are units that (FL) is considering and later on it is said that there are many manuscripts and discussions of these kinds of with these kinds of units also in late 18th, 19th century there are supposedly many families are (FL) players and musician will have written secrets for (FL) for their children and for their (FL) in small small monograms. But many of them are suppose to be around and most of them are not.

There is one due to this (FL) brothers etc., so there are not been still being studied but (FL) has given a complete discussion with this 4 units. Of course in the older times even in (FL) time there used to be another unit called (FL).

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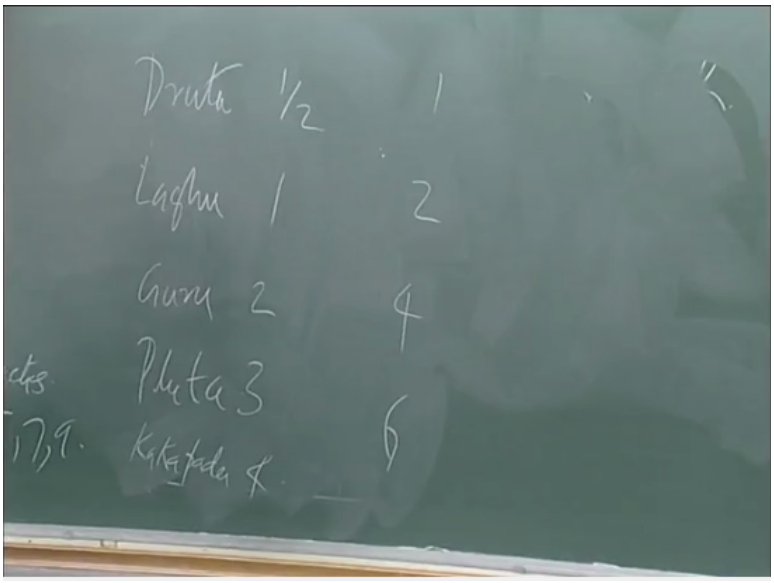

That was this (FL) is also bough I mean till this entry in complicated (FL) known as (FL) and all that. So, this kinds of units are also used in current use okay. So, now let us go along with (FL) people who want to update his theory for current (FL) units are welcome to make there at a there is professor (FL) in Hyderabad who has written quite a few articles and monographs on this subject, he is a violinist okay.

So, now the thing that is done is indeed generalization of the (FL) what is done is a total (FL) amount of say 15 units how many (FL) can be there made up of the LGP which will give you a total of 15, so this is similar to the (FL) where you had only just the (FL) and (FL) with 1 and 2, so to the 1 and 2 which now we will call them 2 and 4 you are adding a 1 and 6 and you are

enlarging the problem of partitions these are ordered partitions of numbers made out of this where repetitions are also allowed.

So, the theory of (FL) is indeed a generalization of the theory of the (FL), now in mathematics literature in India in (FL) Narayapanditha did consider he try to make a generalization Narayabanditha tries to make a generalization of all the earlier discussions into a larger mathematical frame work.

So, he generalizes (FL) which is binary representation into a representation in for an arbitrary ridings for an arbitrary base. Similarly he generalized the (FL) of which was in more to considering any number of units say LGP (FL) and (FL) with values 1,2,3 are with values 1, 2, 3, 4 or with values up to 1,2, 3 up to q all the values appearing but that does not include the peculiar kind of formulation the (FL) was made where the units are 1,2,4 and 6.

So, what theory Narayana has given, so (FL) theory is indeed somewhat more complex and general this appears in a music book about 150 years earlier or 100 years earlier the Narayabanditha but this indeed a more general term at least Narayana generalization does not include this, okay.

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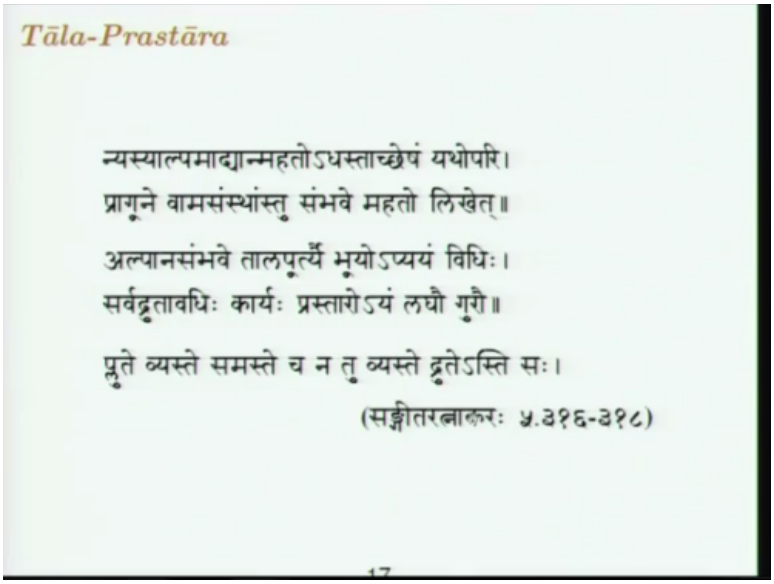

So, now the (FL) of this (FL), so if you have (FL) only that is 1 (FL) just that is all.

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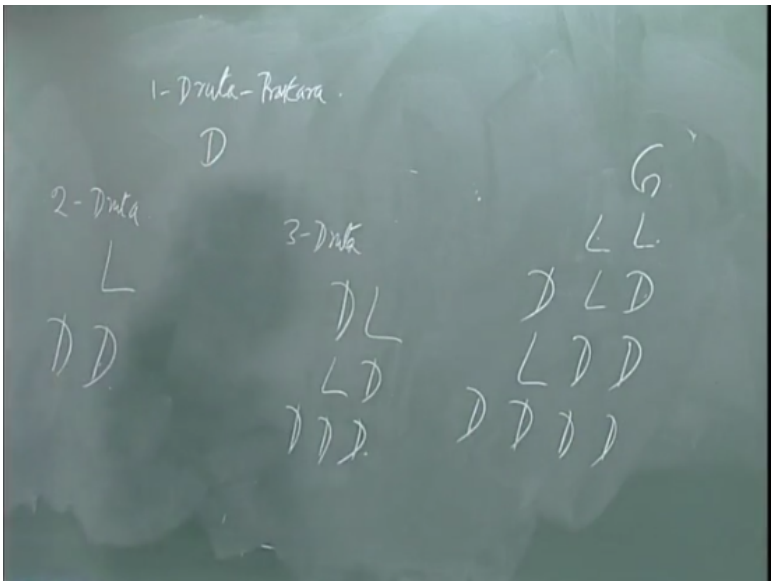

Next is (FL) that will just have 2 right, 2 (FL) this can be 1 (FL) meaning 1 unit of time in the f unit, 2 units of time, so first you write a (FL) then you can write 2 . So, what about the 3 (FL) it can be like this right, so this will be the 3 (FL) I am not written them note so I am not good I am writing them on the board, so this will be 2 (FL), so this will be 3 (FL). So, once you have 4 (FL) a guru is the starting point LL, so then below that D there will be an L you are right.

So, then this will be DD L, then there will be DDD this is how, this will be the 4 (FL).

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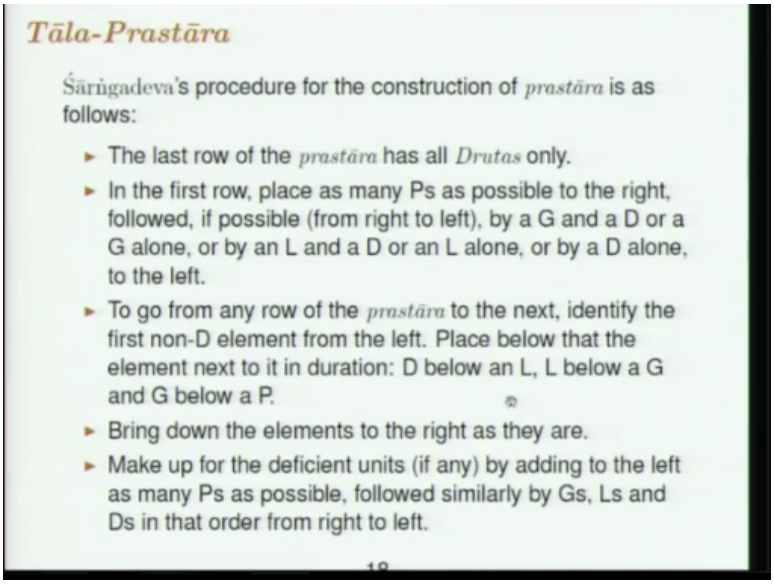

So, the rule is last rows the (FL) is (FL) only, last row depending on the number of units put as many (FL).

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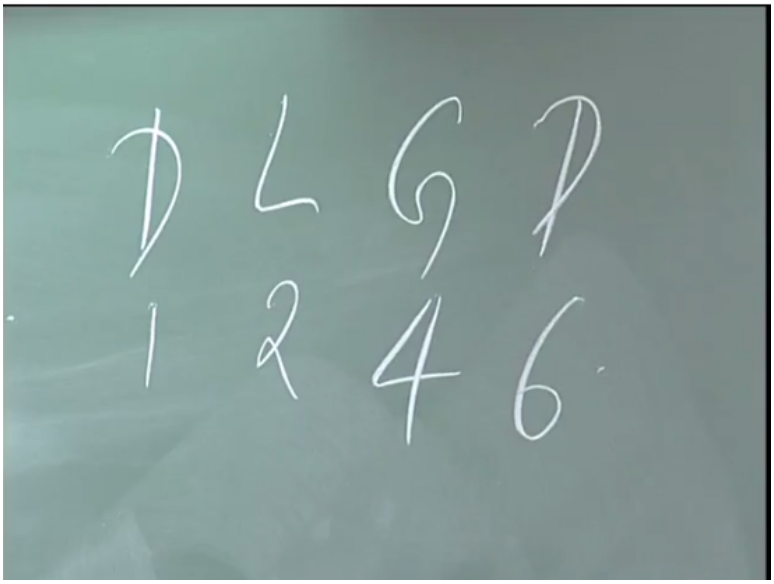

So, the basic units are D is 1, L is 2, G is 4, P is 6 remember that, so in the first row put as many (FL) as possible follow that as many gurus as possible follow that as many L's as possible, follow it by D's that is the first row the last row will have only D's then same way to go from any row to the next row start from the left whenever a non D element appears first below that write the highest element that can be written below that. If you see a P write a G below that.

If you a G write an L below that, if you see an L write a D below that then bring down the elements to the right as they are then on the left again fill up starting from P's if possible then G's, then L's, then D's to fill in the total number of units that are which you are writing the (FL), so the same rule so we can read the verse of (FL) above the (FL) above the greater one (FL) from the beginning from the left wherever there is a greater one.

(FL) put the next one below that (FL) once to the right have to be brought down as they are (FL), so here in the left whatever big things you can write you write (FL) keep filling it up till the number of (FL) are filled up. So, let us take the 6, 6th (FL) 6th (FL) is (FL).

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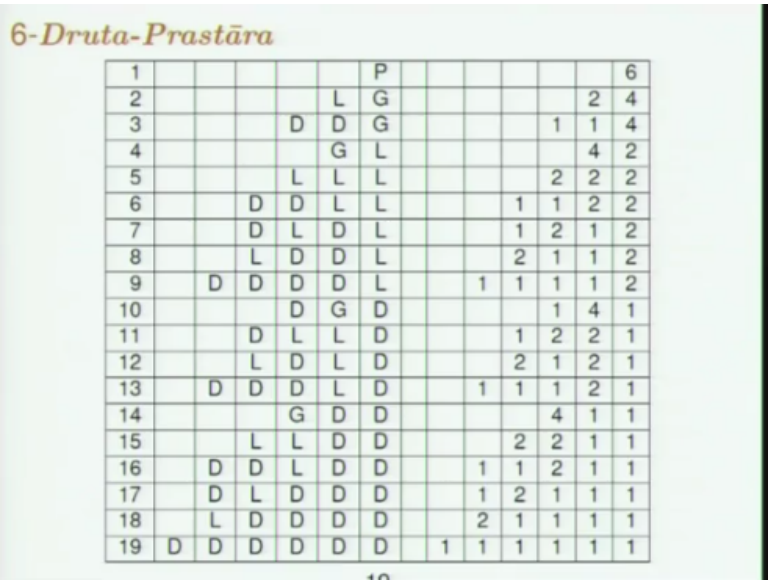

So, below that write (FL) write the guru on the left (FL) from the left below the (FL) write a (FL) bring down the guru as it is you need to make of 6, so put a D again start looking from the left below the G you have to write an L, so to fill up 6 a G is possible, so write that again from the left below this G you have to write an L bring down the L as it is an L is possible, so LLL now below this L write the D bring down these 2 L's as they are in the left write the D.

Below this L write the D bring down this L (FL) 3 (FL) are available. So, write an L first then (FL) like that you go on, so I have written it both in terms of PAGL and in terms of numbers 1, 2, 4 and 6, so this has 19. Now you can straight away see this has 1 row with ending with P, 2 rows ending with G, 1, 2, 3, 4, 5, 6 rows ending with an L, and 1, 2, 3, 4,5, 6,7, 8, 9, 10, 11, 13 I think it is what 10 rows ending with the D.

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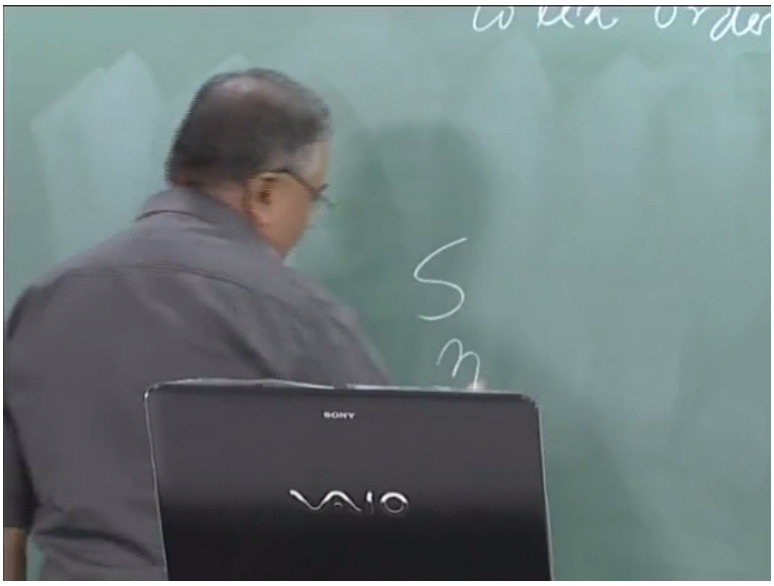

And you can straight away guess what is going to appear, that in the (FL) of n (FL) that you are doing.

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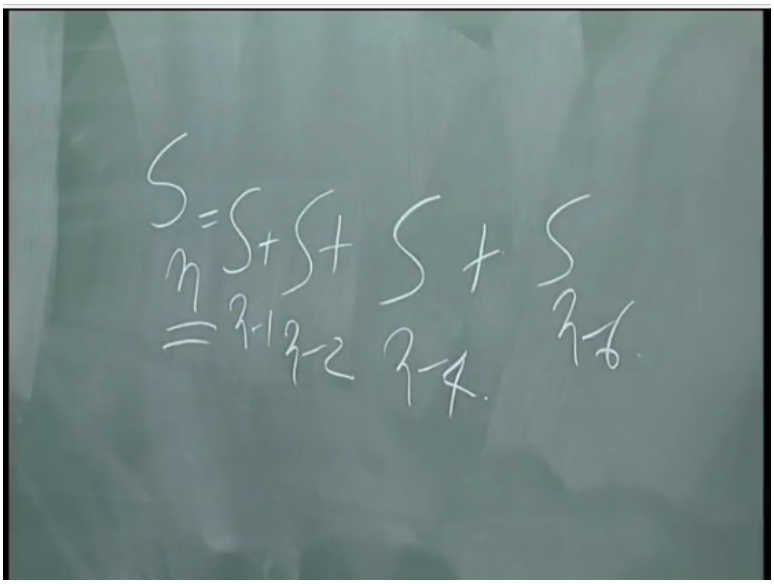

You first take the (FL) of n-6 (FL) put a P in the end for all of them right in the (FL) of value n (FL) total duration n you first start with (FL) of (FL) of length or (FL) of duration n-6 to that you add a (FL) immediately that will become n, then below that you take a (FL) of units n-4 to that you put a guru on the right. Then again you are finished your n, next you do (FL) of n-2 units and to them in the last row you put a (FL) and finally you do (FL) of n-1 units in the end put a (FL).

So, the recurrence relation for the number of rows in the (FL) is Sn is Sn-1+Sn-2+Sn-4+Sn-6 the (FL) of n (FL) units is constructed, so you can see this here in the 6th (FL) there is a above is it is easy because it is below 6 there will be come complications because one of other of them may not exist. So, it is best to see it in the 7 (FL).

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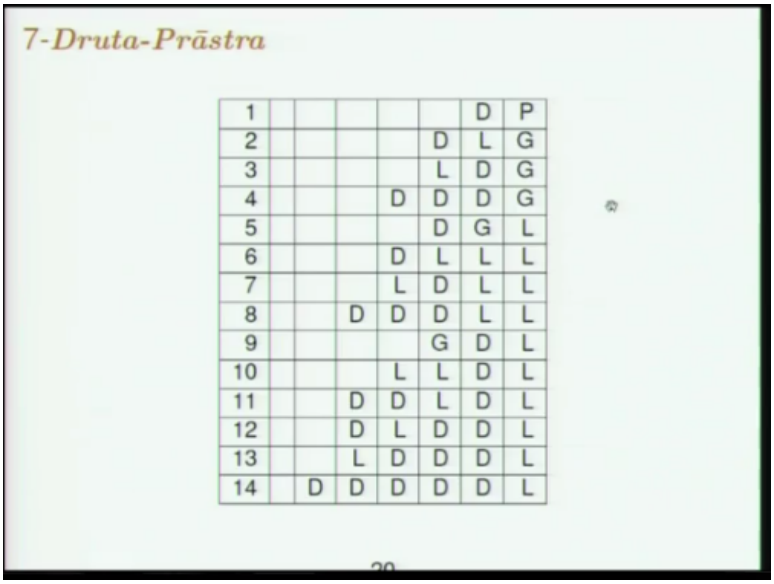

In 7 (FL) you can see 7-6 is 1 there is one row which ends with a (FL), 7-4 is 3 and the (FL) of 3 I have already written it has 3 rows only, so there are 3 rows which end with a G and fact this is DLLDDD whatever written here you write them and put a G to the right. And this is S1 this is the (FL) of 1 (FL) unit which we wrote us you put a P on the right. Next Sn-2 that is 5 unfortunately we did not write down the 5, we can write it down.

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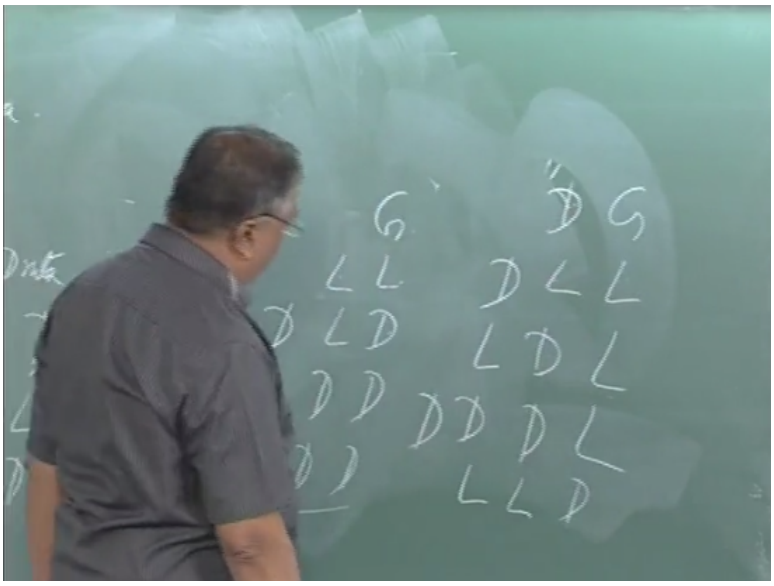

So, what will 5 units look like you start with a G with a D on the left right, so now you have to put an L there an LD will appear below the take a D bring down the L as it is an L can come here no .

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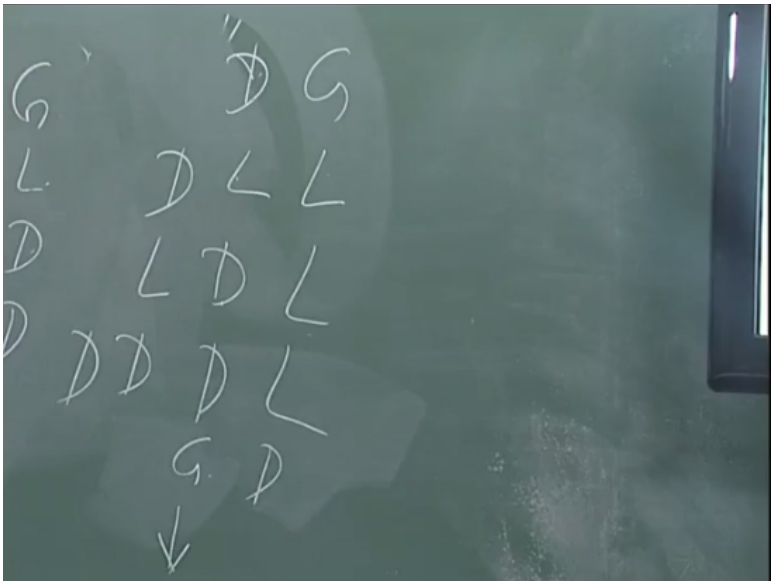

Here a DG will appear, so like that it will go on let you write few more rows, so you can see it here DG all this things having L like the end DGDLLLDLDDLGD I have come up to that point you can go on. So, you put all the and put the L below this is (FL) of length 5 to which you put an L below L to the right, so it will become (FL) of 7 and finally this is (FL) of 6 units which was done here this 19 rows you take this 19 rows.

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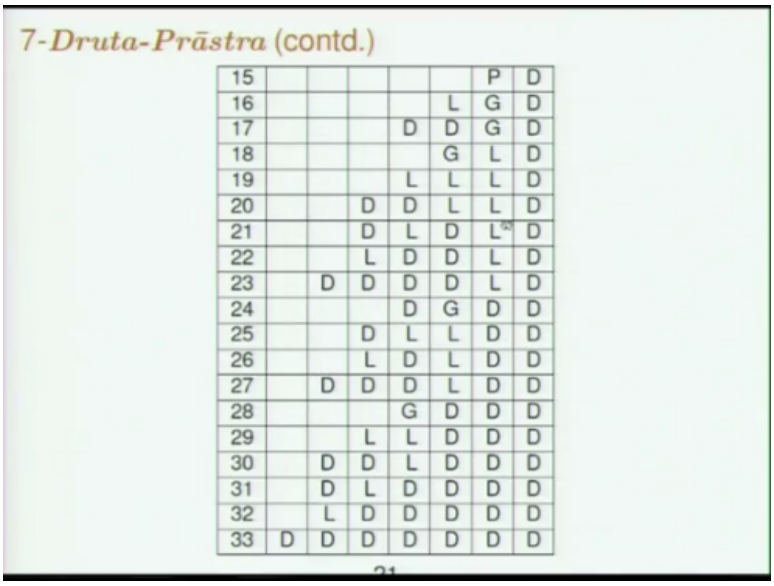

And put a D on the right, so that will give you a (FL) of 7 units. So, in (FL) the rule is (FL) of n (FL) units is made up of first n-6 (FL) unit (FL) with a P on the right n-4 unit (FL) with a G on the right n-2 (FL) with an L on the right n-1 (FL) with a D on the right and therefore the number of rows in the (FL) is Sn is equal to Sn-1, so you can now write down.

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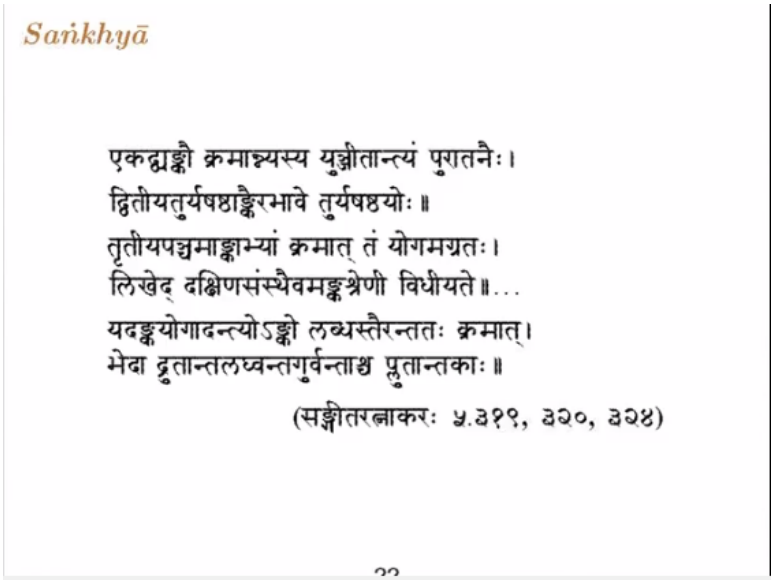

So, this is what (FL) is saying (FL), so basically he is saying (FL) is the second previous (FL) is the fourth previous (FL) is the sixth previous.

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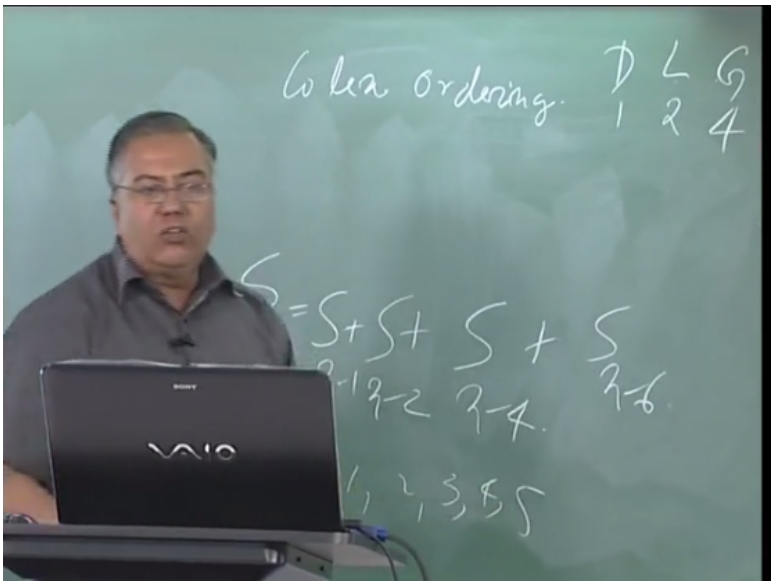

So, (FL) I think till 1 to 5 meaning 1, 2, 3, 4, 5you will have 6 is absent here, 4 is absent here, so there you use the third and the fifth (FL) the 4 numbers of which a given number is a sum or indeed those (FL) which are ending in (FL) and (FL), so that is what he is saying.

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Saṅkhyā

Śārṅgadeva makes the observation that among all the *tāla*-forms which appear in the *n-druta-prastāra*, S_{n-1} end in a D, S_{n-2} in a L, S_{n-4} in a G and S_{n-6} end in a P, and hence the total number of forms *tāla*-forms S_n in the *n-druta-prastāra* is just the sum of these four numbers. Thus,

$$S_n = S_{n-1} + S_{n-2} + S_{n-4} + S_{n-6}$$

Noting $S_1 = 1, S_2 = 2$, we get the Śārṅgadeva sequence of *saṅkhyāṅkas*:

n	1	2	3	4	5	6	7	8	9	10
S_n	1	2	3	6	10	19	33	60	106	191

And so this numbers can now be tabulated, so this is the recurrence relation, so we can call this numbers as (FL) sequence of numbers, so the numbers are 1, 2, 3, 6, 10, 19, 33, 60, so this 60 here is equal to 33+ this Sn-1+19 Sn-2+6 Sn-4+2Sn-6. Similarly this 106 is 60+33+10+3 so like that each one is composed this is all generalized (FL) sequence, so this is the (FL) sequence. Now the rest is doing all complicated calculations of (FL) and (FL) with this sequence.

So, I will just give you a glimpse of how it is we **we** will not be able to going to the details of it, we are already running little bit sort of time on the, so I will just tell you how the (FL) is to be done that will give you.

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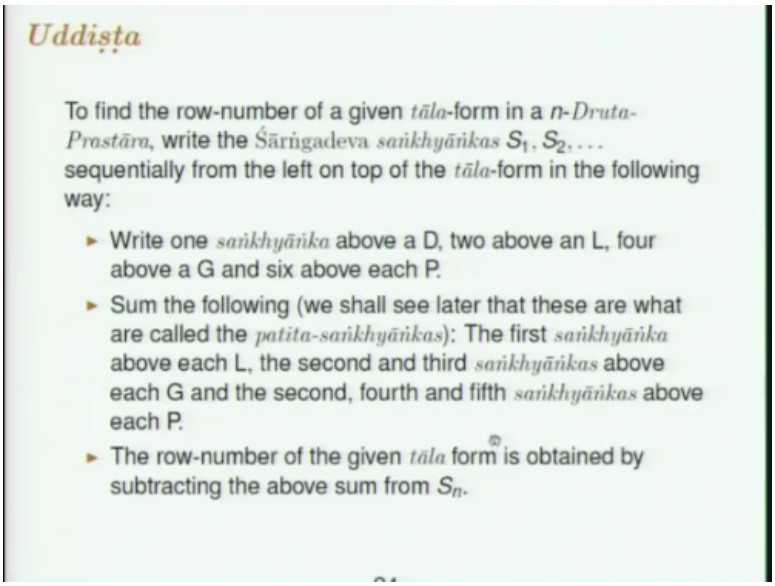

This is the generalization of the way (FL) is done for the (FL) what we do on (FL) these are right the (FL) numbers for a guru write 1 above and 1 below for (FL) write 1 above sum the numbers above the gurus that is the (FL) process for the (FL). Now what is the (FL) process for the (FL). So, let us take some problem now I think I have a examples putting here, so what you do write 1 (FL) above D, 2 above L, 4 above a G and 6 above a P.

Sum the following (FL) later on we will see this (FL) which are summed are called (FL) the first (FL) above each L, the second and third (FL) above each G, second, fourth and fifth (FL) above each P. so, let us see the row number of the (FL) is obtained by subtracting the above sum from Sn.

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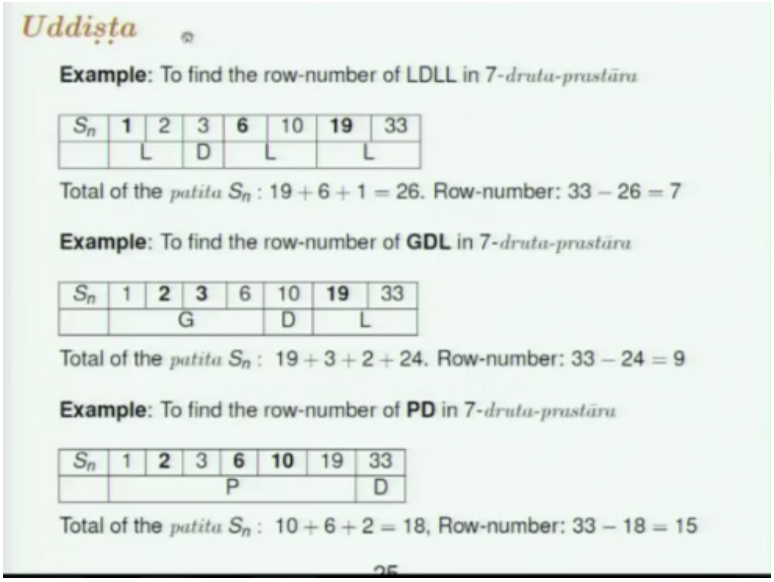

So, to find the row number of LDLL in 7 (FL), so first write LDLL write 2 (FL) above L, 1 above D, 2 above L, 2 above L nothing above the D is to be summed. The first number above the L is to be sum, so 1+6+19 that is 26, 33-26 is 7, so LDLLL is seventh row in 7 (FL) LDLL, LDLL is seventh row in (FL). Another example, GDL write 4 (FL) 1, 2, 3, 6, 10, 19, 33 these are the (FL), so write 4 (FL) above G 1 above D, 2 above L sum the second and third above G and the first above L that is what he says.

(FL) above each L, second and third (FL) above each G and second, fourth and fifth (FL) above each p, so second and third above G, first above L, so you get 24, 33-24 is 9, GDL is ninth row of the 7 (FL). Last example PD, so again above the (FL) write 6 (FL) 1,2, 3,6, 10, 19 above (FL) write one (FL) sum the second, fourth and fifth above the P, so that is 18, 33-18 is 15, so 15th row of 7 (FL) should be PD, so that is the PD that you are seen.

So, the reverse will be the same numbers will come us (FL) what we did in (FL) was got subtracting.

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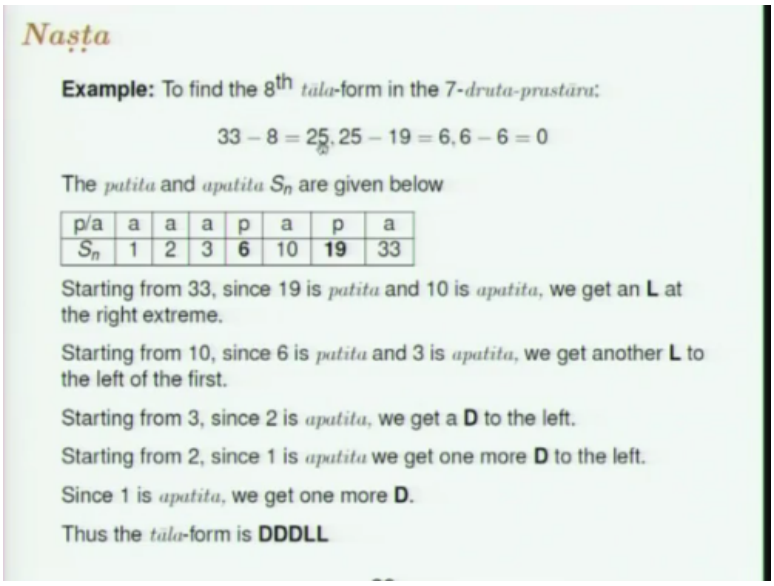

If you want to find out the eighth row in the 7 (FL), subtract 8 from 33 you get 25, then write the (FL) see which can be subtracted from 25, 19 can be subtracted, so 25-19 is 6 and then 6 can be subtracted 6-6 is 0, so this 19 and 6 are (FL) this 33, 10 and 3, 2, 1 are (FL).

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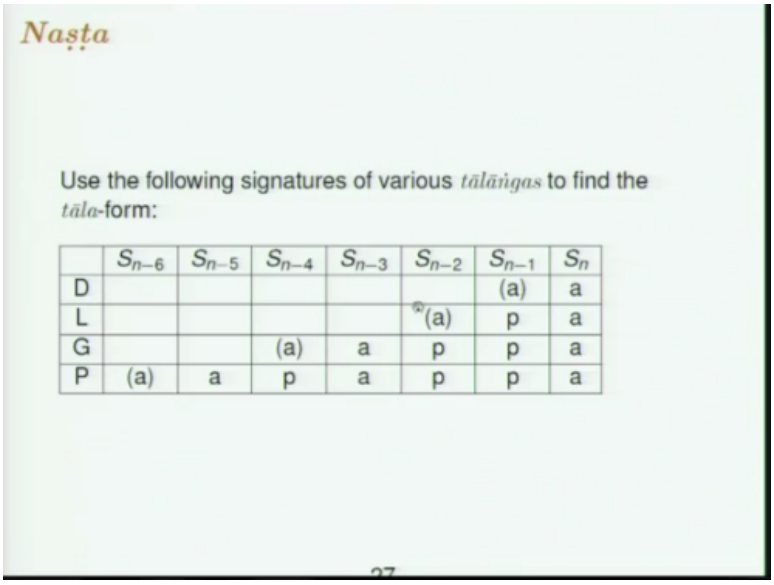

And then you have this what is called the template with which you compare if you have an a it is (FL), if you have ap it is a (FL), if you have an appa it is a guru, if you have an appapa it is a (FL), of course that followed by (FL), so you use that template immediately you will get DDDLL. So, it is all very nicely worked out in the mathematics in terms of the (FL), the idea of (FL) these the generalization of what was done in the matra (FL).

Another example 28th (FL) in 7 (FL), 33-28 is 5 only 3 can be subtracted from 5, 5-3 is 2, next 2 can be subtracted, so only 3 and 2 are (FL) rest of all (FL), so you have an a ppa that is the sign of a guru the rest of the all (FL), so the (FL) forum is a GDDD.

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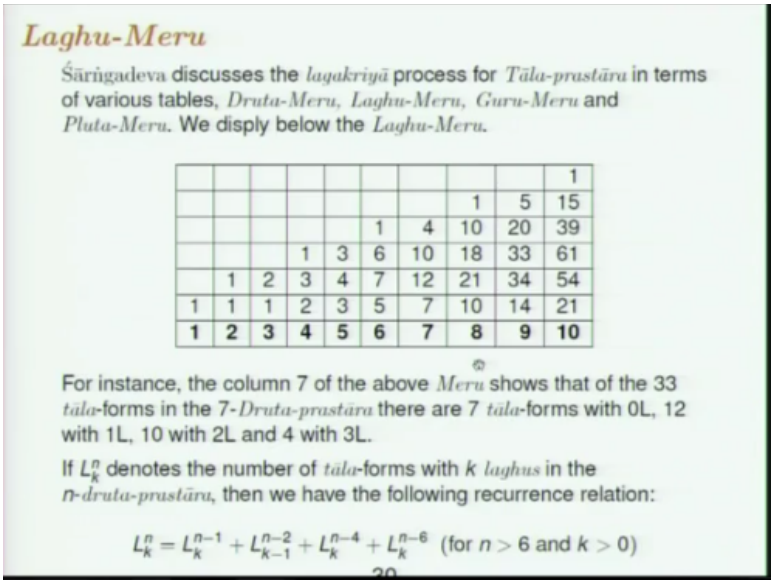

									1
							1	5	15
					1	4	10	20	39
			1	3	6	10	18	33	61
	1	2	3	4	7	12	21	34	54
1	1	1	2	3	5	7	10	14	21
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Finally (FL) also constructs, he construct a (FL) and a guru meru, because given a (FL) given say 15 matra 15 (FL) how many (FL) forums are there with 1 (FL), 2 (FL), 3 (FL) etc., how many (FL) are there with 1 (FL), 2 (FL) etc., 1 guru, 2 guru, now you can even ask more complicated question 2 gurus and 1 (FL), because you have 4 elements now earlier you are only having 2 elements.

So, only one choice was made, so all these are there they have very this entire (FL) section of (FL) runs about verses. He constructs something called (FL) these are the kind of recursion relations which are at the basis of these merus.

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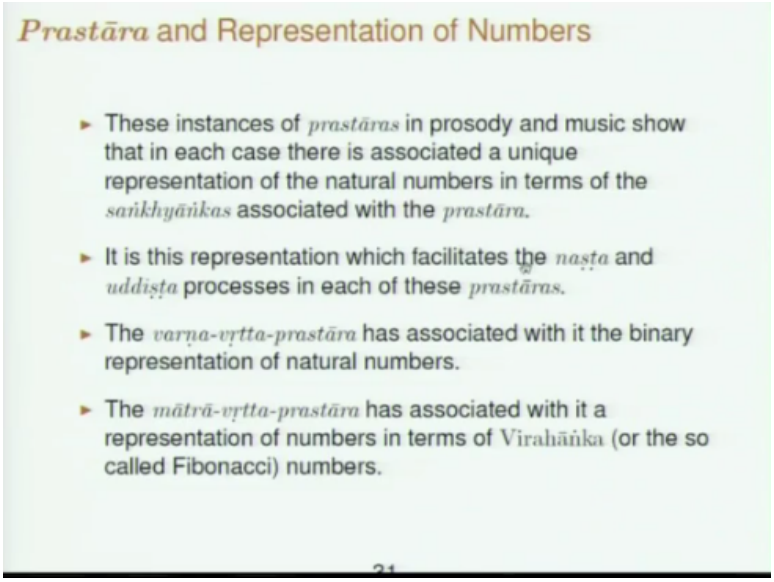

So, in all it is a very nice mathematical exercise, so ultimately in all this kind of combinatorics that we discuss what is the message, so in any enumeration any objects which can be enumerated you have first the (FL) an array and then once you have an array you can ask for questions like low number and what is the object that is sitting in that row number you want an association then you have something called (FL) how many rows satisfy a particular property.

So, these were the mathematical questions that (FL) identify but below this there is a mathematical issue also because once you have to identify a row number with a forum and a forum with a row number you will immediately see with a number a set of representation will immediately be associated. So, in (FL) we had every number written as a sum of 2 to the power various powers of 2, in matra (FL) you saw every number was written as a sum of (FL) numbers are the so called (FL) numbers.

In (FL) you saw every number was represented in terms of factorials and in (FL) every number can be written as a sum of what I showed just now as the (FL) numbers later on we will discuss (FL) of combinations (FL) of permutations has been given by (FL) of combinations can also be done, Narayapanditha does that in (FL) that gives you a representation of all numbers in terms of binomial coefficient, so once we have this array and associating numbers with forums immediately you will get a mathematical formulation relating natural numbers with a certain representations involving this (FL) of that array.

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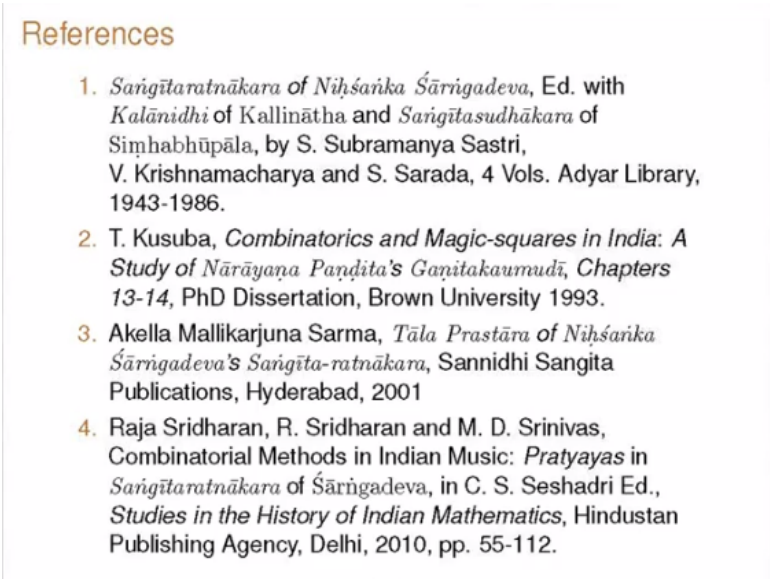

And this is at the bottom of all this mathematics that we have discussed thank you very much.